Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/1.2.3.2-d-x-^m-a+b-x^n+c-x^-2-n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [664]. This is test number [46].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (664)	% 0. (0)
Mathematica	% 99.7 (662)	% 0.3 (2)
Maple	% 74.7 (496)	% 25.3 (168)
Maxima	% 36.3 (241)	% 63.7 (423)
Fricas	% 80.57 (535)	% 19.43 (129)
Sympy	% 43.52 (289)	% 56.48 (375)
Giac	% 54.67 (363)	% 45.33 (301)

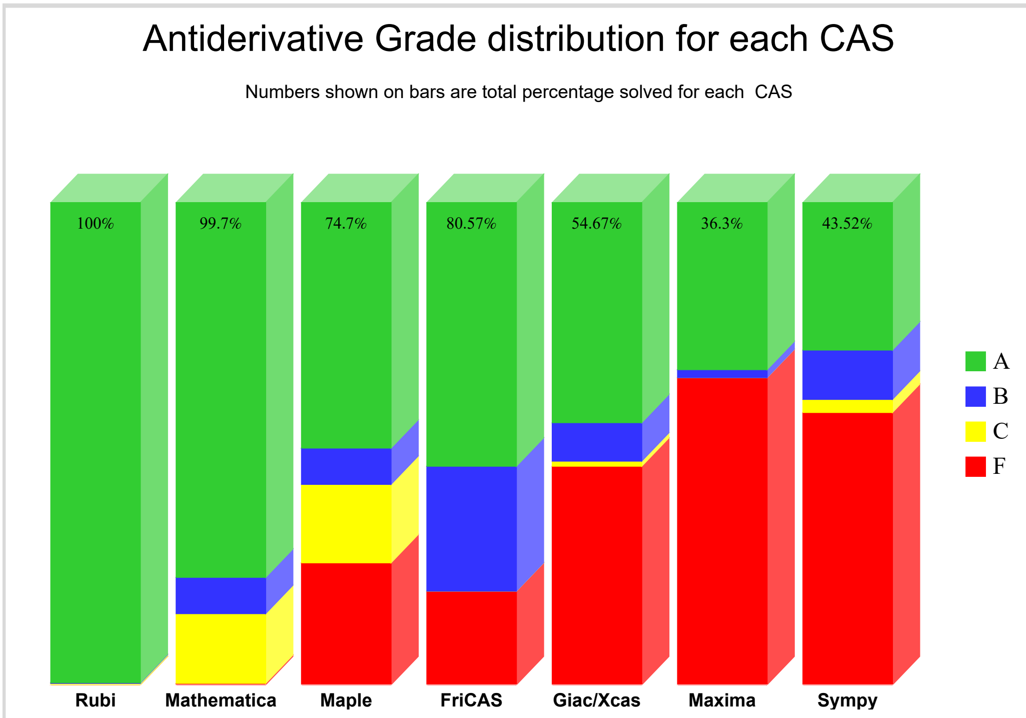
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

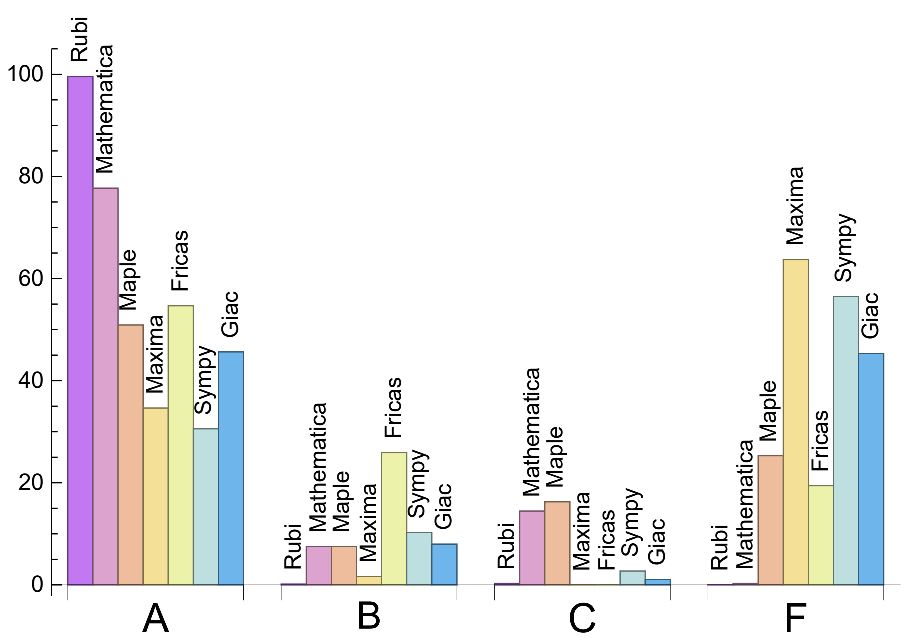
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.55	0.15	0.3	0.
Mathematica	77.71	7.53	14.46	0.3
Maple	50.9	7.53	16.27	25.3
Maxima	34.64	1.66	0.	63.7
Fricas	54.67	25.9	0.	19.43
Sympy	30.57	10.24	2.71	56.48
Giac	45.63	7.98	1.05	45.33

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	157.95	1.	137.	1.
Mathematica	0.24	137.17	0.99	83.	0.92
Maple	0.02	283.22	1.68	78.	0.79
Maxima	1.19	85.85	0.99	57.	1.02
Fricas	2.22	1757.15	7.69	379.	3.77
Sympy	4.87	244.8	2.04	65.	0.94
Giac	1.68	297.16	2.1	113.	1.15

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {132, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 229, 230, 231, 232, 233, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 309, 327, 329, 331, 333, 338, 340, 341, 343, 344, 346, 347, 351, 353, 367, 385, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 594, 595, 602, 603, 604, 605, 606}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

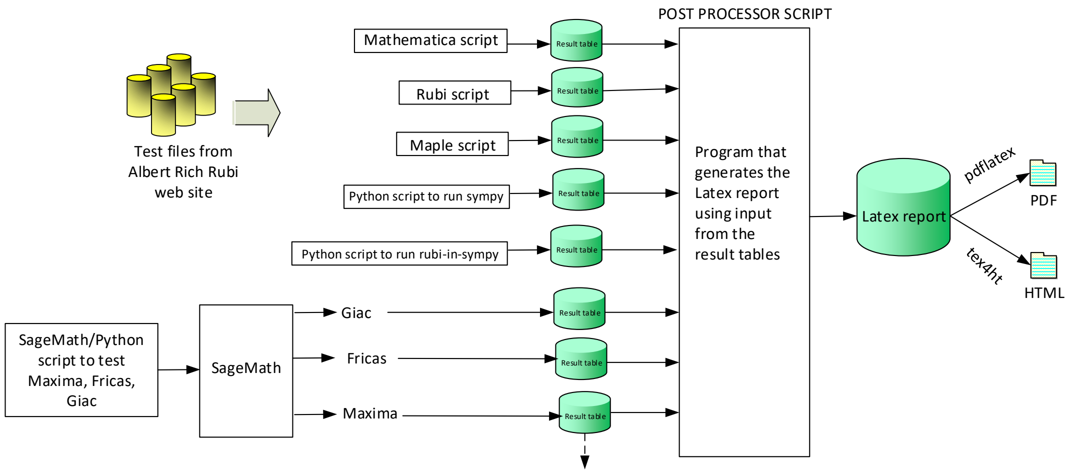
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664 }

B grade: { 154 }

C grade: { 176, 478 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 241, 242, 248, 249, 253, 254, 255, 256, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 328, 330, 332, 339, 342, 345, 348, 349, 350, 352, 355, 368, 369, 370, 371, 372, 373, 374, 376, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 562, 563, 564, 565, 566, 567, 572, 579, 582, 583, 584, 585, 586, 587, 588, 593, 596, 597, 598, 599, 604, 606, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663 }

B grade: { 61, 154, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 232, 233, 243, 244, 245, 246, 247, 252, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 589, 590, 591, 592, 594, 595, 600, 601, 602, 603, 605, 607, 608, 609, 610, 611, 612, 664 }

C grade: { 132, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 184, 250, 251, 257, 258, 309, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 331, 333, 334, 335, 336, 337, 338, 340, 341, 343, 344, 346, 347, 351, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 409, 410, 411, 457, 458, 478, 500, 501, 502, 503, 504, 505, 546, 547, 559, 560, 561 }

F grade: { 661, 662}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 111, 113, 114, 117, 118, 119, 120, 125, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 237, 238, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 313, 314, 315, 316, 317, 318, 328, 329, 330, 331, 332, 333, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 368, 370, 372, 373, 374, 376, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 424, 425, 426, 427, 436, 437, 440, 441, 442, 443, 444, 445, 446, 447, 448, 451, 452, 453, 454, 455, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 502, 503, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 523, 524, 525, 526, 527, 528, 533, 540, 544, 572, 579 }

B grade: { 61, 82, 107, 109, 110, 112, 115, 116, 154, 248, 311, 334, 336, 369, 371, 375, 377, 409, 411, 422, 423, 428, 429, 430, 431, 432, 433, 434, 435, 438, 439, 449, 450, 456, 549, 550, 551, 552, 553, 554, 555, 565, 607, 608, 609, 610, 611, 612, 663, 664 }

C grade: { 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 319, 320, 321, 322, 323, 324, 325, 326, 358, 359, 360, 361, 362, 363, 364, 365, 366, 378, 379, 380, 381, 382, 383, 384, 457, 458, 500, 501, 504, 505, 515, 522, 548, 556, 557, 558, 559, 560, 561, 596, 597, 598, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660 }

F grade: { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 472, 473, 474, 475, 476, 477, 478, 506, 507, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 543, 545, 546, 547, 562, 563, 564, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25, 26, 28, 29, 31, 32, 34, 35, 37, 38, 40, 41, 43, 44, 46, 47, 49, 50, 51, 53, 54, 56, 57, 59, 60, 62, 63, 65, 66, 68, 69, 71, 72, 74, 75, 77, 78, 80, 81, 83, 84, 86, 87, 91, 100, 108, 111, 118, 119, 120, 125, 126, 127, 130, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 348, 350, 352, 354, 356, 368, 370, 372, 374, 376, 386, 387, 388, 390, 392, 393, 394, 395, 405, 406, 407, 408, 409, 410, 440, 441, 442, 443, 444, 445, 446, 447, 448, 455, 460, 461, 466, 467, 468, 469, 470, 471, 473, 474, 475, 479, 480, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 533, 540, 548

B grade: { 154, 389, 391, 607, 608, 609, 610, 611, 612, 663, 664 }

C grade: { }

F grade: { 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 52, 55, 58, 61, 64, 67, 70, 73, 76, 79, 82, 85, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 351, 353, 355, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 371, 373, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 396, 397, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 462, 463, 464, 465, 472, 476, 477, 478, 481, 500, 501, 502, 503, 504, 505, 507, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 234, 235, 236, 237, 238, 240, 241, 242, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 294, 296, 298, 310, 312, 314, 316, 318, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 348, 349, 350, 352, 354, 355, 356, 359, 362, 365, 368, 370, 374, 376, 386, 388, 392, 394, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 455, 460, 462, 463, 464, 465, 466, 467, 468, 469, 473, 474, 475, 478, 479, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 533, 540, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 565, 572, 579, 586, 616, 617, 639, 641, 642 }

B grade: { 82, 143, 144, 145, 146, 147, 148, 149, 150, 154, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 239, 248, 293, 295, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 313, 315, 317, 319, 320, 321, 322, 323, 324, 325, 326, 344, 346, 351, 353, 357, 358, 360, 361, 363, 364, 366, 369, 371, 372, 373, 375, 377, 378, 379, 380, 381, 382, 383, 384, 387, 389, 390, 391, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 454, 456, 457, 458, 470, 471, 552, 556, 557, 558, 559, 560, 561, 593, 596, 597, 598, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 640, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663, 664 }

C grade: { }

F grade: { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 229, 230, 231, 232, 233, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 459, 461, 472, 476, 477, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 562, 563, 564, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

2.1.6 Sympy

A grade: { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 89, 90, 91, 92, 93, 94, 95, 96, 97, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 248, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 313, 315, 317, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 342, 345, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 388, 390, 392, 394, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 440, 441, 442, 443, 444, 445, 446, 447, 448, 456, 457, 458, 460, 493, 494, 495, 496, 498, 613, 615, 618, 620, 638, 640, 643, 645 }

B grade: { 138, 139, 140, 141, 142, 310, 312, 314, 316, 387, 389, 391, 393, 395, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 607, 608, 609, 610, 611, 612, 614, 616, 617, 619, 621, 622, 623, 624, 625, 626, 639, 641, 642, 644, 646, 647, 648, 649, 663, 664 }

C grade: { 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 338, 340, 341, 343, 344, 346 }

F grade: { 1, 2, 3, 4, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 318, 327, 347, 367, 385, 449, 450, 451, 452, 453, 454, 455, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 497, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662 }

2.1.7 Giac

A grade: { 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 120, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 171, 174, 177, 180, 183, 188, 189, 204, 205, 222, 223, 237, 238, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 314, 316, 318, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 342, 345, 348, 350, 352, 354, 356, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 455, 460, 461, 462, 463, 464, 465, 466, 467, 475, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 496, 515, 516, 517, 518, 523, 524, 525, 552, 619, 628, 637, 663, 664 }

B grade: { 45, 61, 82, 118, 119, 125, 154, 170, 172, 173, 175, 176, 178, 179, 181, 248, 249, 349, 351, 355, 357, 389, 391, 473, 474, 522, 597, 598, 607, 608, 609, 610, 611, 612, 614, 616, 622, 624, 626, 631, 633, 635, 639, 641, 644, 647, 649, 650, 652, 655, 657, 658, 660 }

C grade: { 311, 313, 315, 317, 456, 618, 643 }

F grade: { 3, 4, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 182, 184, 185, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 338, 340, 341, 343, 344, 346, 347, 353, 367, 385, 449, 450, 451, 452, 453, 454, 457, 458, 459, 468, 469, 470, 471, 472, 476, 477, 478, 479, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 519, 520, 521, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 599, 600, 601, 602, 603, 604, 605, 606, 613, 615, 617, 620, 621, 623, 625, 627, 629, 630, 632, 634, 636, 638, 640, 642, 645, 646, 648, 651, 653, 654, 656, 659, 661, 662 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	42	39	62	117	0	107
normalized size	1	1.	0.81	0.75	1.19	2.25	0.	2.06
time (sec)	N/A	0.048	0.025	0.029	1.022	2.205	0.	1.147

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	19	63	0	19
normalized size	1	1.	1.	1.16	0.76	2.52	0.	0.76
time (sec)	N/A	0.005	0.008	0.004	1.001	1.968	0.	1.112

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	23	43	0	0
normalized size	1	1.	1.	1.17	1.	1.87	0.	0.
time (sec)	N/A	0.005	0.008	0.005	1.1	1.998	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	46	46	51	107	0	0
normalized size	1	1.	0.6	0.6	0.66	1.39	0.	0.
time (sec)	N/A	0.057	0.013	0.006	1.044	1.992	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	38	50	138	48	51
normalized size	1	1.	1.	0.79	1.04	2.88	1.	1.06
time (sec)	N/A	0.023	0.016	0.029	1.445	1.939	0.135	1.104

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	0	31	12	31
normalized size	1	1.	0.49	0.46	0.	0.39	0.15	0.39
time (sec)	N/A	0.023	0.014	0.004	0.	1.772	0.102	1.12

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	18	31	12	39
normalized size	1	1.	0.49	0.46	0.23	0.39	0.15	0.49
time (sec)	N/A	0.023	0.008	0.004	1.126	1.701	0.101	1.131

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	18	31	12	39
normalized size	1	1.	0.49	0.46	0.23	0.39	0.15	0.49
time (sec)	N/A	0.024	0.007	0.003	1.085	1.715	0.099	1.108

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	31	12	30
normalized size	1	1.	1.06	0.97	0.	0.86	0.33	0.83
time (sec)	N/A	0.028	0.009	0.003	0.	1.676	0.1	1.12

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	18	31	12	39
normalized size	1	1.	0.49	0.46	0.23	0.39	0.15	0.49
time (sec)	N/A	0.018	0.009	0.008	1.17	1.625	0.099	1.109

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	36	33	14	23	8	27
normalized size	1	1.	0.49	0.45	0.19	0.31	0.11	0.36
time (sec)	N/A	0.012	0.007	0.003	1.022	1.709	0.096	1.102

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	37	34	0	30	10	38
normalized size	1	1.	0.49	0.45	0.	0.4	0.13	0.51
time (sec)	N/A	0.02	0.011	0.053	0.	1.75	0.116	1.104

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	38	36	19	28	8	39
normalized size	1	1.	0.49	0.47	0.25	0.36	0.1	0.51
time (sec)	N/A	0.021	0.01	0.003	1.011	1.779	0.266	1.113

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	37	34	20	31	8	35
normalized size	1	1.	0.5	0.46	0.27	0.42	0.11	0.47
time (sec)	N/A	0.02	0.008	0.003	1.018	1.738	0.274	1.101

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	39	38	0	41	10	58
normalized size	1	1.	0.52	0.51	0.	0.55	0.13	0.77
time (sec)	N/A	0.021	0.011	0.008	0.	1.72	0.297	1.108

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	37	34	18	32	14	41
normalized size	1	1.	0.48	0.44	0.23	0.42	0.18	0.53
time (sec)	N/A	0.021	0.008	0.004	1.026	1.805	0.308	1.134

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	36	15	42
normalized size	1	1.	0.49	0.46	0.25	0.46	0.19	0.53
time (sec)	N/A	0.022	0.008	0.003	1.032	1.735	0.311	1.102

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	37	34	0	32	14	41
normalized size	1	1.	0.47	0.43	0.	0.41	0.18	0.52
time (sec)	N/A	0.022	0.008	0.003	0.	1.642	0.317	1.118

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	36	15	42
normalized size	1	1.	0.49	0.46	0.25	0.46	0.19	0.53
time (sec)	N/A	0.022	0.008	0.003	1.013	1.723	0.328	1.115

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	36	15	42
normalized size	1	1.	0.49	0.46	0.25	0.46	0.19	0.53
time (sec)	N/A	0.021	0.008	0.003	1.06	1.713	0.332	1.121

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	0	36	15	42
normalized size	1	1.	0.49	0.46	0.	0.46	0.19	0.53
time (sec)	N/A	0.022	0.008	0.003	0.	1.676	0.342	1.094

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	39	15	42
normalized size	1	1.	0.49	0.46	0.25	0.49	0.19	0.53
time (sec)	N/A	0.022	0.008	0.003	1.019	1.799	0.348	1.127

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	90	0	90
normalized size	1	1.	0.37	0.35	0.28	0.54	0.	0.54
time (sec)	N/A	0.043	0.02	0.006	1.021	1.658	0.	1.127

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	167	61	58	0	85	0	90
normalized size	1	1.4	0.51	0.49	0.	0.71	0.	0.76
time (sec)	N/A	0.053	0.018	0.007	0.	1.722	0.	1.115

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	88	0	90
normalized size	1	1.	0.37	0.35	0.28	0.53	0.	0.54
time (sec)	N/A	0.043	0.018	0.006	1.034	1.724	0.	1.097

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	88	0	90
normalized size	1	1.	0.37	0.35	0.28	0.53	0.	0.54
time (sec)	N/A	0.041	0.016	0.006	1.059	1.778	0.	1.109

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	61	58	0	84	0	61
normalized size	1	1.	0.78	0.74	0.	1.08	0.	0.78
time (sec)	N/A	0.05	0.019	0.006	0.	1.653	0.	1.096

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	85	0	90
normalized size	1	1.	0.37	0.35	0.28	0.51	0.	0.54
time (sec)	N/A	0.041	0.016	0.005	1.011	1.705	0.	1.102

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	85	0	90
normalized size	1	1.	0.37	0.35	0.28	0.51	0.	0.54
time (sec)	N/A	0.041	0.017	0.004	1.022	1.715	0.	1.11

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	60	57	0	82	0	59
normalized size	1	1.	1.67	1.58	0.	2.28	0.	1.64
time (sec)	N/A	0.03	0.018	0.006	0.	1.677	0.	1.112

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	82	0	90
normalized size	1	1.	0.37	0.35	0.28	0.49	0.	0.54
time (sec)	N/A	0.038	0.016	0.004	1.021	1.784	0.	1.102

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	59	56	43	74	0	86
normalized size	1	1.	0.36	0.35	0.27	0.46	0.	0.53
time (sec)	N/A	0.033	0.015	0.003	1.054	1.81	0.	1.122

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	60	57	0	73	0	88
normalized size	1	1.	0.38	0.36	0.	0.46	0.	0.55
time (sec)	N/A	0.048	0.021	0.007	0.	1.768	0.	1.122

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	50	80	0	90
normalized size	1	1.	0.37	0.35	0.3	0.48	0.	0.55
time (sec)	N/A	0.042	0.017	0.006	1.059	1.639	0.	1.115

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	50	82	0	88
normalized size	1	1.	0.37	0.36	0.31	0.5	0.	0.54
time (sec)	N/A	0.042	0.018	0.005	1.051	1.722	0.	1.115

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	62	59	0	85	0	115
normalized size	1	1.	0.39	0.37	0.	0.53	0.	0.71
time (sec)	N/A	0.048	0.022	0.012	0.	1.655	0.	1.124

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	50	81	0	93
normalized size	1	1.	0.37	0.35	0.3	0.49	0.	0.56
time (sec)	N/A	0.042	0.02	0.004	1.018	1.817	0.	1.124

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	50	81	0	92
normalized size	1	1.	0.37	0.36	0.31	0.5	0.	0.56
time (sec)	N/A	0.041	0.021	0.005	1.052	1.684	0.	1.126

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	61	60	0	85	0	116
normalized size	1	1.	0.38	0.37	0.	0.52	0.	0.72
time (sec)	N/A	0.047	0.017	0.012	0.	1.698	0.	1.123

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	50	82	0	95
normalized size	1	1.	0.37	0.35	0.3	0.5	0.	0.58
time (sec)	N/A	0.041	0.013	0.006	1.036	1.682	0.	1.125

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	61	58	50	82	0	90
normalized size	1	1.	0.38	0.36	0.31	0.51	0.	0.56
time (sec)	N/A	0.041	0.015	0.004	1.024	1.792	0.	1.118

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	63	60	0	90	0	115
normalized size	1	1.	0.39	0.37	0.	0.56	0.	0.71
time (sec)	N/A	0.046	0.022	0.011	0.	1.726	0.	1.115

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	50	90	0	93
normalized size	1	1.	0.37	0.35	0.3	0.55	0.	0.56
time (sec)	N/A	0.04	0.013	0.005	1.038	1.65	0.	1.146

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	92	0	93
normalized size	1	1.	0.37	0.35	0.3	0.55	0.	0.56
time (sec)	N/A	0.041	0.016	0.006	1.019	1.688	0.	1.134

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	59	56	0	78	0	92
normalized size	1	1.	1.44	1.37	0.	1.9	0.	2.24
time (sec)	N/A	0.018	0.013	0.008	0.	1.775	0.	1.087

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	95	0	93
normalized size	1	1.	0.37	0.35	0.3	0.57	0.	0.56
time (sec)	N/A	0.041	0.014	0.006	1.07	1.767	0.	1.124

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	96	0	93
normalized size	1	1.	0.37	0.35	0.3	0.57	0.	0.56
time (sec)	N/A	0.041	0.016	0.006	0.989	1.616	0.	1.113

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	61	58	0	85	0	93
normalized size	1	1.	0.73	0.69	0.	1.01	0.	1.11
time (sec)	N/A	0.04	0.014	0.007	0.	1.738	0.	1.117

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	99	0	93
normalized size	1	1.	0.37	0.35	0.3	0.59	0.	0.56
time (sec)	N/A	0.04	0.015	0.007	1.021	1.769	0.	1.098

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	144	0	142
normalized size	1	1.	0.33	0.31	0.3	0.56	0.	0.56
time (sec)	N/A	0.064	0.026	0.009	1.06	1.657	0.	1.101

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	144	0	142
normalized size	1	1.	0.33	0.31	0.3	0.56	0.	0.56
time (sec)	N/A	0.058	0.021	0.006	1.007	1.78	0.	1.105

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	83	80	0	143	0	142
normalized size	1	1.	0.52	0.5	0.	0.89	0.	0.89
time (sec)	N/A	0.12	0.022	0.008	0.	1.697	0.	1.097

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	144	0	142
normalized size	1	1.	0.33	0.31	0.3	0.56	0.	0.56
time (sec)	N/A	0.061	0.026	0.006	1.021	1.795	0.	1.127

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	144	0	142
normalized size	1	1.	0.33	0.31	0.3	0.56	0.	0.56
time (sec)	N/A	0.057	0.024	0.006	0.966	1.696	0.	1.124

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	83	80	0	139	0	142
normalized size	1	1.	0.7	0.67	0.	1.17	0.	1.19
time (sec)	N/A	0.09	0.024	0.009	0.	1.743	0.	1.123

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	140	0	142
normalized size	1	1.	0.33	0.31	0.3	0.55	0.	0.56
time (sec)	N/A	0.06	0.022	0.007	1.025	1.665	0.	1.095

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	140	0	142
normalized size	1	1.	0.33	0.31	0.3	0.55	0.	0.56
time (sec)	N/A	0.059	0.021	0.006	1.012	1.751	0.	1.094

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	83	80	0	136	0	90
normalized size	1	1.	1.06	1.03	0.	1.74	0.	1.15
time (sec)	N/A	0.056	0.021	0.007	0.	1.718	0.	1.118

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	139	0	142
normalized size	1	1.	0.33	0.31	0.3	0.55	0.	0.56
time (sec)	N/A	0.059	0.02	0.008	1.063	1.762	0.	1.111

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	76	134	0	140
normalized size	1	1.	0.33	0.32	0.3	0.53	0.	0.56
time (sec)	N/A	0.057	0.02	0.007	1.027	1.725	0.	1.114

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	82	79	0	135	0	89
normalized size	1	1.	2.28	2.19	0.	3.75	0.	2.47
time (sec)	N/A	0.029	0.021	0.009	0.	1.707	0.	1.101

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	76	132	0	140
normalized size	1	1.	0.33	0.32	0.3	0.52	0.	0.56
time (sec)	N/A	0.054	0.02	0.004	1.016	1.752	0.	1.136

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	81	78	72	123	0	136
normalized size	1	1.	0.33	0.32	0.29	0.5	0.	0.55
time (sec)	N/A	0.051	0.021	0.004	1.027	1.765	0.	1.112

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	82	79	0	134	0	140
normalized size	1	1.	0.33	0.31	0.	0.53	0.	0.56
time (sec)	N/A	0.069	0.026	0.009	0.	1.636	0.	1.098

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	80	138	0	142
normalized size	1	1.	0.33	0.32	0.32	0.55	0.	0.57
time (sec)	N/A	0.061	0.021	0.006	1.182	1.715	0.	1.127

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	80	138	0	139
normalized size	1	1.	0.33	0.32	0.32	0.55	0.	0.55
time (sec)	N/A	0.059	0.023	0.006	1.005	1.739	0.	1.121

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	0	143	0	167
normalized size	1	1.	0.34	0.33	0.	0.57	0.	0.66
time (sec)	N/A	0.074	0.026	0.011	0.	1.815	0.	1.11

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	83	80	80	135	0	144
normalized size	1	1.	0.33	0.32	0.32	0.54	0.	0.58
time (sec)	N/A	0.059	0.025	0.007	1.099	1.705	0.	1.131

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	80	135	0	143
normalized size	1	1.	0.33	0.32	0.32	0.54	0.	0.57
time (sec)	N/A	0.058	0.022	0.008	1.126	1.655	0.	1.111

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	0	140	0	170
normalized size	1	1.	0.34	0.33	0.	0.56	0.	0.67
time (sec)	N/A	0.073	0.024	0.012	0.	1.605	0.	1.133

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	83	80	80	132	0	144
normalized size	1	1.	0.33	0.32	0.32	0.53	0.	0.58
time (sec)	N/A	0.058	0.02	0.006	1.07	1.771	0.	1.104

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	80	132	0	142
normalized size	1	1.	0.34	0.32	0.32	0.53	0.	0.57
time (sec)	N/A	0.059	0.021	0.006	1.113	1.764	0.	1.142

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	0	140	0	171
normalized size	1	1.	0.34	0.33	0.	0.56	0.	0.68
time (sec)	N/A	0.071	0.028	0.011	0.	1.733	0.	1.126

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	80	136	0	146
normalized size	1	1.	0.33	0.32	0.32	0.54	0.	0.58
time (sec)	N/A	0.062	0.016	0.006	1.061	1.776	0.	1.118

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	80	136	0	143
normalized size	1	1.	0.34	0.32	0.32	0.55	0.	0.58
time (sec)	N/A	0.06	0.016	0.006	1.014	1.722	0.	1.142

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	0	144	0	169
normalized size	1	1.	0.34	0.33	0.	0.57	0.	0.67
time (sec)	N/A	0.071	0.02	0.013	0.	1.747	0.	1.146

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	80	139	0	146
normalized size	1	1.	0.33	0.32	0.32	0.55	0.	0.58
time (sec)	N/A	0.059	0.017	0.005	1.071	1.723	0.	1.126

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	83	80	80	142	0	142
normalized size	1	1.	0.33	0.32	0.32	0.57	0.	0.57
time (sec)	N/A	0.06	0.018	0.006	1.008	1.682	0.	1.12

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	85	82	0	150	0	166
normalized size	1	1.	0.34	0.33	0.	0.6	0.	0.66
time (sec)	N/A	0.069	0.027	0.012	0.	1.671	0.	1.114

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	80	150	0	144
normalized size	1	1.	0.33	0.32	0.32	0.6	0.	0.57
time (sec)	N/A	0.058	0.017	0.006	1.018	1.746	0.	1.104

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	80	153	0	144
normalized size	1	1.	0.33	0.32	0.32	0.6	0.	0.57
time (sec)	N/A	0.058	0.019	0.007	0.994	1.683	0.	1.123

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	81	78	0	128	0	143
normalized size	1	1.	1.98	1.9	0.	3.12	0.	3.49
time (sec)	N/A	0.018	0.016	0.005	0.	1.806	0.	1.121

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	80	159	0	144
normalized size	1	1.	0.33	0.32	0.32	0.63	0.	0.57
time (sec)	N/A	0.061	0.019	0.007	1.025	1.788	0.	1.113

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	162	0	144
normalized size	1	1.	0.33	0.31	0.31	0.64	0.	0.56
time (sec)	N/A	0.056	0.018	0.007	1.037	1.72	0.	1.114

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	83	80	0	136	0	144
normalized size	1	1.	0.99	0.95	0.	1.62	0.	1.71
time (sec)	N/A	0.04	0.017	0.009	0.	1.73	0.	1.113

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	165	0	144
normalized size	1	1.	0.33	0.31	0.31	0.65	0.	0.56
time (sec)	N/A	0.058	0.017	0.007	1.005	1.655	0.	1.129

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	169	0	144
normalized size	1	1.	0.33	0.31	0.31	0.66	0.	0.56
time (sec)	N/A	0.056	0.018	0.007	1.037	1.692	0.	1.126

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	83	80	0	142	0	144
normalized size	1	1.	0.65	0.62	0.	1.11	0.	1.12
time (sec)	N/A	0.057	0.02	0.007	0.	1.732	0.	1.117

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	131	113	0	298	32	197
normalized size	1	1.	0.55	0.47	0.	1.24	0.13	0.82
time (sec)	N/A	0.124	0.046	0.009	0.	1.735	0.359	1.151

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	128	110	0	254	22	193
normalized size	1	1.	0.54	0.47	0.	1.08	0.09	0.82
time (sec)	N/A	0.114	0.031	0.007	0.	1.797	0.355	1.106

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	35	32	23	30	10	30
normalized size	1	1.	0.8	0.73	0.52	0.68	0.23	0.68
time (sec)	N/A	0.035	0.008	0.007	1.071	1.598	0.168	1.129

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	109	97	0	744	24	184
normalized size	1	1.	0.54	0.48	0.	3.68	0.12	0.91
time (sec)	N/A	0.085	0.025	0.004	0.	1.913	0.174	1.125

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	109	95	0	749	20	165
normalized size	1	1.	0.54	0.47	0.	3.71	0.1	0.82
time (sec)	N/A	0.117	0.023	0.004	0.	1.745	0.182	1.121

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	42	39	0	49	15	43
normalized size	1	1.	0.52	0.49	0.	0.61	0.19	0.54
time (sec)	N/A	0.034	0.011	0.007	0.	1.882	0.262	1.124

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	133	111	0	262	29	177
normalized size	1	1.	0.56	0.47	0.	1.1	0.12	0.74
time (sec)	N/A	0.108	0.031	0.007	0.	1.831	0.384	1.114

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	140	118	0	335	32	169
normalized size	1	1.	0.58	0.49	0.	1.4	0.13	0.7
time (sec)	N/A	0.107	0.033	0.01	0.	1.839	0.416	1.105

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	54	51	0	80	31	68
normalized size	1	1.	0.44	0.42	0.	0.66	0.25	0.56
time (sec)	N/A	0.051	0.016	0.01	0.	1.733	0.531	1.129

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	235	299	0	1152	0	0
normalized size	1	1.	0.84	1.07	0.	4.11	0.	0.
time (sec)	N/A	0.135	0.075	0.016	0.	1.823	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	235	297	0	1152	0	0
normalized size	1	1.	0.85	1.08	0.	4.17	0.	0.
time (sec)	N/A	0.133	0.064	0.016	0.	1.888	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	24	51	0	0
normalized size	1	1.	0.71	0.63	0.63	1.34	0.	0.
time (sec)	N/A	0.029	0.01	0.006	1.141	1.735	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	237	301	0	1164	0	0
normalized size	1	1.	0.86	1.09	0.	4.2	0.	0.
time (sec)	N/A	0.14	0.071	0.009	0.	1.9	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	235	299	0	1162	0	0
normalized size	1	1.	0.82	1.05	0.	4.06	0.	0.
time (sec)	N/A	0.147	0.069	0.007	0.	1.875	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	74	107	0	196	0	0
normalized size	1	1.	0.5	0.73	0.	1.33	0.	0.
time (sec)	N/A	0.083	0.028	0.02	0.	1.826	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	260	316	0	467	0	0
normalized size	1	1.	0.82	1.	0.	1.48	0.	0.
time (sec)	N/A	0.16	0.085	0.016	0.	1.746	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	266	322	0	540	0	0
normalized size	1	1.	0.84	1.02	0.	1.71	0.	0.
time (sec)	N/A	0.159	0.088	0.015	0.	1.771	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	97	133	0	247	0	0
normalized size	1	1.	0.52	0.71	0.	1.31	0.	0.
time (sec)	N/A	0.097	0.034	0.02	0.	1.798	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	218	519	0	1632	0	0
normalized size	1	1.	0.61	1.45	0.	4.55	0.	0.
time (sec)	N/A	0.185	0.126	0.017	0.	1.932	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	39	32	65	119	0	0
normalized size	1	1.	0.5	0.41	0.83	1.53	0.	0.
time (sec)	N/A	0.053	0.016	0.007	1.028	1.684	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	229	521	0	1632	0	0
normalized size	1	1.	0.62	1.42	0.	4.43	0.	0.
time (sec)	N/A	0.191	0.141	0.017	0.	1.841	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	221	519	0	1636	0	0
normalized size	1	1.	0.61	1.44	0.	4.54	0.	0.
time (sec)	N/A	0.18	0.14	0.016	0.	1.856	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	24	97	0	0
normalized size	1	1.	0.71	0.63	0.63	2.55	0.	0.
time (sec)	N/A	0.03	0.01	0.006	1.055	1.749	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	219	521	0	1651	0	0
normalized size	1	1.	0.61	1.45	0.	4.6	0.	0.
time (sec)	N/A	0.193	0.12	0.008	0.	1.85	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	211	519	0	1652	0	0
normalized size	1	1.	0.58	1.43	0.	4.54	0.	0.
time (sec)	N/A	0.199	0.114	0.01	0.	1.876	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	96	193	0	382	0	0
normalized size	1	1.	0.43	0.87	0.	1.71	0.	0.
time (sec)	N/A	0.124	0.042	0.019	0.	1.551	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	242	536	0	724	0	0
normalized size	1	1.	0.61	1.35	0.	1.82	0.	0.
time (sec)	N/A	0.216	0.13	0.02	0.	1.594	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	234	542	0	799	0	0
normalized size	1	1.	0.59	1.36	0.	2.01	0.	0.
time (sec)	N/A	0.211	0.135	0.022	0.	1.575	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	119	219	0	446	0	0
normalized size	1	1.	0.44	0.81	0.	1.66	0.	0.
time (sec)	N/A	0.143	0.052	0.02	0.	1.614	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	111	453	328	886	0	1215
normalized size	1	1.	0.35	1.45	1.05	2.83	0.	3.88
time (sec)	N/A	0.138	0.1	0.007	1.036	1.598	0.	1.201

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	131	199	161	358	0	518
normalized size	1	1.	0.64	0.97	0.79	1.75	0.	2.53
time (sec)	N/A	0.085	0.071	0.006	1.1	1.584	0.	1.127

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	53	56	47	77	0	112
normalized size	1	1.	0.55	0.58	0.48	0.79	0.	1.15
time (sec)	N/A	0.038	0.023	0.005	1.016	1.518	0.	1.124

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	62	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.018	0.054	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.02	0.022	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.019	0.028	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	66	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.017	0.152	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	110	150	155	336	0	506
normalized size	1	1.	0.64	0.87	0.9	1.95	0.	2.94
time (sec)	N/A	0.113	0.056	0.008	1.152	1.584	0.	1.132

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	77	96	107	223	0	317
normalized size	1	1.	0.59	0.74	0.82	1.72	0.	2.44
time (sec)	N/A	0.079	0.033	0.007	1.029	1.622	0.	1.126

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	51	60	73	142	0	178
normalized size	1	1.	0.61	0.71	0.87	1.69	0.	2.12
time (sec)	N/A	0.055	0.02	0.007	1.073	1.559	0.	1.12

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.007	0.048	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.006	0.034	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	32	40	41	80	0	78
normalized size	1	1.	0.78	0.98	1.	1.95	0.	1.9
time (sec)	N/A	0.028	0.006	0.004	1.044	1.552	0.	1.115

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	60	51	0	0	0	0	0
normalized size	1	1.03	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.006	0.026	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	55	204	0	0	0	0	0
normalized size	1	1.04	3.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.167	0.013	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	54	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.011	0.02	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	49	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.006	0.033	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.006	0.036	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.011	0.039	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.006	0.044	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	556	316	101
normalized size	1	1.	0.96	1.37	0.	6.86	3.9	1.25
time (sec)	N/A	0.083	0.054	0.004	0.	1.598	2.305	1.411

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	443	223	80
normalized size	1	1.	0.98	0.95	0.	7.03	3.54	1.27
time (sec)	N/A	0.058	0.023	0.003	0.	1.513	1.29	1.429

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	296	131	49
normalized size	1	1.	1.11	0.97	0.	7.79	3.45	1.29
time (sec)	N/A	0.036	0.009	0.002	0.	1.473	0.632	1.395

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	66	0	510	253	89
normalized size	1	1.	0.96	0.96	0.	7.39	3.67	1.29
time (sec)	N/A	0.07	0.023	0.006	0.	1.618	3.692	1.436

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	92	119	0	664	345	126
normalized size	1	1.	1.03	1.34	0.	7.46	3.88	1.42
time (sec)	N/A	0.125	0.029	0.008	0.	1.877	80.389	1.414

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	636	636	70	61	0	12034	279	0
normalized size	1	1.	0.11	0.1	0.	18.92	0.44	0.
time (sec)	N/A	1.249	0.03	0.089	0.	7.79	3.941	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	631	70	59	0	11169	196	0
normalized size	1	1.	0.11	0.09	0.	17.7	0.31	0.
time (sec)	N/A	1.024	0.03	0.003	0.	5.375	2.661	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	44	43	0	8007	175	0
normalized size	1	1.	0.08	0.08	0.	14.35	0.31	0.
time (sec)	N/A	0.52	0.017	0.003	0.	2.936	1.757	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	42	43	0	5501	122	0
normalized size	1	1.	0.08	0.08	0.	9.86	0.22	0.
time (sec)	N/A	0.574	0.016	0.003	0.	1.991	1.897	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	43	41	0	6175	158	0
normalized size	1	1.	0.08	0.07	0.	11.07	0.28	0.
time (sec)	N/A	0.472	0.018	0.002	0.	2.12	1.529	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	45	40	0	8374	155	0
normalized size	1	1.	0.08	0.07	0.	15.01	0.28	0.
time (sec)	N/A	0.595	0.018	0.002	0.	2.948	2.825	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	610	610	71	61	0	11187	252	0
normalized size	1	1.	0.12	0.1	0.	18.34	0.41	0.
time (sec)	N/A	0.818	0.031	0.006	0.	6.211	2.544	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	612	612	75	62	0	12411	241	0
normalized size	1	1.	0.12	0.1	0.	20.28	0.39	0.
time (sec)	N/A	0.815	0.033	0.006	0.	5.721	5.786	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	36	77	29	39
normalized size	1	1.	1.	0.8	1.03	2.2	0.83	1.11
time (sec)	N/A	0.024	0.006	0.005	1.226	1.419	0.121	1.095

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	30	63	22	32
normalized size	1	1.	1.	0.82	1.07	2.25	0.79	1.14
time (sec)	N/A	0.019	0.005	0.003	1.184	1.477	0.128	1.113

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	50	15	26
normalized size	1	1.	1.	0.86	1.1	2.38	0.71	1.24
time (sec)	N/A	0.014	0.004	0.004	1.307	1.482	0.116	1.092

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	21	21	18	23	51	15	26
normalized size	1	2.1	2.1	1.8	2.3	5.1	1.5	2.6
time (sec)	N/A	0.014	0.004	0.004	1.388	1.461	0.112	1.091

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	31	31	69	20	32
normalized size	1	1.	1.	1.15	1.15	2.56	0.74	1.19
time (sec)	N/A	0.019	0.006	0.013	1.195	1.439	0.143	1.157

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	36	38	96	29	49
normalized size	1	1.	1.	1.06	1.12	2.82	0.85	1.44
time (sec)	N/A	0.032	0.007	0.008	1.162	1.422	0.17	1.102

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	41	47	109	34	55
normalized size	1	1.	1.	1.	1.15	2.66	0.83	1.34
time (sec)	N/A	0.036	0.006	0.009	1.05	1.486	0.202	1.086

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	118	94	127	358	144	0
normalized size	1	1.	0.95	0.76	1.02	2.89	1.16	0.
time (sec)	N/A	0.114	0.054	0.009	1.633	1.52	0.637	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	114	92	124	305	129	0
normalized size	1	1.	0.93	0.75	1.02	2.5	1.06	0.
time (sec)	N/A	0.094	0.03	0.007	1.674	1.558	0.645	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	111	89	120	327	134	0
normalized size	1	1.	0.93	0.75	1.01	2.75	1.13	0.
time (sec)	N/A	0.082	0.027	0.006	1.7	1.524	0.668	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	111	85	115	331	126	0
normalized size	1	1.	0.98	0.75	1.02	2.93	1.12	0.
time (sec)	N/A	0.074	0.026	0.005	1.63	1.434	0.625	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	107	84	113	392	134	0
normalized size	1	1.	0.96	0.75	1.01	3.5	1.2	0.
time (sec)	N/A	0.068	0.038	0.007	1.627	1.515	0.62	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	106	84	113	342	110	0
normalized size	1	1.	0.95	0.75	1.01	3.05	0.98	0.
time (sec)	N/A	0.069	0.024	0.007	1.566	1.542	0.654	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	108	84	113	289	119	0
normalized size	1	1.	0.96	0.75	1.01	2.58	1.06	0.
time (sec)	N/A	0.069	0.038	0.007	1.656	1.499	1.31	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	107	84	113	450	124	0
normalized size	1	1.	0.96	0.75	1.01	4.02	1.11	0.
time (sec)	N/A	0.065	0.026	0.006	1.68	1.51	1.37	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	118	89	120	409	139	0
normalized size	1	1.	0.99	0.75	1.01	3.44	1.17	0.
time (sec)	N/A	0.082	0.044	0.008	1.674	1.495	1.283	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	113	89	120	378	128	0
normalized size	1	1.	0.95	0.75	1.01	3.18	1.08	0.
time (sec)	N/A	0.078	0.056	0.007	1.618	1.489	1.302	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	118	94	130	339	141	0
normalized size	1	1.	0.94	0.75	1.03	2.69	1.12	0.
time (sec)	N/A	0.103	0.051	0.008	1.608	1.474	1.305	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	118	94	130	510	136	0
normalized size	1	1.	0.94	0.75	1.03	4.05	1.08	0.
time (sec)	N/A	0.099	0.061	0.01	1.698	1.523	1.285	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	59	44	0	3906	26	861
normalized size	1	1.	0.14	0.11	0.	9.48	0.06	2.09
time (sec)	N/A	0.428	0.014	0.008	0.	1.875	0.177	1.195

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	43	95	37	43
normalized size	1	1.	1.	0.85	1.1	2.44	0.95	1.1
time (sec)	N/A	0.035	0.01	0.003	1.598	1.436	0.13	1.091

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	41	40	0	5936	26	1112
normalized size	1	1.	0.1	0.1	0.	14.44	0.06	2.71
time (sec)	N/A	0.281	0.01	0.004	0.	2.302	0.18	1.181

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	39	40	0	3903	24	860
normalized size	1	1.	0.09	0.1	0.	9.5	0.06	2.09
time (sec)	N/A	0.278	0.009	0.004	0.	1.911	0.179	1.151

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	61	27	24
normalized size	1	1.	1.	0.83	1.04	2.65	1.17	1.04
time (sec)	N/A	0.023	0.006	0.001	1.701	1.45	0.124	1.14

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	40	38	0	5936	26	1096
normalized size	1	1.	0.11	0.1	0.	15.83	0.07	2.92
time (sec)	N/A	0.25	0.009	0.004	0.	2.356	0.173	1.162

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	375	42	37	0	3900	20	849
normalized size	1	2.02	0.23	0.2	0.	20.97	0.11	4.56
time (sec)	N/A	0.242	0.01	0.005	0.	1.894	0.18	1.174

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	35	51	107	41	47
normalized size	1	1.	1.34	0.85	1.24	2.61	1.	1.15
time (sec)	N/A	0.039	0.012	0.004	1.612	1.506	0.149	1.113

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	61	50	0	5952	24	1115
normalized size	1	1.	0.15	0.12	0.	14.31	0.06	2.68
time (sec)	N/A	0.3	0.012	0.007	0.	2.266	0.205	1.165

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	65	50	0	3947	31	867
normalized size	1	1.	0.16	0.12	0.	9.44	0.07	2.07
time (sec)	N/A	0.342	0.013	0.007	0.	1.942	0.207	1.194

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	40	58	143	48	61
normalized size	1	1.	1.06	0.83	1.21	2.98	1.	1.27
time (sec)	N/A	0.052	0.014	0.008	1.561	1.447	0.174	1.128

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	54	51	0	5994	37	1129
normalized size	1	1.	0.13	0.12	0.	14.17	0.09	2.67
time (sec)	N/A	0.369	0.015	0.007	0.	2.294	0.215	1.157

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	38	33	0	7470	24	0
normalized size	1	1.	0.1	0.09	0.	19.61	0.06	0.
time (sec)	N/A	0.403	0.009	0.003	0.	5.279	0.154	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	62	27	24
normalized size	1	1.	1.	0.83	1.04	2.7	1.17	1.04
time (sec)	N/A	0.024	0.008	0.003	1.557	1.453	0.118	1.202

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	37	36	0	5416	24	0
normalized size	1	1.	0.09	0.09	0.	13.57	0.06	0.
time (sec)	N/A	0.305	0.008	0.003	0.	2.023	0.145	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	208	0	0	1071	0	0
normalized size	1	1.	0.9	0.	0.	4.64	0.	0.
time (sec)	N/A	0.3	0.165	0.033	0.	1.752	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	164	0	0	863	0	0
normalized size	1	1.	0.96	0.	0.	5.05	0.	0.
time (sec)	N/A	0.152	0.137	0.022	0.	1.707	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	136	0	0	699	0	0
normalized size	1	1.	0.89	0.	0.	4.57	0.	0.
time (sec)	N/A	0.136	0.072	0.019	0.	1.648	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	99	0	0	554	0	132
normalized size	1	1.	0.92	0.	0.	5.13	0.	1.22
time (sec)	N/A	0.085	0.076	0.018	0.	1.648	0.	1.168

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	87	0	0	463	0	103
normalized size	1	1.	1.05	0.	0.	5.58	0.	1.24
time (sec)	N/A	0.06	0.011	0.017	0.	1.547	0.	1.16

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	106	0	0	1335	0	0
normalized size	1	1.	0.97	0.	0.	12.25	0.	0.
time (sec)	N/A	0.111	0.043	0.014	0.	1.83	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	0	0	1411	0	0
normalized size	1	1.	1.	0.	0.	12.6	0.	0.
time (sec)	N/A	0.114	0.049	0.031	0.	1.876	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	502	0	0
normalized size	1	1.	1.01	0.	0.	5.7	0.	0.
time (sec)	N/A	0.073	0.045	0.024	0.	1.766	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	108	0	0	595	0	0
normalized size	1	1.	0.93	0.	0.	5.13	0.	0.
time (sec)	N/A	0.095	0.086	0.028	0.	1.913	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	139	0	0	749	0	0
normalized size	1	1.	0.86	0.	0.	4.65	0.	0.
time (sec)	N/A	0.147	0.102	0.036	0.	2.068	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	173	0	0	910	0	0
normalized size	1	1.	0.87	0.	0.	4.57	0.	0.
time (sec)	N/A	0.227	0.17	0.043	0.	2.646	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	358	0	0	0	0	0
normalized size	1	1.	2.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	0.577	0.029	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	337	0	0	0	0	0
normalized size	1	1.	2.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.422	0.025	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	335	0	0	0	0	0
normalized size	1	1.	2.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.343	0.02	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	340	0	0	0	0	0
normalized size	1	1.	2.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.386	0.021	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	340	0	0	0	0	0
normalized size	1	1.	2.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.331	0.022	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	241	0	0	1570	0	0
normalized size	1	1.	0.82	0.	0.	5.36	0.	0.
time (sec)	N/A	0.402	0.359	0.023	0.	1.891	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	192	0	0	1268	0	0
normalized size	1	1.	0.86	0.	0.	5.69	0.	0.
time (sec)	N/A	0.206	0.196	0.02	0.	2.071	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	175	0	0	1064	0	0
normalized size	1	1.	0.86	0.	0.	5.22	0.	0.
time (sec)	N/A	0.188	0.17	0.018	0.	1.767	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	149	0	0	840	0	232
normalized size	1	1.	0.99	0.	0.	5.6	0.	1.55
time (sec)	N/A	0.116	0.152	0.018	0.	1.657	0.	1.159

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	126	0	0	684	0	182
normalized size	1	1.	1.02	0.	0.	5.52	0.	1.47
time (sec)	N/A	0.086	0.088	0.018	0.	1.633	0.	1.148

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	143	0	0	1717	0	0
normalized size	1	1.	0.92	0.	0.	11.08	0.	0.
time (sec)	N/A	0.179	0.15	0.013	0.	2.498	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	134	0	0	1667	0	0
normalized size	1	1.	0.89	0.	0.	11.11	0.	0.
time (sec)	N/A	0.17	0.11	0.04	0.	2.257	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	134	0	0	1670	0	0
normalized size	1	1.	0.89	0.	0.	11.06	0.	0.
time (sec)	N/A	0.162	0.176	0.033	0.	2.205	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	149	0	0	1806	0	0
normalized size	1	1.	0.91	0.	0.	11.08	0.	0.
time (sec)	N/A	0.176	0.214	0.041	0.	2.476	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	138	0	0	732	0	0
normalized size	1	1.	1.04	0.	0.	5.5	0.	0.
time (sec)	N/A	0.108	0.176	0.035	0.	2.027	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	167	0	0	890	0	0
normalized size	1	1.	1.03	0.	0.	5.49	0.	0.
time (sec)	N/A	0.146	0.143	0.044	0.	2.737	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	206	0	0	1111	0	0
normalized size	1	1.	0.95	0.	0.	5.14	0.	0.
time (sec)	N/A	0.207	0.213	0.059	0.	3.908	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	243	0	0	1318	0	0
normalized size	1	1.	0.95	0.	0.	5.17	0.	0.
time (sec)	N/A	0.312	0.506	0.084	0.	4.654	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	453	0	0	0	0	0
normalized size	1	1.	3.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.903	0.031	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	410	0	0	0	0	0
normalized size	1	1.	2.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.731	0.031	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	408	0	0	0	0	0
normalized size	1	1.	3.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.662	0.025	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	379	0	0	0	0	0
normalized size	1	1.	2.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.514	0.031	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	379	0	0	0	0	0
normalized size	1	1.	2.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.499	0.027	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	137	0	0	717	0	0
normalized size	1	1.	0.8	0.	0.	4.19	0.	0.
time (sec)	N/A	0.216	0.114	0.023	0.	1.642	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	104	0	0	568	0	0
normalized size	1	1.	0.86	0.	0.	4.69	0.	0.
time (sec)	N/A	0.105	0.053	0.02	0.	1.588	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	88	0	0	475	0	0
normalized size	1	1.	0.85	0.	0.	4.57	0.	0.
time (sec)	N/A	0.089	0.033	0.018	0.	1.611	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	0	0	385	0	82
normalized size	1	1.	1.	0.	0.	5.66	0.	1.21
time (sec)	N/A	0.057	0.016	0.017	0.	1.573	0.	1.16

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	285	0	54
normalized size	1	1.	1.	0.	0.	6.63	0.	1.26
time (sec)	N/A	0.034	0.006	0.01	0.	1.522	0.	1.184

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	294	0	0
normalized size	1	1.	1.	0.	0.	6.68	0.	0.
time (sec)	N/A	0.04	0.011	0.01	0.	1.589	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	424	0	0
normalized size	1	1.	1.	0.	0.	5.89	0.	0.
time (sec)	N/A	0.06	0.024	0.022	0.	1.638	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	92	0	0	512	0	0
normalized size	1	1.	0.85	0.	0.	4.74	0.	0.
time (sec)	N/A	0.101	0.062	0.024	0.	1.741	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	112	0	0	613	0	0
normalized size	1	1.	0.77	0.	0.	4.23	0.	0.
time (sec)	N/A	0.158	0.079	0.028	0.	1.859	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	141	0	0	764	0	0
normalized size	1	1.	0.73	0.	0.	3.98	0.	0.
time (sec)	N/A	0.233	0.102	0.029	0.	2.133	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	168	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.095	0.011	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	168	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.085	0.011	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	163	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.06	0.008	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	343	0	0	0	0	0
normalized size	1	1.	2.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.371	0.026	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	342	0	0	0	0	0
normalized size	1	1.	2.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.333	0.023	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	181	0	0	1272	0	0
normalized size	1	1.	0.93	0.	0.	6.52	0.	0.
time (sec)	N/A	0.229	0.192	0.045	0.	2.276	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	137	0	0	981	0	0
normalized size	1	1.	1.	0.	0.	7.16	0.	0.
time (sec)	N/A	0.111	0.119	0.036	0.	1.995	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	107	0	0	833	0	0
normalized size	1	1.	0.89	0.	0.	6.94	0.	0.
time (sec)	N/A	0.09	0.109	0.013	0.	1.822	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	41	38	0	144	0	61
normalized size	1	1.	1.05	0.97	0.	3.69	0.	1.56
time (sec)	N/A	0.029	0.099	0.005	0.	1.688	0.	1.458

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	37	0	146	0	61
normalized size	1	1.	1.	0.97	0.	3.84	0.	1.61
time (sec)	N/A	0.025	0.024	0.007	0.	1.69	0.	1.489

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	0	0	837	0	0
normalized size	1	1.	1.	0.	0.	9.1	0.	0.
time (sec)	N/A	0.078	0.125	0.018	0.	2.009	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	137	0	0	1026	0	0
normalized size	1	1.	0.96	0.	0.	7.23	0.	0.
time (sec)	N/A	0.124	0.085	0.038	0.	2.139	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	179	0	0	1319	0	0
normalized size	1	1.	0.9	0.	0.	6.66	0.	0.
time (sec)	N/A	0.204	0.129	0.046	0.	2.672	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	223	0	0	1550	0	0
normalized size	1	1.	0.87	0.	0.	6.05	0.	0.
time (sec)	N/A	0.286	0.182	0.057	0.	3.658	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	340	0	0	0	0	0
normalized size	1	1.	2.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	0.332	0.013	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	362	0	0	0	0	0
normalized size	1	1.	2.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.496	0.013	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	359	0	0	0	0	0
normalized size	1	1.	2.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.468	0.013	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	407	0	0	0	0	0
normalized size	1	1.	2.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.761	0.044	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	405	0	0	0	0	0
normalized size	1	1.	2.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.648	0.036	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	70	301	0	599	1510	606
normalized size	1	1.	0.69	2.98	0.	5.93	14.95	6.
time (sec)	N/A	0.061	0.072	0.007	0.	1.582	6.029	1.168

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	35	78	0	162	314	161
normalized size	1	1.	0.67	1.5	0.	3.12	6.04	3.1
time (sec)	N/A	0.021	0.03	0.003	0.	1.575	1.507	1.092

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	59	0	0	0	0	0
normalized size	1	1.	0.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.238	0.043	0.028	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	315	315	78	0	0	0	0	0
normalized size	1	1.	0.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.713	0.078	0.02	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	357	0	0	0	0	0
normalized size	1	1.	2.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	0.284	0.01	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	0.113	0.008	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.121	0.011	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	221	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	0.177	0.008	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	179	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.249	0.056	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	224	162	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.243	0.203	0.02	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	162	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.218	0.028	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	138	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.099	0.02	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.173	0.026	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.183	0.022	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.194	0.016	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.179	0.016	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	157	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.204	0.014	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	164	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.175	0.023	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.199	0.025	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	162	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.216	0.027	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.201	0.03	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.179	0.033	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	164	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.234	0.033	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	34	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.007	0.017	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	24	25	32	77	22	32
normalized size	1	1.	0.8	0.83	1.07	2.57	0.73	1.07
time (sec)	N/A	0.012	0.014	0.006	1.5	1.466	0.127	1.119

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	24	59	15	24
normalized size	1	1.	0.82	0.86	1.09	2.68	0.68	1.09
time (sec)	N/A	0.01	0.007	0.007	0.971	1.436	0.11	1.128

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	62	15	26
normalized size	1	1.	1.	0.87	1.13	2.7	0.65	1.13
time (sec)	N/A	0.009	0.01	0.004	1.537	1.428	0.121	1.097

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	22	8	12
normalized size	1	1.	1.	0.91	1.09	2.	0.73	1.09
time (sec)	N/A	0.003	0.002	0.004	0.959	1.405	0.099	1.108

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	26	61	15	26
normalized size	1	1.	0.87	0.87	1.13	2.65	0.65	1.13
time (sec)	N/A	0.007	0.006	0.004	1.456	1.464	0.121	1.092

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	32	89	19	39
normalized size	1	1.	1.	0.88	1.33	3.71	0.79	1.62
time (sec)	N/A	0.012	0.009	0.013	1.035	1.412	0.131	1.108

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	34	78	26	34
normalized size	1	1.	1.	0.83	1.13	2.6	0.87	1.13
time (sec)	N/A	0.012	0.012	0.008	1.497	1.423	0.148	1.117

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	45	111	29	45
normalized size	1	1.	1.	0.85	1.36	3.36	0.88	1.36
time (sec)	N/A	0.016	0.012	0.013	1.002	1.439	0.159	1.077

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	41	96	29	42
normalized size	1	1.	0.89	0.76	1.11	2.59	0.78	1.14
time (sec)	N/A	0.016	0.011	0.011	1.486	1.427	0.19	1.141

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	94	69	112	392	90	112
normalized size	1	1.	0.9	0.66	1.08	3.77	0.87	1.08
time (sec)	N/A	0.055	0.07	0.007	1.491	1.515	0.186	1.104

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	93	70	113	382	90	113
normalized size	1	1.	0.94	0.71	1.14	3.86	0.91	1.14
time (sec)	N/A	0.051	0.06	0.006	1.455	1.641	0.177	1.116

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	90	68	111	371	82	111
normalized size	1	1.	0.93	0.7	1.14	3.82	0.85	1.14
time (sec)	N/A	0.051	0.072	0.005	1.462	1.805	0.173	1.097

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	92	70	113	373	83	113
normalized size	1	1.	0.93	0.71	1.14	3.77	0.84	1.14
time (sec)	N/A	0.05	0.05	0.005	1.48	1.651	0.178	1.107

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	91	68	111	379	88	111
normalized size	1	1.	0.94	0.7	1.14	3.91	0.91	1.14
time (sec)	N/A	0.047	0.05	0.005	1.506	1.589	0.181	1.105

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	98	75	119	390	95	119
normalized size	1	1.	0.92	0.71	1.12	3.68	0.9	1.12
time (sec)	N/A	0.052	0.07	0.008	1.479	1.567	0.213	1.09

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	96	73	122	406	97	117
normalized size	1	1.	0.91	0.69	1.15	3.83	0.92	1.1
time (sec)	N/A	0.052	0.074	0.008	1.499	1.525	0.219	1.12

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	103	80	128	424	102	130
normalized size	1	1.	0.91	0.71	1.13	3.75	0.9	1.15
time (sec)	N/A	0.054	0.078	0.01	1.475	1.571	0.245	1.123

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	101	78	128	433	102	127
normalized size	1	1.	0.89	0.69	1.13	3.83	0.9	1.12
time (sec)	N/A	0.055	0.081	0.007	1.496	1.642	0.249	1.145

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	32	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	0.006	0.017	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	39	41	46	115	34	47
normalized size	1	1.	1.22	1.28	1.44	3.59	1.06	1.47
time (sec)	N/A	0.013	0.024	0.009	1.017	1.456	0.127	1.107

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	22	19	24	59	15	26
normalized size	1	1.	0.85	0.73	0.92	2.27	0.58	1.
time (sec)	N/A	0.013	0.007	0.006	0.975	1.448	0.111	1.109

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	33	36	39	100	26	41
normalized size	1	1.	1.32	1.44	1.56	4.	1.04	1.64
time (sec)	N/A	0.01	0.011	0.007	0.991	1.635	0.123	1.09

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	12	22	8	12
normalized size	1	1.	0.85	0.77	0.92	1.69	0.62	0.92
time (sec)	N/A	0.003	0.003	0.001	1.001	1.671	0.101	1.097

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	33	36	39	100	26	41
normalized size	1	1.	1.32	1.44	1.56	4.	1.04	1.64
time (sec)	N/A	0.008	0.008	0.009	1.005	1.522	0.126	1.129

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	47	32	89	19	41
normalized size	1	1.	0.93	1.68	1.14	3.18	0.68	1.46
time (sec)	N/A	0.015	0.008	0.012	0.988	1.478	0.132	1.085

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	41	50	50	119	36	51
normalized size	1	1.	1.28	1.56	1.56	3.72	1.12	1.59
time (sec)	N/A	0.013	0.017	0.017	1.019	1.422	0.156	1.122

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	54	47	111	29	49
normalized size	1	1.	0.95	1.46	1.27	3.	0.78	1.32
time (sec)	N/A	0.019	0.013	0.016	0.974	1.459	0.161	1.111

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	49	55	57	140	41	57
normalized size	1	1.	1.26	1.41	1.46	3.59	1.05	1.46
time (sec)	N/A	0.017	0.015	0.017	1.013	1.466	0.187	1.12

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	38	43	38	144	32	41
normalized size	1	1.	1.12	1.26	1.12	4.24	0.94	1.21
time (sec)	N/A	0.009	0.016	0.013	1.474	1.561	0.159	1.131

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	35	42	39	134	32	42
normalized size	1	1.	1.21	1.45	1.34	4.62	1.1	1.45
time (sec)	N/A	0.008	0.016	0.011	1.497	1.458	0.157	1.103

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	31	42	36	126	26	39
normalized size	1	1.	1.15	1.56	1.33	4.67	0.96	1.44
time (sec)	N/A	0.007	0.014	0.012	1.546	1.506	0.149	1.091

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	33	42	39	128	27	42
normalized size	1	1.	1.14	1.45	1.34	4.41	0.93	1.45
time (sec)	N/A	0.008	0.012	0.01	1.501	1.517	0.149	1.111

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	42	36	130	31	39
normalized size	1	1.	1.22	1.56	1.33	4.81	1.15	1.44
time (sec)	N/A	0.005	0.01	0.012	1.513	1.488	0.16	1.117

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	40	47	47	143	37	50
normalized size	1	1.	1.11	1.31	1.31	3.97	1.03	1.39
time (sec)	N/A	0.01	0.017	0.014	1.491	1.503	0.185	1.138

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	47	50	157	39	46
normalized size	1	1.	1.06	1.31	1.39	4.36	1.08	1.28
time (sec)	N/A	0.009	0.019	0.015	1.602	1.469	0.187	1.114

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	51	52	57	171	44	58
normalized size	1	1.	1.19	1.21	1.33	3.98	1.02	1.35
time (sec)	N/A	0.012	0.021	0.016	1.511	1.475	0.223	1.103

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	52	57	182	44	55
normalized size	1	1.	1.	1.21	1.33	4.23	1.02	1.28
time (sec)	N/A	0.012	0.019	0.015	1.575	1.517	0.229	1.073

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	58	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	0.044	0.014	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	556	316	101
normalized size	1	1.	0.96	1.37	0.	6.86	3.9	1.25
time (sec)	N/A	0.084	0.052	0.004	0.	1.907	2.909	7.445

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	210	360	0	2192	134	1721
normalized size	1	1.	1.09	1.88	0.	11.42	0.7	8.96
time (sec)	N/A	0.339	0.133	0.035	0.	1.701	3.207	8.028

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	443	223	80
normalized size	1	1.	0.98	0.95	0.	7.03	3.54	1.27
time (sec)	N/A	0.058	0.025	0.003	0.	1.612	1.725	7.948

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	171	216	0	1214	76	1403
normalized size	1	1.	1.08	1.36	0.	7.64	0.48	8.82
time (sec)	N/A	0.126	0.093	0.016	0.	1.527	2.211	7.826

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	296	131	49
normalized size	1	1.	1.11	0.97	0.	7.79	3.45	1.29
time (sec)	N/A	0.034	0.012	0.003	0.	1.513	0.698	7.504

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	133	120	0	1345	88	1370
normalized size	1	1.	0.86	0.78	0.	8.73	0.57	8.9
time (sec)	N/A	0.094	0.085	0.013	0.	1.583	2.459	7.99

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	66	0	510	253	92
normalized size	1	1.	0.96	0.96	0.	7.39	3.67	1.33
time (sec)	N/A	0.068	0.023	0.007	0.	1.801	5.468	7.705

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	75	240	0	2314	153	1721
normalized size	1	1.	0.41	1.3	0.	12.58	0.83	9.35
time (sec)	N/A	0.224	0.032	0.017	0.	1.82	3.647	7.448

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	92	119	0	664	0	127
normalized size	1	1.	1.03	1.34	0.	7.46	0.	1.43
time (sec)	N/A	0.124	0.031	0.009	0.	2.579	0.	7.506

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	70	63	0	13450	360	0
normalized size	1	1.	0.18	0.17	0.	35.3	0.94	0.
time (sec)	N/A	0.647	0.041	0.037	0.	11.078	46.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	70	59	0	10701	218	0
normalized size	1	1.	0.19	0.16	0.	28.46	0.58	0.
time (sec)	N/A	0.574	0.038	0.004	0.	4.351	15.839	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	44	43	0	8154	230	0
normalized size	1	1.	0.14	0.13	0.	25.09	0.71	0.
time (sec)	N/A	0.307	0.024	0.003	0.	2.929	7.066	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	42	43	0	5315	126	0
normalized size	1	1.	0.13	0.13	0.	16.35	0.39	0.
time (sec)	N/A	0.296	0.022	0.002	0.	2.045	3.812	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	43	43	0	5895	172	0
normalized size	1	1.	0.14	0.14	0.	18.71	0.55	0.
time (sec)	N/A	0.29	0.023	0.003	0.	2.294	3.493	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	45	40	0	8162	177	0
normalized size	1	1.	0.14	0.13	0.	25.91	0.56	0.
time (sec)	N/A	0.304	0.025	0.003	0.	3.011	6.577	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	71	63	0	10831	304	0
normalized size	1	1.	0.2	0.17	0.	29.84	0.84	0.
time (sec)	N/A	0.412	0.038	0.006	0.	6.467	16.949	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	75	62	0	13495	277	0
normalized size	1	1.	0.21	0.17	0.	36.97	0.76	0.
time (sec)	N/A	0.399	0.043	0.006	0.	7.611	39.699	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	212	0	0	0	0	0
normalized size	1	1.	1.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.403	0.017	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	47	109	42	47
normalized size	1	1.	1.	0.82	1.07	2.48	0.95	1.07
time (sec)	N/A	0.037	0.011	0.002	1.471	1.458	0.135	1.092

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	98	43	57	128	51	57
normalized size	1	1.	1.81	0.8	1.06	2.37	0.94	1.06
time (sec)	N/A	0.058	0.174	0.005	1.495	1.433	0.138	1.107

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	41	97	37	41
normalized size	1	1.	1.	0.84	1.11	2.62	1.	1.11
time (sec)	N/A	0.031	0.008	0.003	1.5	1.489	0.131	1.094

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	94	62	82	193	76	82
normalized size	1	1.	1.25	0.83	1.09	2.57	1.01	1.09
time (sec)	N/A	0.077	0.121	0.003	1.509	1.528	0.191	1.106

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	61	26	24
normalized size	1	1.	1.	0.83	1.04	2.65	1.13	1.04
time (sec)	N/A	0.021	0.006	0.002	1.518	1.459	0.117	1.166

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	79	62	82	193	76	82
normalized size	1	1.	1.05	0.83	1.09	2.57	1.01	1.09
time (sec)	N/A	0.064	0.05	0.004	1.51	1.467	0.191	1.101

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	138	87	49	109	41	49
normalized size	1	1.	3.54	2.23	1.26	2.79	1.05	1.26
time (sec)	N/A	0.035	0.086	0.009	1.532	1.474	0.15	1.088

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	100	57	57	135	53	57
normalized size	1	1.	1.85	1.06	1.06	2.5	0.98	1.06
time (sec)	N/A	0.052	0.051	0.006	1.515	1.466	0.162	1.111

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	141	94	55	143	48	62
normalized size	1	1.	2.94	1.96	1.15	2.98	1.	1.29
time (sec)	N/A	0.051	0.104	0.009	1.473	1.455	0.181	1.109

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	142	95	99	234	88	99
normalized size	1	1.	1.6	1.07	1.11	2.63	0.99	1.11
time (sec)	N/A	0.096	0.112	0.008	1.515	1.541	0.243	1.092

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	139	110	0	720	192	0
normalized size	1	1.	0.99	0.78	0.	5.11	1.36	0.
time (sec)	N/A	0.097	0.282	0.036	0.	1.606	0.693	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	68	67	0	212	82	97
normalized size	1	1.	0.77	0.76	0.	2.41	0.93	1.1
time (sec)	N/A	0.058	0.022	0.01	0.	1.505	0.17	1.094

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	135	109	0	716	197	0
normalized size	1	1.	0.96	0.78	0.	5.11	1.41	0.
time (sec)	N/A	0.107	0.166	0.013	0.	1.637	0.7	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	135	109	0	716	214	0
normalized size	1	1.	0.96	0.78	0.	5.11	1.53	0.
time (sec)	N/A	0.083	0.162	0.009	0.	1.632	0.694	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	68	67	0	212	82	97
normalized size	1	1.	0.77	0.76	0.	2.41	0.93	1.1
time (sec)	N/A	0.052	0.017	0.009	0.	1.482	0.171	1.114

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	140	114	0	720	218	0
normalized size	1	1.	0.97	0.79	0.	4.97	1.5	0.
time (sec)	N/A	0.112	0.217	0.01	0.	1.684	0.722	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	148	114	0	744	197	0
normalized size	1	1.	1.01	0.78	0.	5.06	1.34	0.
time (sec)	N/A	0.099	0.307	0.011	0.	1.894	0.734	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	95	75	0	255	94	113
normalized size	1	1.	0.97	0.77	0.	2.6	0.96	1.15
time (sec)	N/A	0.084	0.038	0.013	0.	1.461	0.214	1.105

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	171	119	0	774	209	0
normalized size	1	1.	1.11	0.77	0.	5.03	1.36	0.
time (sec)	N/A	0.142	0.356	0.011	0.	1.622	0.75	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	52	0	0	0	0	0
normalized size	1	1.	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.037	0.017	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	38	50	109	42	50
normalized size	1	1.	1.	0.83	1.09	2.37	0.91	1.09
time (sec)	N/A	0.04	0.013	0.003	1.478	1.446	0.139	1.138

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	44	0	117	48	348
normalized size	1	1.	0.96	0.77	0.	2.05	0.84	6.11
time (sec)	N/A	0.043	0.016	0.006	0.	1.455	0.126	1.275

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	43	96	37	43
normalized size	1	1.	1.	0.85	1.1	2.46	0.95	1.1
time (sec)	N/A	0.033	0.008	0.002	1.487	1.499	0.132	1.145

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	98	65	0	545	70	342
normalized size	1	1.	1.2	0.79	0.	6.65	0.85	4.17
time (sec)	N/A	0.071	0.137	0.006	0.	1.55	0.2	1.262

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	61	26	24
normalized size	1	1.	1.	0.83	1.04	2.65	1.13	1.04
time (sec)	N/A	0.022	0.006	0.001	1.5	1.475	0.124	1.113

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	83	65	0	545	70	0
normalized size	1	1.	1.01	0.79	0.	6.65	0.85	0.
time (sec)	N/A	0.057	0.056	0.008	0.	1.597	0.195	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	35	51	108	41	51
normalized size	1	1.	1.34	0.85	1.24	2.63	1.	1.24
time (sec)	N/A	0.038	0.013	0.007	1.483	1.463	0.15	1.135

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	44	0	123	49	348
normalized size	1	1.	0.96	0.77	0.	2.16	0.86	6.11
time (sec)	N/A	0.042	0.016	0.01	0.	1.429	0.155	1.277

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	40	58	143	48	65
normalized size	1	1.	1.06	0.83	1.21	2.98	1.	1.35
time (sec)	N/A	0.054	0.013	0.005	1.546	1.504	0.19	1.138

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	56	75	0	576	82	358
normalized size	1	1.	0.58	0.78	0.	6.	0.85	3.73
time (sec)	N/A	0.094	0.018	0.01	0.	1.573	0.257	1.273

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	59	44	0	2319	26	343
normalized size	1	1.	0.17	0.12	0.	6.51	0.07	0.96
time (sec)	N/A	0.333	0.014	0.007	0.	1.833	1.258	1.148

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	41	32	0	608	165	277
normalized size	1	1.	0.15	0.12	0.	2.21	0.6	1.01
time (sec)	N/A	0.243	0.011	0.008	0.	1.594	0.209	1.158

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	39	40	0	1913	24	342
normalized size	1	1.	0.11	0.12	0.	5.51	0.07	0.99
time (sec)	N/A	0.209	0.011	0.005	0.	1.751	1.286	1.189

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	40	40	0	1914	26	342
normalized size	1	1.	0.11	0.11	0.	5.39	0.07	0.96
time (sec)	N/A	0.2	0.011	0.007	0.	1.808	1.298	1.145

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	42	30	0	608	165	277
normalized size	1	1.	0.15	0.11	0.	2.21	0.6	1.01
time (sec)	N/A	0.211	0.011	0.005	0.	1.609	0.24	1.148

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	61	52	0	2327	29	348
normalized size	1	1.	0.17	0.14	0.	6.46	0.08	0.97
time (sec)	N/A	0.239	0.016	0.009	0.	1.824	1.291	1.2

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	65	50	0	2361	31	348
normalized size	1	1.	0.18	0.14	0.	6.38	0.08	0.94
time (sec)	N/A	0.236	0.014	0.01	0.	1.801	1.305	1.16

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	54	43	0	653	180	293
normalized size	1	1.	0.19	0.15	0.	2.28	0.63	1.02
time (sec)	N/A	0.241	0.016	0.008	0.	1.555	0.272	1.143

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	54	51	0	1990	36	358
normalized size	1	1.	0.14	0.14	0.	5.28	0.1	0.95
time (sec)	N/A	0.286	0.015	0.01	0.	1.753	1.378	1.168

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	54	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.041	0.016	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	38	68	157	60	68
normalized size	1	1.	0.92	0.61	1.1	2.53	0.97	1.1
time (sec)	N/A	0.056	0.032	0.003	1.497	1.483	0.142	1.221

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	97	117	0	456	54	89
normalized size	1	1.	1.08	1.3	0.	5.07	0.6	0.99
time (sec)	N/A	0.142	0.146	0.041	0.	1.585	0.198	1.23

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	53	33	61	143	53	61
normalized size	1	1.	0.96	0.6	1.11	2.6	0.96	1.11
time (sec)	N/A	0.033	0.023	0.003	1.484	1.429	0.135	1.236

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	75	110	0	517	49	63
normalized size	1	1.	0.93	1.36	0.	6.38	0.6	0.78
time (sec)	N/A	0.084	0.047	0.017	0.	1.593	0.194	1.276

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	38	19	42	107	42	42
normalized size	1	1.	1.65	0.83	1.83	4.65	1.83	1.83
time (sec)	N/A	0.026	0.01	0.	1.494	1.456	0.122	1.293

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	74	60	0	417	49	55
normalized size	1	1.	0.99	0.8	0.	5.56	0.65	0.73
time (sec)	N/A	0.058	0.038	0.014	0.	1.54	0.195	1.306

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	35	69	155	58	69
normalized size	1	1.	0.96	0.61	1.21	2.72	1.02	1.21
time (sec)	N/A	0.034	0.029	0.007	1.489	1.481	0.153	1.249

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	65	117	0	460	56	92
normalized size	1	1.	0.73	1.31	0.	5.17	0.63	1.03
time (sec)	N/A	0.072	0.018	0.021	0.	1.591	0.23	1.257

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	60	42	76	193	65	85
normalized size	1	1.	0.91	0.64	1.15	2.92	0.98	1.29
time (sec)	N/A	0.067	0.034	0.007	1.473	1.662	0.193	1.231

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	73	122	0	564	65	104
normalized size	1	1.	0.75	1.26	0.	5.81	0.67	1.07
time (sec)	N/A	0.134	0.019	0.016	0.	1.771	0.275	1.242

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	440	58	46	0	3565	29	343
normalized size	1	0.96	0.13	0.1	0.	7.75	0.06	0.75
time (sec)	N/A	0.416	0.014	0.006	0.	1.913	1.175	1.356

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	41	40	0	2326	26	342
normalized size	1	1.	0.1	0.09	0.	5.4	0.06	0.79
time (sec)	N/A	0.295	0.012	0.007	0.	1.806	1.146	1.362

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	39	40	0	2758	24	342
normalized size	1	1.	0.09	0.09	0.	6.12	0.05	0.76
time (sec)	N/A	0.28	0.011	0.006	0.	1.869	1.141	1.339

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	431	40	40	0	3101	26	342
normalized size	1	1.01	0.09	0.09	0.	7.26	0.06	0.8
time (sec)	N/A	0.259	0.01	0.005	0.	1.86	1.14	1.361

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	42	37	0	2304	26	342
normalized size	1	1.	0.1	0.09	0.	5.57	0.06	0.83
time (sec)	N/A	0.257	0.011	0.006	0.	1.774	1.107	1.243

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	61	52	0	3545	32	348
normalized size	1	1.	0.15	0.12	0.	8.52	0.08	0.84
time (sec)	N/A	0.285	0.017	0.009	0.	1.852	1.245	1.329

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	466	65	50	0	3754	34	348
normalized size	1	1.	0.14	0.11	0.	8.06	0.07	0.75
time (sec)	N/A	0.367	0.015	0.007	0.	2.171	1.232	1.376

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	191	0	0	0	0	0
normalized size	1	1.	1.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.172	0.017	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	38	68	157	58	72
normalized size	1	1.	0.9	0.61	1.1	2.53	0.94	1.16
time (sec)	N/A	0.046	0.033	0.003	1.467	1.713	0.137	1.15

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	103	67	124	290	170	131
normalized size	1	1.	1.14	0.74	1.38	3.22	1.89	1.46
time (sec)	N/A	0.072	0.051	0.005	1.465	1.71	0.462	1.178

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	53	33	61	143	53	65
normalized size	1	1.	0.96	0.6	1.11	2.6	0.96	1.18
time (sec)	N/A	0.032	0.021	0.002	1.478	1.725	0.137	1.149

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	91	62	117	277	165	124
normalized size	1	1.	1.12	0.77	1.44	3.42	2.04	1.53
time (sec)	N/A	0.053	0.035	0.003	1.501	1.777	0.448	1.168

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	38	19	42	107	42	45
normalized size	1	1.	1.65	0.83	1.83	4.65	1.83	1.96
time (sec)	N/A	0.027	0.01	0.002	1.502	1.643	0.119	1.163

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	91	62	117	277	165	124
normalized size	1	1.	1.21	0.83	1.56	3.69	2.2	1.65
time (sec)	N/A	0.038	0.029	0.003	1.486	1.743	0.437	1.142

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	64	69	155	58	73
normalized size	1	1.	0.96	1.12	1.21	2.72	1.02	1.28
time (sec)	N/A	0.03	0.031	0.01	1.484	1.775	0.156	1.165

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	103	67	124	306	172	131
normalized size	1	1.	1.16	0.75	1.39	3.44	1.93	1.47
time (sec)	N/A	0.058	0.06	0.009	1.48	1.758	0.477	1.17

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	61	71	76	193	66	89
normalized size	1	1.	0.92	1.08	1.15	2.92	1.	1.35
time (sec)	N/A	0.063	0.034	0.008	1.507	1.705	0.196	1.146

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	111	72	134	327	197	140
normalized size	1	1.	1.14	0.74	1.38	3.37	2.03	1.44
time (sec)	N/A	0.09	0.074	0.01	1.477	1.714	0.516	1.17

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	160	205	0	988	58	200
normalized size	1	1.	0.94	1.21	0.	5.81	0.34	1.18
time (sec)	N/A	0.114	0.274	0.053	0.	2.01	0.921	1.222

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	160	206	0	844	53	198
normalized size	1	1.	0.96	1.23	0.	5.05	0.32	1.19
time (sec)	N/A	0.078	0.15	0.024	0.	1.961	0.897	1.217

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	132	206	0	932	49	198
normalized size	1	1.	0.76	1.19	0.	5.39	0.28	1.14
time (sec)	N/A	0.075	0.197	0.034	0.	1.919	0.884	1.248

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	166	131	110	0	903	53	198
normalized size	1	1.14	0.9	0.76	0.	6.23	0.37	1.37
time (sec)	N/A	0.06	0.045	0.02	0.	1.927	0.891	1.217

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	160	206	0	834	53	198
normalized size	1	1.	0.95	1.22	0.	4.93	0.31	1.17
time (sec)	N/A	0.06	0.172	0.021	0.	1.877	0.908	1.206

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	174	211	0	981	63	205
normalized size	1	1.	1.01	1.23	0.	5.7	0.37	1.19
time (sec)	N/A	0.089	0.271	0.024	0.	1.934	0.948	1.232

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	166	209	0	1037	63	205
normalized size	1	1.	0.91	1.15	0.	5.7	0.35	1.13
time (sec)	N/A	0.118	0.267	0.029	0.	1.905	0.975	1.225

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	189	216	0	913	71	215
normalized size	1	1.	1.09	1.25	0.	5.28	0.41	1.24
time (sec)	N/A	0.14	0.294	0.033	0.	1.947	0.983	1.227

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	189	216	0	1077	68	215
normalized size	1	1.	1.	1.14	0.	5.7	0.36	1.14
time (sec)	N/A	0.163	0.276	0.036	0.	1.955	0.997	1.202

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	51	15	23
normalized size	1	1.	1.	0.86	1.1	2.43	0.71	1.1
time (sec)	N/A	0.013	0.004	0.006	0.97	1.693	0.114	1.14

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	30	58	19	30
normalized size	1	1.	1.	0.88	1.15	2.23	0.73	1.15
time (sec)	N/A	0.019	0.005	0.006	1.004	1.682	0.126	1.106

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	41	100	37	41
normalized size	1	1.	1.	0.84	1.11	2.7	1.	1.11
time (sec)	N/A	0.034	0.012	0.002	1.478	1.669	0.141	2.655

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	62	27	24
normalized size	1	1.	1.	0.83	1.04	2.7	1.17	1.04
time (sec)	N/A	0.022	0.006	0.001	1.516	1.688	0.127	2.653

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	197	66	49	112	41	45
normalized size	1	1.	5.05	1.69	1.26	2.87	1.05	1.15
time (sec)	N/A	0.035	0.036	0.037	1.517	1.701	0.165	1.115

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	208	73	55	144	48	61
normalized size	1	1.	4.33	1.52	1.15	3.	1.	1.27
time (sec)	N/A	0.051	0.041	0.021	1.491	1.599	0.219	1.115

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	197	66	0	112	41	45
normalized size	1	1.	5.05	1.69	0.	2.87	1.05	1.15
time (sec)	N/A	0.036	0.019	0.02	0.	1.649	0.166	1.1

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	140	236	0	1006	600	196
normalized size	1	1.	0.95	1.61	0.	6.84	4.08	1.33
time (sec)	N/A	0.137	0.122	0.006	0.	1.824	1.04	1.107

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	112	190	0	829	496	153
normalized size	1	1.	0.95	1.61	0.	7.03	4.2	1.3
time (sec)	N/A	0.105	0.084	0.004	0.	1.821	0.964	1.126

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	84	132	0	653	381	116
normalized size	1	1.	0.94	1.48	0.	7.34	4.28	1.3
time (sec)	N/A	0.086	0.108	0.004	0.	1.725	0.787	1.142

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	73	101	0	537	306	90
normalized size	1	1.	1.04	1.44	0.	7.67	4.37	1.29
time (sec)	N/A	0.049	0.065	0.001	0.	1.689	0.638	1.153

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	427	216	74
normalized size	1	1.	1.02	1.	0.	7.62	3.86	1.32
time (sec)	N/A	0.036	0.031	0.003	0.	1.697	0.311	1.159

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	277	124	46
normalized size	1	1.	1.06	0.97	0.	7.69	3.44	1.28
time (sec)	N/A	0.032	0.007	0.	0.	1.776	0.203	1.129

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	62	0	494	564	84
normalized size	1	1.	0.98	1.	0.	7.97	9.1	1.35
time (sec)	N/A	0.048	0.068	0.005	0.	1.893	1.906	1.134

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	112	0	626	862	107
normalized size	1	1.	0.95	1.38	0.	7.73	10.64	1.32
time (sec)	N/A	0.102	0.079	0.008	0.	1.826	3.495	1.115

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	102	150	0	798	1525	142
normalized size	1	1.	0.98	1.44	0.	7.67	14.66	1.37
time (sec)	N/A	0.153	0.136	0.007	0.	2.118	5.231	1.125

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	131	214	0	979	2105	184
normalized size	1	1.	0.96	1.56	0.	7.15	15.36	1.34
time (sec)	N/A	0.193	0.101	0.008	0.	2.262	8.597	1.156

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	163	434	0	2201	1012	254
normalized size	1	1.	0.83	2.21	0.	11.23	5.16	1.3
time (sec)	N/A	0.205	0.229	0.01	0.	1.794	2.099	1.127

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	132	352	0	1760	842	217
normalized size	1	1.	0.88	2.35	0.	11.73	5.61	1.45
time (sec)	N/A	0.148	0.184	0.01	0.	1.832	1.603	1.107

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	109	209	0	1374	729	169
normalized size	1	1.	0.96	1.83	0.	12.05	6.39	1.48
time (sec)	N/A	0.1	0.148	0.007	0.	1.752	1.236	1.121

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	81	97	0	826	280	119
normalized size	1	1.	1.14	1.37	0.	11.63	3.94	1.68
time (sec)	N/A	0.042	0.097	0.007	0.	1.806	0.753	1.115

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	740	252	103
normalized size	1	1.	1.05	1.06	0.	11.21	3.82	1.56
time (sec)	N/A	0.034	0.067	0.003	0.	1.704	0.714	1.164

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	745	265	103
normalized size	1	1.	1.06	1.03	0.	11.29	4.02	1.56
time (sec)	N/A	0.033	0.08	0.003	0.	1.771	0.8	1.122

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	107	237	0	1685	2236	170
normalized size	1	1.	0.99	2.19	0.	15.6	20.7	1.57
time (sec)	N/A	0.145	0.188	0.012	0.	2.28	7.442	1.137

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	131	328	0	2049	2672	231
normalized size	1	1.	0.89	2.22	0.	13.84	18.05	1.56
time (sec)	N/A	0.182	0.293	0.014	0.	2.892	11.488	1.133

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	175	418	0	2606	4083	309
normalized size	1	1.	0.87	2.07	0.	12.9	20.21	1.53
time (sec)	N/A	0.239	0.368	0.018	0.	3.635	18.667	1.158

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	260	1040	0	4120	1714	381
normalized size	1	1.	1.09	4.37	0.	17.31	7.2	1.6
time (sec)	N/A	0.291	0.401	0.016	0.	2.029	4.28	1.158

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	221	530	0	3429	1510	331
normalized size	1	1.	1.16	2.79	0.	18.05	7.95	1.74
time (sec)	N/A	0.279	0.337	0.013	0.	2.055	2.834	1.131

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	174	260	0	2006	547	273
normalized size	1	1.	1.57	2.34	0.	18.07	4.93	2.46
time (sec)	N/A	0.067	0.182	0.012	0.	2.052	1.75	1.151

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	126	223	0	1848	510	220
normalized size	1	1.	1.18	2.08	0.	17.27	4.77	2.06
time (sec)	N/A	0.05	0.213	0.009	0.	1.835	1.517	1.129

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	131	262	0	1893	570	208
normalized size	1	1.	1.14	2.28	0.	16.46	4.96	1.81
time (sec)	N/A	0.068	0.148	0.009	0.	1.795	1.45	1.157

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	102	130	0	1692	479	182
normalized size	1	1.	0.99	1.26	0.	16.43	4.65	1.77
time (sec)	N/A	0.041	0.103	0.003	0.	1.794	1.358	1.14

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	97	129	0	1685	474	184
normalized size	1	1.	0.96	1.28	0.	16.68	4.69	1.82
time (sec)	N/A	0.041	0.102	0.004	0.	1.908	1.378	1.17

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	178	781	0	4228	4862	323
normalized size	1	1.	0.96	4.22	0.	22.85	26.28	1.75
time (sec)	N/A	0.22	0.365	0.016	0.	4.551	21.887	1.157

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	221	954	0	4849	5722	417
normalized size	1	1.	0.92	3.99	0.	20.29	23.94	1.74
time (sec)	N/A	0.277	0.472	0.02	0.	6.046	52.609	1.141

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	41	108	34	43
normalized size	1	1.	1.	0.78	1.02	2.7	0.85	1.08
time (sec)	N/A	0.022	0.006	0.006	1.056	1.667	0.152	1.192

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	34	85	27	36
normalized size	1	1.	1.	0.79	1.03	2.58	0.82	1.09
time (sec)	N/A	0.02	0.004	0.006	1.081	1.714	0.145	1.202

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	66	20	30
normalized size	1	1.	1.	0.81	1.04	2.54	0.77	1.15
time (sec)	N/A	0.015	0.004	0.005	1.065	1.665	0.142	1.195

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	54	17	26
normalized size	1	1.	1.	0.86	1.1	2.57	0.81	1.24
time (sec)	N/A	0.011	0.003	0.003	1.072	1.645	0.106	1.116

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	18	23	50	15	26
normalized size	1	1.	0.91	0.78	1.	2.17	0.65	1.13
time (sec)	N/A	0.014	0.003	0.004	1.011	1.679	0.147	1.099

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	70	24	32
normalized size	1	1.	1.	0.81	1.04	2.59	0.89	1.19
time (sec)	N/A	0.017	0.005	0.007	1.107	1.764	0.187	1.118

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	35	90	31	39
normalized size	1	1.	1.	0.79	1.03	2.65	0.91	1.15
time (sec)	N/A	0.031	0.004	0.008	1.074	1.715	0.22	1.095

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	42	117	36	46
normalized size	1	1.	1.	0.78	1.02	2.85	0.88	1.12
time (sec)	N/A	0.035	0.005	0.008	1.08	1.728	0.161	1.126

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	49	138	41	53
normalized size	1	1.	1.	0.77	1.02	2.88	0.85	1.1
time (sec)	N/A	0.042	0.005	0.009	1.084	1.752	0.246	1.136

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	213	701	0	2272	0	0
normalized size	1	1.	1.04	3.44	0.	11.14	0.	0.
time (sec)	N/A	0.231	0.539	0.013	0.	3.944	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	163	334	0	1689	0	0
normalized size	1	1.	1.12	2.3	0.	11.65	0.	0.
time (sec)	N/A	0.134	0.258	0.006	0.	2.571	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	128	121	0	1407	0	0
normalized size	1	1.	1.22	1.15	0.	13.4	0.	0.
time (sec)	N/A	0.083	0.085	0.005	0.	2.185	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	89	88	0	410	0	0
normalized size	1	1.	1.33	1.31	0.	6.12	0.	0.
time (sec)	N/A	0.041	0.054	0.006	0.	1.854	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	138	197	0	1002	0	0
normalized size	1	1.	1.04	1.48	0.	7.53	0.	0.
time (sec)	N/A	0.099	0.174	0.009	0.	2.305	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	256	376	0	2286	0	0
normalized size	1	1.	1.16	1.71	0.	10.39	0.	0.
time (sec)	N/A	0.194	0.401	0.01	0.	4.327	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	32	40	11	22	0	39
normalized size	1	1.	0.44	0.55	0.15	0.3	0.	0.53
time (sec)	N/A	0.036	0.02	0.009	1.	2.025	0.	1.115

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	202	343	0	2168	129	4593
normalized size	1	1.	1.13	1.92	0.	12.11	0.72	25.66
time (sec)	N/A	0.291	0.131	0.01	0.	2.11	1.65	2.326

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	631	70	59	0	11169	196	0
normalized size	1	1.	0.11	0.09	0.	17.7	0.31	0.
time (sec)	N/A	1.17	0.037	0.009	0.	6.329	2.929	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	70	59	0	10701	218	0
normalized size	1	1.	0.19	0.16	0.	28.46	0.58	0.
time (sec)	N/A	0.666	0.044	0.002	0.	5.53	20.029	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	84	0	0	0	0
normalized size	1	1.	1.	0.79	0.	0.	0.	0.
time (sec)	N/A	0.092	0.058	0.007	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	52	73	126	51	66
normalized size	1	1.	0.72	1.3	1.82	3.15	1.27	1.65
time (sec)	N/A	0.019	0.026	0.003	1.065	1.995	0.412	1.119

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	50	50	31	0	0	74
normalized size	1	1.	0.67	0.67	0.41	0.	0.	0.99
time (sec)	N/A	0.04	0.03	0.01	1.035	0.	0.	1.163

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	56	109	0	197	0	189
normalized size	1	1.	0.41	0.8	0.	1.44	0.	1.38
time (sec)	N/A	0.081	0.05	0.012	0.	2.133	0.	1.17

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	56	87	0	140	0	138
normalized size	1	1.	0.41	0.64	0.	1.02	0.	1.01
time (sec)	N/A	0.071	0.037	0.002	0.	2.03	0.	1.131

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	65	65	0	82	0	86
normalized size	1	1.	0.47	0.47	0.	0.6	0.	0.63
time (sec)	N/A	0.056	0.033	0.003	0.	1.956	0.	1.135

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	43	43	0	28	0	35
normalized size	1	1.	0.49	0.49	0.	0.32	0.	0.4
time (sec)	N/A	0.04	0.009	0.001	0.	2.019	0.	1.116

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	65	103	62	89	0	82
normalized size	1	1.	0.44	0.7	0.42	0.61	0.	0.56
time (sec)	N/A	0.07	0.038	0.016	1.064	2.063	0.	1.134

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	72	92	88	243	0	86
normalized size	1	1.	0.55	0.71	0.68	1.87	0.	0.66
time (sec)	N/A	0.072	0.046	0.009	1.078	1.866	0.	1.175

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	58	54	85	285	0	0
normalized size	1	1.	0.43	0.4	0.63	2.11	0.	0.
time (sec)	N/A	0.075	0.037	0.006	1.098	1.871	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	58	54	85	477	0	0
normalized size	1	1.	0.42	0.39	0.62	3.48	0.	0.
time (sec)	N/A	0.078	0.037	0.007	1.104	1.93	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	58	54	85	643	0	0
normalized size	1	1.	0.42	0.39	0.62	4.69	0.	0.
time (sec)	N/A	0.079	0.037	0.007	1.069	1.957	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	58	54	85	842	0	0
normalized size	1	1.	0.42	0.39	0.62	6.15	0.	0.
time (sec)	N/A	0.078	0.038	0.007	1.016	2.041	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	68	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.03	0.033	0.	0.	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	468	207	0	489	1354	0	2111
normalized size	1	1.	0.44	0.	1.04	2.89	0.	4.51
time (sec)	N/A	0.224	0.224	0.007	1.063	3.189	0.	1.184

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	143	0	267	653	0	1006
normalized size	1	1.	0.45	0.	0.85	2.07	0.	3.19
time (sec)	N/A	0.14	0.185	0.006	0.994	2.592	0.	1.142

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	83	0	104	240	0	309
normalized size	1	1.	0.58	0.	0.73	1.69	0.	2.18
time (sec)	N/A	0.068	0.049	0.005	0.973	2.124	0.	1.163

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.013	0.007	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	61	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.015	0.007	0.	0.	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	162	101	0	0	186	0	0
normalized size	1	1.11	0.69	0.	0.	1.27	0.	0.
time (sec)	N/A	0.099	0.088	0.01	0.	2.692	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	93	114	200	315	0	0
normalized size	1	1.	0.53	0.65	1.14	1.79	0.	0.
time (sec)	N/A	0.104	0.068	0.016	1.012	21.128	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	121	174	161	0	0	320
normalized size	1	1.	0.45	0.65	0.6	0.	0.	1.19
time (sec)	N/A	0.151	0.1	0.014	1.063	0.	0.	1.538

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	66	68	0	0	0	108
normalized size	1	1.	0.37	0.38	0.	0.	0.	0.6
time (sec)	N/A	0.092	0.034	0.026	0.	0.	0.	1.188

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	125	115	107	0	0	234
normalized size	1	1.	0.32	0.29	0.27	0.	0.	0.6
time (sec)	N/A	0.187	0.073	0.022	0.978	0.	0.	1.3

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	99	91	77	0	0	173
normalized size	1	1.	0.34	0.31	0.26	0.	0.	0.59
time (sec)	N/A	0.137	0.059	0.009	1.04	0.	0.	1.261

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	77	69	41	0	0	107
normalized size	1	1.	0.41	0.37	0.22	0.	0.	0.57
time (sec)	N/A	0.091	0.029	0.005	1.033	0.	0.	1.171

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	49	50	14	0	0	46
normalized size	1	1.	0.56	0.57	0.16	0.	0.	0.52
time (sec)	N/A	0.053	0.015	0.004	0.968	0.	0.	1.156

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	86	78	59	0	0	104
normalized size	1	1.	0.45	0.41	0.31	0.	0.	0.55
time (sec)	N/A	0.118	0.041	0.006	1.019	0.	0.	1.205

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	126	141	131	0	0	163
normalized size	1	1.	0.42	0.47	0.44	0.	0.	0.54
time (sec)	N/A	0.187	0.09	0.01	1.024	0.	0.	1.229

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	152	199	188	0	0	190
normalized size	1	1.	0.37	0.49	0.46	0.	0.	0.46
time (sec)	N/A	0.268	0.139	0.011	1.007	0.	0.	1.212

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	98	94	77	0	0	170
normalized size	1	1.	0.34	0.33	0.27	0.	0.	0.59
time (sec)	N/A	0.138	0.053	0.032	0.977	0.	0.	1.237

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	103	91	70	0	0	169
normalized size	1	1.	0.35	0.31	0.24	0.	0.	0.58
time (sec)	N/A	0.137	0.051	0.02	0.99	0.	0.	1.181

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	98	152	134	703	0	113
normalized size	1	1.	0.44	0.68	0.6	3.17	0.	0.51
time (sec)	N/A	0.126	0.097	0.011	1.013	2.193	0.	1.133

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	124	116	107	0	0	232
normalized size	1	1.	0.32	0.3	0.27	0.	0.	0.59
time (sec)	N/A	0.18	0.07	0.024	0.994	0.	0.	1.364

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	38	62	61	84	42	0
normalized size	1	1.	0.83	1.35	1.33	1.83	0.91	0.
time (sec)	N/A	0.036	0.029	0.023	0.976	1.93	31.294	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	33	43	49	26	0
normalized size	1	1.	0.93	1.18	1.54	1.75	0.93	0.
time (sec)	N/A	0.026	0.014	0.02	0.98	1.911	81.362	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	26	30	37	0
normalized size	1	1.	1.	1.2	1.73	2.	2.47	0.
time (sec)	N/A	0.012	0.004	0.017	0.988	1.903	14.01	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	26	36	47	42	34
normalized size	1	1.	0.96	1.13	1.57	2.04	1.83	1.48
time (sec)	N/A	0.019	0.007	0.02	0.99	1.851	14.695	1.1

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	69	78	128	0	0
normalized size	1	1.	0.86	1.21	1.37	2.25	0.	0.
time (sec)	N/A	0.042	0.065	0.022	0.963	1.763	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	62	88	96	159	73	0
normalized size	1	1.	0.82	1.16	1.26	2.09	0.96	0.
time (sec)	N/A	0.047	0.079	0.025	1.067	1.573	144.718	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	75	105	113	190	0	0
normalized size	1	1.	0.81	1.13	1.22	2.04	0.	0.
time (sec)	N/A	0.054	0.106	0.026	1.004	1.563	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	34	54	0	625	0	0
normalized size	1	1.	0.14	0.23	0.	2.65	0.	0.
time (sec)	N/A	0.206	0.009	0.087	0.	1.704	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	34	54	0	502	0	0
normalized size	1	1.	0.21	0.34	0.	3.14	0.	0.
time (sec)	N/A	0.132	0.01	0.058	0.	1.71	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	32	79	0	323	0	0
normalized size	1	1.	0.64	1.58	0.	6.46	0.	0.
time (sec)	N/A	0.035	0.007	0.062	0.	1.667	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	34	97	0	379	0	0
normalized size	1	1.	0.5	1.43	0.	5.57	0.	0.
time (sec)	N/A	0.041	0.008	0.075	0.	1.706	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	34	73	0	427	0	0
normalized size	1	1.	0.19	0.41	0.	2.43	0.	0.
time (sec)	N/A	0.142	0.008	0.065	0.	1.675	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	34	73	0	605	0	0
normalized size	1	1.	0.13	0.29	0.	2.4	0.	0.
time (sec)	N/A	0.225	0.008	0.075	0.	1.722	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	38	0	58	126	0	0
normalized size	1	1.	1.03	0.	1.57	3.41	0.	0.
time (sec)	N/A	0.04	0.017	0.097	1.08	1.589	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	43	0	0	124	0	0
normalized size	1	1.	1.13	0.	0.	3.26	0.	0.
time (sec)	N/A	0.039	0.021	0.065	0.	1.561	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	40	208	100	165	0	0
normalized size	1	1.	0.36	1.86	0.89	1.47	0.	0.
time (sec)	N/A	0.042	0.057	0.049	1.006	1.598	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	40	135	65	107	0	0
normalized size	1	1.	0.36	1.21	0.58	0.96	0.	0.
time (sec)	N/A	0.04	0.038	0.023	0.986	1.6	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	44	64	30	47	0	0
normalized size	1	1.	0.44	0.65	0.3	0.47	0.	0.
time (sec)	N/A	0.031	0.023	0.02	1.013	1.575	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	46	71	43	49	0	0
normalized size	1	1.	0.51	0.79	0.48	0.54	0.	0.
time (sec)	N/A	0.044	0.032	0.03	1.019	1.59	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	35	37	55	86	0	0
normalized size	1	1.	0.73	0.77	1.15	1.79	0.	0.
time (sec)	N/A	0.027	0.017	0.023	0.99	1.558	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	40	37	93	147	0	0
normalized size	1	1.	0.45	0.42	1.06	1.67	0.	0.
time (sec)	N/A	0.052	0.033	0.024	1.028	1.566	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	40	37	131	211	0	0
normalized size	1	1.	0.45	0.42	1.49	2.4	0.	0.
time (sec)	N/A	0.051	0.034	0.032	1.02	1.55	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	55	132	63	155	0	234
normalized size	1	1.	0.51	1.22	0.58	1.44	0.	2.17
time (sec)	N/A	0.042	0.032	0.041	1.055	1.638	0.	1.117

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	46	61	34	61	0	72
normalized size	1	1.	0.49	0.66	0.37	0.66	0.	0.77
time (sec)	N/A	0.028	0.023	0.015	1.019	1.533	0.	1.126

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	46	61	34	61	0	72
normalized size	1	1.	0.49	0.66	0.37	0.66	0.	0.77
time (sec)	N/A	0.025	0.022	0.013	0.998	1.572	0.	1.113

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	39	56	26	45	0	34
normalized size	1	1.	0.44	0.64	0.3	0.51	0.	0.39
time (sec)	N/A	0.019	0.014	0.013	0.961	1.591	0.	1.102

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	37	54	18	32	0	0
normalized size	1	1.	0.44	0.64	0.21	0.38	0.	0.
time (sec)	N/A	0.026	0.016	0.019	1.003	1.627	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	42	61	30	43	0	0
normalized size	1	1.	0.45	0.65	0.32	0.46	0.	0.
time (sec)	N/A	0.033	0.025	0.018	1.005	1.605	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	47	61	30	57	0	0
normalized size	1	1.	0.49	0.64	0.31	0.59	0.	0.
time (sec)	N/A	0.029	0.026	0.019	0.983	1.548	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	90	532	373	887	0	3671
normalized size	1	1.	0.38	2.24	1.57	3.73	0.	15.42
time (sec)	N/A	0.098	0.109	0.055	1.012	1.633	0.	1.398

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	123	146	146	304	0	394
normalized size	1	1.	0.58	0.69	0.69	1.43	0.	1.86
time (sec)	N/A	0.062	0.08	0.016	1.016	1.558	0.	1.15

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	124	145	147	306	0	394
normalized size	1	1.	0.59	0.69	0.7	1.45	0.	1.87
time (sec)	N/A	0.058	0.073	0.016	1.004	1.57	0.	1.142

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	122	138	136	277	0	355
normalized size	1	1.	0.59	0.67	0.66	1.34	0.	1.72
time (sec)	N/A	0.052	0.076	0.014	0.988	1.563	0.	1.158

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	68	127	58	99	0	0
normalized size	1	1.	0.35	0.65	0.3	0.51	0.	0.
time (sec)	N/A	0.052	0.039	0.016	0.983	1.569	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	124	147	136	270	0	0
normalized size	1	1.	0.58	0.69	0.64	1.27	0.	0.
time (sec)	N/A	0.07	0.094	0.024	1.037	1.589	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	124	145	136	292	0	0
normalized size	1	1.	0.57	0.67	0.62	1.34	0.	0.
time (sec)	N/A	0.07	0.103	0.023	1.013	1.606	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	62	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.022	0.056	0.	0.	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	53	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.015	0.018	0.	0.	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	53	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.014	0.02	0.	0.	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	44	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.01	0.019	0.	0.	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	42	66	36	47	0	0
normalized size	1	1.	0.49	0.78	0.42	0.55	0.	0.
time (sec)	N/A	0.036	0.015	0.018	0.994	1.597	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.014	0.017	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	53	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.013	0.019	0.	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.021	0.036	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.018	0.069	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.017	0.057	0.	0.	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	46	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.012	0.053	0.	0.	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	76	104	95	244	0	0
normalized size	1	1.	0.48	0.65	0.6	1.53	0.	0.
time (sec)	N/A	0.083	0.067	0.02	0.962	1.87	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	53	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.015	0.059	0.	0.	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	55	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.016	0.063	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	58	0	0	163	0	0
normalized size	1	1.	1.12	0.	0.	3.13	0.	0.
time (sec)	N/A	0.02	0.024	0.243	0.	1.651	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	51	0	93	0	0
normalized size	1	1.	0.74	1.19	0.	2.16	0.	0.
time (sec)	N/A	0.013	0.061	0.035	0.	1.644	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	80	0	0	231	0	0
normalized size	1	1.	0.62	0.	0.	1.78	0.	0.
time (sec)	N/A	0.055	0.065	0.241	0.	1.693	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	59	0	0	171	0	0
normalized size	1	1.	0.58	0.	0.	1.68	0.	0.
time (sec)	N/A	0.044	0.044	0.092	0.	1.673	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	124	75	0	0	401	0	0
normalized size	1	1.06	0.64	0.	0.	3.43	0.	0.
time (sec)	N/A	0.064	0.033	0.05	0.	1.622	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	54	148	80	161	0	0
normalized size	1	1.	0.52	1.44	0.78	1.56	0.	0.
time (sec)	N/A	0.065	0.029	0.071	1.001	1.652	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	93	973	0	761	0	0
normalized size	1	1.	0.84	8.77	0.	6.86	0.	0.
time (sec)	N/A	0.122	0.203	0.138	0.	1.665	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	80	664	0	633	0	0
normalized size	1	1.	0.92	7.63	0.	7.28	0.	0.
time (sec)	N/A	0.074	0.128	0.091	0.	1.693	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	402	0	517	0	0
normalized size	1	1.	0.91	5.91	0.	7.6	0.	0.
time (sec)	N/A	0.051	0.067	0.076	0.	1.627	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	113	0	351	0	53
normalized size	1	1.	1.	2.9	0.	9.	0.	1.36
time (sec)	N/A	0.033	0.062	0.046	0.	1.574	0.	1.099

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	135	658	0	738	0	0
normalized size	1	1.	1.38	6.71	0.	7.53	0.	0.
time (sec)	N/A	0.126	0.615	0.114	0.	1.71	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	112	958	0	938	0	0
normalized size	1	1.	0.89	7.6	0.	7.44	0.	0.
time (sec)	N/A	0.171	0.339	0.135	0.	1.675	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	143	1300	0	1130	0	0
normalized size	1	1.	0.87	7.93	0.	6.89	0.	0.
time (sec)	N/A	0.231	0.458	0.16	0.	1.704	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	340	280	0	9276	0	0
normalized size	1	1.	0.96	0.79	0.	26.28	0.	0.
time (sec)	N/A	0.627	0.879	0.256	0.	4.15	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	610	610	526	260	0	9999	0	0
normalized size	1	1.	0.86	0.43	0.	16.39	0.	0.
time (sec)	N/A	1.152	0.773	0.215	0.	4.08	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	145	114	0	1705	0	0
normalized size	1	1.	0.86	0.67	0.	10.09	0.	0.
time (sec)	N/A	0.192	0.279	0.113	0.	1.799	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	127	268	0	2527	0	0
normalized size	1	1.	0.62	1.31	0.	12.33	0.	0.
time (sec)	N/A	0.395	0.204	0.201	0.	1.999	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	699	699	127	534	0	13524	0	0
normalized size	1	1.	0.18	0.76	0.	19.35	0.	0.
time (sec)	N/A	1.493	0.15	0.398	0.	14.367	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	127	630	0	12146	0	0
normalized size	1	1.	0.31	1.52	0.	29.34	0.	0.
time (sec)	N/A	0.787	0.142	0.549	0.	8.345	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	129	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.128	0.033	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	129	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.106	0.025	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	119	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.092	0.02	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	397	0	590	0	0
normalized size	1	1.	1.	5.36	0.	7.97	0.	0.
time (sec)	N/A	0.066	0.115	0.066	0.	1.609	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	129	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.122	0.028	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	129	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.123	0.03	0.	0.	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	365	0	0	0	0	0
normalized size	1	1.	2.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.178	0.789	0.075	0.	0.	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	366	0	0	0	0	0
normalized size	1	1.	2.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	0.737	0.059	0.	0.	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	364	0	0	0	0	0
normalized size	1	1.	2.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.681	0.059	0.	0.	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	351	0	0	0	0	0
normalized size	1	1.	2.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.619	0.059	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	110	125	0	1554	0	0
normalized size	1	1.	0.92	1.05	0.	13.06	0.	0.
time (sec)	N/A	0.095	0.149	0.108	0.	2.01	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	365	0	0	0	0	0
normalized size	1	1.	2.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	0.662	0.064	0.	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	365	0	0	0	0	0
normalized size	1	1.	2.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	0.643	0.063	0.	0.	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	518	0	0	0	0	0
normalized size	1	1.	3.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	1.557	0.05	0.	0.	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	524	0	0	0	0	0
normalized size	1	1.	3.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	1.521	0.048	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	520	0	0	0	0	0
normalized size	1	1.	3.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	1.529	0.05	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	513	0	0	0	0	0
normalized size	1	1.	3.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	1.537	0.049	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	158	209	0	1955	0	0
normalized size	1	1.	0.91	1.21	0.	11.3	0.	0.
time (sec)	N/A	0.159	0.293	0.045	0.	3.164	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	526	0	0	0	0	0
normalized size	1	1.	3.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	1.396	0.057	0.	0.	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	520	0	0	0	0	0
normalized size	1	1.	3.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	1.393	0.056	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	0.17	0.017	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.153	0.015	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.141	0.016	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	166	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.087	0.017	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	347	0	0
normalized size	1	1.	1.	0.	0.	7.38	0.	0.
time (sec)	N/A	0.033	0.066	0.033	0.	1.861	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	173	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	0.152	0.016	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	175	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.16	0.015	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0
normalized size	1	1.	2.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	0.885	0.012	0.	0.	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0
normalized size	1	1.	2.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	0.859	0.013	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0
normalized size	1	1.	2.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.847	0.013	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	384	0	0	0	0	0
normalized size	1	1.	2.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.973	0.012	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	94	0	0	984	0	0
normalized size	1	1.	0.96	0.	0.	10.04	0.	0.
time (sec)	N/A	0.073	0.315	0.013	0.	2.566	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	395	0	0	0	0	0
normalized size	1	1.	2.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.806	0.011	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	399	0	0	0	0	0
normalized size	1	1.	2.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.843	0.014	0.	0.	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	137	3798	0	5146	0	0
normalized size	1	1.	0.75	20.87	0.	28.27	0.	0.
time (sec)	N/A	0.156	0.323	0.105	0.	2.265	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	86	1065	0	1665	0	7363
normalized size	1	1.	0.74	9.1	0.	14.23	0.	62.93
time (sec)	N/A	0.069	0.136	0.067	0.	1.849	0.	1.223

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	41	205	0	371	0	752
normalized size	1	1.	0.71	3.53	0.	6.4	0.	12.97
time (sec)	N/A	0.024	0.062	0.042	0.	1.891	0.	1.109

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	143	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	0.148	0.029	0.	0.	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	1511	0	0	0	0	0
normalized size	1	1.	4.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.961	2.289	0.036	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	615	615	7827	0	0	0	0	0
normalized size	1	1.	12.73	0.	0.	0.	0.	0.
time (sec)	N/A	10.724	6.722	0.047	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	618	0	0	0	0	0
normalized size	1	1.	3.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	3.318	0.253	0.	0.	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	388	0	0	0	0	0
normalized size	1	1.	2.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	0.771	0.14	0.	0.	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	183	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	0.191	0.03	0.	0.	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	428	0	0	0	0	0
normalized size	1	1.	2.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.173	1.521	0.02	0.	0.	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	181	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.378	0.049	0.	0.	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	150	298	192	393	178	224
normalized size	1	1.	3.26	6.48	4.17	8.54	3.87	4.87
time (sec)	N/A	0.054	0.038	0.001	1.031	1.484	0.093	1.079

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	401	1314	544	1258	559	730
normalized size	1	1.	4.51	14.76	6.11	14.13	6.28	8.2
time (sec)	N/A	0.187	0.116	0.002	1.028	1.46	0.164	1.079

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	797	7550	1177	3015	1314	1708
normalized size	1	1.	5.78	54.71	8.53	21.85	9.52	12.38
time (sec)	N/A	0.373	0.29	0.002	1.049	1.542	0.289	1.116

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	154	349	224	490	240	297
normalized size	1	1.	2.8	6.35	4.07	8.91	4.36	5.4
time (sec)	N/A	0.053	0.021	0.	1.014	1.454	0.1	1.092

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	405	1413	593	1517	722	925
normalized size	1	1.	3.89	13.59	5.7	14.59	6.94	8.89
time (sec)	N/A	0.164	0.076	0.001	0.983	1.554	0.182	1.114

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	801	7697	1242	3555	1654	2113
normalized size	1	1.	5.04	48.41	7.81	22.36	10.4	13.29
time (sec)	N/A	0.315	0.041	0.001	1.005	1.568	0.318	1.152

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	219	158	0	2539	178	0
normalized size	1	1.	1.13	0.82	0.	13.16	0.92	0.
time (sec)	N/A	0.438	0.154	0.065	0.	2.001	2.741	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	151	0	956	280	375
normalized size	1	1.	0.95	1.86	0.	11.8	3.46	4.63
time (sec)	N/A	0.129	0.044	0.006	0.	1.849	1.812	1.407

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	175	140	0	1524	104	0
normalized size	1	1.	1.07	0.85	0.	9.29	0.63	0.
time (sec)	N/A	0.155	0.101	0.004	0.	1.756	1.57	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	129	0	598	168	247
normalized size	1	1.	1.07	3.	0.	13.91	3.91	5.74
time (sec)	N/A	0.061	0.017	0.005	0.	1.755	1.025	1.383

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	128	184	0	1040	320	0
normalized size	1	1.	1.36	1.96	0.	11.06	3.4	0.
time (sec)	N/A	0.132	0.08	0.008	0.	1.992	4.866	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	206	168	0	2753	211	4884
normalized size	1	1.	1.06	0.86	0.	14.12	1.08	25.05
time (sec)	N/A	0.286	0.367	0.007	0.	2.28	3.663	3.041

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	154	213	0	1782	464	138
normalized size	1	1.	1.27	1.76	0.	14.73	3.83	1.14
time (sec)	N/A	0.198	0.13	0.012	0.	2.469	14.425	1.234

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	235	188	0	4215	347	0
normalized size	1	1.	1.05	0.84	0.	18.82	1.55	0.
time (sec)	N/A	0.497	0.217	0.01	0.	2.175	12.059	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	263	323	0	5245	571	0
normalized size	1	1.	0.97	1.2	0.	19.43	2.11	0.
time (sec)	N/A	0.579	0.549	0.017	0.	2.135	48.544	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	100	276	0	2192	493	490
normalized size	1	1.	1.03	2.85	0.	22.6	5.08	5.05
time (sec)	N/A	0.135	0.139	0.02	0.	1.969	38.446	3.219

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	247	319	0	5304	578	0
normalized size	1	1.	0.97	1.26	0.	20.88	2.28	0.
time (sec)	N/A	0.388	1.084	0.018	0.	2.274	35.26	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	98	270	0	2214	495	494
normalized size	1	1.	1.02	2.81	0.	23.06	5.16	5.15
time (sec)	N/A	0.122	0.133	0.02	0.	2.024	25.69	3.041

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	271	364	0	6835	740	0
normalized size	1	1.	0.91	1.22	0.	22.86	2.47	0.
time (sec)	N/A	0.702	1.002	0.017	0.	2.621	24.53	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	235	693	0	5200	1091	784
normalized size	1	1.	1.45	4.28	0.	32.1	6.73	4.84
time (sec)	N/A	0.294	0.429	0.03	0.	3.371	167.586	3.25

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	339	1304	0	9384	0	0
normalized size	1	1.	0.97	3.75	0.	26.97	0.	0.
time (sec)	N/A	1.653	1.756	0.027	0.	3.339	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	284	1014	0	9488	0	302
normalized size	1	1.	1.33	4.76	0.	44.54	0.	1.42
time (sec)	N/A	0.39	0.51	0.036	0.	5.091	0.	2.834

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	384	1518	0	12725	0	0
normalized size	1	1.	0.94	3.72	0.	31.19	0.	0.
time (sec)	N/A	3.676	3.256	0.034	0.	4.719	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	328	704	0	14348	0	0
normalized size	1	1.	0.96	2.06	0.	42.08	0.	0.
time (sec)	N/A	0.951	4.782	0.042	0.	3.93	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	146	544	0	7896	0	873
normalized size	1	1.	0.97	3.63	0.	52.64	0.	5.82
time (sec)	N/A	0.196	0.223	0.038	0.	3.283	0.	17.367

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	382	885	0	16612	0	0
normalized size	1	1.	1.05	2.44	0.	45.76	0.	0.
time (sec)	N/A	1.039	5.057	0.043	0.	4.944	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	147	541	0	7795	0	873
normalized size	1	1.	0.98	3.61	0.	51.97	0.	5.82
time (sec)	N/A	0.185	0.183	0.04	0.	3.314	0.	17.123

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	463	1010	0	18502	0	0
normalized size	1	1.	1.06	2.31	0.	42.34	0.	0.
time (sec)	N/A	5.361	6.167	0.043	0.	6.205	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	391	4477	0	20893	0	1802
normalized size	1	1.	1.53	17.56	0.	81.93	0.	7.07
time (sec)	N/A	0.486	3.991	0.068	0.	11.127	0.	17.448

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	484	560	6821	0	22920	0	0
normalized size	1	1.	1.16	14.09	0.	47.36	0.	0.
time (sec)	N/A	1.227	6.232	0.062	0.	8.978	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	491	5575	0	32173	0	509
normalized size	1	1.	1.51	17.15	0.	98.99	0.	1.57
time (sec)	N/A	0.585	6.184	0.077	0.	17.597	0.	17.045

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	222	164	0	2772	219	0
normalized size	1	1.	1.1	0.81	0.	13.72	1.08	0.
time (sec)	N/A	0.362	0.147	0.003	0.	1.656	3.704	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	80	154	0	977	332	387
normalized size	1	1.	0.92	1.77	0.	11.23	3.82	4.45
time (sec)	N/A	0.127	0.042	0.004	0.	1.564	2.154	1.472

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	178	143	0	1646	124	0
normalized size	1	1.	1.05	0.84	0.	9.68	0.73	0.
time (sec)	N/A	0.153	0.095	0.003	0.	1.611	1.917	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	47	130	0	603	189	250
normalized size	1	1.	1.07	2.95	0.	13.7	4.3	5.68
time (sec)	N/A	0.063	0.017	0.005	0.	1.556	1.356	1.43

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	131	190	0	1045	348	0
normalized size	1	1.	1.27	1.84	0.	10.15	3.38	0.
time (sec)	N/A	0.138	0.075	0.007	0.	1.718	6.014	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	209	174	0	3001	258	5022
normalized size	1	1.	1.02	0.85	0.	14.71	1.26	24.62
time (sec)	N/A	0.273	0.347	0.004	0.	1.724	4.568	3.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	157	222	0	1814	532	618
normalized size	1	1.	1.18	1.67	0.	13.64	4.	4.65
time (sec)	N/A	0.195	0.133	0.007	0.	2.416	17.918	1.463

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	238	197	0	4533	411	0
normalized size	1	1.	1.01	0.83	0.	19.21	1.74	0.
time (sec)	N/A	0.488	0.213	0.009	0.	1.897	14.61	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	266	695	0	5463	639	0
normalized size	1	1.	0.95	2.49	0.	19.58	2.29	0.
time (sec)	N/A	0.539	0.535	0.016	0.	2.118	50.054	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	103	500	0	2290	554	522
normalized size	1	1.	1.	4.85	0.	22.23	5.38	5.07
time (sec)	N/A	0.138	0.135	0.017	0.	1.679	39.026	2.376

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	250	693	0	5496	646	0
normalized size	1	1.	0.95	2.63	0.	20.9	2.46	0.
time (sec)	N/A	0.368	1.035	0.016	0.	2.067	39.205	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	99	484	0	2268	525	506
normalized size	1	1.	1.01	4.94	0.	23.14	5.36	5.16
time (sec)	N/A	0.127	0.129	0.019	0.	1.756	26.748	2.405

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	238	714	0	5227	0	848
normalized size	1	1.	1.37	4.1	0.	30.04	0.	4.87
time (sec)	N/A	0.295	0.425	0.027	0.	4.083	0.	2.415

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	342	1346	0	9735	0	0
normalized size	1	1.	0.95	3.74	0.	27.04	0.	0.
time (sec)	N/A	1.599	1.713	0.026	0.	2.761	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	287	1047	0	9563	0	1160
normalized size	1	1.	1.26	4.59	0.	41.94	0.	5.09
time (sec)	N/A	0.369	0.492	0.03	0.	10.347	0.	2.463

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	387	1569	0	13152	0	0
normalized size	1	1.	0.91	3.71	0.	31.09	0.	0.
time (sec)	N/A	3.55	3.306	0.028	0.	3.815	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	331	3432	0	14596	0	0
normalized size	1	1.	0.94	9.72	0.	41.35	0.	0.
time (sec)	N/A	0.87	4.613	0.04	0.	3.694	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	149	2181	0	8080	0	979
normalized size	1	1.	0.94	13.72	0.	50.82	0.	6.16
time (sec)	N/A	0.2	0.217	0.04	0.	2.966	0.	11.698

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	385	4751	0	16847	0	0
normalized size	1	1.	1.03	12.67	0.	44.93	0.	0.
time (sec)	N/A	0.972	4.963	0.04	0.	4.167	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	148	2132	0	7892	0	909
normalized size	1	1.	0.97	13.93	0.	51.58	0.	5.94
time (sec)	N/A	0.192	0.182	0.042	0.	3.328	0.	11.762

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	394	4606	0	20941	0	1887
normalized size	1	1.	1.46	17.06	0.	77.56	0.	6.99
time (sec)	N/A	0.496	3.998	0.063	0.	14.791	0.	12.045

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	499	575	7019	0	23385	0	0
normalized size	1	1.	1.15	14.07	0.	46.86	0.	0.
time (sec)	N/A	1.094	6.219	0.059	0.	7.836	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	509	5737	0	32292	0	2847
normalized size	1	1.	1.48	16.73	0.	94.15	0.	8.3
time (sec)	N/A	0.589	6.168	0.076	0.	37.718	0.	16.478

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	340	340	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.691	1.136	0.023	0.	0.	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	398	398	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.687	0.43	0.042	0.	0.	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	105	140	532	107	38
normalized size	1	1.	1.	3.09	4.12	15.65	3.15	1.12
time (sec)	N/A	0.036	0.013	0.002	0.97	1.077	0.092	1.081

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	188	175	235	1289	187	62
normalized size	1	1.	3.36	3.12	4.2	23.02	3.34	1.11
time (sec)	N/A	0.096	0.012	0.003	1.019	1.009	0.134	1.102

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [411] had the largest ratio of [0.8]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.	15	0.133
2	A	1	1	1.	15	0.067
3	A	1	1	1.	15	0.067
4	A	3	3	1.	15	0.2
5	A	8	8	1.	11	0.727
6	A	3	2	1.	26	0.077
7	A	3	2	1.	26	0.077
8	A	3	2	1.	26	0.077
9	A	2	2	1.	26	0.077
10	A	3	2	1.	24	0.083
11	A	2	1	1.	22	0.045
12	A	3	2	1.	26	0.077
13	A	3	2	1.	26	0.077
14	A	3	2	1.	26	0.077
15	A	3	2	1.	26	0.077
16	A	3	2	1.	26	0.077
17	A	3	2	1.	26	0.077
18	A	3	2	1.	26	0.077
19	A	3	2	1.	26	0.077
20	A	3	2	1.	26	0.077
21	A	3	2	1.	26	0.077
22	A	3	2	1.	26	0.077
23	A	3	2	1.	26	0.077
24	A	4	3	1.4	26	0.115
25	A	3	2	1.	26	0.077
26	A	3	2	1.	26	0.077
27	A	4	3	1.	26	0.115
28	A	3	2	1.	26	0.077
29	A	3	2	1.	26	0.077
30	A	2	2	1.	26	0.077
31	A	3	2	1.	24	0.083
32	A	3	2	1.	22	0.091
33	A	4	3	1.	26	0.115
34	A	3	2	1.	26	0.077
35	A	3	2	1.	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	4	3	1.	26	0.115
37	A	3	2	1.	26	0.077
38	A	3	2	1.	26	0.077
39	A	4	3	1.	26	0.115
40	A	3	2	1.	26	0.077
41	A	3	2	1.	26	0.077
42	A	4	3	1.	26	0.115
43	A	3	2	1.	26	0.077
44	A	3	2	1.	26	0.077
45	A	2	2	1.	26	0.077
46	A	3	2	1.	26	0.077
47	A	3	2	1.	26	0.077
48	A	4	4	1.	26	0.154
49	A	3	2	1.	26	0.077
50	A	3	2	1.	26	0.077
51	A	3	2	1.	26	0.077
52	A	4	3	1.	26	0.115
53	A	3	2	1.	26	0.077
54	A	3	2	1.	26	0.077
55	A	4	3	1.	26	0.115
56	A	3	2	1.	26	0.077
57	A	3	2	1.	26	0.077
58	A	4	3	1.	26	0.115
59	A	3	2	1.	26	0.077
60	A	3	2	1.	26	0.077
61	A	2	2	1.	26	0.077
62	A	3	2	1.	24	0.083
63	A	3	2	1.	22	0.091
64	A	4	3	1.	26	0.115
65	A	3	2	1.	26	0.077
66	A	3	2	1.	26	0.077
67	A	4	3	1.	26	0.115
68	A	3	2	1.	26	0.077
69	A	3	2	1.	26	0.077
70	A	4	3	1.	26	0.115
71	A	3	2	1.	26	0.077
72	A	3	2	1.	26	0.077
73	A	4	3	1.	26	0.115
74	A	3	2	1.	26	0.077
75	A	3	2	1.	26	0.077
76	A	4	3	1.	26	0.115
77	A	3	2	1.	26	0.077
78	A	3	2	1.	26	0.077
79	A	4	3	1.	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	3	2	1.	26	0.077
81	A	3	2	1.	26	0.077
82	A	2	2	1.	26	0.077
83	A	3	2	1.	26	0.077
84	A	3	2	1.	26	0.077
85	A	4	4	1.	26	0.154
86	A	3	2	1.	26	0.077
87	A	3	2	1.	26	0.077
88	A	5	4	1.	26	0.154
89	A	8	8	1.	26	0.308
90	A	8	8	1.	26	0.308
91	A	3	3	1.	26	0.115
92	A	7	7	1.	24	0.292
93	A	7	7	1.	22	0.318
94	A	5	5	1.	26	0.192
95	A	8	8	1.	26	0.308
96	A	8	8	1.	26	0.308
97	A	4	3	1.	26	0.115
98	A	9	9	1.	26	0.346
99	A	9	9	1.	26	0.346
100	A	2	2	1.	26	0.077
101	A	9	8	1.	24	0.333
102	A	9	8	1.	22	0.364
103	A	4	3	1.	26	0.115
104	A	10	9	1.	26	0.346
105	A	10	9	1.	26	0.346
106	A	4	3	1.	26	0.115
107	A	11	9	1.	26	0.346
108	A	4	3	1.	26	0.115
109	A	11	9	1.	26	0.346
110	A	11	9	1.	26	0.346
111	A	2	2	1.	26	0.077
112	A	11	8	1.	24	0.333
113	A	11	8	1.	22	0.364
114	A	4	3	1.	26	0.115
115	A	12	9	1.	26	0.346
116	A	12	9	1.	26	0.346
117	A	4	3	1.	26	0.115
118	A	3	2	1.	28	0.071
119	A	3	2	1.	28	0.071
120	A	3	2	1.	28	0.071
121	A	2	2	1.	28	0.071
122	A	2	2	1.	28	0.071
123	A	2	2	1.	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
124	A	2	2	1.	26	0.077
125	A	4	3	1.	24	0.125
126	A	4	3	1.	24	0.125
127	A	4	3	1.	24	0.125
128	A	2	2	1.	24	0.083
129	A	2	2	1.	24	0.083
130	A	2	2	1.	24	0.083
131	A	2	2	1.03	22	0.091
132	A	3	3	1.04	20	0.15
133	A	3	3	1.	24	0.125
134	A	2	2	1.	24	0.083
135	A	2	2	1.	24	0.083
136	A	3	3	1.	24	0.125
137	A	2	2	1.	24	0.083
138	A	6	6	1.	18	0.333
139	A	5	5	1.	18	0.278
140	A	3	3	1.	18	0.167
141	A	7	7	1.	18	0.389
142	A	8	7	1.	18	0.389
143	A	14	8	1.	18	0.444
144	A	14	8	1.	18	0.444
145	A	13	7	1.	18	0.389
146	A	13	7	1.	18	0.389
147	A	13	7	1.	16	0.438
148	A	13	7	1.	14	0.5
149	A	14	8	1.	18	0.444
150	A	14	8	1.	18	0.444
151	A	6	4	1.	16	0.25
152	A	5	4	1.	16	0.25
153	A	4	3	1.	16	0.188
154	B	4	3	2.1	16	0.188
155	A	6	5	1.	16	0.312
156	A	4	3	1.	16	0.188
157	A	4	3	1.	16	0.188
158	A	15	10	1.	16	0.625
159	A	15	10	1.	16	0.625
160	A	14	9	1.	16	0.562
161	A	14	9	1.	16	0.562
162	A	13	8	1.	16	0.5
163	A	13	8	1.	16	0.5
164	A	13	8	1.	14	0.571
165	A	13	8	1.	12	0.667
166	A	14	9	1.	16	0.562
167	A	14	9	1.	16	0.562

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
168	A	15	10	1.	16	0.625
169	A	15	10	1.	16	0.625
170	A	14	8	1.	16	0.5
171	A	5	5	1.	16	0.312
172	A	13	7	1.	16	0.438
173	A	13	7	1.	16	0.438
174	A	3	3	1.	16	0.188
175	A	13	7	1.	14	0.5
176	C	13	7	2.02	12	0.583
177	A	7	7	1.	16	0.438
178	A	14	8	1.	16	0.5
179	A	14	8	1.	16	0.5
180	A	8	7	1.	16	0.438
181	A	16	10	1.	16	0.625
182	A	13	7	1.	10	0.7
183	A	3	3	1.	14	0.214
184	A	13	7	1.	14	0.5
185	A	7	7	1.	20	0.35
186	A	6	6	1.	20	0.3
187	A	6	6	1.	20	0.3
188	A	5	5	1.	20	0.25
189	A	4	4	1.	20	0.2
190	A	7	6	1.	20	0.3
191	A	7	6	1.	20	0.3
192	A	4	4	1.	20	0.2
193	A	5	5	1.	20	0.25
194	A	6	6	1.	20	0.3
195	A	7	7	1.	20	0.35
196	A	2	2	1.	20	0.1
197	A	2	2	1.	18	0.111
198	A	2	2	1.	16	0.125
199	A	2	2	1.	20	0.1
200	A	2	2	1.	20	0.1
201	A	8	7	1.	20	0.35
202	A	7	6	1.	20	0.3
203	A	7	6	1.	20	0.3
204	A	6	5	1.	20	0.25
205	A	5	4	1.	20	0.2
206	A	8	7	1.	20	0.35
207	A	8	7	1.	20	0.35
208	A	8	7	1.	20	0.35
209	A	8	7	1.	20	0.35
210	A	5	4	1.	20	0.2
211	A	6	5	1.	20	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
212	A	7	6	1.	20	0.3
213	A	8	7	1.	20	0.35
214	A	2	2	1.	20	0.1
215	A	2	2	1.	18	0.111
216	A	2	2	1.	16	0.125
217	A	2	2	1.	20	0.1
218	A	2	2	1.	20	0.1
219	A	6	6	1.	20	0.3
220	A	5	5	1.	20	0.25
221	A	5	5	1.	20	0.25
222	A	4	4	1.	20	0.2
223	A	3	3	1.	20	0.15
224	A	3	3	1.	20	0.15
225	A	4	4	1.	20	0.2
226	A	5	5	1.	20	0.25
227	A	6	6	1.	20	0.3
228	A	7	6	1.	20	0.3
229	A	2	2	1.	20	0.1
230	A	2	2	1.	18	0.111
231	A	2	2	1.	16	0.125
232	A	2	2	1.	20	0.1
233	A	2	2	1.	20	0.1
234	A	6	6	1.	20	0.3
235	A	5	5	1.	20	0.25
236	A	5	5	1.	20	0.25
237	A	2	2	1.	20	0.1
238	A	2	2	1.	20	0.1
239	A	5	5	1.	20	0.25
240	A	5	5	1.	20	0.25
241	A	6	6	1.	20	0.3
242	A	7	6	1.	20	0.3
243	A	2	2	1.	20	0.1
244	A	2	2	1.	18	0.111
245	A	2	2	1.	16	0.125
246	A	2	2	1.	20	0.1
247	A	2	2	1.	20	0.1
248	A	2	1	1.	20	0.05
249	A	2	1	1.	18	0.056
250	A	3	2	1.	20	0.1
251	A	4	3	1.	20	0.15
252	A	2	2	1.	22	0.091
253	A	2	2	1.	22	0.091
254	A	2	2	1.	22	0.091
255	A	2	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
256	A	2	2	1.	20	0.1
257	A	4	4	1.	18	0.222
258	A	3	3	1.	18	0.167
259	A	2	2	1.	18	0.111
260	A	2	2	1.	18	0.111
261	A	2	2	1.	18	0.111
262	A	2	2	1.	16	0.125
263	A	2	2	1.	14	0.143
264	A	3	3	1.	18	0.167
265	A	2	2	1.	18	0.111
266	A	2	2	1.	18	0.111
267	A	3	3	1.	18	0.167
268	A	2	2	1.	18	0.111
269	A	2	2	1.	18	0.111
270	A	3	3	1.	18	0.167
271	A	2	2	1.	16	0.125
272	A	5	5	1.	16	0.312
273	A	4	3	1.	16	0.188
274	A	4	4	1.	16	0.25
275	A	2	2	1.	16	0.125
276	A	4	4	1.	14	0.286
277	A	4	3	1.	16	0.188
278	A	5	5	1.	16	0.312
279	A	4	3	1.	16	0.188
280	A	6	5	1.	16	0.312
281	A	12	9	1.	16	0.562
282	A	11	8	1.	16	0.5
283	A	11	8	1.	16	0.5
284	A	11	8	1.	16	0.5
285	A	11	8	1.	12	0.667
286	A	12	9	1.	16	0.562
287	A	12	9	1.	16	0.562
288	A	13	9	1.	16	0.562
289	A	13	9	1.	16	0.562
290	A	2	2	1.	16	0.125
291	A	5	5	1.	16	0.312
292	A	4	3	1.	16	0.188
293	A	4	4	1.	16	0.25
294	A	2	2	1.	16	0.125
295	A	4	4	1.	14	0.286
296	A	4	3	1.	16	0.188
297	A	5	5	1.	16	0.312
298	A	4	3	1.	16	0.188
299	A	6	5	1.	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
300	A	6	6	1.	16	0.375
301	A	5	5	1.	16	0.312
302	A	5	5	1.	16	0.312
303	A	5	5	1.	16	0.312
304	A	5	5	1.	12	0.417
305	A	6	6	1.	16	0.375
306	A	6	6	1.	16	0.375
307	A	7	6	1.	16	0.375
308	A	7	6	1.	16	0.375
309	A	3	2	1.	18	0.111
310	A	6	6	1.	18	0.333
311	A	5	4	1.	18	0.222
312	A	5	5	1.	18	0.278
313	A	4	3	1.	18	0.167
314	A	3	3	1.	18	0.167
315	A	4	3	1.	16	0.188
316	A	7	7	1.	18	0.389
317	A	5	4	1.	18	0.222
318	A	8	7	1.	18	0.389
319	A	8	5	1.	18	0.278
320	A	8	5	1.	18	0.278
321	A	7	4	1.	18	0.222
322	A	7	4	1.	18	0.222
323	A	7	4	1.	18	0.222
324	A	7	4	1.	14	0.286
325	A	8	5	1.	18	0.278
326	A	8	5	1.	18	0.278
327	A	3	2	1.	14	0.143
328	A	6	6	1.	14	0.429
329	A	7	5	1.	14	0.357
330	A	5	5	1.	14	0.357
331	A	10	7	1.	14	0.5
332	A	3	3	1.	14	0.214
333	A	10	6	1.	12	0.5
334	A	7	7	1.	14	0.5
335	A	7	5	1.	14	0.357
336	A	8	7	1.	14	0.5
337	A	13	10	1.	14	0.714
338	A	20	7	1.	14	0.5
339	A	9	6	1.	14	0.429
340	A	19	7	1.	14	0.5
341	A	19	6	1.	14	0.429
342	A	9	6	1.	10	0.6
343	A	20	8	1.	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
344	A	20	7	1.	14	0.5
345	A	12	9	1.	14	0.643
346	A	22	10	1.	14	0.714
347	A	3	2	1.	16	0.125
348	A	6	6	1.	16	0.375
349	A	5	4	1.	16	0.25
350	A	5	5	1.	16	0.312
351	A	10	7	1.	16	0.438
352	A	3	3	1.	16	0.188
353	A	10	6	1.	14	0.429
354	A	7	7	1.	16	0.438
355	A	5	4	1.	16	0.25
356	A	8	7	1.	16	0.438
357	A	13	10	1.	16	0.625
358	A	20	7	1.	16	0.438
359	A	19	6	1.	16	0.375
360	A	19	7	1.	16	0.438
361	A	19	6	1.	16	0.375
362	A	19	6	1.	12	0.5
363	A	22	8	1.	16	0.5
364	A	20	7	1.	16	0.438
365	A	22	9	1.	16	0.562
366	A	22	10	1.	16	0.625
367	A	3	2	1.	16	0.125
368	A	5	4	1.	16	0.25
369	A	5	4	1.	16	0.25
370	A	4	3	1.	16	0.188
371	A	4	3	1.	16	0.188
372	A	3	3	1.	16	0.188
373	A	4	3	1.	14	0.214
374	A	6	5	1.	16	0.312
375	A	5	4	1.	16	0.25
376	A	7	5	1.	16	0.312
377	A	6	5	1.	16	0.312
378	A	20	8	0.96	16	0.5
379	A	19	7	1.	16	0.438
380	A	19	7	1.	16	0.438
381	A	19	7	1.01	16	0.438
382	A	19	7	1.	12	0.583
383	A	20	8	1.	16	0.5
384	A	20	8	1.	16	0.5
385	A	3	2	1.	16	0.125
386	A	5	4	1.	16	0.25
387	A	5	4	1.	16	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
388	A	4	3	1.	16	0.188
389	A	4	3	1.	16	0.188
390	A	3	3	1.	16	0.188
391	A	4	3	1.	14	0.214
392	A	6	5	1.	16	0.312
393	A	5	4	1.	16	0.25
394	A	7	5	1.	16	0.312
395	A	6	5	1.	16	0.312
396	A	8	5	1.	16	0.312
397	A	7	4	1.	16	0.25
398	A	7	4	1.	16	0.25
399	A	7	4	1.14	16	0.25
400	A	7	4	1.	12	0.333
401	A	8	5	1.	16	0.312
402	A	8	5	1.	16	0.312
403	A	9	6	1.	16	0.375
404	A	9	6	1.	16	0.375
405	A	4	3	1.	16	0.188
406	A	5	4	1.	16	0.25
407	A	5	5	1.	14	0.357
408	A	3	3	1.	14	0.214
409	A	7	7	1.	14	0.5
410	A	8	7	1.	14	0.5
411	A	8	8	1.	10	0.8
412	A	7	6	1.	18	0.333
413	A	7	6	1.	18	0.333
414	A	7	6	1.	16	0.375
415	A	6	6	1.	14	0.429
416	A	5	5	1.	18	0.278
417	A	3	3	1.	18	0.167
418	A	7	7	1.	18	0.389
419	A	8	7	1.	18	0.389
420	A	8	7	1.	18	0.389
421	A	8	7	1.	18	0.389
422	A	8	7	1.	16	0.438
423	A	8	7	1.	14	0.5
424	A	7	7	1.	18	0.389
425	A	4	4	1.	18	0.222
426	A	4	4	1.	18	0.222
427	A	4	4	1.	18	0.222
428	A	8	7	1.	18	0.389
429	A	8	7	1.	18	0.389
430	A	8	7	1.	18	0.389
431	A	9	8	1.	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
432	A	8	8	1.	18	0.444
433	A	5	4	1.	18	0.222
434	A	5	5	1.	18	0.278
435	A	5	5	1.	18	0.278
436	A	5	5	1.	18	0.278
437	A	5	4	1.	18	0.222
438	A	9	8	1.	18	0.444
439	A	9	8	1.	18	0.444
440	A	6	4	1.	18	0.222
441	A	6	4	1.	16	0.25
442	A	5	4	1.	14	0.286
443	A	4	3	1.	18	0.167
444	A	4	3	1.	18	0.167
445	A	6	5	1.	18	0.278
446	A	4	3	1.	18	0.167
447	A	4	3	1.	18	0.167
448	A	4	3	1.	18	0.167
449	A	9	7	1.	16	0.438
450	A	8	7	1.	16	0.438
451	A	7	6	1.	16	0.375
452	A	4	4	1.	16	0.25
453	A	5	5	1.	16	0.312
454	A	6	6	1.	16	0.375
455	A	4	3	1.	22	0.136
456	A	5	4	1.	14	0.286
457	A	15	9	1.	14	0.643
458	A	9	6	1.	14	0.429
459	A	7	6	1.	20	0.3
460	A	4	3	1.	23	0.13
461	A	4	4	1.	22	0.182
462	A	4	3	1.	26	0.115
463	A	4	3	1.	26	0.115
464	A	3	2	1.	26	0.077
465	A	4	3	1.	26	0.115
466	A	4	3	1.	26	0.115
467	A	4	3	1.	26	0.115
468	A	4	3	1.	26	0.115
469	A	4	3	1.	26	0.115
470	A	4	3	1.	26	0.115
471	A	4	3	1.	26	0.115
472	A	4	4	1.	30	0.133
473	A	4	3	1.	28	0.107
474	A	4	3	1.	26	0.115
475	A	4	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
476	A	3	3	1.	28	0.107
477	A	3	3	1.	28	0.107
478	C	7	3	1.11	77	0.039
479	A	4	3	1.	26	0.115
480	A	4	3	1.	26	0.115
481	A	5	4	1.	24	0.167
482	A	5	4	1.	26	0.154
483	A	5	4	1.	26	0.154
484	A	5	4	1.	26	0.154
485	A	4	3	1.	26	0.115
486	A	5	4	1.	26	0.154
487	A	5	4	1.	26	0.154
488	A	5	4	1.	26	0.154
489	A	5	4	1.	26	0.154
490	A	5	4	1.	26	0.154
491	A	4	3	1.	26	0.115
492	A	5	4	1.	26	0.154
493	A	4	3	1.	23	0.13
494	A	4	3	1.	23	0.13
495	A	2	2	1.	23	0.087
496	A	5	5	1.	21	0.238
497	A	4	3	1.	23	0.13
498	A	4	3	1.	23	0.13
499	A	4	3	1.	23	0.13
500	A	12	9	1.	25	0.36
501	A	9	9	1.	25	0.36
502	A	5	5	1.	25	0.2
503	A	6	6	1.	25	0.24
504	A	11	11	1.	25	0.44
505	A	14	11	1.	25	0.44
506	A	1	1	1.	26	0.038
507	A	1	1	1.	28	0.036
508	A	4	3	1.	32	0.094
509	A	4	3	1.	32	0.094
510	A	3	2	1.	32	0.062
511	A	4	3	1.	32	0.094
512	A	2	2	1.	32	0.062
513	A	4	3	1.	32	0.094
514	A	4	3	1.	32	0.094
515	A	5	4	1.	30	0.133
516	A	3	2	1.	28	0.071
517	A	3	2	1.	26	0.077
518	A	2	1	1.	24	0.042
519	A	3	2	1.	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
520	A	3	2	1.	28	0.071
521	A	3	2	1.	28	0.071
522	A	9	4	1.	30	0.133
523	A	3	2	1.	28	0.071
524	A	3	2	1.	26	0.077
525	A	3	2	1.	24	0.083
526	A	4	3	1.	28	0.107
527	A	3	2	1.	28	0.071
528	A	3	2	1.	28	0.071
529	A	2	2	1.	30	0.067
530	A	2	2	1.	28	0.071
531	A	2	2	1.	26	0.077
532	A	2	2	1.	24	0.083
533	A	5	5	1.	28	0.179
534	A	2	2	1.	28	0.071
535	A	2	2	1.	28	0.071
536	A	2	2	1.	30	0.067
537	A	2	2	1.	28	0.071
538	A	2	2	1.	26	0.077
539	A	2	2	1.	24	0.083
540	A	4	3	1.	28	0.107
541	A	2	2	1.	28	0.071
542	A	2	2	1.	28	0.071
543	A	2	2	1.	36	0.056
544	A	2	2	1.	31	0.065
545	A	3	3	1.	34	0.088
546	A	3	3	1.	33	0.091
547	A	3	3	1.06	35	0.086
548	A	4	3	1.	30	0.1
549	A	7	6	1.	24	0.25
550	A	6	6	1.	24	0.25
551	A	5	5	1.	24	0.208
552	A	3	3	1.	22	0.136
553	A	8	7	1.	24	0.292
554	A	8	7	1.	24	0.292
555	A	8	7	1.	24	0.292
556	A	8	5	1.	26	0.192
557	A	14	8	1.	26	0.308
558	A	4	3	1.	26	0.115
559	A	6	5	1.	26	0.192
560	A	16	10	1.	26	0.385
561	A	10	7	1.	26	0.269
562	A	3	2	1.	20	0.1
563	A	3	2	1.	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
564	A	3	2	1.	16	0.125
565	A	7	7	1.	20	0.35
566	A	3	2	1.	20	0.1
567	A	3	2	1.	20	0.1
568	A	2	2	1.	22	0.091
569	A	2	2	1.	22	0.091
570	A	2	2	1.	20	0.1
571	A	2	2	1.	18	0.111
572	A	7	6	1.	22	0.273
573	A	2	2	1.	22	0.091
574	A	2	2	1.	22	0.091
575	A	2	2	1.	22	0.091
576	A	2	2	1.	22	0.091
577	A	2	2	1.	20	0.1
578	A	2	2	1.	18	0.111
579	A	8	7	1.	22	0.318
580	A	2	2	1.	22	0.091
581	A	2	2	1.	22	0.091
582	A	2	2	1.	22	0.091
583	A	2	2	1.	22	0.091
584	A	2	2	1.	20	0.1
585	A	2	2	1.	18	0.111
586	A	3	3	1.	22	0.136
587	A	2	2	1.	22	0.091
588	A	2	2	1.	22	0.091
589	A	2	2	1.	22	0.091
590	A	2	2	1.	22	0.091
591	A	2	2	1.	20	0.1
592	A	2	2	1.	18	0.111
593	A	5	5	1.	22	0.227
594	A	2	2	1.	22	0.091
595	A	2	2	1.	22	0.091
596	A	14	3	1.	22	0.136
597	A	10	3	1.	22	0.136
598	A	6	3	1.	20	0.15
599	A	3	2	1.	22	0.091
600	A	5	3	1.	22	0.136
601	A	6	4	1.	22	0.182
602	A	2	2	1.	24	0.083
603	A	2	2	1.	24	0.083
604	A	2	2	1.	24	0.083
605	A	2	2	1.	24	0.083
606	A	2	2	1.	22	0.091
607	A	3	2	1.	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
608	A	4	3	1.	30	0.1
609	A	4	3	1.	30	0.1
610	A	3	2	1.	31	0.065
611	A	4	3	1.	33	0.091
612	A	4	3	1.	33	0.091
613	A	5	4	1.	30	0.133
614	A	6	6	1.	30	0.2
615	A	4	3	1.	30	0.1
616	A	4	4	1.	28	0.143
617	A	8	8	1.	30	0.267
618	A	5	4	1.	30	0.133
619	A	9	8	1.	30	0.267
620	A	6	5	1.	30	0.167
621	A	5	4	1.	30	0.133
622	A	5	5	1.	30	0.167
623	A	5	4	1.	30	0.133
624	A	5	5	1.	28	0.179
625	A	5	4	1.	22	0.182
626	A	9	8	1.	30	0.267
627	A	6	5	1.	30	0.167
628	A	9	8	1.	30	0.267
629	A	7	5	1.	30	0.167
630	A	6	5	1.	30	0.167
631	A	6	6	1.	30	0.2
632	A	6	5	1.	30	0.167
633	A	6	5	1.	28	0.179
634	A	6	5	1.	22	0.227
635	A	10	9	1.	30	0.3
636	A	7	6	1.	30	0.2
637	A	10	9	1.	30	0.3
638	A	5	4	1.	33	0.121
639	A	6	6	1.	33	0.182
640	A	4	3	1.	33	0.091
641	A	4	4	1.	31	0.129
642	A	8	8	1.	33	0.242
643	A	5	4	1.	33	0.121
644	A	9	8	1.	33	0.242
645	A	6	5	1.	33	0.152
646	A	5	4	1.	33	0.121
647	A	5	5	1.	33	0.152
648	A	5	4	1.	33	0.121
649	A	5	5	1.	31	0.161
650	A	9	8	1.	33	0.242
651	A	6	5	1.	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
652	A	9	8	1.	33	0.242
653	A	7	5	1.	33	0.152
654	A	6	5	1.	33	0.152
655	A	6	6	1.	33	0.182
656	A	6	5	1.	33	0.152
657	A	6	5	1.	31	0.161
658	A	10	9	1.	33	0.273
659	A	7	6	1.	33	0.182
660	A	10	9	1.	33	0.273
661	A	7	6	1.	26	0.231
662	A	10	9	1.	28	0.321
663	A	3	2	1.	24	0.083
664	A	4	3	1.	26	0.115

Chapter 3

Listing of integrals

3.1 $\int \left(ax^3+bx^6\right)^{5/3}\,dx$

Optimal. Leaf size=52

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}$$

```
[Out] (-3*a*(a*x^3 + b*x^6)^(8/3))/(88*b^2*x^8) + (a*x^3 + b*x^6)^(8/3)/(11*b*x^5
)
```

Rubi [A] time = 0.0482703, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2002, 2014}

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(5/3),x]
```

```
[Out] (-3*a*(a*x^3 + b*x^6)^(8/3))/(88*b^2*x^8) + (a*x^3 + b*x^6)^(8/3)/(11*b*x^5
)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \left(ax^3+bx^6\right)^{5/3}\,dx = \frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5} - \frac{(3a)\int \frac{\left(ax^3+bx^6\right)^{5/3}}{x^3}\,dx}{11b}$$
$$= -\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8} + \frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}$$

Mathematica [A] time = 0.0248177, size = 42, normalized size = 0.81

$$\frac{x\left(a+bx^3\right)^3\left(8bx^3-3a\right)}{88b^2\sqrt[3]{x^3\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(5/3),x]
```

```
[Out] (x*(a + b*x^3)^3*(-3*a + 8*b*x^3))/(88*b^2*(x^3*(a + b*x^3))^(1/3))
```

Maple [A] time = 0.029, size = 39, normalized size = 0.8

$$-\frac{\left(bx^3+a\right)\left(-8\,bx^3+3\,a\right)}{88\,b^2x^5}\left(bx^6+ax^3\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^6+a*x^3)^(5/3),x)
```

```
[Out] -1/88*(b*x^3+a)*(-8*b*x^3+3*a)*(b*x^6+a*x^3)^(5/3)/b^2/x^5
```

Maxima [A] time = 1.02154, size = 62, normalized size = 1.19

$$\frac{\left(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3\right)\left(bx^3+a\right)^{\frac{2}{3}}}{88\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
```

```
[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^3 + a)^(2/3)/b^2
```

Fricas [A] time = 2.20542, size = 117, normalized size = 2.25

$$\frac{\left(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3\right)\left(bx^6+ax^3\right)^{\frac{2}{3}}}{88\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
```

```
[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^6 + a*x^3)^(2/3)
/(b^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ax^3 + bx^6\right)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**6+a*x**3)**(5/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(5/3), x)
```

Giac [A] time = 1.14666, size = 107, normalized size = 2.06

$$\frac{\frac{11\left(5\left(bx^3+a\right)^{\frac{8}{3}}-8\left(bx^3+a\right)^{\frac{5}{3}}a\right)a}{b}+\frac{2\left(20\left(bx^3+a\right)^{\frac{11}{3}}-55\left(bx^3+a\right)^{\frac{8}{3}}a+44\left(bx^3+a\right)^{\frac{5}{3}}a^2\right)}{b}}{440\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="giac")
```

```
[Out] 1/440*(11*(5*(b*x^3 + a)^(8/3) - 8*(b*x^3 + a)^(5/3)*a)*a/b + 2*(20*(b*x^3
+ a)^(11/3) - 55*(b*x^3 + a)^(8/3)*a + 44*(b*x^3 + a)^(5/3)*a^2)/b)/b
```

3.2 $\int \left(ax^3+bx^6\right)^{2/3} dx$

Optimal. Leaf size=25

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

```
[Out] (a*x^3 + b*x^6)^(5/3)/(5*b*x^5)
```

Rubi [A] time = 0.0048654, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(2/3),x]
```

```
[Out] (a*x^3 + b*x^6)^(5/3)/(5*b*x^5)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \left(ax^3+bx^6\right)^{2/3} dx = \frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

Mathematica [A] time = 0.0076133, size = 25, normalized size = 1.

$$\frac{\left(x^3\left(a+bx^3\right)\right)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(2/3),x]
```

```
[Out] (x^3*(a + b*x^3))^(5/3)/(5*b*x^5)
```

Maple [A] time = 0.004, size = 29, normalized size = 1.2

$$\frac{bx^3+a}{5\,bx^2}\left(bx^6+ax^3\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^6+a*x^3)^(2/3),x)
```

```
[Out] 1/5*(b*x^3+a)/b/x^2*(b*x^6+a*x^3)^(2/3)
```

Maxima [A] time = 1.00102, size = 19, normalized size = 0.76

$$\frac{\left(bx^3+a\right)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^3 + a)^(5/3)/b
```

Fricas [A] time = 1.96835, size = 63, normalized size = 2.52

$$\frac{\left(bx^6+ax^3\right)^{\frac{2}{3}}\left(bx^3+a\right)}{5\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/5*(b*x^6 + a*x^3)^(2/3)*(b*x^3 + a)/(b*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ax^3+bx^6\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**6+a*x**3)**(2/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(2/3), x)
```

Giac [A] time = 1.11211, size = 19, normalized size = 0.76

$$\frac{\left(bx^3+a\right)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="giac")
```

```
[Out] 1/5*(b*x^3 + a)^(5/3)/b
```

3.3 $\int \frac{1}{\left(ax^3+bx^6\right)^{2/3}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

```
[Out] -((a*x^3 + b*x^6)^(1/3)/(a*x^2))
```

Rubi [A] time = 0.0049352, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(-2/3),x]
```

```
[Out] -((a*x^3 + b*x^6)^(1/3)/(a*x^2))
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(ax^3+bx^6\right)^{2/3}}\,dx=-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Mathematica [A] time = 0.0081178, size = 23, normalized size = 1.

$$-\frac{\sqrt[3]{x^3\left(a+bx^3\right)}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(-2/3),x]
```

```
[Out] -((x^3*(a + b*x^3))^(1/3)/(a*x^2))
```

Maple [A] time = 0.005, size = 27, normalized size = 1.2

$$-\frac{x\left(bx^3+a\right)}{a}\left(bx^6+ax^3\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a*x^3)^(2/3),x)
```

```
[Out] -x*(b*x^3+a)/a/(b*x^6+a*x^3)^(2/3)
```

Maxima [A] time = 1.09956, size = 23, normalized size = 1.

$$-\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -(b*x^3 + a)^(1/3)/(a*x)
```

Fricas [A] time = 1.99811, size = 43, normalized size = 1.87

$$-\frac{\left(bx^6+ax^3\right)^{\frac{1}{3}}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -(b*x^6 + a*x^3)^(1/3)/(a*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax^3+bx^6\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a*x**3)**(2/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+ax^3\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^6 + a*x^3)^(-2/3), x)
```

3.4 $\int \frac{1}{\left(ax^3+bx^6\right)^{5/3}}\,dx$

Optimal. Leaf size=77

$$\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}$$

```
[Out] 1/(2*a*x^2*(a*x^3 + b*x^6)^(2/3)) - (3*(a*x^3 + b*x^6)^(1/3))/(4*a^2*x^5) +
 (9*b*(a*x^3 + b*x^6)^(1/3))/(4*a^3*x^2)
```

Rubi [A] time = 0.0566138, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2001, 2016, 2000}

$$\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(-5/3),x]
```

```
[Out] 1/(2*a*x^2*(a*x^3 + b*x^6)^(2/3)) - (3*(a*x^3 + b*x^6)^(1/3))/(4*a^2*x^5) +
 (9*b*(a*x^3 + b*x^6)^(1/3))/(4*a^3*x^2)
```

Rule 2001

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
, j, n}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1
)/(n - j)], 0] && LtQ[p, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(ax^3+bx^6\right)^{5/3}}\,dx = \frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}} + \frac{3\int \frac{1}{x^3\left(ax^3+bx^6\right)^{2/3}}\,dx}{a}$$
$$= \frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}} - \frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5} - \frac{(9b)\int \frac{1}{\left(ax^3+bx^6\right)^{2/3}}\,dx}{4a^2}$$
$$= \frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}} - \frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5} + \frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}$$

Mathematica [A] time = 0.013138, size = 46, normalized size = 0.6

$$\frac{-a^2+6abx^3+9b^2x^6}{4a^3x^2\left(x^3\left(a+bx^3\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(-5/3),x]
```

```
[Out] (-a^2 + 6*a*b*x^3 + 9*b^2*x^6)/(4*a^3*x^2*(x^3*(a + b*x^3))^(2/3))
```

Maple [A] time = 0.006, size = 46, normalized size = 0.6

$$-\frac{x\left(bx^3+a\right)\left(-9\,b^2x^6-6\,bx^3a+a^2\right)}{4\,a^3}\left(bx^6+ax^3\right)^{-\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a*x^3)^(5/3),x)
```

```
[Out] -1/4*x*(b*x^3+a)*(-9*b^2*x^6-6*a*b*x^3+a^2)/a^3/(b*x^6+a*x^3)^(5/3)
```

Maxima [A] time = 1.04392, size = 51, normalized size = 0.66

$$\frac{9\,b^2x^6+6\,abx^3-a^2}{4\left(bx^3+a\right)^{\frac{2}{3}}a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
```

```
[Out] 1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)/((b*x^3 + a)^(2/3)*a^3*x^4)
```

Fricas [A] time = 1.99237, size = 107, normalized size = 1.39

$$\frac{\left(9\,b^2x^6+6\,abx^3-a^2\right)\left(bx^6+ax^3\right)^{\frac{1}{3}}}{4\left(a^3bx^8+a^4x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
```

```
[Out] 1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)*(b*x^6 + a*x^3)^(1/3)/(a^3*b*x^8 + a^4*x^
5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax^3 + bx^6\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a*x**3)**(5/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(-5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6 + ax^3\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^6 + a*x^3)^(-5/3), x)
```

3.5 $\int \frac{1}{-x^3+x^6}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Rubi [A] time = 0.023464, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {1593, 325, 200, 31, 634, 618, 204, 628}

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-x^3+x^6}\,dx &= \int \frac{1}{x^3\left(-1+x^3\right)}\,dx\\
&= \frac{1}{2x^2}+\int \frac{1}{-1+x^3}\,dx\\
&= \frac{1}{2x^2}+\frac{1}{3}\int \frac{1}{-1+x}\,dx+\frac{1}{3}\int \frac{-2-x}{1+x+x^2}\,dx\\
&= \frac{1}{2x^2}+\frac{1}{3}\log(1-x)-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= \frac{1}{2x^2}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0159592, size = 48, normalized size = 1.

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Maple [A] time = 0.029, size = 38, normalized size = 0.8

$$-\frac{\ln\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln(x-1)}{3}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-x^3),x)
```

```
[Out] -1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/3*ln(x-1)+1/2/x^
2
```

Maxima [A] time = 1.44545, size = 50, normalized size = 1.04

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(x - 1)
```

Fricas [A] time = 1.93914, size = 138, normalized size = 2.88

$$-\frac{2\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x^2\log\left(x^2+x+1\right)-2x^2\log(x-1)-3}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) + x^2*log(x^2 + x + 1) -
2*x^2*log(x - 1) - 3)/x^2
```

Sympy [A] time = 0.135273, size = 48, normalized size = 1.

$$\frac{\log(x-1)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-x**3),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3 + 1/(2*x**2)
```

Giac [A] time = 1.10437, size = 51, normalized size = 1.06

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(abs(x - 1))
```

3.6 $\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}$$

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (b*x^9*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3))
```

Rubi [A] time = 0.0230703, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (b*x^9*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^5\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(abx^5+b^2x^8\right)dx}{ab+b^2x^3}\\ &= \frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.0136005, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(3ax^6+2bx^9\right)}{18\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(3*a*x^6 + 2*b*x^9))/(18*(a + b*x^3))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.5

$$\frac{x^6\left(2\,bx^3+3\,a\right)}{18\,bx^3+18\,a}\sqrt{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/18*x^6*(2*b*x^3+3*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77177, size = 31, normalized size = 0.39

$$\frac{1}{9}\,bx^9+\frac{1}{6}\,ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*b*x^9 + 1/6*a*x^6
```

Sympy [A] time = 0.10151, size = 12, normalized size = 0.15

$$\frac{ax^6}{6}+\frac{bx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**6/6 + b*x**9/9
```

Giac [A] time = 1.12008, size = 31, normalized size = 0.39

$$\frac{1}{18}\left(2\,bx^{9}+3\,ax^{6}\right)\operatorname{sgn}\left(bx^{3}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/18*(2*b*x^9 + 3*a*x^6)*sgn(b*x^3 + a)
```

3.7 $\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}$$

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b*x^8*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rubi [A] time = 0.0233265, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b*x^8*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(abx^4+b^2x^7\right)dx}{ab+b^2x^3}\\ &= \frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}+\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}\end{aligned}$$

Mathematica [A] time = 0.0077349, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(8ax^5+5bx^8\right)}{40\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(8*a*x^5 + 5*b*x^8))/(40*(a + b*x^3))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.5

$$\frac{x^5\left(5\,bx^3+8\,a\right)}{40\,bx^3+40\,a}\sqrt{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/40*x^5*(5*b*x^3+8*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A] time = 1.12601, size = 18, normalized size = 0.23

$$\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*b*x^8 + 1/5*a*x^5
```

Fricas [A] time = 1.70126, size = 31, normalized size = 0.39

$$\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*b*x^8 + 1/5*a*x^5
```

Sympy [A] time = 0.101234, size = 12, normalized size = 0.15

$$\frac{ax^5}{5}+\frac{bx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**5/5 + b*x**8/8
```

Giac [A] time = 1.13147, size = 39, normalized size = 0.49

$$\frac{1}{8}\,bx^8\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,ax^5\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*b*x^8*sgn(b*x^3 + a) + 1/5*a*x^5*sgn(b*x^3 + a)
```

3.8 $\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

```
[Out] (a*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b*x^7*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rubi [A] time = 0.0235534, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b*x^7*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(abx^3+b^2x^6\right)dx}{ab+b^2x^3}\\
&= \frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.0072048, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(7ax^4+4bx^7\right)}{28\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(7*a*x^4 + 4*b*x^7))/(28*(a + b*x^3))
```

Maple [A] time = 0.003, size = 36, normalized size = 0.5

$$\frac{x^{4}\left(4\,b x^{3}+7\,a\right)}{28\,b x^{3}+28\,a}\sqrt{\left(b x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/28*x^4*(4*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A] time = 1.08544, size = 18, normalized size = 0.23

$$\frac{1}{7}\,b x^{7}+\frac{1}{4}\,a x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

Fricas [A] time = 1.71453, size = 31, normalized size = 0.39

$$\frac{1}{7}\,b x^{7}+\frac{1}{4}\,a x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

Sympy [A] time = 0.098831, size = 12, normalized size = 0.15

$$\frac{a x^{4}}{4}+\frac{b x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**4/4 + b*x**7/7
```

Giac [A] time = 1.10824, size = 39, normalized size = 0.49

$$\frac{1}{7}\,bx^7\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,ax^4\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7*b*x^7*sgn(b*x^3 + a) + 1/4*a*x^4*sgn(b*x^3 + a)
```

3.9 $\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

```
[Out] ((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*b)
```

Rubi [A] time = 0.0275715, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 609}

$$\frac{\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*b)
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^3\right)\\ &= \frac{\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{6b}\end{aligned}$$

Mathematica [A] time = 0.009384, size = 38, normalized size = 1.06

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(2ax^3+bx^6\right)}{6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(2*a*x^3 + b*x^6))/(6*(a + b*x^3))
```

Maple [A] time = 0.003, size = 35, normalized size = 1.

$$\frac{x^{3} \left(b x^{3}+2 a \right) \sqrt{\left(b x^{3}+a \right)^{2}}}{6 b x^{3}+6 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*x^3*(b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67638, size = 31, normalized size = 0.86

$$\frac{1}{6} b x^{6} + \frac{1}{3} a x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/3*a*x^3
```

Sympy [A] time = 0.099698, size = 12, normalized size = 0.33

$$\frac{a x^{3}}{3} + \frac{b x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**3/3 + b*x**6/6
```

Giac [A] time = 1.11975, size = 30, normalized size = 0.83

$$\frac{1}{6} \left(b x^{6} + 2 a x^{3}\right) \mathrm{sgn}\left(b x^{3} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(b*x^6 + 2*a*x^3)*sgn(b*x^3 + a)
```

3.10 $\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b*x^5*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
```

Rubi [A] time = 0.0180364, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1355, 14}

$$\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b*x^5*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(abx+b^2x^4\right)dx}{ab+b^2x^3}\\&=\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.0089468, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(5ax^2+2bx^5\right)}{10\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(5*a*x^2 + 2*b*x^5))/(10*(a + b*x^3))
```

Maple [A] time = 0.008, size = 36, normalized size = 0.5

$$\frac{x^2\left(2\,bx^3+5\,a\right)}{10\,bx^3+10\,a}\sqrt{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/10*x^2*(2*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A] time = 1.17017, size = 18, normalized size = 0.23

$$\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/2*a*x^2
```

Fricas [A] time = 1.62523, size = 31, normalized size = 0.39

$$\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*b*x^5 + 1/2*a*x^2
```

Sympy [A] time = 0.098539, size = 12, normalized size = 0.15

$$\frac{ax^2}{2}+\frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**2/2 + b*x**5/5
```

Giac [A] time = 1.10858, size = 39, normalized size = 0.49

$$\frac{1}{5}bx^5\text{sgn}\left(bx^3+a\right)+\frac{1}{2}ax^2\text{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5*sgn(b*x^3 + a) + 1/2*a*x^2*sgn(b*x^3 + a)
```

3.11 $\int \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=74

$$\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b*x^4*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rubi [A] time = 0.0123292, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1343}

$$\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b*x^4*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(2ab+2b^2x^3\right)dx}{2ab+2b^2x^3}\\ &= \frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.0069461, size = 36, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(4ax+bx^4\right)}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(4*a*x + b*x^4))/(4*(a + b*x^3))
```

Maple [A] time = 0.003, size = 33, normalized size = 0.5

$$\frac{x\left(bx^3+4a\right)}{4bx^3+4a}\sqrt{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/4*x*(b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A] time = 1.02239, size = 14, normalized size = 0.19

$$\frac{1}{4}bx^4+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + a*x
```

Fricas [A] time = 1.7091, size = 23, normalized size = 0.31

$$\frac{1}{4}bx^4+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + a*x
```

Sympy [A] time = 0.096229, size = 8, normalized size = 0.11

$$ax+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x + b*x**4/4
```

Giac [A] time = 1.10233, size = 27, normalized size = 0.36

$$\frac{1}{4}\left(bx^4+4ax\right)\operatorname{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(b*x^4 + 4*a*x)*sgn(b*x^3 + a)
```

3.12 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx$

Optimal. Leaf size=75

$$\frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] (b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (a*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.0195575, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

```
[Out] (b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (a*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x}+b^2x^2\right)dx}{ab+b^2x^3}\\
&= \frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}
\end{aligned}$$

Mathematica [A] time = 0.0113884, size = 37, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(3a\log(x)+bx^3\right)}{3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3 + 3*a*Log[x]))/(3*(a + b*x^3))
```

Maple [A] time = 0.053, size = 34, normalized size = 0.5

$$\frac{bx^3+3a\ln(x)}{3bx^3+3a}\sqrt{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x,x)
```

```
[Out] 1/3*((b*x^3+a)^2)^(1/2)*(b*x^3+3*a*ln(x))/(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74956, size = 30, normalized size = 0.4

$$\frac{1}{3}bx^3+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + a*log(x)
```

Sympy [A] time = 0.116376, size = 10, normalized size = 0.13

$$a\log(x)+\frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x,x)
```

```
[Out] a*log(x) + b*x**3/3
```

Giac [A] time = 1.10445, size = 38, normalized size = 0.51

$$\frac{1}{3}bx^3\mathrm{sgn}\left(bx^3+a\right)+a\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*sgn(b*x^3 + a) + a*log(abs(x))*sgn(b*x^3 + a)
```

3.13 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2} dx$

Optimal. Leaf size=77

$$\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (b*x^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
```

Rubi [A] time = 0.021185, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (b*x^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^2} dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^2}+b^2x\right) dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.0103088, size = 38, normalized size = 0.49

$$\frac{\left(bx^3-2a\right)\sqrt{\left(a+bx^3\right)^2}}{2x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

```
[Out] ((-2*a + b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x*(a + b*x^3))
```

Maple [A] time = 0.003, size = 36, normalized size = 0.5

$$-\frac{-bx^3+2a}{2x\left(bx^3+a\right)}\sqrt{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(-b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

Maxima [A] time = 1.01067, size = 19, normalized size = 0.25

$$\frac{bx^3-2a}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^3 - 2*a)/x
```

Fricas [A] time = 1.77874, size = 28, normalized size = 0.36

$$\frac{bx^3-2a}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^3 - 2*a)/x
```

Sympy [A] time = 0.266181, size = 8, normalized size = 0.1

$$-\frac{a}{x}+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**2,x)
```

```
[Out] -a/x + b*x**2/2
```

Giac [A] time = 1.11263, size = 39, normalized size = 0.51

$$\frac{1}{2}bx^2\mathrm{sgn}\left(bx^3+a\right)-\frac{a\mathrm{sgn}\left(bx^3+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*sgn(b*x^3 + a) - a*sgn(b*x^3 + a)/x
```

3.14 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx$

Optimal. Leaf size=74

$$\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b*x*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rubi [A] time = 0.020172, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b*x*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^3}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2+\frac{ab}{x^3}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.0075755, size = 37, normalized size = 0.5

$$-\frac{\left(a-2bx^3\right)\sqrt{\left(a+bx^3\right)^2}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
```

```
[Out] -((a - 2*b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x^2*(a + b*x^3))
```

Maple [A] time = 0.003, size = 34, normalized size = 0.5

$$-\frac{-2\,b x^3+a}{2\,x^2\left(b x^3+a\right)}\sqrt{\left(b x^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^3,x)
```

```
[Out] -1/2*(-2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)
```

Maxima [A] time = 1.01841, size = 20, normalized size = 0.27

$$\frac{2\,b x^3-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^3 - a)/x^2
```

Fricas [A] time = 1.73779, size = 31, normalized size = 0.42

$$\frac{2\,b x^3-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^3 - a)/x^2
```

Sympy [A] time = 0.273665, size = 8, normalized size = 0.11

$$-\frac{a}{2x^2}+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*x
```

Giac [A] time = 1.10113, size = 35, normalized size = 0.47

$$bx\operatorname{sgn}\left(bx^3+a\right)-\frac{a\operatorname{sgn}\left(bx^3+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] b*x*sgn(b*x^3 + a) - 1/2*a*sgn(b*x^3 + a)/x^2
```

3.15 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx$

Optimal. Leaf size=75

$$\frac{b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.0209114, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^4}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^4}+\frac{b^2}{x}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.0112664, size = 39, normalized size = 0.52

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a-3bx^3\log(x)\right)}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a - 3*b*x^3*Log[x]))/(3*x^3*(a + b*x^3))
```

Maple [A] time = 0.008, size = 38, normalized size = 0.5

$$\frac{3\,b\ln(x)\,x^3 - a}{\left(3\,bx^3 + 3\,a\right)x^3}\sqrt{\left(bx^3 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^4,x)
```

```
[Out] 1/3*((b*x^3+a)^2)^(1/2)*(3*b*ln(x)*x^3-a)/(b*x^3+a)/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72024, size = 41, normalized size = 0.55

$$\frac{3\,bx^3\log(x) - a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x^3*log(x) - a)/x^3
```

Sympy [A] time = 0.296766, size = 10, normalized size = 0.13

$$-\frac{a}{3x^3} + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**4,x)
```

```
[Out] -a/(3*x**3) + b*log(x)
```

Giac [A] time = 1.10787, size = 58, normalized size = 0.77

$$b\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{bx^3\mathrm{sgn}\left(bx^3+a\right)+a\mathrm{sgn}\left(bx^3+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] b*log(abs(x))*sgn(b*x^3 + a) - 1/3*(b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a)
)/x^3
```

3.16 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx$

Optimal. Leaf size=77

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))
```

Rubi [A] time = 0.0209954, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^5}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^5}+\frac{b^2}{x^2}\right)dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.008404, size = 37, normalized size = 0.48

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a+4bx^3\right)}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a + 4*b*x^3))/(4*x^4*(a + b*x^3))
```

Maple [A] time = 0.004, size = 34, normalized size = 0.4

$$-\frac{4\,b x^{3}+a}{4\,x^{4}\left(b x^{3}+a\right)}\sqrt{\left(b x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^5,x)
```

```
[Out] -1/4*(4*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)
```

Maxima [A] time = 1.02588, size = 18, normalized size = 0.23

$$-\frac{4\,b x^{3}+a}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b*x^3 + a)/x^4
```

Fricas [A] time = 1.80482, size = 32, normalized size = 0.42

$$-\frac{4\,b x^{3}+a}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*x^3 + a)/x^4
```

Sympy [A] time = 0.308046, size = 14, normalized size = 0.18

$$-\frac{a+4bx^{3}}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**5,x)
```

```
[Out] -(a + 4*b*x**3)/(4*x**4)
```

Giac [A] time = 1.13396, size = 41, normalized size = 0.53

$$-\frac{4\,bx^3\mathrm{sgn}\left(bx^3+a\right)+a\mathrm{sgn}\left(bx^3+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(4*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^4
```

3.17 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))
```

Rubi [A] time = 0.0221565, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^6}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^6}+\frac{b^2}{x^3}\right)dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.0076367, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+5bx^3\right)}{10x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(2*a + 5*b*x^3))/(10*x^5*(a + b*x^3))
```

Maple [A] time = 0.003, size = 36, normalized size = 0.5

$$-\frac{5\,b x^{3}+2\,a}{10\,x^{5}\left(b x^{3}+a\right)}\sqrt{\left(b x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^6,x)
```

```
[Out] -1/10*(5*b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)
```

Maxima [A] time = 1.03187, size = 20, normalized size = 0.25

$$-\frac{5\,b x^{3}+2\,a}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

Fricas [A] time = 1.73477, size = 36, normalized size = 0.46

$$-\frac{5\,b x^{3}+2\,a}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

Sympy [A] time = 0.31084, size = 15, normalized size = 0.19

$$-\frac{2a+5bx^{3}}{10x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**6,x)
```

```
[Out] -(2*a + 5*b*x**3)/(10*x**5)
```

Giac [A] time = 1.10216, size = 42, normalized size = 0.53

$$-\frac{5\,bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a\mathrm{sgn}\left(bx^3+a\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/10*(5*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^5
```

3.18 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3))
```

Rubi [A] time = 0.0216519, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^7}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^7}+\frac{b^2}{x^4}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.0084849, size = 37, normalized size = 0.47

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a+2bx^3\right)}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a + 2*b*x^3))/(6*x^6*(a + b*x^3))
```

Maple [A] time = 0.003, size = 34, normalized size = 0.4

$$-\frac{2bx^3+a}{6x^6\left(bx^3+a\right)}\sqrt{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^7,x)
```

```
[Out] -1/6*(2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6425, size = 32, normalized size = 0.41

$$-\frac{2bx^3+a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*x^3 + a)/x^6
```

Sympy [A] time = 0.317141, size = 14, normalized size = 0.18

$$-\frac{a+2bx^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**7,x)
```

```
[Out] -(a + 2*b*x**3)/(6*x**6)
```

Giac [A] time = 1.11759, size = 41, normalized size = 0.52

$$-\frac{2\,bx^3\mathrm{sgn}\left(bx^3+a\right)+a\mathrm{sgn}\left(bx^3+a\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(2*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^6
```

3.19 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))
```

Rubi [A] time = 0.0223182, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^8}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^8}+\frac{b^2}{x^5}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.0081179, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(4a+7bx^3\right)}{28x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(4*a + 7*b*x^3))/(28*x^7*(a + b*x^3))
```

Maple [A] time = 0.003, size = 36, normalized size = 0.5

$$-\frac{7\,b x^{3}+4\,a}{28\,x^{7}\left(b x^{3}+a\right)}\sqrt{\left(b x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^8,x)
```

```
[Out] -1/28*(7*b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

Maxima [A] time = 1.01311, size = 20, normalized size = 0.25

$$-\frac{7\,b x^{3}+4\,a}{28\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

Fricas [A] time = 1.72289, size = 36, normalized size = 0.46

$$-\frac{7\,b x^{3}+4\,a}{28\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

Sympy [A] time = 0.328466, size = 15, normalized size = 0.19

$$-\frac{4a+7bx^{3}}{28x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**8,x)
```

```
[Out] -(4*a + 7*b*x**3)/(28*x**7)
```

Giac [A] time = 1.11539, size = 42, normalized size = 0.53

$$-\frac{7\,bx^3\mathrm{sgn}\left(bx^3+a\right)+4\,a\mathrm{sgn}\left(bx^3+a\right)}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] -1/28*(7*b*x^3*sgn(b*x^3 + a) + 4*a*sgn(b*x^3 + a))/x^7
```

3.20 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))
```

Rubi [A] time = 0.0211998, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^9}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^9}+\frac{b^2}{x^6}\right)dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.0083746, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(5a+8bx^3\right)}{40x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(5*a + 8*b*x^3))/(40*x^8*(a + b*x^3))
```

Maple [A] time = 0.003, size = 36, normalized size = 0.5

$$-\frac{8\,b x^3+5\,a}{40\,x^8\left(b x^3+a\right)}\sqrt{\left(b x^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^9,x)
```

```
[Out] -1/40*(8*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)
```

Maxima [A] time = 1.05952, size = 20, normalized size = 0.25

$$-\frac{8\,b x^3+5\,a}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/40*(8*b*x^3 + 5*a)/x^8
```

Fricas [A] time = 1.71312, size = 36, normalized size = 0.46

$$-\frac{8\,b x^3+5\,a}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/40*(8*b*x^3 + 5*a)/x^8
```

Sympy [A] time = 0.332222, size = 15, normalized size = 0.19

$$-\frac{5a+8bx^3}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**9,x)
```

```
[Out] -(5*a + 8*b*x**3)/(40*x**8)
```

Giac [A] time = 1.12057, size = 42, normalized size = 0.53

$$-\frac{8\,bx^3\mathrm{sgn}\left(bx^3+a\right)+5\,a\mathrm{sgn}\left(bx^3+a\right)}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/40*(8*b*x^3*sgn(b*x^3 + a) + 5*a*sgn(b*x^3 + a))/x^8
```

3.21 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3))
```

Rubi [A] time = 0.0216881, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^{10}}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^{10}}+\frac{b^2}{x^7}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.0081457, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+3bx^3\right)}{18x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(2*a + 3*b*x^3))/(18*x^9*(a + b*x^3))
```

Maple [A] time = 0.003, size = 36, normalized size = 0.5

$$-\frac{3\,b x^{3}+2\,a}{18\,x^{9}\left(b x^{3}+a\right)}\sqrt{\left(b x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^10,x)
```

```
[Out] -1/18*(3*b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67632, size = 36, normalized size = 0.46

$$-\frac{3\,b x^{3}+2\,a}{18\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/18*(3*b*x^3 + 2*a)/x^9
```

Sympy [A] time = 0.341909, size = 15, normalized size = 0.19

$$-\frac{2a+3bx^{3}}{18x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**10,x)
```

```
[Out] -(2*a + 3*b*x**3)/(18*x**9)
```

Giac [A] time = 1.09434, size = 42, normalized size = 0.53

$$-\frac{3\,bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a\mathrm{sgn}\left(bx^3+a\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/18*(3*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^9
```

3.22 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rubi [A] time = 0.0222281, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^{11}}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^{11}}+\frac{b^2}{x^8}\right)dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.0082613, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(7a+10bx^3\right)}{70x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(7*a + 10*b*x^3))/(70*x^10*(a + b*x^3))
```

Maple [A] time = 0.003, size = 36, normalized size = 0.5

$$-\frac{10\,b x^{3}+7\,a}{70\,x^{10}\left(b x^{3}+a\right)}\sqrt{\left(b x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^11,x)
```

```
[Out] -1/70*(10*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)
```

Maxima [A] time = 1.01896, size = 20, normalized size = 0.25

$$-\frac{10\,b x^{3}+7\,a}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/70*(10*b*x^3 + 7*a)/x^10
```

Fricas [A] time = 1.79878, size = 39, normalized size = 0.49

$$-\frac{10\,b x^{3}+7\,a}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/70*(10*b*x^3 + 7*a)/x^10
```

Sympy [A] time = 0.347625, size = 15, normalized size = 0.19

$$-\frac{7a+10bx^{3}}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**11,x)
```

```
[Out] -(7*a + 10*b*x**3)/(70*x**10)
```

Giac [A] time = 1.1271, size = 42, normalized size = 0.53

$$-\frac{10\,bx^3\mathrm{sgn}\left(bx^3+a\right)+7\,a\mathrm{sgn}\left(bx^3+a\right)}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/70*(10*b*x^3*sgn(b*x^3 + a) + 7*a*sgn(b*x^3 + a))/x^10
```

3.23 $\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}$$

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a^2*b*x^13
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (3*a*b^2*x^16*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (b^3*x^19*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(19*(a + b*x^3))
```

Rubi [A] time = 0.042637, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a^2*b*x^13
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (3*a*b^2*x^16*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (b^3*x^19*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(19*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^9\left(ab+b^2x^3\right)^3 dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x^9+3a^2b^4x^{12}+3ab^5x^{15}+b^6x^{18}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.0204177, size = 61, normalized size = 0.37

$$\frac{x^{10}\sqrt{\left(a+bx^3\right)^2}\left(4560a^2bx^3+1976a^3+3705ab^2x^6+1040b^3x^9\right)}{19760\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^3)^2]*(1976*a^3 + 4560*a^2*b*x^3 + 3705*a*b^2*x^6 + 104
0*b^3*x^9))/(19760*(a + b*x^3))
```

Maple [A] time = 0.006, size = 58, normalized size = 0.4

$$\frac{x^{10}\left(1040\,b^3x^9+3705\,ab^2x^6+4560\,a^2bx^3+1976\,a^3\right)}{19760\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/19760*x^10*(1040*b^3*x^9+3705*a*b^2*x^6+4560*a^2*b*x^3+1976*a^3)*((b*x^3+
a)^2)^(3/2)/(b*x^3+a)^3
```

Maxima [A] time = 1.02062, size = 47, normalized size = 0.28

$$\frac{1}{19}\,b^3x^{19}+\frac{3}{16}\,ab^2x^{16}+\frac{3}{13}\,a^2bx^{13}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

Fricas [A] time = 1.65848, size = 90, normalized size = 0.54

$$\frac{1}{19}\,b^3x^{19}+\frac{3}{16}\,ab^2x^{16}+\frac{3}{13}\,a^2bx^{13}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^9\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**9*((a + b*x**3)**2)**(3/2), x)
```

Giac [A] time = 1.12727, size = 90, normalized size = 0.54

$$\frac{1}{19}\,b^3x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{16}\,ab^2x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{13}\,a^2bx^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{10}\,a^3x^{10}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/19*b^3*x^19*sgn(b*x^3 + a) + 3/16*a*b^2*x^16*sgn(b*x^3 + a) + 3/13*a^2*b*
x^13*sgn(b*x^3 + a) + 1/10*a^3*x^10*sgn(b*x^3 + a)
```

3.24 $\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^4}{15b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^3}{12b^3}$$

```
[Out] (a^2*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*b^3) - (2*a*(a + b*
x^3)^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*b^3) + ((a + b*x^3)^5*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(18*b^3)
```

Rubi [A] time = 0.0532489, antiderivative size = 167, normalized size of antiderivative = 1.4, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^3x^{18}\sqrt{a^2+2abx^3+b^2x^6}}{18\left(a+bx^3\right)}+\frac{ab^2x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{a^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (a^2*b*x^12*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (a*b^2*x^15*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^18*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(18*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^8 \left(ab+b^2x^3\right)^3 dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x^2\left(ab+b^2x\right)^3 dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(a^3b^3x^2+3a^2b^4x^3+3ab^5x^4+b^6x^5\right) dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= \frac{a^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)} + \frac{a^2bx^{12}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)} + \frac{ab^2x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)} +$$

Mathematica [A] time = 0.0178823, size = 61, normalized size = 0.51

$$\frac{x^9\sqrt{\left(a+bx^3\right)^2}\left(45a^2bx^3+20a^3+36ab^2x^6+10b^3x^9\right)}{180\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^3)^2]*(20*a^3 + 45*a^2*b*x^3 + 36*a*b^2*x^6 + 10*b^3*x^9
))/(180*(a + b*x^3))
```

Maple [A] time = 0.007, size = 58, normalized size = 0.5

$$\frac{x^9\left(10\,b^3x^9+36\,ab^2x^6+45\,a^2bx^3+20\,a^3\right)}{180\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/180*x^9*(10*b^3*x^9+36*a*b^2*x^6+45*a^2*b*x^3+20*a^3)*((b*x^3+a)^2)^(3/2)
/(b*x^3+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72202, size = 85, normalized size = 0.71

$$\frac{1}{18}\,b^3x^{18} + \frac{1}{5}\,ab^2x^{15} + \frac{1}{4}\,a^2bx^{12} + \frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**8*((a + b*x**3)**2)**(3/2), x)
```

Giac [A] time = 1.11471, size = 90, normalized size = 0.76

$$\frac{1}{18}\,b^3x^{18}\mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{5}\,ab^2x^{15}\mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{4}\,a^2bx^{12}\mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{9}\,a^3x^9\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/18*b^3*x^18*sgn(b*x^3 + a) + 1/5*a*b^2*x^15*sgn(b*x^3 + a) + 1/4*a^2*b*x^
12*sgn(b*x^3 + a) + 1/9*a^3*x^9*sgn(b*x^3 + a)
```

3.25 $\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$

Optimal. Leaf size=167

$$\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a^2*b*x^11*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (3*a*b^2*x^14*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (b^3*x^17*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(17*(a + b*x^3))
```

Rubi [A] time = 0.0426231, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a^2*b*x^11*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (3*a*b^2*x^14*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (b^3*x^17*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(17*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^7\left(ab+b^2x^3\right)^3 \, dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^3b^3x^7+3a^2b^4x^{10}+3ab^5x^{13}+b^6x^{16}\right) dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.0176902, size = 61, normalized size = 0.37

$$\frac{x^8\sqrt{\left(a+bx^3\right)^2}\left(2856a^2bx^3+1309a^3+2244ab^2x^6+616b^3x^9\right)}{10472\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^3)^2]*(1309*a^3 + 2856*a^2*b*x^3 + 2244*a*b^2*x^6 + 616*
b^3*x^9))/(10472*(a + b*x^3))
```

Maple [A] time = 0.006, size = 58, normalized size = 0.4

$$\frac{x^8\left(616\,b^3x^9+2244\,ab^2x^6+2856\,a^2bx^3+1309\,a^3\right)}{10472\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/10472*x^8*(616*b^3*x^9+2244*a*b^2*x^6+2856*a^2*b*x^3+1309*a^3)*((b*x^3+a)
^2)^(3/2)/(b*x^3+a)^3
```

Maxima [A] time = 1.03374, size = 47, normalized size = 0.28

$$\frac{1}{17}\,b^3x^{17}+\frac{3}{14}\,ab^2x^{14}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

Fricas [A] time = 1.72384, size = 88, normalized size = 0.53

$$\frac{1}{17}\,b^3x^{17}+\frac{3}{14}\,ab^2x^{14}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**7*((a + b*x**3)**2)**(3/2), x)
```

Giac [A] time = 1.09728, size = 90, normalized size = 0.54

$$\frac{1}{17}b^3x^{17}\operatorname{sgn}\left(bx^3+a\right)+\frac{3}{14}ab^2x^{14}\operatorname{sgn}\left(bx^3+a\right)+\frac{3}{11}a^2bx^{11}\operatorname{sgn}\left(bx^3+a\right)+\frac{1}{8}a^3x^8\operatorname{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/17*b^3*x^17*sgn(b*x^3 + a) + 3/14*a*b^2*x^14*sgn(b*x^3 + a) + 3/11*a^2*b*
x^11*sgn(b*x^3 + a) + 1/8*a^3*x^8*sgn(b*x^3 + a)
```

3.26 $\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a^2*b*x^10*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (b^3*x^16*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(16*(a + b*x^3))
```

Rubi [A] time = 0.0408195, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a^2*b*x^10*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (b^3*x^16*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(16*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^6\left(a^2+2abx^3+b^2x^6\right)^{3/2}dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^6\left(ab+b^2x^3\right)^3dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x^6+3a^2b^4x^9+3ab^5x^{12}+b^6x^{15}\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b}}{13\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.0164577, size = 61, normalized size = 0.37

$$\frac{x^7\sqrt{\left(a+bx^3\right)^2}\left(2184a^2bx^3+1040a^3+1680ab^2x^6+455b^3x^9\right)}{7280\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^3)^2]*(1040*a^3 + 2184*a^2*b*x^3 + 1680*a*b^2*x^6 + 455*
b^3*x^9))/(7280*(a + b*x^3))
```

Maple [A] time = 0.006, size = 58, normalized size = 0.4

$$\frac{x^7\left(455\,b^3x^9+1680\,ab^2x^6+2184\,a^2bx^3+1040\,a^3\right)}{7280\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/7280*x^7*(455*b^3*x^9+1680*a*b^2*x^6+2184*a^2*b*x^3+1040*a^3)*((b*x^3+a)^
2)^(3/2)/(b*x^3+a)^3
```

Maxima [A] time = 1.05866, size = 47, normalized size = 0.28

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{13}\,ab^2x^{13}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

Fricas [A] time = 1.7783, size = 88, normalized size = 0.53

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{13}\,ab^2x^{13}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**6*((a + b*x**3)**2)**(3/2), x)
```

Giac [A] time = 1.10918, size = 90, normalized size = 0.54

$$\frac{1}{16}\,b^3x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{13}\,ab^2x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{10}\,a^2bx^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{7}\,a^3x^7\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*b^3*x^16*sgn(b*x^3 + a) + 3/13*a*b^2*x^13*sgn(b*x^3 + a) + 3/10*a^2*b*
x^10*sgn(b*x^3 + a) + 1/7*a^3*x^7*sgn(b*x^3 + a)
```

3.27 $\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=78

$$\frac{\left(a+bx^3\right)^4 \sqrt{a^2+2abx^3+b^2x^6}}{15b^2}-\frac{a\left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12b^2}$$

```
[Out] -(a*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*b^2) + ((a + b*x^3)^
4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*b^2)
```

Rubi [A] time = 0.050472, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{\left(a+bx^3\right)^4 \sqrt{a^2+2abx^3+b^2x^6}}{15b^2}-\frac{a\left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -(a*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*b^2) + ((a + b*x^3)^
4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*b^2)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^5 \left(ab+b^2x^3\right)^3 dx}{b^2 \left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x \left(ab+b^2x\right)^3 dx, x, x^3\right)}{3b^2 \left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(-\frac{a\left(ab+b^2x\right)^3}{b}+\frac{\left(ab+b^2x\right)^4}{b^2}\right) dx, x, x^3\right)}{3b^2 \left(ab+b^2x^3\right)}$$
$$= -\frac{a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^2}+\frac{\left(a+bx^3\right)^4\sqrt{a^2+2abx^3+b^2x^6}}{15b^2}$$

Mathematica [A] time = 0.0190837, size = 61, normalized size = 0.78

$$\frac{x^6\sqrt{\left(a+bx^3\right)^2}\left(20a^2bx^3+10a^3+15ab^2x^6+4b^3x^9\right)}{60\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^3)^2]*(10*a^3 + 20*a^2*b*x^3 + 15*a*b^2*x^6 + 4*b^3*x^9)
)/(60*(a + b*x^3))
```

Maple [A] time = 0.006, size = 58, normalized size = 0.7

$$\frac{x^6\left(4\,b^3x^9+15\,ab^2x^6+20\,a^2bx^3+10\,a^3\right)}{60\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/60*x^6*(4*b^3*x^9+15*a*b^2*x^6+20*a^2*b*x^3+10*a^3)*((b*x^3+a)^2)^(3/2)/(
b*x^3+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65279, size = 84, normalized size = 1.08

$$\frac{1}{15}\,b^{3}x^{15}+\frac{1}{4}\,ab^{2}x^{12}+\frac{1}{3}\,a^{2}bx^{9}+\frac{1}{6}\,a^{3}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{5}\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**5*((a + b*x**3)**2)**(3/2), x)
```

Giac [A] time = 1.09632, size = 61, normalized size = 0.78

$$\frac{1}{60}\,\left(4\,b^{3}x^{15}+15\,ab^{2}x^{12}+20\,a^{2}bx^{9}+10\,a^{3}x^{6}\right)\mathrm{sgn}\left(bx^{3}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/60*(4*b^3*x^15 + 15*a*b^2*x^12 + 20*a^2*b*x^9 + 10*a^3*x^6)*sgn(b*x^3 + a
)
```

3.28 $\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$

Optimal. Leaf size=167

$$\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a^2*b*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a*b^2*x^11*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (b^3*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(14*(a + b*x^3))
```

Rubi [A] time = 0.0413745, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a^2*b*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a*b^2*x^11*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (b^3*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(14*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^4 \left(ab+b^2x^3\right)^3 \, dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(a^3b^3x^4+3a^2b^4x^7+3ab^5x^{10}+b^6x^{13}\right) \, dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2}}{11\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.0160818, size = 61, normalized size = 0.37

$$\frac{x^5\sqrt{\left(a+bx^3\right)^2}\left(1155a^2bx^3+616a^3+840ab^2x^6+220b^3x^9\right)}{3080\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)^2]*(616*a^3 + 1155*a^2*b*x^3 + 840*a*b^2*x^6 + 220*b^
3*x^9))/(3080*(a + b*x^3))
```

Maple [A] time = 0.005, size = 58, normalized size = 0.4

$$\frac{x^5\left(220\,b^3x^9+840\,ab^2x^6+1155\,a^2bx^3+616\,a^3\right)}{3080\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/3080*x^5*(220*b^3*x^9+840*a*b^2*x^6+1155*a^2*b*x^3+616*a^3)*((b*x^3+a)^2)
^(3/2)/(b*x^3+a)^3
```

Maxima [A] time = 1.0113, size = 47, normalized size = 0.28

$$\frac{1}{14}\,b^3x^{14}+\frac{3}{11}\,ab^2x^{11}+\frac{3}{8}\,a^2bx^8+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Fricas [A] time = 1.70544, size = 85, normalized size = 0.51

$$\frac{1}{14}\,b^3x^{14}+\frac{3}{11}\,ab^2x^{11}+\frac{3}{8}\,a^2bx^8+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**4*((a + b*x**3)**2)**(3/2), x)
```

Giac [A] time = 1.10193, size = 90, normalized size = 0.54

$$\frac{1}{14}b^3x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{11}ab^2x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{8}a^2bx^8\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}a^3x^5\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*sgn(b*x^3 + a) + 3/11*a*b^2*x^11*sgn(b*x^3 + a) + 3/8*a^2*b*x
^8*sgn(b*x^3 + a) + 1/5*a^3*x^5*sgn(b*x^3 + a)
```

3.29 $\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

```
[Out] (a^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (3*a^2*b*x^7*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a*b^2*x^10*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(13*(a + b*x^3))
```

Rubi [A] time = 0.0409833, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```



```
[Out] (a^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (3*a^2*b*x^7*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a*b^2*x^10*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(13*(a + b*x^3))
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(ab+b^2x^3\right)^3dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x^3+3a^2b^4x^6+3ab^5x^9+b^6x^{12}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.0165013, size = 61, normalized size = 0.37

$$\frac{x^4\sqrt{\left(a+bx^3\right)^2}\left(780a^2bx^3+455a^3+546ab^2x^6+140b^3x^9\right)}{1820\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)^2]*(455*a^3 + 780*a^2*b*x^3 + 546*a*b^2*x^6 + 140*b^3
*x^9))/(1820*(a + b*x^3))
```

Maple [A] time = 0.004, size = 58, normalized size = 0.4

$$\frac{x^4\left(140\,b^3x^9+546\,ab^2x^6+780\,a^2bx^3+455\,a^3\right)}{1820\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/1820*x^4*(140*b^3*x^9+546*a*b^2*x^6+780*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)^
(3/2)/(b*x^3+a)^3
```

Maxima [A] time = 1.02238, size = 47, normalized size = 0.28

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{7}\,a^2bx^7+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Fricas [A] time = 1.71503, size = 85, normalized size = 0.51

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{7}\,a^2bx^7+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**3*((a + b*x**3)**2)**(3/2), x)
```

Giac [A] time = 1.11049, size = 90, normalized size = 0.54

$$\frac{1}{13}b^3x^{13}\operatorname{sgn}\left(bx^3+a\right)+\frac{3}{10}ab^2x^{10}\operatorname{sgn}\left(bx^3+a\right)+\frac{3}{7}a^2bx^7\operatorname{sgn}\left(bx^3+a\right)+\frac{1}{4}a^3x^4\operatorname{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*sgn(b*x^3 + a) + 3/10*a*b^2*x^10*sgn(b*x^3 + a) + 3/7*a^2*b*x
^7*sgn(b*x^3 + a) + 1/4*a^3*x^4*sgn(b*x^3 + a)
```

3.30 $\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(12*b)
```

Rubi [A] time = 0.02958, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 609}

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(12*b)
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx&=\frac{1}{3}\operatorname{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx,x,x^3\right)\\&=\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}\end{aligned}$$

Mathematica [A] time = 0.0182238, size = 60, normalized size = 1.67

$$\frac{x^3\sqrt{\left(a+bx^3\right)^2}\left(6a^2bx^3+4a^3+4ab^2x^6+b^3x^9\right)}{12\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(4*a^3 + 6*a^2*b*x^3 + 4*a*b^2*x^6 + b^3*x^9))/(12
*(a + b*x^3))
```

Maple [A] time = 0.006, size = 57, normalized size = 1.6

$$\frac{x^3\left(b^3x^9+4\,ab^2x^6+6\,a^2bx^3+4\,a^3\right)}{12\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/12*x^3*(b^3*x^9+4*a*b^2*x^6+6*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/(b*x^3
+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67701, size = 82, normalized size = 2.28

$$\frac{1}{12}\,b^3x^{12}+\frac{1}{3}\,ab^2x^9+\frac{1}{2}\,a^2bx^6+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*b^3*x^12 + 1/3*a*b^2*x^9 + 1/2*a^2*b*x^6 + 1/3*a^3*x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x**3)**2)**(3/2), x)
```

Giac [A] time = 1.11209, size = 59, normalized size = 1.64

$$\frac{1}{12}\left(b^3x^{12} + 4\,ab^2x^9 + 6\,a^2bx^6 + 4\,a^3x^3\right)\operatorname{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*(b^3*x^12 + 4*a*b^2*x^9 + 6*a^2*b*x^6 + 4*a^3*x^3)*sgn(b*x^3 + a)
```

3.31 $\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a^2*b*x^5*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a*b^2*x^8*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(11*(a + b*x^3))
```

Rubi [A] time = 0.0378921, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1355, 270}

$$\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a^2*b*x^5*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a*b^2*x^8*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(11*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x+3a^2b^4x^4+3ab^5x^7+b^6x^{10}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\end{aligned}$$

Mathematica [A] time = 0.015788, size = 61, normalized size = 0.37

$$\frac{x^2\sqrt{\left(a+bx^3\right)^2}\left(264a^2bx^3+220a^3+165ab^2x^6+40b^3x^9\right)}{440\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^3)^2]*(220*a^3 + 264*a^2*b*x^3 + 165*a*b^2*x^6 + 40*b^3*
x^9))/(440*(a + b*x^3))
```

Maple [A] time = 0.004, size = 58, normalized size = 0.4

$$\frac{x^2\left(40\,b^3x^9+165\,ab^2x^6+264\,a^2bx^3+220\,a^3\right)}{440\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/440*x^2*(40*b^3*x^9+165*a*b^2*x^6+264*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(3
/2)/(b*x^3+a)^3
```

Maxima [A] time = 1.02098, size = 47, normalized size = 0.28

$$\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Fricas [A] time = 1.78368, size = 82, normalized size = 0.49

$$\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**(3/2), x)
```

Giac [A] time = 1.1024, size = 90, normalized size = 0.54

$$\frac{1}{11}b^3x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{8}ab^2x^8\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{5}a^2bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}a^3x^2\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11*sgn(b*x^3 + a) + 3/8*a*b^2*x^8*sgn(b*x^3 + a) + 3/5*a^2*b*x^5
*sgn(b*x^3 + a) + 1/2*a^3*x^2*sgn(b*x^3 + a)
```

3.32 $\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx$

Optimal. Leaf size=162

$$\frac{b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{10\left(a+bx^3\right)^3}+\frac{3ab^2x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{7\left(a+bx^3\right)^3}+\frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3}+\frac{a^3x\left(a^2+2abx^3+b\right.}{\left(a+bx^3\right)^3}$$

```
[Out] (a^3*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(a + b*x^3)^3 + (3*a^2*b*x^4*(a^2
 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(4*(a + b*x^3)^3) + (3*a*b^2*x^7*(a^2 + 2*a*
b*x^3 + b^2*x^6)^(3/2))/(7*(a + b*x^3)^3) + (b^3*x^10*(a^2 + 2*a*b*x^3 + b^
2*x^6)^(3/2))/(10*(a + b*x^3)^3)
```

Rubi [A] time = 0.0327202, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1343, 194}

$$\frac{b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{10\left(a+bx^3\right)^3}+\frac{3ab^2x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{7\left(a+bx^3\right)^3}+\frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3}+\frac{a^3x\left(a^2+2abx^3+b\right.}{\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(a + b*x^3)^3 + (3*a^2*b*x^4*(a^2
 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(4*(a + b*x^3)^3) + (3*a*b^2*x^7*(a^2 + 2*a*
b*x^3 + b^2*x^6)^(3/2))/(7*(a + b*x^3)^3) + (b^3*x^10*(a^2 + 2*a*b*x^3 + b^
2*x^6)^(3/2))/(10*(a + b*x^3)^3)
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx &= \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}\int \left(2ab+2b^2x^3\right)^3\,dx}{\left(2ab+2b^2x^3\right)^3}\\ &= \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}\int \left(8a^3b^3+24a^2b^4x^3+24ab^5x^6+8b^6x^9\right)\,dx}{\left(2ab+2b^2x^3\right)^3}\\ &= \frac{a^3x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{\left(a+bx^3\right)^3}+\frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3}+\frac{3ab^2x^7\left(a^2+2abx^3+b\right.}{7\left(a+bx^3\right)^3}\end{aligned}$$

Mathematica [A] time = 0.0153953, size = 59, normalized size = 0.36

$$\frac{x\sqrt{(a+bx^3)^2}\,(105a^2bx^3+140a^3+60ab^2x^6+14b^3x^9)}{140\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^3)^2]*(140*a^3 + 105*a^2*b*x^3 + 60*a*b^2*x^6 + 14*b^3*x^9
))/(140*(a + b*x^3))
```

Maple [A] time = 0.003, size = 56, normalized size = 0.4

$$\frac{x\,(14\,b^3x^9+60\,ab^2x^6+105\,a^2bx^3+140\,a^3)}{140\,(bx^3+a)^3}\left((bx^3+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/140*x*(14*b^3*x^9+60*a*b^2*x^6+105*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/2)
/(b*x^3+a)^3
```

Maxima [A] time = 1.05412, size = 43, normalized size = 0.27

$$\frac{1}{10}\,b^3x^{10}+\frac{3}{7}\,ab^2x^7+\frac{3}{4}\,a^2bx^4+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Fricas [A] time = 1.81017, size = 74, normalized size = 0.46

$$\frac{1}{10}\,b^3x^{10}+\frac{3}{7}\,ab^2x^7+\frac{3}{4}\,a^2bx^4+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2), x)
```

Giac [A] time = 1.12193, size = 86, normalized size = 0.53

$$\frac{1}{10}b^3x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{7}ab^2x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{4}a^2bx^4\mathrm{sgn}\left(bx^3+a\right)+a^3x\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/10*b^3*x^10*sgn(b*x^3 + a) + 3/7*a*b^2*x^7*sgn(b*x^3 + a) + 3/4*a^2*b*x^4
*sgn(b*x^3 + a) + a^3*x*sgn(b*x^3 + a)
```

3.33 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=160

$$\frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{a^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] (a^2*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (a*b^2*x^6*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(9*(a + b*x^3)) + (a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/
(a + b*x^3)
```

Rubi [A] time = 0.0477448, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{a^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

```
[Out] (a^2*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (a*b^2*x^6*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(9*(a + b*x^3)) + (a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/
(a + b*x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x}\,dx}{b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x}\,dx, x, x^3\right)}{3b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)dx, x, x^3\right)}{3b^2(ab+b^2x^3)}$$
$$= \frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} +$$

Mathematica [A] time = 0.0206155, size = 60, normalized size = 0.38

$$\frac{\sqrt{(a+bx^3)^2}\left(bx^3(18a^2+9abx^3+2b^2x^6)+18a^3\log(x)\right)}{18(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(18*a^2 + 9*a*b*x^3 + 2*b^2*x^6) + 18*a^3*Log[x
]))/(18*(a + b*x^3))
```

Maple [A] time = 0.007, size = 57, normalized size = 0.4

$$\frac{2\,b^3x^9+9\,ab^2x^6+18\,a^2bx^3+18\,a^3\ln(x)}{18\,(bx^3+a)^3}\left((bx^3+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(3/2)*(2*b^3*x^9+9*a*b^2*x^6+18*a^2*b*x^3+18*a^3*ln(x))/
(b*x^3+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76796, size = 73, normalized size = 0.46

$$\frac{1}{9}b^3x^9 + \frac{1}{2}ab^2x^6 + a^2bx^3 + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x, x)
```

Giac [A] time = 1.12158, size = 88, normalized size = 0.55

$$\frac{1}{9}b^3x^9\mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{2}ab^2x^6\mathrm{sgn}\left(bx^3 + a\right) + a^2bx^3\mathrm{sgn}\left(bx^3 + a\right) + a^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*sgn(b*x^3 + a) + 1/2*a*b^2*x^6*sgn(b*x^3 + a) + a^2*b*x^3*sgn(b
*x^3 + a) + a^3*log(abs(x))*sgn(b*x^3 + a)
```

3.34 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=165

$$\frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (3*a^2*b*x^2*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a*b^2*x^5*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(8*(a + b*x^3))
```

Rubi [A] time = 0.0424294, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (3*a^2*b*x^2*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a*b^2*x^5*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(8*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^2} \, dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^2}+3a^2b^4x+3ab^5x^4+b^6x^7\right) dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\end{aligned}$$

Mathematica [A] time = 0.0165717, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(60a^2bx^3-40a^3+24ab^2x^6+5b^3x^9\right)}{40x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-40*a^3 + 60*a^2*b*x^3 + 24*a*b^2*x^6 + 5*b^3*x^9))/(
40*x*(a + b*x^3))
```

Maple [A] time = 0.006, size = 58, normalized size = 0.4

$$-\frac{-5\,b^3x^9-24\,ab^2x^6-60\,a^2bx^3+40\,a^3}{40\,x\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x)
```

```
[Out] -1/40*(-5*b^3*x^9-24*a*b^2*x^6-60*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)/x/(
b*x^3+a)^3
```

Maxima [A] time = 1.05854, size = 50, normalized size = 0.3

$$\frac{5\,b^3x^9+24\,ab^2x^6+60\,a^2bx^3-40\,a^3}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

Fricas [A] time = 1.6392, size = 80, normalized size = 0.48

$$\frac{5\,b^3x^9+24\,ab^2x^6+60\,a^2bx^3-40\,a^3}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**2, x)
```

Giac [A] time = 1.11469, size = 90, normalized size = 0.55

$$\frac{1}{8}\, b^3x^8\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{5}\, ab^2x^5\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{2}\, a^2bx^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{a^3\mathrm{sgn}\left(bx^3 + a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*b^3*x^8*sgn(b*x^3 + a) + 3/5*a*b^2*x^5*sgn(b*x^3 + a) + 3/2*a^2*b*x^2*s
gn(b*x^3 + a) - a^3*sgn(b*x^3 + a)/x
```

3.35 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=163

$$\frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a^2*b*x*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (3*a*b^2*x^4*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(7*(a + b*x^3))
```

Rubi [A] time = 0.042097, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a^2*b*x*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (3*a*b^2*x^4*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(7*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^3}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(3a^2b^4+\frac{a^3b^3}{x^3}+3ab^5x^3+b^6x^6\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{b^3}{}\end{aligned}$$

Mathematica [A] time = 0.0178124, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(84a^2bx^3-14a^3+21ab^2x^6+4b^3x^9\right)}{28x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^3 + 84*a^2*b*x^3 + 21*a*b^2*x^6 + 4*b^3*x^9))/(
28*x^2*(a + b*x^3))
```

Maple [A] time = 0.005, size = 58, normalized size = 0.4

$$-\frac{-4\,b^3x^9-21\,ab^2x^6-84\,a^2bx^3+14\,a^3}{28\,x^2\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x)
```

```
[Out] -1/28*(-4*b^3*x^9-21*a*b^2*x^6-84*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/x^2
/(b*x^3+a)^3
```

Maxima [A] time = 1.05122, size = 50, normalized size = 0.31

$$\frac{4\,b^3x^9+21\,ab^2x^6+84\,a^2bx^3-14\,a^3}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

Fricas [A] time = 1.72203, size = 82, normalized size = 0.5

$$\frac{4\,b^3x^9+21\,ab^2x^6+84\,a^2bx^3-14\,a^3}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**3, x)
```

Giac [A] time = 1.11464, size = 88, normalized size = 0.54

$$\frac{1}{7}\,b^3x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{4}\,ab^2x^4\mathrm{sgn}\left(bx^3+a\right)+3\,a^2bx\mathrm{sgn}\left(bx^3+a\right)-\frac{a^3\mathrm{sgn}\left(bx^3+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7*sgn(b*x^3 + a) + 3/4*a*b^2*x^4*sgn(b*x^3 + a) + 3*a^2*b*x*sgn(b
*x^3 + a) - 1/2*a^3*sgn(b*x^3 + a)/x^2
```

3.36 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=161

$$\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (a*b^2*x^3*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^6*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(6*(a + b*x^3)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x
])/(a + b*x^3)
```

Rubi [A] time = 0.0475871, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (a*b^2*x^3*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^6*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(6*(a + b*x^3)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x
])/(a + b*x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^4}\,dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^2}\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(3ab^5+\frac{a^3b^3}{x^2}+\frac{3a^2b^4}{x}+b^6x\right)dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{3a^2b^{\cdot}}{}$$

Mathematica [A] time = 0.0215773, size = 62, normalized size = 0.39

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(18a^2bx^3\log(x)-2a^3+6ab^2x^6+b^3x^9\right)}{6x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^3 + 6*a*b^2*x^6 + b^3*x^9 + 18*a^2*b*x^3*Log[x])
)/(6*x^3*(a + b*x^3))
```

Maple [A] time = 0.012, size = 59, normalized size = 0.4

$$\frac{b^3x^9+6\,ab^2x^6+18\,a^2b\ln(x)\,x^3-2\,a^3}{6\,\left(bx^3+a\right)^3x^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x)
```

```
[Out] 1/6*((b*x^3+a)^2)^(3/2)*(b^3*x^9+6*a*b^2*x^6+18*a^2*b*ln(x)*x^3-2*a^3)/(b*x
^3+a)^3/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65512, size = 85, normalized size = 0.53

$$\frac{b^3x^9 + 6\,ab^2x^6 + 18\,a^2bx^3\log(x) - 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^9 + 6*a*b^2*x^6 + 18*a^2*b*x^3*log(x) - 2*a^3)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**4, x)
```

Giac [A] time = 1.12446, size = 115, normalized size = 0.71

$$\frac{1}{6}\,b^3x^6\mathrm{sgn}\left(bx^3 + a\right) + ab^2x^3\mathrm{sgn}\left(bx^3 + a\right) + 3\,a^2b\log\left(|x|\right)\mathrm{sgn}\left(bx^3 + a\right) - \frac{3\,a^2bx^3\mathrm{sgn}\left(bx^3 + a\right) + a^3\mathrm{sgn}\left(bx^3 + a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*sgn(b*x^3 + a) + a*b^2*x^3*sgn(b*x^3 + a) + 3*a^2*b*log(abs(x))
*sgn(b*x^3 + a) - 1/3*(3*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^3
```

3.37 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=165

$$\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (3*a*b^2*x^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(5*(a + b*x^3))
```

Rubi [A] time = 0.041809, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (3*a*b^2*x^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(5*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^5}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^5}+\frac{3a^2b^4}{x^2}+3ab^5x+b^6x^4\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^5}{}\end{aligned}$$

Mathematica [A] time = 0.0202044, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-60a^2bx^3-5a^3+30ab^2x^6+4b^3x^9\right)}{20x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-5*a^3 - 60*a^2*b*x^3 + 30*a*b^2*x^6 + 4*b^3*x^9))/(2
0*x^4*(a + b*x^3))
```

Maple [A] time = 0.004, size = 58, normalized size = 0.4

$$-\frac{-4\,b^3x^9-30\,ab^2x^6+60\,a^2bx^3+5\,a^3}{20\,x^4\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x)
```

```
[Out] -1/20*(-4*b^3*x^9-30*a*b^2*x^6+60*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/x^4/
(b*x^3+a)^3
```

Maxima [A] time = 1.01822, size = 50, normalized size = 0.3

$$\frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

Fricas [A] time = 1.81665, size = 81, normalized size = 0.49

$$\frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**5, x)
```

Giac [A] time = 1.12378, size = 93, normalized size = 0.56

$$\frac{1}{5}\, b^3x^5\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{2}\, ab^2x^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{12\, a^2bx^3\mathrm{sgn}\left(bx^3 + a\right) + a^3\mathrm{sgn}\left(bx^3 + a\right)}{4\, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5*sgn(b*x^3 + a) + 3/2*a*b^2*x^2*sgn(b*x^3 + a) - 1/4*(12*a^2*b*x
^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^4
```

3.38 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=163

$$\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a*b^2*x*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(4*(a + b*x^3))
```

Rubi [A] time = 0.0407628, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a*b^2*x*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(4*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^6}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(3ab^5+\frac{a^3b^3}{x^6}+\frac{3a^2b^4}{x^3}+b^6x^3\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3 \cdots}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.0206669, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-30a^2bx^3-4a^3+60ab^2x^6+5b^3x^9\right)}{20x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-4*a^3 - 30*a^2*b*x^3 + 60*a*b^2*x^6 + 5*b^3*x^9))/(2
0*x^5*(a + b*x^3))
```

Maple [A] time = 0.005, size = 58, normalized size = 0.4

$$-\frac{-5\,b^3x^9-60\,ab^2x^6+30\,a^2bx^3+4\,a^3}{20\,x^5\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x)
```

```
[Out] -1/20*(-5*b^3*x^9-60*a*b^2*x^6+30*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^5/
(b*x^3+a)^3
```

Maxima [A] time = 1.05179, size = 50, normalized size = 0.31

$$\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

Fricas [A] time = 1.68448, size = 81, normalized size = 0.5

$$\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**6, x)
```

Giac [A] time = 1.12634, size = 92, normalized size = 0.56

$$\frac{1}{4}\, b^3x^4\mathrm{sgn}\left(bx^3+a\right)+3\, ab^2x\mathrm{sgn}\left(bx^3+a\right)-\frac{15\, a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+2\, a^3\mathrm{sgn}\left(bx^3+a\right)}{10\, x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*sgn(b*x^3 + a) + 3*a*b^2*x*sgn(b*x^3 + a) - 1/10*(15*a^2*b*x^3*
sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^5
```

3.39 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=162

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (a^2*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(3*(a + b*x^3)) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log
[x])/(a + b*x^3)
```

Rubi [A] time = 0.0465453, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (a^2*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(3*(a + b*x^3)) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log
[x])/(a + b*x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^7}\,dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^3}\,dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(b^6+\frac{a^3b^3}{x^3}+\frac{3a^2b^4}{x^2}+\frac{3ab^5}{x}\right)dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)} - \frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)} + \frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)} + \frac{3ab^2}{}$$

Mathematica [A] time = 0.017146, size = 61, normalized size = 0.38

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(6a^2bx^3+a^3-18ab^2x^6\log(x)-2b^3x^9\right)}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a^3 + 6*a^2*b*x^3 - 2*b^3*x^9 - 18*a*b^2*x^6*Log[x])
)/(6*x^6*(a + b*x^3))
```

Maple [A] time = 0.012, size = 60, normalized size = 0.4

$$\frac{2\,b^3x^9+18\,ab^2\ln(x)\,x^6-6\,a^2bx^3-a^3}{6\,\left(bx^3+a\right)^3x^6}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x)
```

```
[Out] 1/6*((b*x^3+a)^2)^(3/2)*(2*b^3*x^9+18*a*b^2*ln(x)*x^6-6*a^2*b*x^3-a^3)/(b*x
^3+a)^3/x^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69841, size = 85, normalized size = 0.52

$$\frac{2\,b^{3}x^{9}+18\,ab^{2}x^{6}\log\left(x\right)-6\,a^{2}bx^{3}-a^{3}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^9 + 18*a*b^2*x^6*log(x) - 6*a^2*b*x^3 - a^3)/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**7, x)
```

Giac [A] time = 1.12255, size = 116, normalized size = 0.72

$$\frac{1}{3}\,b^{3}x^{3}\mathrm{sgn}\left(bx^{3}+a\right)+3\,ab^{2}\log\left(|x|\right)\mathrm{sgn}\left(bx^{3}+a\right)-\frac{9\,ab^{2}x^{6}\mathrm{sgn}\left(bx^{3}+a\right)+6\,a^{2}bx^{3}\mathrm{sgn}\left(bx^{3}+a\right)+a^{3}\mathrm{sgn}\left(bx^{3}+a\right)}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*sgn(b*x^3 + a) + 3*a*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(9*a*
b^2*x^6*sgn(b*x^3 + a) + 6*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x
^6
```

3.40 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=165

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(2*(a + b*x^3))
```

Rubi [A] time = 0.040594, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(2*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^8}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^8}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^5}+\frac{3ab^5}{x^2}+b^6x\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{b^3x
\end{aligned}$$

Mathematica [A] time = 0.0133269, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\,(21a^2bx^3+4a^3+84ab^2x^6-14b^3x^9)}{28x^7\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(4*a^3 + 21*a^2*b*x^3 + 84*a*b^2*x^6 - 14*b^3*x^9))/(
28*x^7*(a + b*x^3))
```

Maple [A] time = 0.006, size = 58, normalized size = 0.4

$$-\frac{-14\,b^3x^9+84\,ab^2x^6+21\,a^2bx^3+4\,a^3}{28\,x^7\,(bx^3+a)^3}\left((bx^3+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x)
```

```
[Out] -1/28*(-14*b^3*x^9+84*a*b^2*x^6+21*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^7
/(b*x^3+a)^3
```

Maxima [A] time = 1.03561, size = 50, normalized size = 0.3

$$\frac{14\,b^3x^9-84\,ab^2x^6-21\,a^2bx^3-4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

Fricas [A] time = 1.68245, size = 82, normalized size = 0.5

$$\frac{14\,b^3x^9-84\,ab^2x^6-21\,a^2bx^3-4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**8, x)
```

Giac [A] time = 1.1253, size = 95, normalized size = 0.58

$$\frac{1}{2}\,b^3x^2\operatorname{sgn}\left(bx^3+a\right)-\frac{84\,ab^2x^6\operatorname{sgn}\left(bx^3+a\right)+21\,a^2bx^3\operatorname{sgn}\left(bx^3+a\right)+4\,a^3\operatorname{sgn}\left(bx^3+a\right)}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x^3 + a) - 1/28*(84*a*b^2*x^6*sgn(b*x^3 + a) + 21*a^2*b*x
^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^7
```

3.41 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=162

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(a + b*x^3)
```

Rubi [A] time = 0.0409398, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(a + b*x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^9}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^9}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^6+\frac{a^3b^3}{x^9}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^3}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^3x\sqrt{a}}{}\end{aligned}$$

Mathematica [A] time = 0.01548, size = 61, normalized size = 0.38

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(24a^2bx^3+5a^3+60ab^2x^6-40b^3x^9\right)}{40x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(5*a^3 + 24*a^2*b*x^3 + 60*a*b^2*x^6 - 40*b^3*x^9))/(
40*x^8*(a + b*x^3))
```

Maple [A] time = 0.004, size = 58, normalized size = 0.4

$$-\frac{-40\,b^3x^9+60\,ab^2x^6+24\,a^2bx^3+5\,a^3}{40\,x^8\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x)
```

```
[Out] -1/40*(-40*b^3*x^9+60*a*b^2*x^6+24*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/x^8
/(b*x^3+a)^3
```

Maxima [A] time = 1.02377, size = 50, normalized size = 0.31

$$\frac{40\,b^3x^9-60\,ab^2x^6-24\,a^2bx^3-5\,a^3}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

Fricas [A] time = 1.79179, size = 82, normalized size = 0.51

$$\frac{40\,b^3x^9-60\,ab^2x^6-24\,a^2bx^3-5\,a^3}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{9}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**9, x)
```

Giac [A] time = 1.1176, size = 90, normalized size = 0.56

$$b^{3}x\mathrm{sgn}\left(bx^{3} + a\right) - \frac{60\, ab^{2}x^{6}\mathrm{sgn}\left(bx^{3} + a\right) + 24\, a^{2}bx^{3}\mathrm{sgn}\left(bx^{3} + a\right) + 5\, a^{3}\mathrm{sgn}\left(bx^{3} + a\right)}{40\, x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] b^3*x*sgn(b*x^3 + a) - 1/40*(60*a*b^2*x^6*sgn(b*x^3 + a) + 24*a^2*b*x^3*sgn
(b*x^3 + a) + 5*a^3*sgn(b*x^3 + a))/x^8
```

3.42 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=161

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6\left(a+bx^3\right)}-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (a^2*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(2*x^6*(a + b*x^3)) - (a*b^2*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x
])/(a + b*x^3)
```

Rubi [A] time = 0.0464384, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6\left(a+bx^3\right)}-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (a^2*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(2*x^6*(a + b*x^3)) - (a*b^2*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x
])/(a + b*x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{10}}\,dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^4}\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^3}+\frac{3ab^5}{x^2}+\frac{b^6}{x}\right)dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)} - \frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6\left(a+bx^3\right)} - \frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)} + \frac{b^3\sqrt{a^2+}}{}$$

Mathematica [A] time = 0.0220902, size = 63, normalized size = 0.39

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a\left(2a^2+9abx^3+18b^2x^6\right)-18b^3x^9\log(x)\right)}{18x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a*(2*a^2 + 9*a*b*x^3 + 18*b^2*x^6) - 18*b^3*x^9*Log[
x]))/(18*x^9*(a + b*x^3))
```

Maple [A] time = 0.011, size = 60, normalized size = 0.4

$$\frac{18\,b^3\ln(x)\,x^9-18\,ab^2x^6-9\,a^2bx^3-2\,a^3}{18\,\left(bx^3+a\right)^3x^9}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(3/2)*(18*b^3*ln(x)*x^9-18*a*b^2*x^6-9*a^2*b*x^3-2*a^3)/
(b*x^3+a)^3/x^9
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7255, size = 90, normalized size = 0.56

$$\frac{18\,b^3x^9\log\left(x\right)-18\,ab^2x^6-9\,a^2bx^3-2\,a^3}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(18*b^3*x^9*log(x) - 18*a*b^2*x^6 - 9*a^2*b*x^3 - 2*a^3)/x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**10, x)
```

Giac [A] time = 1.11498, size = 115, normalized size = 0.71

$$b^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{11\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+18\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+9\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a^3\mathrm{sgn}\left(bx^3+a\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x))*sgn(b*x^3 + a) - 1/18*(11*b^3*x^9*sgn(b*x^3 + a) + 18*a*b^2
*x^6*sgn(b*x^3 + a) + 9*a^2*b*x^3*sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^
9
```

3.43 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=165

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(x*(a + b*x^3))
```

Rubi [A] time = 0.0402338, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```


```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(x*(a + b*x^3))
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{11}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{11}}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{11}}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^5}+\frac{b^6}{x^2}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2}}{x}\end{aligned}$$

Mathematica [A] time = 0.0128461, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(60a^2bx^3+14a^3+105ab^2x^6+140b^3x^9\right)}{140x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(14*a^3 + 60*a^2*b*x^3 + 105*a*b^2*x^6 + 140*b^3*x^9)
)/(140*x^10*(a + b*x^3))
```

Maple [A] time = 0.005, size = 58, normalized size = 0.4

$$-\frac{140\,b^3x^9+105\,ab^2x^6+60\,a^2bx^3+14\,a^3}{140\,x^{10}\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x)
```

```
[Out] -1/140*(140*b^3*x^9+105*a*b^2*x^6+60*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/
x^10/(b*x^3+a)^3
```

Maxima [A] time = 1.03771, size = 50, normalized size = 0.3

$$-\frac{140\,b^3x^9+105\,ab^2x^6+60\,a^2bx^3+14\,a^3}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

Fricas [A] time = 1.64957, size = 90, normalized size = 0.55

$$-\frac{140\,b^3x^9+105\,ab^2x^6+60\,a^2bx^3+14\,a^3}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**11, x)
```

Giac [A] time = 1.14586, size = 93, normalized size = 0.56

$$-\frac{140\,b^3x^9\text{sgn}\left(bx^3+a\right)+105\,ab^2x^6\text{sgn}\left(bx^3+a\right)+60\,a^2bx^3\text{sgn}\left(bx^3+a\right)+14\,a^3\text{sgn}\left(bx^3+a\right)}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/140*(140*b^3*x^9*sgn(b*x^3 + a) + 105*a*b^2*x^6*sgn(b*x^3 + a) + 60*a^2*
b*x^3*sgn(b*x^3 + a) + 14*a^3*sgn(b*x^3 + a))/x^10
```

3.44 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{12}} dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(2*x^2*(a + b*x^3))
```

Rubi [A] time = 0.040581, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(2*x^2*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{12}} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{12}} dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{12}}+\frac{3a^2b^4}{x^9}+\frac{3ab^5}{x^6}+\frac{b^6}{x^3}\right) dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b^3\sqrt{}}{}
\end{aligned}$$

Mathematica [A] time = 0.0159287, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(165a^2bx^3+40a^3+264ab^2x^6+220b^3x^9\right)}{440x^{11}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(40*a^3 + 165*a^2*b*x^3 + 264*a*b^2*x^6 + 220*b^3*x^9
))/(440*x^11*(a + b*x^3))
```

Maple [A] time = 0.006, size = 58, normalized size = 0.4

$$-\frac{220\,b^3x^9+264\,ab^2x^6+165\,a^2bx^3+40\,a^3}{440\,x^{11}\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x)
```

```
[Out] -1/440*(220*b^3*x^9+264*a*b^2*x^6+165*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)
/x^11/(b*x^3+a)^3
```

Maxima [A] time = 1.01859, size = 50, normalized size = 0.3

$$-\frac{220\,b^3x^9+264\,ab^2x^6+165\,a^2bx^3+40\,a^3}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11
```

Fricas [A] time = 1.68789, size = 92, normalized size = 0.55

$$-\frac{220\,b^3x^9+264\,ab^2x^6+165\,a^2bx^3+40\,a^3}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**12, x)
```

Giac [A] time = 1.13448, size = 93, normalized size = 0.56

$$-\frac{220\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+264\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+165\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+40\,a^3\mathrm{sgn}\left(bx^3+a\right)}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/440*(220*b^3*x^9*sgn(b*x^3 + a) + 264*a*b^2*x^6*sgn(b*x^3 + a) + 165*a^2
*b*x^3*sgn(b*x^3 + a) + 40*a^3*sgn(b*x^3 + a))/x^11
```

3.45 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{13}} \, dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

```
[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*a*x^12)
```

Rubi [A] time = 0.0182668, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 264}

$$-\frac{\left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

```
[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*a*x^12)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{13}} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{\left(ab+b^2x^3\right)^3}{x^{13}} \, dx}{b^2\left(ab+b^2x^3\right)} \\
&= -\frac{\left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}
\end{aligned}$$

Mathematica [A] time = 0.0129933, size = 59, normalized size = 1.44

$$-\frac{\sqrt{\left(a+bx^3\right)^2} \left(4a^2bx^3+a^3+6ab^2x^6+4b^3x^9\right)}{12x^{12}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a^3 + 4*a^2*b*x^3 + 6*a*b^2*x^6 + 4*b^3*x^9))/(12*x^
12*(a + b*x^3))
```

Maple [A] time = 0.008, size = 56, normalized size = 1.4

$$-\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x)
```

```
[Out] -1/12*(4*b^3*x^9+6*a*b^2*x^6+4*a^2*b*x^3+a^3)*((b*x^3+a)^2)^(3/2)/x^12/(b*x
^3+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77512, size = 78, normalized size = 1.9

$$-\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**13, x)
```

Giac [B] time = 1.08694, size = 92, normalized size = 2.24

$$-\frac{4\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+6\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+4\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\mathrm{sgn}\left(bx^3+a\right)}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(4*b^3*x^9*sgn(b*x^3 + a) + 6*a*b^2*x^6*sgn(b*x^3 + a) + 4*a^2*b*x^3*
sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^12
```

3.46 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(4*x^4*(a + b*x^3))
```

Rubi [A] time = 0.0412958, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(4*x^4*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{14}}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{14}}+\frac{3a^2b^4}{x^{11}}+\frac{3ab^5}{x^8}+\frac{b^6}{x^5}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b^3\sqrt{}}{}\end{aligned}$$

Mathematica [A] time = 0.0135445, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(546a^2bx^3+140a^3+780ab^2x^6+455b^3x^9\right)}{1820x^{13}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(140*a^3 + 546*a^2*b*x^3 + 780*a*b^2*x^6 + 455*b^3*x^
9))/(1820*x^13*(a + b*x^3))
```

Maple [A] time = 0.006, size = 58, normalized size = 0.4

$$-\frac{455\,b^3x^9+780\,ab^2x^6+546\,a^2bx^3+140\,a^3}{1820\,x^{13}\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x)
```

```
[Out] -1/1820*(455*b^3*x^9+780*a*b^2*x^6+546*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/
2)/x^13/(b*x^3+a)^3
```

Maxima [A] time = 1.07021, size = 50, normalized size = 0.3

$$-\frac{455\,b^3x^9+780\,ab^2x^6+546\,a^2bx^3+140\,a^3}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13
```

Fricas [A] time = 1.76716, size = 95, normalized size = 0.57

$$-\frac{455\,b^3x^9+780\,ab^2x^6+546\,a^2bx^3+140\,a^3}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**14, x)
```

Giac [A] time = 1.12411, size = 93, normalized size = 0.56

$$-\frac{455\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+780\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+546\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+140\,a^3\mathrm{sgn}\left(bx^3+a\right)}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/1820*(455*b^3*x^9*sgn(b*x^3 + a) + 780*a*b^2*x^6*sgn(b*x^3 + a) + 546*a^
2*b*x^3*sgn(b*x^3 + a) + 140*a^3*sgn(b*x^3 + a))/x^13
```

3.47 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(5*x^5*(a + b*x^3))
```

Rubi [A] time = 0.0409204, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```


```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(5*x^5*(a + b*x^3))
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{15}}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{15}}+\frac{3a^2b^4}{x^{12}}+\frac{3ab^5}{x^9}+\frac{b^6}{x^6}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2}}{5x}\end{aligned}$$

Mathematica [A] time = 0.0159379, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\,(840a^2bx^3+220a^3+1155ab^2x^6+616b^3x^9)}{3080x^{14}\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(220*a^3 + 840*a^2*b*x^3 + 1155*a*b^2*x^6 + 616*b^3*x
^9))/(3080*x^14*(a + b*x^3))
```

Maple [A] time = 0.006, size = 58, normalized size = 0.4

$$-\frac{616\,b^3x^9+1155\,ab^2x^6+840\,a^2bx^3+220\,a^3}{3080\,x^{14}\,(bx^3+a)^3}\left((bx^3+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x)
```

```
[Out] -1/3080*(616*b^3*x^9+1155*a*b^2*x^6+840*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(3
/2)/x^14/(b*x^3+a)^3
```

Maxima [A] time = 0.989244, size = 50, normalized size = 0.3

$$-\frac{616\,b^3x^9+1155\,ab^2x^6+840\,a^2bx^3+220\,a^3}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14
```

Fricas [A] time = 1.61594, size = 96, normalized size = 0.57

$$-\frac{616\,b^3x^9+1155\,ab^2x^6+840\,a^2bx^3+220\,a^3}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**15, x)
```

Giac [A] time = 1.11327, size = 93, normalized size = 0.56

$$-\frac{616\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+1155\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+840\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+220\,a^3\mathrm{sgn}\left(bx^3+a\right)}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/3080*(616*b^3*x^9*sgn(b*x^3 + a) + 1155*a*b^2*x^6*sgn(b*x^3 + a) + 840*a
^2*b*x^3*sgn(b*x^3 + a) + 220*a^3*sgn(b*x^3 + a))/x^14
```

3.48 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=84

$$\frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}$$

```
[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*a*x^15) + (b*(a + b*x^
3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(60*a^2*x^12)
```

Rubi [A] time = 0.0396448, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1355, 266, 45, 37}

$$\frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

```
[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*a*x^15) + (b*(a + b*x^
3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(60*a^2*x^12)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{16}}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^6}\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}\\
&= -\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}-\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^5}\,dx,x,x^3\right)}{15ab\left(ab+b^2x^3\right)}\\
&= -\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}+\frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}
\end{aligned}$$

Mathematica [A] time = 0.0140864, size = 61, normalized size = 0.73

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(15a^2bx^3+4a^3+20ab^2x^6+10b^3x^9\right)}{60x^{15}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(4*a^3 + 15*a^2*b*x^3 + 20*a*b^2*x^6 + 10*b^3*x^9))/(
60*x^15*(a + b*x^3))
```

Maple [A] time = 0.007, size = 58, normalized size = 0.7

$$-\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x)
```

```
[Out] -1/60*(10*b^3*x^9+20*a*b^2*x^6+15*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^15
/(b*x^3+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73807, size = 85, normalized size = 1.01

$$-\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**16, x)
```

Giac [A] time = 1.11682, size = 93, normalized size = 1.11

$$-\frac{10\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+20\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+15\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+4\,a^3\mathrm{sgn}\left(bx^3+a\right)}{60\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/60*(10*b^3*x^9*sgn(b*x^3 + a) + 20*a*b^2*x^6*sgn(b*x^3 + a) + 15*a^2*b*x
^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^15
```

3.49 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(7*x^7*(a + b*x^3))
```

Rubi [A] time = 0.0401521, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(7*x^7*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{17}}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{17}}+\frac{3a^2b^4}{x^{14}}+\frac{3ab^5}{x^{11}}+\frac{b^6}{x^8}\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2}}{7x}
\end{aligned}$$

Mathematica [A] time = 0.01461, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(1680a^2bx^3+455a^3+2184ab^2x^6+1040b^3x^9\right)}{7280x^{16}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(455*a^3 + 1680*a^2*b*x^3 + 2184*a*b^2*x^6 + 1040*b^3
*x^9))/(7280*x^16*(a + b*x^3))
```

Maple [A] time = 0.007, size = 58, normalized size = 0.4

$$-\frac{1040\,b^3x^9+2184\,ab^2x^6+1680\,a^2bx^3+455\,a^3}{7280\,x^{16}\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x)
```

```
[Out] -1/7280*(1040*b^3*x^9+2184*a*b^2*x^6+1680*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)^
(3/2)/x^16/(b*x^3+a)^3
```

Maxima [A] time = 1.02071, size = 50, normalized size = 0.3

$$-\frac{1040\,b^3x^9+2184\,ab^2x^6+1680\,a^2bx^3+455\,a^3}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16
```

Fricas [A] time = 1.76863, size = 99, normalized size = 0.59

$$-\frac{1040\,b^3x^9+2184\,ab^2x^6+1680\,a^2bx^3+455\,a^3}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**17, x)
```

Giac [A] time = 1.09777, size = 93, normalized size = 0.56

$$-\frac{1040\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+2184\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+1680\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+455\,a^3\mathrm{sgn}\left(bx^3+a\right)}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/7280*(1040*b^3*x^9*sgn(b*x^3 + a) + 2184*a*b^2*x^6*sgn(b*x^3 + a) + 1680
*a^2*b*x^3*sgn(b*x^3 + a) + 455*a^3*sgn(b*x^3 + a))/x^16
```

3.50 $\int x^{13} \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29\left(a+bx^3\right)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx^3+b^2}}{2\left(a+bx^3\right)}$$

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (5*a^4*b*x^17
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^3*b^2*x^20*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^23*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(23*(a + b*x^3)) + (5*a*b^4*x^26*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(26*(a + b*x^3)) + (b^5*x^29*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(29*
(a + b*x^3))
```

Rubi [A] time = 0.0636303, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29\left(a+bx^3\right)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx^3+b^2}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```


```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (5*a^4*b*x^17
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^3*b^2*x^20*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^23*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(23*(a + b*x^3)) + (5*a*b^4*x^26*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(26*(a + b*x^3)) + (b^5*x^29*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(29*
(a + b*x^3))
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{13}\left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^{13}+5a^4b^6x^{16}+10a^3b^7x^{19}+10a^2b^8x^{22}+5ab^9x^{25}+b^{10}x\right.}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx^3+b^2x}}{2\left(a+bx^3\right)}$$

Mathematica [A] time = 0.0255999, size = 83, normalized size = 0.33

$$\frac{x^{14}\sqrt{\left(a+bx^3\right)^2}\left(897260a^2b^3x^9+1031849a^3b^2x^6+606970a^4bx^3+147407a^5+396865ab^4x^{12}+71162b^5x^{15}\right)}{2063698\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]

[Out] (x^14*Sqrt[(a + b*x^3)^2]*(147407*a^5 + 606970*a^4*b*x^3 + 1031849*a^3*b^2*
x^6 + 897260*a^2*b^3*x^9 + 396865*a*b^4*x^12 + 71162*b^5*x^15))/(2063698*(a
 + b*x^3))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.3

$$\frac{x^{14}\left(71162\,b^5x^{15}+396865\,ab^4x^{12}+897260\,a^2b^3x^9+1031849\,a^3b^2x^6+606970\,a^4bx^3+147407\,a^5\right)}{2063698\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] 1/2063698*x^14*(71162*b^5*x^15+396865*a*b^4*x^12+897260*a^2*b^3*x^9+1031849
*a^3*b^2*x^6+606970*a^4*b*x^3+147407*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 1.06016, size = 77, normalized size = 0.3

$$\frac{1}{29}\,b^5x^{29}+\frac{5}{26}\,ab^4x^{26}+\frac{10}{23}\,a^2b^3x^{23}+\frac{1}{2}\,a^3b^2x^{20}+\frac{5}{17}\,a^4bx^{17}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^2*x^20 + 5
/17*a^4*b*x^17 + 1/14*a^5*x^14
```

Fricas [A] time = 1.65686, size = 144, normalized size = 0.56

$$\frac{1}{29}b^5x^{29} + \frac{5}{26}ab^4x^{26} + \frac{10}{23}a^2b^3x^{23} + \frac{1}{2}a^3b^2x^{20} + \frac{5}{17}a^4bx^{17} + \frac{1}{14}a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^2*x^20 + 5
/17*a^4*b*x^17 + 1/14*a^5*x^14
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{13}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**13*((a + b*x**3)**2)**(5/2), x)
```

Giac [A] time = 1.10066, size = 142, normalized size = 0.56

$$\frac{1}{29}b^5x^{29}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{26}ab^4x^{26}\mathrm{sgn}\left(bx^3+a\right) + \frac{10}{23}a^2b^3x^{23}\mathrm{sgn}\left(bx^3+a\right) + \frac{1}{2}a^3b^2x^{20}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{17}a^4bx^{17}\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/29*b^5*x^29*sgn(b*x^3 + a) + 5/26*a*b^4*x^26*sgn(b*x^3 + a) + 10/23*a^2*b
^3*x^23*sgn(b*x^3 + a) + 1/2*a^3*b^2*x^20*sgn(b*x^3 + a) + 5/17*a^4*b*x^17*
sgn(b*x^3 + a) + 1/14*a^5*x^14*sgn(b*x^3 + a)
```

3.51 $\int x^{12} \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28\left(a+bx^3\right)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}$$

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^4*b*x^16
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (10*a^3*b^2*x^19*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a^2*b^3*x^22*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (a*b^4*x^25*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(5*(a + b*x^3)) + (b^5*x^28*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(28*
(a + b*x^3))
```

Rubi [A] time = 0.0584196, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28\left(a+bx^3\right)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```


```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^4*b*x^16
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (10*a^3*b^2*x^19*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a^2*b^3*x^22*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (a*b^4*x^25*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(5*(a + b*x^3)) + (b^5*x^28*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(28*
(a + b*x^3))
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{12}\left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^5b^5x^{12}+5a^4b^6x^{15}+10a^3b^7x^{18}+10a^2b^8x^{21}+5ab^9x^{24}+b\right.}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3}}{19\left(a+bx^3\right)}$$

Mathematica [A] time = 0.0210903, size = 83, normalized size = 0.33

$$\frac{x^{13}\sqrt{\left(a+bx^3\right)^2}\left(691600a^2b^3x^9+800800a^3b^2x^6+475475a^4bx^3+117040a^5+304304ab^4x^{12}+54340b^5x^{15}\right)}{1521520\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^13*Sqrt[(a + b*x^3)^2]*(117040*a^5 + 475475*a^4*b*x^3 + 800800*a^3*b^2*x
^6 + 691600*a^2*b^3*x^9 + 304304*a*b^4*x^12 + 54340*b^5*x^15))/(1521520*(a
+ b*x^3))
```

Maple [A] time = 0.006, size = 80, normalized size = 0.3

$$\frac{x^{13}\left(54340\,b^5x^{15}+304304\,ab^4x^{12}+691600\,a^2b^3x^9+800800\,a^3b^2x^6+475475\,a^4bx^3+117040\,a^5\right)}{1521520\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/1521520*x^13*(54340*b^5*x^15+304304*a*b^4*x^12+691600*a^2*b^3*x^9+800800*
a^3*b^2*x^6+475475*a^4*b*x^3+117040*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 1.00733, size = 77, normalized size = 0.3

$$\frac{1}{28}b^5x^{28}+\frac{1}{5}ab^4x^{25}+\frac{5}{11}a^2b^3x^{22}+\frac{10}{19}a^3b^2x^{19}+\frac{5}{16}a^4bx^{16}+\frac{1}{13}a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^2*x^19 + 5
/16*a^4*b*x^16 + 1/13*a^5*x^13
```

Fricas [A] time = 1.77969, size = 144, normalized size = 0.56

$$\frac{1}{28}\,b^{5}x^{28}+\frac{1}{5}\,ab^{4}x^{25}+\frac{5}{11}\,a^{2}b^{3}x^{22}+\frac{10}{19}\,a^{3}b^{2}x^{19}+\frac{5}{16}\,a^{4}bx^{16}+\frac{1}{13}\,a^{5}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^2*x^19 + 5
/16*a^4*b*x^16 + 1/13*a^5*x^13
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{12}\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**12*((a + b*x**3)**2)**(5/2), x)
```

Giac [A] time = 1.10475, size = 142, normalized size = 0.56

$$\frac{1}{28}\,b^{5}x^{28}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{1}{5}\,ab^{4}x^{25}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{5}{11}\,a^{2}b^{3}x^{22}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{10}{19}\,a^{3}b^{2}x^{19}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{5}{16}\,a^{4}bx^{16}\mathrm{sgn}\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/28*b^5*x^28*sgn(b*x^3 + a) + 1/5*a*b^4*x^25*sgn(b*x^3 + a) + 5/11*a^2*b^3
*x^22*sgn(b*x^3 + a) + 10/19*a^3*b^2*x^19*sgn(b*x^3 + a) + 5/16*a^4*b*x^16*
sgn(b*x^3 + a) + 1/13*a^5*x^13*sgn(b*x^3 + a)
```

3.52 $\int x^{11} \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^8}{27b^4}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{8b^4}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{7b^4}-\frac{a^3\sqrt{a^2+2ab}}{}$$

```
[Out] -(a^3*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^4) + (a^2*(a + b
*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*b^4) - (a*(a + b*x^3)^7*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(8*b^4) + ((a + b*x^3)^8*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(27*b^4)
```

Rubi [A] time = 0.120288, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^8}{27b^4}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{8b^4}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{7b^4}-\frac{a^3\sqrt{a^2+2ab}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -(a^3*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^4) + (a^2*(a + b
*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*b^4) - (a*(a + b*x^3)^7*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(8*b^4) + ((a + b*x^3)^8*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(27*b^4)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{11} \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^{11} \left(ab+b^2x^3\right)^5 dx}{b^4 \left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \operatorname{Subst}\left(\int x^3 \left(ab+b^2x\right)^5 dx, x, x^3\right)}{3b^4 \left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \operatorname{Subst}\left(\int \left(-\frac{a^3(ab+b^2x)^5}{b^3} + \frac{3a^2(ab+b^2x)^6}{b^4} - \frac{3a(ab+b^2x)^7}{b^5} + \frac{(ab+b^2x)^8}{b^6}\right) dx,\right.}{3b^4 \left(ab+b^2x^3\right)}$$

$$= -\frac{a^3 \left(a+bx^3\right)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^4} + \frac{a^2 \left(a+bx^3\right)^6 \sqrt{a^2+2abx^3+b^2x^6}}{7b^4} - \frac{a \left(a+bx^3\right)}{}$$

Mathematica [A] time = 0.0220392, size = 83, normalized size = 0.52

$$\frac{x^{12}\sqrt{\left(a+bx^3\right)^2} \left(720a^2b^3x^9+840a^3b^2x^6+504a^4bx^3+126a^5+315ab^4x^{12}+56b^5x^{15}\right)}{1512 \left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^12*Sqrt[(a + b*x^3)^2]*(126*a^5 + 504*a^4*b*x^3 + 840*a^3*b^2*x^6 + 720*
a^2*b^3*x^9 + 315*a*b^4*x^12 + 56*b^5*x^15))/(1512*(a + b*x^3))
```

Maple [A] time = 0.008, size = 80, normalized size = 0.5

$$\frac{x^{12} \left(56\,b^5x^{15}+315\,ab^4x^{12}+720\,a^2b^3x^9+840\,a^3b^2x^6+504\,a^4bx^3+126\,a^5\right)}{1512\,\left(bx^3+a\right)^5} \left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/1512*x^12*(56*b^5*x^15+315*a*b^4*x^12+720*a^2*b^3*x^9+840*a^3*b^2*x^6+504
*a^4*b*x^3+126*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69744, size = 143, normalized size = 0.89

$$\frac{1}{27}b^5x^{27} + \frac{5}{24}ab^4x^{24} + \frac{10}{21}a^2b^3x^{21} + \frac{5}{9}a^3b^2x^{18} + \frac{1}{3}a^4bx^{15} + \frac{1}{12}a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^2*x^18 + 1
/3*a^4*b*x^15 + 1/12*a^5*x^12
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{11}\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**11*((a + b*x**3)**2)**(5/2), x)
```

Giac [A] time = 1.09734, size = 142, normalized size = 0.89

$$\frac{1}{27}b^5x^{27}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{24}ab^4x^{24}\mathrm{sgn}\left(bx^3 + a\right) + \frac{10}{21}a^2b^3x^{21}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{9}a^3b^2x^{18}\mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{3}a^4bx^{15}\mathrm{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/27*b^5*x^27*sgn(b*x^3 + a) + 5/24*a*b^4*x^24*sgn(b*x^3 + a) + 10/21*a^2*b
^3*x^21*sgn(b*x^3 + a) + 5/9*a^3*b^2*x^18*sgn(b*x^3 + a) + 1/3*a^4*b*x^15*s
gn(b*x^3 + a) + 1/12*a^5*x^12*sgn(b*x^3 + a)
```

3.53 $\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}$$

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^4*b*x^14
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (10*a^3*b^2*x^17*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^2*b^3*x^20*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^23*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(23*(a + b*x^3)) + (b^5*x^26*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(26*
(a + b*x^3))
```

Rubi [A] time = 0.0608434, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```


```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^4*b*x^14
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (10*a^3*b^2*x^17*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^2*b^3*x^20*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^23*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(23*(a + b*x^3)) + (b^5*x^26*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(26*
(a + b*x^3))
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{10}\left(ab+b^2x^3\right)^5dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^{10}+5a^4b^6x^{13}+10a^3b^7x^{16}+10a^2b^8x^{19}+5ab^9x^{22}+b\right.}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3}}{17\left(a+bx^3\right)}$$

Mathematica [A] time = 0.0262638, size = 83, normalized size = 0.33

$$\frac{x^{11}\sqrt{\left(a+bx^3\right)^2}\left(391391a^2b^3x^9+460460a^3b^2x^6+279565a^4bx^3+71162a^5+170170ab^4x^{12}+30107b^5x^{15}\right)}{782782\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^11*Sqrt[(a + b*x^3)^2]*(71162*a^5 + 279565*a^4*b*x^3 + 460460*a^3*b^2*x^
6 + 391391*a^2*b^3*x^9 + 170170*a*b^4*x^12 + 30107*b^5*x^15))/(782782*(a +
b*x^3))
```

Maple [A] time = 0.006, size = 80, normalized size = 0.3

$$\frac{x^{11}\left(30107\,b^5x^{15}+170170\,ab^4x^{12}+391391\,a^2b^3x^9+460460\,a^3b^2x^6+279565\,a^4bx^3+71162\,a^5\right)}{782782\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/782782*x^11*(30107*b^5*x^15+170170*a*b^4*x^12+391391*a^2*b^3*x^9+460460*a
^3*b^2*x^6+279565*a^4*b*x^3+71162*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 1.02142, size = 77, normalized size = 0.3

$$\frac{1}{26}\,b^5x^{26}+\frac{5}{23}\,ab^4x^{23}+\frac{1}{2}\,a^2b^3x^{20}+\frac{10}{17}\,a^3b^2x^{17}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^2*x^17 + 5
/14*a^4*b*x^14 + 1/11*a^5*x^11
```

Fricas [A] time = 1.79483, size = 144, normalized size = 0.56

$$\frac{1}{26}\,b^5x^{26} + \frac{5}{23}\,ab^4x^{23} + \frac{1}{2}\,a^2b^3x^{20} + \frac{10}{17}\,a^3b^2x^{17} + \frac{5}{14}\,a^4bx^{14} + \frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^2*x^17 + 5
/14*a^4*b*x^14 + 1/11*a^5*x^11
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{10} \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**10*((a + b*x**3)**2)**(5/2), x)
```

Giac [A] time = 1.12712, size = 142, normalized size = 0.56

$$\frac{1}{26}\,b^5x^{26}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{23}\,ab^4x^{23}\mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{2}\,a^2b^3x^{20}\mathrm{sgn}\left(bx^3 + a\right) + \frac{10}{17}\,a^3b^2x^{17}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{14}\,a^4bx^{14}\mathrm{sgn}\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/26*b^5*x^26*sgn(b*x^3 + a) + 5/23*a*b^4*x^23*sgn(b*x^3 + a) + 1/2*a^2*b^3
*x^20*sgn(b*x^3 + a) + 10/17*a^3*b^2*x^17*sgn(b*x^3 + a) + 5/14*a^4*b*x^14*
sgn(b*x^3 + a) + 1/11*a^5*x^11*sgn(b*x^3 + a)
```

3.54 $\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\left(a+bx^3\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

```
[Out] (a^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (5*a^4*b*x^13
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^3*b^2*x^16*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^2*b^3*x^19*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(22*(a + b*x^3)) + (b^5*x^25*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2
5*(a + b*x^3))
```

Rubi [A] time = 0.0573355, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\left(a+bx^3\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (5*a^4*b*x^13
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^3*b^2*x^16*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^2*b^3*x^19*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(22*(a + b*x^3)) + (b^5*x^25*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2
5*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^9 \left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^5b^5x^9+5a^4b^6x^{12}+10a^3b^7x^{15}+10a^2b^8x^{18}+5ab^9x^{21}+b^{10}x^{24}\right.}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)} + \frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)} + \frac{5a^3b^2x^{16}\sqrt{a^2+2abx^3+b^2}}{8\left(a+bx^3\right)}$$

Mathematica [A] time = 0.0243075, size = 83, normalized size = 0.33

$$\frac{x^{10}\sqrt{\left(a+bx^3\right)^2}\left(286000a^2b^3x^9+339625a^3b^2x^6+209000a^4bx^3+54340a^5+123500ab^4x^{12}+21736b^5x^{15}\right)}{543400\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^3)^2]*(54340*a^5 + 209000*a^4*b*x^3 + 339625*a^3*b^2*x^
6 + 286000*a^2*b^3*x^9 + 123500*a*b^4*x^12 + 21736*b^5*x^15))/(543400*(a +
b*x^3))
```

Maple [A] time = 0.006, size = 80, normalized size = 0.3

$$\frac{x^{10}\left(21736\,b^5x^{15}+123500\,ab^4x^{12}+286000\,a^2b^3x^9+339625\,a^3b^2x^6+209000\,a^4bx^3+54340\,a^5\right)}{543400\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/543400*x^10*(21736*b^5*x^15+123500*a*b^4*x^12+286000*a^2*b^3*x^9+339625*a
^3*b^2*x^6+209000*a^4*b*x^3+54340*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 0.965566, size = 77, normalized size = 0.3

$$\frac{1}{25}\,b^5x^{25}+\frac{5}{22}\,ab^4x^{22}+\frac{10}{19}\,a^2b^3x^{19}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 + 5
/13*a^4*b*x^13 + 1/10*a^5*x^10
```

Fricas [A] time = 1.69586, size = 144, normalized size = 0.56

$$\frac{1}{25}b^5x^{25} + \frac{5}{22}ab^4x^{22} + \frac{10}{19}a^2b^3x^{19} + \frac{5}{8}a^3b^2x^{16} + \frac{5}{13}a^4bx^{13} + \frac{1}{10}a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 + 5
/13*a^4*b*x^13 + 1/10*a^5*x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^9\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**9*((a + b*x**3)**2)**(5/2), x)
```

Giac [A] time = 1.12372, size = 142, normalized size = 0.56

$$\frac{1}{25}b^5x^{25}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{22}ab^4x^{22}\mathrm{sgn}\left(bx^3 + a\right) + \frac{10}{19}a^2b^3x^{19}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{8}a^3b^2x^{16}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{13}a^4bx^{13}\mathrm{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/25*b^5*x^25*sgn(b*x^3 + a) + 5/22*a*b^4*x^22*sgn(b*x^3 + a) + 10/19*a^2*b
^3*x^19*sgn(b*x^3 + a) + 5/8*a^3*b^2*x^16*sgn(b*x^3 + a) + 5/13*a^4*b*x^13*
sgn(b*x^3 + a) + 1/10*a^5*x^10*sgn(b*x^3 + a)
```

3.55 $\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{24b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{21b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^3}$$

```
[Out] (a^2*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^3) - (2*a*(a + b*
x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^3) + ((a + b*x^3)^7*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(24*b^3)
```

Rubi [A] time = 0.0903906, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{24b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{21b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^3) - (2*a*(a + b*
x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^3) + ((a + b*x^3)^7*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(24*b^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^8 \left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x^2\left(ab+b^2x\right)^5 dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(\frac{a^2\left(ab+b^2x\right)^5}{b^2}-\frac{2a\left(ab+b^2x\right)^6}{b^3}+\frac{\left(ab+b^2x\right)^7}{b^4}\right) dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$= \frac{a^2\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^3}-\frac{2a\left(a+bx^3\right)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^3}+\frac{\left(a+bx^3\right)^7}{}$$

Mathematica [A] time = 0.024054, size = 83, normalized size = 0.7

$$\frac{x^9\sqrt{\left(a+bx^3\right)^2}\left(280a^2b^3x^9+336a^3b^2x^6+210a^4bx^3+56a^5+120ab^4x^{12}+21b^5x^{15}\right)}{504\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^3)^2]*(56*a^5 + 210*a^4*b*x^3 + 336*a^3*b^2*x^6 + 280*a^
2*b^3*x^9 + 120*a*b^4*x^12 + 21*b^5*x^15))/(504*(a + b*x^3))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.7

$$\frac{x^9\left(21\,b^5x^{15}+120\,ab^4x^{12}+280\,a^2b^3x^9+336\,a^3b^2x^6+210\,a^4bx^3+56\,a^5\right)}{504\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/504*x^9*(21*b^5*x^15+120*a*b^4*x^12+280*a^2*b^3*x^9+336*a^3*b^2*x^6+210*a
^4*b*x^3+56*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74264, size = 139, normalized size = 1.17

$$\frac{1}{24}b^5x^{24} + \frac{5}{21}ab^4x^{21} + \frac{5}{9}a^2b^3x^{18} + \frac{2}{3}a^3b^2x^{15} + \frac{5}{12}a^4bx^{12} + \frac{1}{9}a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/1
2*a^4*b*x^12 + 1/9*a^5*x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**8*((a + b*x**3)**2)**(5/2), x)
```

Giac [A] time = 1.12256, size = 142, normalized size = 1.19

$$\frac{1}{24}b^5x^{24}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{21}ab^4x^{21}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{9}a^2b^3x^{18}\mathrm{sgn}\left(bx^3 + a\right) + \frac{2}{3}a^3b^2x^{15}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{12}a^4bx^{12}\mathrm{sgn}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24*sgn(b*x^3 + a) + 5/21*a*b^4*x^21*sgn(b*x^3 + a) + 5/9*a^2*b^3
*x^18*sgn(b*x^3 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^3 + a) + 5/12*a^4*b*x^12*sg
n(b*x^3 + a) + 1/9*a^5*x^9*sgn(b*x^3 + a)
```

3.56 $\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2}}{7\left(a+bx^3\right)}$$

```
[Out] (a^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a^4*b*x^11*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^3*b^2*x^14*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (10*a^2*b^3*x^17*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(4*(a + b*x^3)) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a
+ b*x^3))
```

Rubi [A] time = 0.0602653, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2}}{7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a^4*b*x^11*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^3*b^2*x^14*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (10*a^2*b^3*x^17*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(4*(a + b*x^3)) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a
+ b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^7\left(ab+b^2x^3\right)^5dx}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^7+5a^4b^6x^{10}+10a^3b^7x^{13}+10a^2b^8x^{16}+5ab^9x^{19}+b^{10}x^{22}\right.}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2x}}{7\left(a+bx^3\right)}$$

Mathematica [A] time = 0.022395, size = 83, normalized size = 0.33

$$\frac{x^8\sqrt{\left(a+bx^3\right)^2}\left(141680a^2b^3x^9+172040a^3b^2x^6+109480a^4bx^3+30107a^5+60214ab^4x^{12}+10472b^5x^{15}\right)}{240856\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^3)^2]*(30107*a^5 + 109480*a^4*b*x^3 + 172040*a^3*b^2*x^6
 + 141680*a^2*b^3*x^9 + 60214*a*b^4*x^12 + 10472*b^5*x^15))/(240856*(a + b*
x^3))
```

Maple [A] time = 0.007, size = 80, normalized size = 0.3

$$\frac{x^8\left(10472\,b^5x^{15}+60214\,ab^4x^{12}+141680\,a^2b^3x^9+172040\,a^3b^2x^6+109480\,a^4bx^3+30107\,a^5\right)}{240856\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/240856*x^8*(10472*b^5*x^15+60214*a*b^4*x^12+141680*a^2*b^3*x^9+172040*a^3
*b^2*x^6+109480*a^4*b*x^3+30107*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 1.02469, size = 77, normalized size = 0.3

$$\frac{1}{23}\,b^5x^{23}+\frac{1}{4}\,ab^4x^{20}+\frac{10}{17}\,a^2b^3x^{17}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5/
11*a^4*b*x^11 + 1/8*a^5*x^8
```

Fricas [A] time = 1.66509, size = 140, normalized size = 0.55

$$\frac{1}{23}\,b^5x^{23}+\frac{1}{4}\,ab^4x^{20}+\frac{10}{17}\,a^2b^3x^{17}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5/
11*a^4*b*x^11 + 1/8*a^5*x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**7*((a + b*x**3)**2)**(5/2), x)
```

Giac [A] time = 1.09516, size = 142, normalized size = 0.56

$$\frac{1}{23}\,b^5x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,ab^4x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{17}\,a^2b^3x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^3b^2x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}\,a^4bx^{11}\mathrm{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/23*b^5*x^23*sgn(b*x^3 + a) + 1/4*a*b^4*x^20*sgn(b*x^3 + a) + 10/17*a^2*b^
3*x^17*sgn(b*x^3 + a) + 5/7*a^3*b^2*x^14*sgn(b*x^3 + a) + 5/11*a^4*b*x^11*s
gn(b*x^3 + a) + 1/8*a^5*x^8*sgn(b*x^3 + a)
```

3.57 $\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}$$

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^4*b*x^10*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^3*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^2*b^3*x^16*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(19*(a + b*x^3)) + (b^5*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*(a
+ b*x^3))
```

Rubi [A] time = 0.0594273, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^4*b*x^10*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^3*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^2*b^3*x^16*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(19*(a + b*x^3)) + (b^5*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*(a
+ b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^6\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^6\left(ab+b^2x^3\right)^5dx}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^6+5a^4b^6x^9+10a^3b^7x^{12}+10a^2b^8x^{15}+5ab^9x^{18}+b^{10}\right.}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3+}}{13\left(a+bx^3\right)}$$

Mathematica [A] time = 0.0207801, size = 83, normalized size = 0.33

$$\frac{x^7\sqrt{\left(a+bx^3\right)^2}\left(95095a^2b^3x^9+117040a^3b^2x^6+76076a^4bx^3+21736a^5+40040ab^4x^{12}+6916b^5x^{15}\right)}{152152\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^3)^2]*(21736*a^5 + 76076*a^4*b*x^3 + 117040*a^3*b^2*x^6
+ 95095*a^2*b^3*x^9 + 40040*a*b^4*x^12 + 6916*b^5*x^15))/(152152*(a + b*x^3
))
```

Maple [A] time = 0.006, size = 80, normalized size = 0.3

$$\frac{x^7\left(6916\,b^5x^{15}+40040\,ab^4x^{12}+95095\,a^2b^3x^9+117040\,a^3b^2x^6+76076\,a^4bx^3+21736\,a^5\right)}{152152\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/152152*x^7*(6916*b^5*x^15+40040*a*b^4*x^12+95095*a^2*b^3*x^9+117040*a^3*b
^2*x^6+76076*a^4*b*x^3+21736*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 1.01154, size = 77, normalized size = 0.3

$$\frac{1}{22}\,b^5x^{22}+\frac{5}{19}\,ab^4x^{19}+\frac{5}{8}\,a^2b^3x^{16}+\frac{10}{13}\,a^3b^2x^{13}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 + 1
/2*a^4*b*x^10 + 1/7*a^5*x^7
```

Fricas [A] time = 1.75121, size = 140, normalized size = 0.55

$$\frac{1}{22}b^5x^{22} + \frac{5}{19}ab^4x^{19} + \frac{5}{8}a^2b^3x^{16} + \frac{10}{13}a^3b^2x^{13} + \frac{1}{2}a^4bx^{10} + \frac{1}{7}a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 + 1
/2*a^4*b*x^10 + 1/7*a^5*x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**6*((a + b*x**3)**2)**(5/2), x)
```

Giac [A] time = 1.09437, size = 142, normalized size = 0.56

$$\frac{1}{22}b^5x^{22}\text{sgn}\left(bx^3+a\right) + \frac{5}{19}ab^4x^{19}\text{sgn}\left(bx^3+a\right) + \frac{5}{8}a^2b^3x^{16}\text{sgn}\left(bx^3+a\right) + \frac{10}{13}a^3b^2x^{13}\text{sgn}\left(bx^3+a\right) + \frac{1}{2}a^4bx^{10}\text{sgn}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22*sgn(b*x^3 + a) + 5/19*a*b^4*x^19*sgn(b*x^3 + a) + 5/8*a^2*b^3
*x^16*sgn(b*x^3 + a) + 10/13*a^3*b^2*x^13*sgn(b*x^3 + a) + 1/2*a^4*b*x^10*s
gn(b*x^3 + a) + 1/7*a^5*x^7*sgn(b*x^3 + a)
```

3.58 $\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=78

$$\frac{\left(a+bx^3\right)^6 \sqrt{a^2+2abx^3+b^2x^6}}{21b^2}-\frac{a\left(a+bx^3\right)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^2}$$

```
[Out] -(a*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^2) + ((a + b*x^3)^
6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^2)
```

Rubi [A] time = 0.056291, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{\left(a+bx^3\right)^6 \sqrt{a^2+2abx^3+b^2x^6}}{21b^2}-\frac{a\left(a+bx^3\right)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -(a*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^2) + ((a + b*x^3)^
6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^2)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^5 \left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int x\left(ab+b^2x\right)^5 dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^5}{b}+\frac{\left(ab+b^2x\right)^6}{b^2}\right)dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^2}+\frac{\left(a+bx^3\right)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^2}$$

Mathematica [A] time = 0.0206595, size = 83, normalized size = 1.06

$$\frac{x^6\sqrt{\left(a+bx^3\right)^2}\left(84a^2b^3x^9+105a^3b^2x^6+70a^4bx^3+21a^5+35ab^4x^{12}+6b^5x^{15}\right)}{126\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^3)^2]*(21*a^5 + 70*a^4*b*x^3 + 105*a^3*b^2*x^6 + 84*a^2*
b^3*x^9 + 35*a*b^4*x^12 + 6*b^5*x^15))/(126*(a + b*x^3))
```

Maple [A] time = 0.007, size = 80, normalized size = 1.

$$\frac{x^6\left(6\,b^5x^{15}+35\,ab^4x^{12}+84\,a^2b^3x^9+105\,a^3b^2x^6+70\,a^4bx^3+21\,a^5\right)}{126\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/126*x^6*(6*b^5*x^15+35*a*b^4*x^12+84*a^2*b^3*x^9+105*a^3*b^2*x^6+70*a^4*b
*x^3+21*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7183, size = 136, normalized size = 1.74

$$\frac{1}{21}\,b^5x^{21} + \frac{5}{18}\,ab^4x^{18} + \frac{2}{3}\,a^2b^3x^{15} + \frac{5}{6}\,a^3b^2x^{12} + \frac{5}{9}\,a^4bx^9 + \frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*x^12 + 5/9
*a^4*b*x^9 + 1/6*a^5*x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**5*((a + b*x**3)**2)**(5/2), x)
```

Giac [A] time = 1.11771, size = 90, normalized size = 1.15

$$\frac{1}{126}\left(6\,b^5x^{21} + 35\,ab^4x^{18} + 84\,a^2b^3x^{15} + 105\,a^3b^2x^{12} + 70\,a^4bx^9 + 21\,a^5x^6\right)\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/126*(6*b^5*x^21 + 35*a*b^4*x^18 + 84*a^2*b^3*x^15 + 105*a^3*b^2*x^12 + 70
*a^4*b*x^9 + 21*a^5*x^6)*sgn(b*x^3 + a)
```

3.59 $\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20\left(a+bx^3\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}$$

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (5*a^4*b*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^3*b^2*x^11*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^2*b^3*x^14*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(17*(a + b*x^3)) + (b^5*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*(a
 + b*x^3))
```

Rubi [A] time = 0.0588751, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20\left(a+bx^3\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (5*a^4*b*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^3*b^2*x^11*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^2*b^3*x^14*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(17*(a + b*x^3)) + (b^5*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*(a
 + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^4+5a^4b^6x^7+10a^3b^7x^{10}+10a^2b^8x^{13}+5ab^9x^{16}+b^{10}\right.}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)} + \frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)} + \frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3+}}{11\left(a+bx^3\right)}$$

Mathematica [A] time = 0.0199625, size = 83, normalized size = 0.33

$$\frac{x^5\sqrt{\left(a+bx^3\right)^2}\left(37400a^2b^3x^9+47600a^3b^2x^6+32725a^4bx^3+10472a^5+15400ab^4x^{12}+2618b^5x^{15}\right)}{52360\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)^2]*(10472*a^5 + 32725*a^4*b*x^3 + 47600*a^3*b^2*x^6 +
 37400*a^2*b^3*x^9 + 15400*a*b^4*x^12 + 2618*b^5*x^15))/(52360*(a + b*x^3))
```

Maple [A] time = 0.008, size = 80, normalized size = 0.3

$$\frac{x^5\left(2618\,b^5x^{15}+15400\,ab^4x^{12}+37400\,a^2b^3x^9+47600\,a^3b^2x^6+32725\,a^4bx^3+10472\,a^5\right)}{52360\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/52360*x^5*(2618*b^5*x^15+15400*a*b^4*x^12+37400*a^2*b^3*x^9+47600*a^3*b^2
*x^6+32725*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 1.06292, size = 77, normalized size = 0.3

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5
/8*a^4*b*x^8 + 1/5*a^5*x^5
```

Fricas [A] time = 1.7622, size = 139, normalized size = 0.55

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5
/8*a^4*b*x^8 + 1/5*a^5*x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**4*((a + b*x**3)**2)**(5/2), x)
```

Giac [A] time = 1.11075, size = 142, normalized size = 0.56

$$\frac{1}{20}\, b^5x^{20}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{17}\, ab^4x^{17}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{7}\, a^2b^3x^{14}\mathrm{sgn}\left(bx^3 + a\right) + \frac{10}{11}\, a^3b^2x^{11}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{8}\, a^4bx^8\mathrm{sgn}\left(bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20*sgn(b*x^3 + a) + 5/17*a*b^4*x^17*sgn(b*x^3 + a) + 5/7*a^2*b^3
*x^14*sgn(b*x^3 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^3 + a) + 5/8*a^4*b*x^8*sg
n(b*x^3 + a) + 1/5*a^5*x^5*sgn(b*x^3 + a)
```

3.60 $\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2}}{a+bx^3}$$

```
[Out] (a^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (5*a^4*b*x^7*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^3*b^2*x^10*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (10*a^2*b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(13*(a + b*x^3)) + (5*a*b^4*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(16*(a + b*x^3)) + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x
^3))
```

Rubi [A] time = 0.0573269, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (5*a^4*b*x^7*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^3*b^2*x^10*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (10*a^2*b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(13*(a + b*x^3)) + (5*a*b^4*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(16*(a + b*x^3)) + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x
^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3 \left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^5b^5x^3+5a^4b^6x^6+10a^3b^7x^9+10a^2b^8x^{12}+5ab^9x^{15}+b^{10}x^{18}\right)}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)} + \frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)} + \frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Mathematica [A] time = 0.02019, size = 83, normalized size = 0.33

$$\frac{x^4\sqrt{\left(a+bx^3\right)^2}\left(21280a^2b^3x^9+27664a^3b^2x^6+19760a^4bx^3+6916a^5+8645ab^4x^{12}+1456b^5x^{15}\right)}{27664\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)^2]*(6916*a^5 + 19760*a^4*b*x^3 + 27664*a^3*b^2*x^6 +
21280*a^2*b^3*x^9 + 8645*a*b^4*x^12 + 1456*b^5*x^15))/(27664*(a + b*x^3))
```

Maple [A] time = 0.007, size = 80, normalized size = 0.3

$$\frac{x^4\left(1456\,b^5x^{15}+8645\,ab^4x^{12}+21280\,a^2b^3x^9+27664\,a^3b^2x^6+19760\,a^4bx^3+6916\,a^5\right)}{27664\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/27664*x^4*(1456*b^5*x^15+8645*a*b^4*x^12+21280*a^2*b^3*x^9+27664*a^3*b^2*
x^6+19760*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 1.02717, size = 76, normalized size = 0.3

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*a
^4*b*x^7 + 1/4*a^5*x^4
```

Fricas [A] time = 1.72516, size = 134, normalized size = 0.53

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*a
^4*b*x^7 + 1/4*a^5*x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**3*((a + b*x**3)**2)**(5/2), x)
```

Giac [A] time = 1.11393, size = 140, normalized size = 0.56

$$\frac{1}{19} b^5 x^{19} \mathrm{sgn}\left(bx^3+a\right) + \frac{5}{16} ab^4 x^{16} \mathrm{sgn}\left(bx^3+a\right) + \frac{10}{13} a^2 b^3 x^{13} \mathrm{sgn}\left(bx^3+a\right) + a^3 b^2 x^{10} \mathrm{sgn}\left(bx^3+a\right) + \frac{5}{7} a^4 b x^7 \mathrm{sgn}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19*sgn(b*x^3 + a) + 5/16*a*b^4*x^16*sgn(b*x^3 + a) + 10/13*a^2*b
^3*x^13*sgn(b*x^3 + a) + a^3*b^2*x^10*sgn(b*x^3 + a) + 5/7*a^4*b*x^7*sgn(b*
x^3 + a) + 1/4*a^5*x^4*sgn(b*x^3 + a)
```

3.61 $\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}$$

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(18*b)
```

Rubi [A] time = 0.0289644, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 609}

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(18*b)
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2} dx &= \frac{1}{3}\operatorname{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^3\right)\\ &= \frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}\end{aligned}$$

Mathematica [B] time = 0.0214581, size = 82, normalized size = 2.28

$$\frac{x^3\sqrt{\left(a+bx^3\right)^2}\left(15a^2b^3x^9+20a^3b^2x^6+15a^4bx^3+6a^5+6ab^4x^{12}+b^5x^{15}\right)}{18\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(6*a^5 + 15*a^4*b*x^3 + 20*a^3*b^2*x^6 + 15*a^2*b^
3*x^9 + 6*a*b^4*x^12 + b^5*x^15))/(18*(a + b*x^3))
```

Maple [B] time = 0.009, size = 79, normalized size = 2.2

$$\frac{x^3\left(b^5x^{15}+6\,ab^4x^{12}+15\,a^2b^3x^9+20\,a^3b^2x^6+15\,a^4bx^3+6\,a^5\right)}{18\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/18*x^3*(b^5*x^15+6*a*b^4*x^12+15*a^2*b^3*x^9+20*a^3*b^2*x^6+15*a^4*b*x^3+
6*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7066, size = 135, normalized size = 3.75

$$\frac{1}{18}\,b^5x^{18}+\frac{1}{3}\,ab^4x^{15}+\frac{5}{6}\,a^2b^3x^{12}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{6}\,a^4bx^6+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^5*x^18 + 1/3*a*b^4*x^15 + 5/6*a^2*b^3*x^12 + 10/9*a^3*b^2*x^9 + 5/6*
a^4*b*x^6 + 1/3*a^5*x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**2*((a + b*x**3)**2)**(5/2), x)
```

Giac [B] time = 1.1011, size = 89, normalized size = 2.47

$$\frac{1}{18}\left(b^{5}x^{18}+6\,ab^{4}x^{15}+15\,a^{2}b^{3}x^{12}+20\,a^{3}b^{2}x^{9}+15\,a^{4}bx^{6}+6\,a^{5}x^{3}\right)\mathrm{sgn}\left(bx^{3}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/18*(b^5*x^18 + 6*a*b^4*x^15 + 15*a^2*b^3*x^12 + 20*a^3*b^2*x^9 + 15*a^4*b
*x^6 + 6*a^5*x^3)*sgn(b*x^3 + a)
```

3.62 $\int x \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

```
[Out] (a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (a^4*b*x^5*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^8*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(4*(a + b*x^3)) + (10*a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(11*(a + b*x^3)) + (5*a*b^4*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(14*(a + b*x^3)) + (b^5*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^
3))
```

Rubi [A] time = 0.0544888, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1355, 270}

$$\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```


```
[Out] (a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (a^4*b*x^5*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^8*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(4*(a + b*x^3)) + (10*a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(11*(a + b*x^3)) + (5*a*b^4*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(14*(a + b*x^3)) + (b^5*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^
3))
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x+5a^4b^6x^4+10a^3b^7x^7+10a^2b^8x^{10}+5ab^9x^{13}+b^{10}x^{16}\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+$$

Mathematica [A] time = 0.0204332, size = 83, normalized size = 0.33

$$\frac{x^2\sqrt{\left(a+bx^3\right)^2}\left(4760a^2b^3x^9+6545a^3b^2x^6+5236a^4bx^3+2618a^5+1870ab^4x^{12}+308b^5x^{15}\right)}{5236\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^3)^2]*(2618*a^5 + 5236*a^4*b*x^3 + 6545*a^3*b^2*x^6 + 47
60*a^2*b^3*x^9 + 1870*a*b^4*x^12 + 308*b^5*x^15))/(5236*(a + b*x^3))
```

Maple [A] time = 0.004, size = 80, normalized size = 0.3

$$\frac{x^2\left(308\,b^5x^{15}+1870\,ab^4x^{12}+4760\,a^2b^3x^9+6545\,a^3b^2x^6+5236\,a^4bx^3+2618\,a^5\right)}{5236\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/5236*x^2*(308*b^5*x^15+1870*a*b^4*x^12+4760*a^2*b^3*x^9+6545*a^3*b^2*x^6+
5236*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 1.0165, size = 76, normalized size = 0.3

$$\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a^
4*b*x^5 + 1/2*a^5*x^2
```

Fricas [A] time = 1.7524, size = 132, normalized size = 0.52

$$\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a^
4*b*x^5 + 1/2*a^5*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**(5/2), x)
```

Giac [A] time = 1.13603, size = 140, normalized size = 0.56

$$\frac{1}{17}\,b^5x^{17}\operatorname{sgn}\left(bx^3+a\right)+\frac{5}{14}\,ab^4x^{14}\operatorname{sgn}\left(bx^3+a\right)+\frac{10}{11}\,a^2b^3x^{11}\operatorname{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^3b^2x^8\operatorname{sgn}\left(bx^3+a\right)+a^4bx^5\operatorname{sgn}\left(bx^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17*sgn(b*x^3 + a) + 5/14*a*b^4*x^14*sgn(b*x^3 + a) + 10/11*a^2*b
^3*x^11*sgn(b*x^3 + a) + 5/4*a^3*b^2*x^8*sgn(b*x^3 + a) + a^4*b*x^5*sgn(b*x
^3 + a) + 1/2*a^5*x^2*sgn(b*x^3 + a)
```

3.63 $\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^{16}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{16\left(a+bx^3\right)^5}+\frac{5ab^4x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{13\left(a+bx^3\right)^5}+\frac{a^2b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}+\frac{10a^3b^2x^7\left(a^2+2abx^3\right.}{7\left(a+bx^3\right)^5}$$

```
[Out] (a^5*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a^4*b*x^4*(a^2
 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(4*(a + b*x^3)^5) + (10*a^3*b^2*x^7*(a^2 + 2
*a*b*x^3 + b^2*x^6)^(5/2))/(7*(a + b*x^3)^5) + (a^2*b^3*x^10*(a^2 + 2*a*b*x
^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a*b^4*x^13*(a^2 + 2*a*b*x^3 + b^2*x
^6)^(5/2))/(13*(a + b*x^3)^5) + (b^5*x^16*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)
)/(16*(a + b*x^3)^5)
```

Rubi [A] time = 0.0508262, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1343, 194}

$$\frac{b^5x^{16}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{16\left(a+bx^3\right)^5}+\frac{5ab^4x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{13\left(a+bx^3\right)^5}+\frac{a^2b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}+\frac{10a^3b^2x^7\left(a^2+2abx^3\right.}{7\left(a+bx^3\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a^4*b*x^4*(a^2
 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(4*(a + b*x^3)^5) + (10*a^3*b^2*x^7*(a^2 + 2
*a*b*x^3 + b^2*x^6)^(5/2))/(7*(a + b*x^3)^5) + (a^2*b^3*x^10*(a^2 + 2*a*b*x
^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a*b^4*x^13*(a^2 + 2*a*b*x^3 + b^2*x
^6)^(5/2))/(13*(a + b*x^3)^5) + (b^5*x^16*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)
)/(16*(a + b*x^3)^5)
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2} \int \left(2ab+2b^2x^3\right)^5 dx}{\left(2ab+2b^2x^3\right)^5}$$
$$= \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2} \int \left(32a^5b^5+160a^4b^6x^3+320a^3b^7x^6+320a^2b^8x^9+160ab^9x^{12}\right.}{\left(2ab+2b^2x^3\right)^5}$$
$$= \frac{a^5x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5} + \frac{5a^4bx^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{4\left(a+bx^3\right)^5} + \frac{10a^3b^2x^7\left(a^2+2abx^3+\right.}{7\left(a+bx^3\right)^5}$$

Mathematica [A] time = 0.0205875, size = 81, normalized size = 0.33

$$\frac{x\sqrt{\left(a+bx^3\right)^2}\left(1456a^2b^3x^9+2080a^3b^2x^6+1820a^4bx^3+1456a^5+560ab^4x^{12}+91b^5x^{15}\right)}{1456\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^3)^2]*(1456*a^5 + 1820*a^4*b*x^3 + 2080*a^3*b^2*x^6 + 1456
*a^2*b^3*x^9 + 560*a*b^4*x^12 + 91*b^5*x^15))/(1456*(a + b*x^3))
```

Maple [A] time = 0.004, size = 78, normalized size = 0.3

$$\frac{x\left(91\,b^5x^{15}+560\,ab^4x^{12}+1456\,a^2b^3x^9+2080\,a^3b^2x^6+1820\,a^4bx^3+1456\,a^5\right)}{1456\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/1456*x*(91*b^5*x^15+560*a*b^4*x^12+1456*a^2*b^3*x^9+2080*a^3*b^2*x^6+1820
*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 1.02702, size = 72, normalized size = 0.29

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{13}\,ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}\,a^3b^2x^7+\frac{5}{4}\,a^4bx^4+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4
*b*x^4 + a^5*x
```

Fricas [A] time = 1.76481, size = 123, normalized size = 0.5

$$\frac{1}{16}b^5x^{16}+\frac{5}{13}ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^4bx^4+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4
*b*x^4 + a^5*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2), x)
```

Giac [A] time = 1.11208, size = 136, normalized size = 0.55

$$\frac{1}{16}b^5x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{13}ab^4x^{13}\mathrm{sgn}\left(bx^3+a\right)+a^2b^3x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{7}a^3b^2x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}a^4bx^4\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16*sgn(b*x^3 + a) + 5/13*a*b^4*x^13*sgn(b*x^3 + a) + a^2*b^3*x^1
0*sgn(b*x^3 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^3 + a) + 5/4*a^4*b*x^4*sgn(b*x^
3 + a) + a^5*x*sgn(b*x^3 + a)
```

3.64 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15\left(a+bx^3\right)}+\frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2}}{3\left(a+bx^3\right)}$$

```
[Out] (5*a^4*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^3*b^2*
x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (10*a^2*b^3*x^9*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (5*a*b^4*x^12*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(12*(a + b*x^3)) + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(15*(a + b*x^3)) + (a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
b*x^3)
```

Rubi [A] time = 0.0686626, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15\left(a+bx^3\right)}+\frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2}}{3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```


```
[Out] (5*a^4*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^3*b^2*
x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (10*a^2*b^3*x^9*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (5*a*b^4*x^12*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(12*(a + b*x^3)) + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(15*(a + b*x^3)) + (a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
b*x^3)
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x}\,dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \left(5a^4b^6+\frac{a^5b^5}{x}+10a^3b^7x+10a^2b^8x^2+5ab^9x^3+b^{10}x^4\right)dx,\right.}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)} + \frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)} + \frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x}}{9\left(a+bx^3\right)}$$

Mathematica [A] time = 0.0260967, size = 82, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(bx^3\left(200a^2b^2x^6+300a^3bx^3+300a^4+75ab^3x^9+12b^4x^{12}\right)+180a^5\log(x)\right)}{180\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(300*a^4 + 300*a^3*b*x^3 + 200*a^2*b^2*x^6 + 75
*a*b^3*x^9 + 12*b^4*x^12) + 180*a^5*Log[x]))/(180*(a + b*x^3))
```

Maple [A] time = 0.009, size = 79, normalized size = 0.3

$$\frac{12\,b^5x^{15}+75\,ab^4x^{12}+200\,a^2b^3x^9+300\,a^3b^2x^6+300\,a^4bx^3+180\,a^5\ln(x)}{180\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x)
```

```
[Out] 1/180*((b*x^3+a)^2)^(5/2)*(12*b^5*x^15+75*a*b^4*x^12+200*a^2*b^3*x^9+300*a^
3*b^2*x^6+300*a^4*b*x^3+180*a^5*ln(x))/(b*x^3+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63602, size = 134, normalized size = 0.53

$$\frac{1}{15}\,b^{5}x^{15}+\frac{5}{12}\,ab^{4}x^{12}+\frac{10}{9}\,a^{2}b^{3}x^{9}+\frac{5}{3}\,a^{3}b^{2}x^{6}+\frac{5}{3}\,a^{4}bx^{3}+a^{5}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3*
a^4*b*x^3 + a^5*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x, x)
```

Giac [A] time = 1.09832, size = 140, normalized size = 0.56

$$\frac{1}{15}\,b^{5}x^{15}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{5}{12}\,ab^{4}x^{12}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{10}{9}\,a^{2}b^{3}x^{9}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{5}{3}\,a^{3}b^{2}x^{6}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{5}{3}\,a^{4}bx^{3}\mathrm{sgn}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15*sgn(b*x^3 + a) + 5/12*a*b^4*x^12*sgn(b*x^3 + a) + 10/9*a^2*b^
3*x^9*sgn(b*x^3 + a) + 5/3*a^3*b^2*x^6*sgn(b*x^3 + a) + 5/3*a^4*b*x^3*sgn(b
*x^3 + a) + a^5*log(abs(x))*sgn(b*x^3 + a)
```

3.65 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+$$

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (5*a^4*b*x^2*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (2*a^3*b^2*x^5*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(4*(a + b*x^3)) + (5*a*b^4*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(1
1*(a + b*x^3)) + (b^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)
)
```

Rubi [A] time = 0.0605986, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (5*a^4*b*x^2*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (2*a^3*b^2*x^5*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(4*(a + b*x^3)) + (5*a*b^4*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(1
1*(a + b*x^3)) + (b^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)
)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^2}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^2}+5a^4b^6x+10a^3b^7x^4+10a^2b^8x^7+5ab^9x^{10}+b^{10}x^{13}\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Mathematica [A] time = 0.020515, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(385a^2b^3x^9+616a^3b^2x^6+770a^4bx^3-308a^5+140ab^4x^{12}+22b^5x^{15}\right)}{308x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-308*a^5 + 770*a^4*b*x^3 + 616*a^3*b^2*x^6 + 385*a^2*
b^3*x^9 + 140*a*b^4*x^12 + 22*b^5*x^15))/(308*x*(a + b*x^3))
```

Maple [A] time = 0.006, size = 80, normalized size = 0.3

$$-\frac{-22\,b^5x^{15}-140\,ab^4x^{12}-385\,a^2b^3x^9-616\,a^3b^2x^6-770\,a^4bx^3+308\,a^5}{308\,x\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x)
```

```
[Out] -1/308*(-22*b^5*x^15-140*a*b^4*x^12-385*a^2*b^3*x^9-616*a^3*b^2*x^6-770*a^4
*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x/(b*x^3+a)^5
```

Maxima [A] time = 1.18157, size = 80, normalized size = 0.32

$$\frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 7
70*a^4*b*x^3 - 308*a^5)/x
```

Fricas [A] time = 1.71525, size = 138, normalized size = 0.55

$$\frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 7
70*a^4*b*x^3 - 308*a^5)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**2, x)
```

Giac [A] time = 1.12656, size = 142, normalized size = 0.57

$$\frac{1}{14}\,b^5x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}\,ab^4x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^2b^3x^8\mathrm{sgn}\left(bx^3+a\right)+2\,a^3b^2x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^4bx^2\mathrm{sgn}\left(bx^3+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/14*b^5*x^14*sgn(b*x^3 + a) + 5/11*a*b^4*x^11*sgn(b*x^3 + a) + 5/4*a^2*b^3
*x^8*sgn(b*x^3 + a) + 2*a^3*b^2*x^5*sgn(b*x^3 + a) + 5/2*a^4*b*x^2*sgn(b*x^
3 + a) - a^5*sgn(b*x^3 + a)/x
```

3.66 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (5*a^4*b*x*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^4*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(7*(a + b*x^3)) + (a*b^4*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*
(a + b*x^3)) + (b^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3))
```

Rubi [A] time = 0.0589937, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (5*a^4*b*x*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^4*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(7*(a + b*x^3)) + (a*b^4*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*
(a + b*x^3)) + (b^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^3}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(5a^4b^6+\frac{a^5b^5}{x^3}+10a^3b^7x^3+10a^2b^8x^6+5ab^9x^9+b^{10}x^{12}\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+ \ldots$$

Mathematica [A] time = 0.0228377, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(260a^2b^3x^9+455a^3b^2x^6+910a^4bx^3-91a^5+91ab^4x^{12}+14b^5x^{15}\right)}{182x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-91*a^5 + 910*a^4*b*x^3 + 455*a^3*b^2*x^6 + 260*a^2*b
^3*x^9 + 91*a*b^4*x^12 + 14*b^5*x^15))/(182*x^2*(a + b*x^3))
```

Maple [A] time = 0.006, size = 80, normalized size = 0.3

$$-\frac{-14\,b^5x^{15}-91\,ab^4x^{12}-260\,a^2b^3x^9-455\,a^3b^2x^6-910\,a^4bx^3+91\,a^5}{182\,x^2\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x)
```

```
[Out] -1/182*(-14*b^5*x^15-91*a*b^4*x^12-260*a^2*b^3*x^9-455*a^3*b^2*x^6-910*a^4*
b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^2/(b*x^3+a)^5
```

Maxima [A] time = 1.00467, size = 80, normalized size = 0.32

$$\frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 91
0*a^4*b*x^3 - 91*a^5)/x^2
```

Fricas [A] time = 1.73851, size = 138, normalized size = 0.55

$$\frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 91
0*a^4*b*x^3 - 91*a^5)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**3, x)
```

Giac [A] time = 1.12094, size = 139, normalized size = 0.55

$$\frac{1}{13}\,b^5x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,ab^4x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{7}\,a^2b^3x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^3b^2x^4\mathrm{sgn}\left(bx^3+a\right)+5\,a^4bx\mathrm{sgn}\left(bx^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13*sgn(b*x^3 + a) + 1/2*a*b^4*x^10*sgn(b*x^3 + a) + 10/7*a^2*b^3
*x^7*sgn(b*x^3 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^3 + a) + 5*a^4*b*x*sgn(b*x^3
+ a) - 1/2*a^5*sgn(b*x^3 + a)/x^2
```

3.67 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a^3*b^2*x^
3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^2*b^3*x^6*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^9*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(9*(a + b*x^3)) + (b^5*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(12*(a + b*x^3)) + (5*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b
*x^3)
```

Rubi [A] time = 0.0735974, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a^3*b^2*x^
3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^2*b^3*x^6*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^9*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(9*(a + b*x^3)) + (b^5*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(12*(a + b*x^3)) + (5*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b
*x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^4}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^2}\,dx, x, x^3\right)}{3b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(10a^3b^7+\frac{a^5b^5}{x^2}+\frac{5a^4b^6}{x}+10a^2b^8x+5ab^9x^2+b^{10}x^3\right) dx,$$
$$3b^4(ab+b^2x^3)$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x}}{3(a+bx^3)}$$

Mathematica [A] time = 0.0261356, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^3)^2}\,(60a^2b^3x^9+120a^3b^2x^6+180a^4bx^3\log(x)-12a^5+20ab^4x^{12}+3b^5x^{15})}{36x^3(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-12*a^5 + 120*a^3*b^2*x^6 + 60*a^2*b^3*x^9 + 20*a*b^4
*x^12 + 3*b^5*x^15 + 180*a^4*b*x^3*Log[x]))/(36*x^3*(a + b*x^3))
```

Maple [A] time = 0.011, size = 82, normalized size = 0.3

$$\frac{3\,b^5x^{15}+20\,ab^4x^{12}+60\,a^2b^3x^9+120\,a^3b^2x^6+180\,a^4b\ln(x)\,x^3-12\,a^5}{36\,(bx^3+a)^5\,x^3}\left((bx^3+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x)
```

```
[Out] 1/36*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+20*a*b^4*x^12+60*a^2*b^3*x^9+120*a^3*b
^2*x^6+180*a^4*b*ln(x)*x^3-12*a^5)/(b*x^3+a)^5/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81542, size = 143, normalized size = 0.57

$$\frac{3\,b^5x^{15} + 20\,ab^4x^{12} + 60\,a^2b^3x^9 + 120\,a^3b^2x^6 + 180\,a^4bx^3\log(x) - 12\,a^5}{36\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^5*x^15 + 20*a*b^4*x^12 + 60*a^2*b^3*x^9 + 120*a^3*b^2*x^6 + 180*a
^4*b*x^3*log(x) - 12*a^5)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**4, x)
```

Giac [A] time = 1.1099, size = 167, normalized size = 0.66

$$\frac{1}{12}\,b^5x^{12}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{9}\,ab^4x^9\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{3}\,a^2b^3x^6\mathrm{sgn}\left(bx^3 + a\right) + \frac{10}{3}\,a^3b^2x^3\mathrm{sgn}\left(bx^3 + a\right) + 5\,a^4b\log\left(|x|\right)\mathrm{sgn}\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*b^5*x^12*sgn(b*x^3 + a) + 5/9*a*b^4*x^9*sgn(b*x^3 + a) + 5/3*a^2*b^3*x
^6*sgn(b*x^3 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^3 + a) + 5*a^4*b*log(abs(x))*s
gn(b*x^3 + a) - 1/3*(5*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^3
```

3.68 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5} dx$

Optimal. Leaf size=249

$$\frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^3*b^2*x^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(a + b*x^3) + (2*a^2*b^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(a + b*x^3) + (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b
*x^3)) + (b^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3))
```

Rubi [A] time = 0.0593276, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^3*b^2*x^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(a + b*x^3) + (2*a^2*b^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(a + b*x^3) + (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b
*x^3)) + (b^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^5}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^2}+10a^3b^7x+10a^2b^8x^4+5ab^9x^7+b^{10}x^{10}\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)} + \frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + 2a^2$$

Mathematica [A] time = 0.0250369, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(176a^2b^3x^9+440a^3b^2x^6-440a^4bx^3-22a^5+55ab^4x^{12}+8b^5x^{15}\right)}{88x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-22*a^5 - 440*a^4*b*x^3 + 440*a^3*b^2*x^6 + 176*a^2*b
^3*x^9 + 55*a*b^4*x^12 + 8*b^5*x^15))/(88*x^4*(a + b*x^3))
```

Maple [A] time = 0.007, size = 80, normalized size = 0.3

$$-\frac{-8\,b^5x^{15}-55\,ab^4x^{12}-176\,a^2b^3x^9-440\,a^3b^2x^6+440\,a^4bx^3+22\,a^5}{88\,x^4\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x)
```

```
[Out] -1/88*(-8*b^5*x^15-55*a*b^4*x^12-176*a^2*b^3*x^9-440*a^3*b^2*x^6+440*a^4*b*
x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^4/(b*x^3+a)^5
```

Maxima [A] time = 1.09904, size = 80, normalized size = 0.32

$$\frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*
a^4*b*x^3 - 22*a^5)/x^4
```

Fricas [A] time = 1.70535, size = 135, normalized size = 0.54

$$\frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*
a^4*b*x^3 - 22*a^5)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**5, x)
```

Giac [A] time = 1.13103, size = 144, normalized size = 0.58

$$\frac{1}{11}\,b^5x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,ab^4x^8\mathrm{sgn}\left(bx^3+a\right)+2\,a^2b^3x^5\mathrm{sgn}\left(bx^3+a\right)+5\,a^3b^2x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{20\,a^4bx^3\mathrm{sgn}\left(bx^3+\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11*sgn(b*x^3 + a) + 5/8*a*b^4*x^8*sgn(b*x^3 + a) + 2*a^2*b^3*x^5
*sgn(b*x^3 + a) + 5*a^3*b^2*x^2*sgn(b*x^3 + a) - 1/4*(20*a^4*b*x^3*sgn(b*x^
3 + a) + a^5*sgn(b*x^3 + a))/x^4
```

3.69 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (10*a^3*b^2*x*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7
*(a + b*x^3)) + (b^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3))
```

Rubi [A] time = 0.0575716, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (10*a^3*b^2*x*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7
*(a + b*x^3)) + (b^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^6}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(10a^3b^7+\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^3}+10a^2b^8x^3+5ab^9x^6+b^{10}x^9\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)} + \frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} +$$

Mathematica [A] time = 0.0217986, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(175a^2b^3x^9+700a^3b^2x^6-175a^4bx^3-14a^5+50ab^4x^{12}+7b^5x^{15}\right)}{70x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^5 - 175*a^4*b*x^3 + 700*a^3*b^2*x^6 + 175*a^2*b
^3*x^9 + 50*a*b^4*x^12 + 7*b^5*x^15))/(70*x^5*(a + b*x^3))
```

Maple [A] time = 0.008, size = 80, normalized size = 0.3

$$-\frac{-7\,b^5x^{15}-50\,ab^4x^{12}-175\,a^2b^3x^9-700\,a^3b^2x^6+175\,a^4bx^3+14\,a^5}{70\,x^5\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x)
```

```
[Out] -1/70*(-7*b^5*x^15-50*a*b^4*x^12-175*a^2*b^3*x^9-700*a^3*b^2*x^6+175*a^4*b*
x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^5/(b*x^3+a)^5
```

Maxima [A] time = 1.12618, size = 80, normalized size = 0.32

$$\frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*
a^4*b*x^3 - 14*a^5)/x^5
```

Fricas [A] time = 1.65492, size = 135, normalized size = 0.54

$$\frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*
a^4*b*x^3 - 14*a^5)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**6, x)
```

Giac [A] time = 1.11071, size = 143, normalized size = 0.57

$$\frac{1}{10}\,b^5x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,ab^4x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^2b^3x^4\mathrm{sgn}\left(bx^3+a\right)+10\,a^3b^2x\mathrm{sgn}\left(bx^3+a\right)-\frac{25\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*sgn(b*x^3 + a) + 5/7*a*b^4*x^7*sgn(b*x^3 + a) + 5/2*a^2*b^3*x
^4*sgn(b*x^3 + a) + 10*a^3*b^2*x*sgn(b*x^3 + a) - 1/10*(25*a^4*b*x^3*sgn(b*
x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^5
```

3.70 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^7} \, dx$

Optimal. Leaf size=252

$$\frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a^2*b^3*x^3*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^6*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(6*(a + b*x^3)) + (b^5*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9
*(a + b*x^3)) + (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*
x^3)
```

Rubi [A] time = 0.0733688, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a^2*b^3*x^3*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^6*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(6*(a + b*x^3)) + (b^5*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9
*(a + b*x^3)) + (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*
x^3)
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^7}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^7}dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^3}dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(10a^2b^8+\frac{a^5b^5}{x^3}+\frac{5a^4b^6}{x^2}+\frac{10a^3b^7}{x}+5ab^9x+b^{10}x^2\right)dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)} + \frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)} + 5a$$

Mathematica [A] time = 0.0244631, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(60a^2b^3x^9+180a^3b^2x^6\log(x)-30a^4bx^3-3a^5+15ab^4x^{12}+2b^5x^{15}\right)}{18x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-3*a^5 - 30*a^4*b*x^3 + 60*a^2*b^3*x^9 + 15*a*b^4*x^1
2 + 2*b^5*x^15 + 180*a^3*b^2*x^6*Log[x]))/(18*x^6*(a + b*x^3))
```

Maple [A] time = 0.012, size = 82, normalized size = 0.3

$$\frac{2\,b^5x^{15}+15\,ab^4x^{12}+60\,a^2b^3x^9+180\,a^3b^2\ln(x)\,x^6-30\,a^4bx^3-3\,a^5}{18\,\left(bx^3+a\right)^5x^6}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(5/2)*(2*b^5*x^15+15*a*b^4*x^12+60*a^2*b^3*x^9+180*a^3*b
^2*ln(x)*x^6-30*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60496, size = 140, normalized size = 0.56

$$\frac{2\,b^5x^{15} + 15\,ab^4x^{12} + 60\,a^2b^3x^9 + 180\,a^3b^2x^6\log\left(x\right) - 30\,a^4bx^3 - 3\,a^5}{18\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^5*x^15 + 15*a*b^4*x^12 + 60*a^2*b^3*x^9 + 180*a^3*b^2*x^6*log(x)
- 30*a^4*b*x^3 - 3*a^5)/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**7, x)
```

Giac [A] time = 1.13323, size = 170, normalized size = 0.67

$$\frac{1}{9}\,b^5x^9\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{6}\,ab^4x^6\mathrm{sgn}\left(bx^3 + a\right) + \frac{10}{3}\,a^2b^3x^3\mathrm{sgn}\left(bx^3 + a\right) + 10\,a^3b^2\log\left(|x|\right)\mathrm{sgn}\left(bx^3 + a\right) - \frac{30\,a^3b^2x^6\mathrm{sg}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9*sgn(b*x^3 + a) + 5/6*a*b^4*x^6*sgn(b*x^3 + a) + 10/3*a^2*b^3*x^
3*sgn(b*x^3 + a) + 10*a^3*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(30*a^3*b^2*
x^6*sgn(b*x^3 + a) + 10*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^6
```

3.71 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8} dx$

Optimal. Leaf size=248

$$\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{5a^4}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^2*b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(a + b*x^3) + (a*b^4*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b
*x^3) + (b^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rubi [A] time = 0.058232, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{5a^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^2*b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(a + b*x^3) + (a*b^4*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b
*x^3) + (b^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^8}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^8}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^2}+10a^2b^8x+5ab^9x^4+b^{10}x^7\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)} + \frac{5}{}$$

Mathematica [A] time = 0.0202955, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(280a^2b^3x^9-560a^3b^2x^6-70a^4bx^3-8a^5+56ab^4x^{12}+7b^5x^{15}\right)}{56x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-8*a^5 - 70*a^4*b*x^3 - 560*a^3*b^2*x^6 + 280*a^2*b^3
*x^9 + 56*a*b^4*x^12 + 7*b^5*x^15))/(56*x^7*(a + b*x^3))
```

Maple [A] time = 0.006, size = 80, normalized size = 0.3

$$-\frac{-7b^5x^{15}-56ab^4x^{12}-280a^2b^3x^9+560a^3b^2x^6+70a^4bx^3+8a^5}{56x^7\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x)
```

```
[Out] -1/56*(-7*b^5*x^15-56*a*b^4*x^12-280*a^2*b^3*x^9+560*a^3*b^2*x^6+70*a^4*b*x
^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^7/(b*x^3+a)^5
```

Maxima [A] time = 1.07002, size = 80, normalized size = 0.32

$$\frac{7b^5x^{15}+56ab^4x^{12}+280a^2b^3x^9-560a^3b^2x^6-70a^4bx^3-8a^5}{56x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a
^4*b*x^3 - 8*a^5)/x^7
```

Fricas [A] time = 1.77122, size = 132, normalized size = 0.53

$$\frac{7 b^5 x^{15} + 56 a b^4 x^{12} + 280 a^2 b^3 x^9 - 560 a^3 b^2 x^6 - 70 a^4 b x^3 - 8 a^5}{56 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a
^4*b*x^3 - 8*a^5)/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^3\right)^2\right)^{\frac{5}{2}}}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**8, x)
```

Giac [A] time = 1.10424, size = 144, normalized size = 0.58

$$\frac{1}{8} b^5 x^8 \mathrm{sgn}\left(b x^3 + a\right) + a b^4 x^5 \mathrm{sgn}\left(b x^3 + a\right) + 5 a^2 b^3 x^2 \mathrm{sgn}\left(b x^3 + a\right) - \frac{280 a^3 b^2 x^6 \mathrm{sgn}\left(b x^3 + a\right) + 35 a^4 b x^3 \mathrm{sgn}\left(b x^3 + a\right) +}{28 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8*sgn(b*x^3 + a) + a*b^4*x^5*sgn(b*x^3 + a) + 5*a^2*b^3*x^2*sgn(b
*x^3 + a) - 1/28*(280*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(b*x^3 +
 a) + 4*a^5*sgn(b*x^3 + a))/x^7
```

3.72 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}-$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (a^4*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(a + b*x^3) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a +
b*x^3)) + (b^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rubi [A] time = 0.059024, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (a^4*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(a + b*x^3) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a +
b*x^3)) + (b^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^9}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(10a^2b^8+\frac{a^5b^5}{x^9}+\frac{5a^4b^6}{x^6}+\frac{10a^3b^7}{x^3}+5ab^9x^3+b^{10}x^6\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}+\frac{10a^2b^3}{}$$

Mathematica [A] time = 0.0214479, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(560a^2b^3x^9-280a^3b^2x^6-56a^4bx^3-7a^5+70ab^4x^{12}+8b^5x^{15}\right)}{56x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-7*a^5 - 56*a^4*b*x^3 - 280*a^3*b^2*x^6 + 560*a^2*b^3
*x^9 + 70*a*b^4*x^12 + 8*b^5*x^15))/(56*x^8*(a + b*x^3))
```

Maple [A] time = 0.006, size = 80, normalized size = 0.3

$$-\frac{-8\,b^5x^{15}-70\,ab^4x^{12}-560\,a^2b^3x^9+280\,a^3b^2x^6+56\,a^4bx^3+7\,a^5}{56\,x^8\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x)
```

```
[Out] -1/56*(-8*b^5*x^15-70*a*b^4*x^12-560*a^2*b^3*x^9+280*a^3*b^2*x^6+56*a^4*b*x
^3+7*a^5)*((b*x^3+a)^2)^(5/2)/x^8/(b*x^3+a)^5
```

Maxima [A] time = 1.1126, size = 80, normalized size = 0.32

$$\frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a
^4*b*x^3 - 7*a^5)/x^8
```

Fricas [A] time = 1.76352, size = 132, normalized size = 0.53

$$\frac{8 b^5 x^{15} + 70 a b^4 x^{12} + 560 a^2 b^3 x^9 - 280 a^3 b^2 x^6 - 56 a^4 b x^3 - 7 a^5}{56 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a
^4*b*x^3 - 7*a^5)/x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^3\right)^2\right)^{\frac{5}{2}}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**9, x)
```

Giac [A] time = 1.14239, size = 142, normalized size = 0.57

$$\frac{1}{7} b^5 x^7 \mathrm{sgn}\left(b x^3 + a\right) + \frac{5}{4} a b^4 x^4 \mathrm{sgn}\left(b x^3 + a\right) + 10 a^2 b^3 x \mathrm{sgn}\left(b x^3 + a\right) - \frac{40 a^3 b^2 x^6 \mathrm{sgn}\left(b x^3 + a\right) + 8 a^4 b x^3 \mathrm{sgn}\left(b x^3 + a\right.}{8 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7*sgn(b*x^3 + a) + 5/4*a*b^4*x^4*sgn(b*x^3 + a) + 10*a^2*b^3*x*sg
n(b*x^3 + a) - 1/8*(40*a^3*b^2*x^6*sgn(b*x^3 + a) + 8*a^4*b*x^3*sgn(b*x^3 +
 a) + a^5*sgn(b*x^3 + a))/x^8
```

3.73 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a^5\sqrt{a}}{9}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(3*(a + b*x^3)) + (b^5*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6
*(a + b*x^3)) + (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*
x^3)
```

Rubi [A] time = 0.0711647, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a^5\sqrt{a}}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(3*(a + b*x^3)) + (b^5*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6
*(a + b*x^3)) + (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*
x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{10}}dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^4}dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(5ab^9+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^3}+\frac{10a^3b^7}{x^2}+\frac{10a^2b^8}{x}+b^{10}x\right)dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)} + 5$$

Mathematica [A] time = 0.0280952, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-60a^3b^2x^6+180a^2b^3x^9\log(x)-15a^4bx^3-2a^5+30ab^4x^{12}+3b^5x^{15}\right)}{18x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^5 - 15*a^4*b*x^3 - 60*a^3*b^2*x^6 + 30*a*b^4*x^1
2 + 3*b^5*x^15 + 180*a^2*b^3*x^9*Log[x]))/(18*x^9*(a + b*x^3))
```

Maple [A] time = 0.011, size = 82, normalized size = 0.3

$$\frac{3\,b^5x^{15}+30\,ab^4x^{12}+180\,a^2b^3\ln(x)\,x^9-60\,a^3b^2x^6-15\,a^4bx^3-2\,a^5}{18\,\left(bx^3+a\right)^5x^9}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+30*a*b^4*x^12+180*a^2*b^3*ln(x)*x^9-60
*a^3*b^2*x^6-15*a^4*b*x^3-2*a^5)/(b*x^3+a)^5/x^9
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73341, size = 140, normalized size = 0.56

$$\frac{3\,b^5x^{15}+30\,ab^4x^{12}+180\,a^2b^3x^9\log\left(x\right)-60\,a^3b^2x^6-15\,a^4bx^3-2\,a^5}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(3*b^5*x^15 + 30*a*b^4*x^12 + 180*a^2*b^3*x^9*log(x) - 60*a^3*b^2*x^6
- 15*a^4*b*x^3 - 2*a^5)/x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**10, x)
```

Giac [A] time = 1.12624, size = 171, normalized size = 0.68

$$\frac{1}{6}\,b^5x^6\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{3}\,ab^4x^3\mathrm{sgn}\left(bx^3+a\right)+10\,a^2b^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{110\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+60\,a^3b^2x^6\mathrm{sgn}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6*sgn(b*x^3 + a) + 5/3*a*b^4*x^3*sgn(b*x^3 + a) + 10*a^2*b^3*log(
abs(x))*sgn(b*x^3 + a) - 1/18*(110*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*
x^6*sgn(b*x^3 + a) + 15*a^4*b*x^3*sgn(b*x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^
9
```

3.74 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=253

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5ab^4}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(x*(a + b*x^3)) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(2*(a + b*x^3)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3
))
```

Rubi [A] time = 0.0623355, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5ab^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(x*(a + b*x^3)) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(2*(a + b*x^3)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3
))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{11}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{11}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{11}}+\frac{5a^4b^6}{x^8}+\frac{10a^3b^7}{x^5}+\frac{10a^2b^8}{x^2}+5ab^9x+b^{10}x^4\right)dx}{b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)} - 10a^2b$$

Mathematica [A] time = 0.0163053, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(700a^2b^3x^9+175a^3b^2x^6+50a^4bx^3+7a^5-175ab^4x^{12}-14b^5x^{15}\right)}{70x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(7*a^5 + 50*a^4*b*x^3 + 175*a^3*b^2*x^6 + 700*a^2*b^3
*x^9 - 175*a*b^4*x^12 - 14*b^5*x^15))/(70*x^10*(a + b*x^3))
```

Maple [A] time = 0.006, size = 80, normalized size = 0.3

$$-\frac{-14\,b^5x^{15}-175\,ab^4x^{12}+700\,a^2b^3x^9+175\,a^3b^2x^6+50\,a^4bx^3+7\,a^5}{70\,x^{10}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x)
```

```
[Out] -1/70*(-14*b^5*x^15-175*a*b^4*x^12+700*a^2*b^3*x^9+175*a^3*b^2*x^6+50*a^4*b
*x^3+7*a^5)*((b*x^3+a)^2)^(5/2)/x^10/(b*x^3+a)^5
```

Maxima [A] time = 1.06051, size = 80, normalized size = 0.32

$$\frac{14\,b^5x^{15}+175\,ab^4x^{12}-700\,a^2b^3x^9-175\,a^3b^2x^6-50\,a^4bx^3-7\,a^5}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] 1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 50
*a^4*b*x^3 - 7*a^5)/x^10
```

Fricas [A] time = 1.77641, size = 136, normalized size = 0.54

$$\frac{14\,b^5x^{15} + 175\,ab^4x^{12} - 700\,a^2b^3x^9 - 175\,a^3b^2x^6 - 50\,a^4bx^3 - 7\,a^5}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 50
*a^4*b*x^3 - 7*a^5)/x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**11, x)
```

Giac [A] time = 1.11812, size = 146, normalized size = 0.58

$$\frac{1}{5}\,b^5x^5\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{2}\,ab^4x^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{700\,a^2b^3x^9\mathrm{sgn}\left(bx^3 + a\right) + 175\,a^3b^2x^6\mathrm{sgn}\left(bx^3 + a\right) + 50\,a^4bx^3\mathrm{sgn}\left(bx\right.}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5*sgn(b*x^3 + a) + 5/2*a*b^4*x^2*sgn(b*x^3 + a) - 1/70*(700*a^2*b
^3*x^9*sgn(b*x^3 + a) + 175*a^3*b^2*x^6*sgn(b*x^3 + a) + 50*a^4*b*x^3*sgn(b
*x^3 + a) + 7*a^5*sgn(b*x^3 + a))/x^10
```

3.75 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=247

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}+\frac{5ab^4x\sqrt{a}}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (2*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(x^2*(a + b*x^3)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
a + b*x^3) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rubi [A] time = 0.0601738, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}+\frac{5ab^4x\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (2*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(x^2*(a + b*x^3)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
a + b*x^3) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{12}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{12}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(5ab^9+\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^9}+\frac{10a^3b^7}{x^6}+\frac{10a^2b^8}{x^3}+b^{10}x^3\right)dx}{b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)} - \frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)} - 5a$$

Mathematica [A] time = 0.0160328, size = 83, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(440a^2b^3x^9+176a^3b^2x^6+55a^4bx^3+8a^5-440ab^4x^{12}-22b^5x^{15}\right)}{88x^{11}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(8*a^5 + 55*a^4*b*x^3 + 176*a^3*b^2*x^6 + 440*a^2*b^3
*x^9 - 440*a*b^4*x^12 - 22*b^5*x^15))/(88*x^11*(a + b*x^3))
```

Maple [A] time = 0.006, size = 80, normalized size = 0.3

$$-\frac{-22\,b^5x^{15}-440\,ab^4x^{12}+440\,a^2b^3x^9+176\,a^3b^2x^6+55\,a^4bx^3+8\,a^5}{88\,x^{11}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x)
```

```
[Out] -1/88*(-22*b^5*x^15-440*a*b^4*x^12+440*a^2*b^3*x^9+176*a^3*b^2*x^6+55*a^4*b
*x^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^11/(b*x^3+a)^5
```

Maxima [A] time = 1.01425, size = 80, normalized size = 0.32

$$\frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] 1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 55
*a^4*b*x^3 - 8*a^5)/x^11
```

Fricas [A] time = 1.7216, size = 136, normalized size = 0.55

$$\frac{22\,b^{5}x^{15} + 440\,ab^{4}x^{12} - 440\,a^{2}b^{3}x^{9} - 176\,a^{3}b^{2}x^{6} - 55\,a^{4}bx^{3} - 8\,a^{5}}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] 1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 55
*a^4*b*x^3 - 8*a^5)/x^11
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**12, x)
```

Giac [A] time = 1.14235, size = 143, normalized size = 0.58

$$\frac{1}{4}\,b^{5}x^{4}\mathrm{sgn}\left(bx^{3} + a\right) + 5\,ab^{4}x\mathrm{sgn}\left(bx^{3} + a\right) - \frac{440\,a^{2}b^{3}x^{9}\mathrm{sgn}\left(bx^{3} + a\right) + 176\,a^{3}b^{2}x^{6}\mathrm{sgn}\left(bx^{3} + a\right) + 55\,a^{4}bx^{3}\mathrm{sgn}\left(bx^{3} + a\right.}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4*sgn(b*x^3 + a) + 5*a*b^4*x*sgn(b*x^3 + a) - 1/88*(440*a^2*b^3*x
^9*sgn(b*x^3 + a) + 176*a^3*b^2*x^6*sgn(b*x^3 + a) + 55*a^4*b*x^3*sgn(b*x^3
 + a) + 8*a^5*sgn(b*x^3 + a))/x^11
```

3.76 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{13}}\,dx$

Optimal. Leaf size=252

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{b^5x^3}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(3*(a + b*x^3)) + (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b
*x^3)
```

Rubi [A] time = 0.0705895, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{b^5x^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(3*(a + b*x^3)) + (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b
*x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{13}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{13}}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^5}\,dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(b^{10}+\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^3}+\frac{10a^2b^8}{x^2}+\frac{5ab^9}{x}\right) dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)} - \frac{10a^2b}{}$$

Mathematica [A] time = 0.0195072, size = 85, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(120a^2b^3x^9+60a^3b^2x^6+20a^4bx^3+3a^5-180ab^4x^{12}\log(x)-12b^5x^{15}\right)}{36x^{12}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(3*a^5 + 20*a^4*b*x^3 + 60*a^3*b^2*x^6 + 120*a^2*b^3*
x^9 - 12*b^5*x^15 - 180*a*b^4*x^12*Log[x]))/(36*x^12*(a + b*x^3))
```

Maple [A] time = 0.013, size = 82, normalized size = 0.3

$$\frac{12\,b^5x^{15}+180\,ab^4\ln(x)\,x^{12}-120\,a^2b^3x^9-60\,a^3b^2x^6-20\,a^4bx^3-3\,a^5}{36\,\left(bx^3+a\right)^5x^{12}}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x)
```

```
[Out] 1/36*((b*x^3+a)^2)^(5/2)*(12*b^5*x^15+180*a*b^4*ln(x)*x^12-120*a^2*b^3*x^9-
60*a^3*b^2*x^6-20*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^12
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74709, size = 144, normalized size = 0.57

$$\frac{12\,b^5x^{15} + 180\,ab^4x^{12}\log(x) - 120\,a^2b^3x^9 - 60\,a^3b^2x^6 - 20\,a^4bx^3 - 3\,a^5}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b^5*x^15 + 180*a*b^4*x^12*log(x) - 120*a^2*b^3*x^9 - 60*a^3*b^2*x^
6 - 20*a^4*b*x^3 - 3*a^5)/x^12
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**13, x)
```

Giac [A] time = 1.14565, size = 169, normalized size = 0.67

$$\frac{1}{3}\,b^5x^3\mathrm{sgn}\left(bx^3 + a\right) + 5\,ab^4\log\left(|x|\right)\mathrm{sgn}\left(bx^3 + a\right) - \frac{125\,ab^4x^{12}\mathrm{sgn}\left(bx^3 + a\right) + 120\,a^2b^3x^9\mathrm{sgn}\left(bx^3 + a\right) + 60\,a^3b^2x^6\mathrm{s}}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3*sgn(b*x^3 + a) + 5*a*b^4*log(abs(x))*sgn(b*x^3 + a) - 1/36*(125
*a*b^4*x^12*sgn(b*x^3 + a) + 120*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*x^
6*sgn(b*x^3 + a) + 20*a^4*b*x^3*sgn(b*x^3 + a) + 3*a^5*sgn(b*x^3 + a))/x^12
```

3.77 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{14}} dx$

Optimal. Leaf size=253

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2}}{x(}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^10*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(2*x^4*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(x*(a + b*x^3)) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)
)
```

Rubi [A] time = 0.0594934, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2}}{x(}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^10*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(2*x^4*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(x*(a + b*x^3)) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)
)
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{14}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{14}}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{14}}+\frac{5a^4b^6}{x^{11}}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^5}+\frac{5ab^9}{x^2}+b^{10}x\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)} - 5a$$

Mathematica [A] time = 0.0168434, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(455a^2b^3x^9+260a^3b^2x^6+91a^4bx^3+14a^5+910ab^4x^{12}-91b^5x^{15}\right)}{182x^{13}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(14*a^5 + 91*a^4*b*x^3 + 260*a^3*b^2*x^6 + 455*a^2*b^
3*x^9 + 910*a*b^4*x^12 - 91*b^5*x^15))/(182*x^13*(a + b*x^3))
```

Maple [A] time = 0.005, size = 80, normalized size = 0.3

$$-\frac{-91\,b^5x^{15}+910\,ab^4x^{12}+455\,a^2b^3x^9+260\,a^3b^2x^6+91\,a^4bx^3+14\,a^5}{182\,x^{13}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x)
```

```
[Out] -1/182*(-91*b^5*x^15+910*a*b^4*x^12+455*a^2*b^3*x^9+260*a^3*b^2*x^6+91*a^4*
b*x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^13/(b*x^3+a)^5
```

Maxima [A] time = 1.07088, size = 80, normalized size = 0.32

$$\frac{91\,b^5x^{15}-910\,ab^4x^{12}-455\,a^2b^3x^9-260\,a^3b^2x^6-91\,a^4bx^3-14\,a^5}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 - 9
1*a^4*b*x^3 - 14*a^5)/x^13
```

Fricas [A] time = 1.72346, size = 139, normalized size = 0.55

$$\frac{91\, b^{5} x^{15} - 910\, a b^{4} x^{12} - 455\, a^{2} b^{3} x^{9} - 260\, a^{3} b^{2} x^{6} - 91\, a^{4} b x^{3} - 14\, a^{5}}{182\, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 - 9
1*a^4*b*x^3 - 14*a^5)/x^13
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{14}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**14, x)
```

Giac [A] time = 1.12583, size = 146, normalized size = 0.58

$$\frac{1}{2}\, b^{5} x^{2} \mathrm{sgn}\left(b x^{3} + a\right) - \frac{910\, a b^{4} x^{12} \mathrm{sgn}\left(b x^{3} + a\right) + 455\, a^{2} b^{3} x^{9} \mathrm{sgn}\left(b x^{3} + a\right) + 260\, a^{3} b^{2} x^{6} \mathrm{sgn}\left(b x^{3} + a\right) + 91\, a^{4} b x^{3} \mathrm{sgn}\left(b x^{3}\right.}{182\, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2*sgn(b*x^3 + a) - 1/182*(910*a*b^4*x^12*sgn(b*x^3 + a) + 455*a^2
*b^3*x^9*sgn(b*x^3 + a) + 260*a^3*b^2*x^6*sgn(b*x^3 + a) + 91*a^4*b*x^3*sgn
(b*x^3 + a) + 14*a^5*sgn(b*x^3 + a))/x^13
```

3.78 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}} dx$

Optimal. Leaf size=248

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5ab^4\sqrt{}}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(2*x^2*(a + b*x^3)) + (b^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rubi [A] time = 0.0598894, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5ab^4\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(2*x^2*(a + b*x^3)) + (b^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{15}}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^{10}+\frac{a^5b^5}{x^{15}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^9}+\frac{10a^2b^8}{x^6}+\frac{5ab^9}{x^3}\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)} - \frac{2a^2b^3}{}$$

Mathematica [A] time = 0.0180983, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(616a^2b^3x^9+385a^3b^2x^6+140a^4bx^3+22a^5+770ab^4x^{12}-308b^5x^{15}\right)}{308x^{14}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(22*a^5 + 140*a^4*b*x^3 + 385*a^3*b^2*x^6 + 616*a^2*b
^3*x^9 + 770*a*b^4*x^12 - 308*b^5*x^15))/(308*x^14*(a + b*x^3))
```

Maple [A] time = 0.006, size = 80, normalized size = 0.3

$$-\frac{-308\,b^5x^{15}+770\,ab^4x^{12}+616\,a^2b^3x^9+385\,a^3b^2x^6+140\,a^4bx^3+22\,a^5}{308\,x^{14}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x)
```

```
[Out] -1/308*(-308*b^5*x^15+770*a*b^4*x^12+616*a^2*b^3*x^9+385*a^3*b^2*x^6+140*a^
4*b*x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^14/(b*x^3+a)^5
```

Maxima [A] time = 1.00807, size = 80, normalized size = 0.32

$$\frac{308\,b^5x^{15}-770\,ab^4x^{12}-616\,a^2b^3x^9-385\,a^3b^2x^6-140\,a^4bx^3-22\,a^5}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="maxima")
```

```
[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 -
140*a^4*b*x^3 - 22*a^5)/x^14
```

Fricas [A] time = 1.68173, size = 142, normalized size = 0.57

$$\frac{308\,b^5x^{15} - 770\,ab^4x^{12} - 616\,a^2b^3x^9 - 385\,a^3b^2x^6 - 140\,a^4bx^3 - 22\,a^5}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="fricas")
```

```
[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 -
140*a^4*b*x^3 - 22*a^5)/x^14
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**15, x)
```

Giac [A] time = 1.12006, size = 142, normalized size = 0.57

$$b^5x\operatorname{sgn}\left(bx^3 + a\right) - \frac{770\,ab^4x^{12}\operatorname{sgn}\left(bx^3 + a\right) + 616\,a^2b^3x^9\operatorname{sgn}\left(bx^3 + a\right) + 385\,a^3b^2x^6\operatorname{sgn}\left(bx^3 + a\right) + 140\,a^4bx^3\operatorname{sgn}\left(b\right.}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="giac")
```

```
[Out] b^5*x*sgn(b*x^3 + a) - 1/308*(770*a*b^4*x^12*sgn(b*x^3 + a) + 616*a^2*b^3*x
^9*sgn(b*x^3 + a) + 385*a^3*b^2*x^6*sgn(b*x^3 + a) + 140*a^4*b*x^3*sgn(b*x^
3 + a) + 22*a^5*sgn(b*x^3 + a))/x^14
```

3.79 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{16}}\,dx$

Optimal. Leaf size=251

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a}}{3x}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*x^15*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(3*x^3*(a + b*x^3)) + (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
 b*x^3)
```

Rubi [A] time = 0.069104, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*x^15*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(3*x^3*(a + b*x^3)) + (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
 b*x^3)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{16}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^6}\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^3}+\frac{5ab^9}{x^2}+\frac{b^{10}}{x}\right)dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)} - 5$$

Mathematica [A] time = 0.0269838, size = 85, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a\left(200a^2b^2x^6+75a^3bx^3+12a^4+300ab^3x^9+300b^4x^{12}\right)-180b^5x^{15}\log(x)\right)}{180x^{15}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a*(12*a^4 + 75*a^3*b*x^3 + 200*a^2*b^2*x^6 + 300*a*b
^3*x^9 + 300*b^4*x^12) - 180*b^5*x^15*Log[x]))/(180*x^15*(a + b*x^3))
```

Maple [A] time = 0.012, size = 82, normalized size = 0.3

$$\frac{180\,b^5\ln(x)\,x^{15}-300\,ab^4x^{12}-300\,a^2b^3x^9-200\,a^3b^2x^6-75\,a^4bx^3-12\,a^5}{180\,\left(bx^3+a\right)^5x^{15}}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x)
```

```
[Out] 1/180*((b*x^3+a)^2)^(5/2)*(180*b^5*ln(x)*x^15-300*a*b^4*x^12-300*a^2*b^3*x^
9-200*a^3*b^2*x^6-75*a^4*b*x^3-12*a^5)/(b*x^3+a)^5/x^15
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6705, size = 150, normalized size = 0.6

$$\frac{180\, b^{5} x^{15} \log\left(x\right) - 300\, a b^{4} x^{12} - 300\, a^{2} b^{3} x^{9} - 200\, a^{3} b^{2} x^{6} - 75\, a^{4} b x^{3} - 12\, a^{5}}{180\, x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] 1/180*(180*b^5*x^15*log(x) - 300*a*b^4*x^12 - 300*a^2*b^3*x^9 - 200*a^3*b^2
*x^6 - 75*a^4*b*x^3 - 12*a^5)/x^15
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**16, x)
```

Giac [A] time = 1.11427, size = 166, normalized size = 0.66

$$b^{5} \log\left(|x|\right) \mathrm{sgn}\left(b x^{3} + a\right) - \frac{137\, b^{5} x^{15} \mathrm{sgn}\left(b x^{3} + a\right) + 300\, a b^{4} x^{12} \mathrm{sgn}\left(b x^{3} + a\right) + 300\, a^{2} b^{3} x^{9} \mathrm{sgn}\left(b x^{3} + a\right) + 200\, a^{3} b^{2} x^{6} \mathrm{sg}}{180\, x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] b^5*log(abs(x))*sgn(b*x^3 + a) - 1/180*(137*b^5*x^15*sgn(b*x^3 + a) + 300*a
*b^4*x^12*sgn(b*x^3 + a) + 300*a^2*b^3*x^9*sgn(b*x^3 + a) + 200*a^3*b^2*x^6
*sgn(b*x^3 + a) + 75*a^4*b*x^3*sgn(b*x^3 + a) + 12*a^5*sgn(b*x^3 + a))/x^15
```

3.80 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{17}}\,dx$

Optimal. Leaf size=251

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-5ab^4$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(4*x^4*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)
)
```

Rubi [A] time = 0.0581628, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-5ab^4$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(4*x^4*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)
)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{17}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{17}}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{17}}+\frac{5a^4b^6}{x^{14}}+\frac{10a^3b^7}{x^{11}}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^5}+\frac{b^{10}}{x^2}\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)} - \frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)} - \frac{10a^2b^3}{}$$

Mathematica [A] time = 0.0169142, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2080a^2b^3x^9+1456a^3b^2x^6+560a^4bx^3+91a^5+1820ab^4x^{12}+1456b^5x^{15}\right)}{1456x^{16}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(91*a^5 + 560*a^4*b*x^3 + 1456*a^3*b^2*x^6 + 2080*a^2
*b^3*x^9 + 1820*a*b^4*x^12 + 1456*b^5*x^15))/(1456*x^16*(a + b*x^3))
```

Maple [A] time = 0.006, size = 80, normalized size = 0.3

$$-\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x)
```

```
[Out] -1/1456*(1456*b^5*x^15+1820*a*b^4*x^12+2080*a^2*b^3*x^9+1456*a^3*b^2*x^6+56
0*a^4*b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^16/(b*x^3+a)^5
```

Maxima [A] time = 1.01842, size = 80, normalized size = 0.32

$$-\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2*
x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16
```

Fricas [A] time = 1.74611, size = 150, normalized size = 0.6

$$-\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2*
x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**17, x)
```

Giac [A] time = 1.10355, size = 144, normalized size = 0.57

$$-\frac{1456\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+1820\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+2080\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+1456\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+560}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/1456*(1456*b^5*x^15*sgn(b*x^3 + a) + 1820*a*b^4*x^12*sgn(b*x^3 + a) + 20
80*a^2*b^3*x^9*sgn(b*x^3 + a) + 1456*a^3*b^2*x^6*sgn(b*x^3 + a) + 560*a^4*b
*x^3*sgn(b*x^3 + a) + 91*a^5*sgn(b*x^3 + a))/x^16
```

3.81 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{18}}\,dx$

Optimal. Leaf size=253

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2}}{x^5}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(x^5*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b
*x^3))
```

Rubi [A] time = 0.0580731, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2}}{x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(x^5*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b
*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{18}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{18}}+\frac{5a^4b^6}{x^{15}}+\frac{10a^3b^7}{x^{12}}+\frac{10a^2b^8}{x^9}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^3}\right)dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - 5$$

Mathematica [A] time = 0.0193082, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\,(6545a^2b^3x^9+4760a^3b^2x^6+1870a^4bx^3+308a^5+5236ab^4x^{12}+2618b^5x^{15})}{5236x^{17}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(308*a^5 + 1870*a^4*b*x^3 + 4760*a^3*b^2*x^6 + 6545*a
^2*b^3*x^9 + 5236*a*b^4*x^12 + 2618*b^5*x^15))/(5236*x^17*(a + b*x^3))
```

Maple [A] time = 0.007, size = 80, normalized size = 0.3

$$-\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}(bx^3+a)^5}\left((bx^3+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x)
```

```
[Out] -1/5236*(2618*b^5*x^15+5236*a*b^4*x^12+6545*a^2*b^3*x^9+4760*a^3*b^2*x^6+18
70*a^4*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x^17/(b*x^3+a)^5
```

Maxima [A] time = 0.993811, size = 80, normalized size = 0.32

$$-\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2*
x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

Fricas [A] time = 1.68313, size = 153, normalized size = 0.6

$$-\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2*
x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{18}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**18,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**18, x)
```

Giac [A] time = 1.12291, size = 144, normalized size = 0.57

$$-\frac{2618\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+5236\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+6545\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+4760\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+1870\,a^4}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] -1/5236*(2618*b^5*x^15*sgn(b*x^3 + a) + 5236*a*b^4*x^12*sgn(b*x^3 + a) + 65
45*a^2*b^3*x^9*sgn(b*x^3 + a) + 4760*a^3*b^2*x^6*sgn(b*x^3 + a) + 1870*a^4*
b*x^3*sgn(b*x^3 + a) + 308*a^5*sgn(b*x^3 + a))/x^17
```

3.82 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{19}}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*a*x^18)
```

Rubi [A] time = 0.0184203, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 264}

$$-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*a*x^18)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{19}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{19}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}
\end{aligned}$$

Mathematica [A] time = 0.0164692, size = 81, normalized size = 1.98

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(20a^2b^3x^9+15a^3b^2x^6+6a^4bx^3+a^5+15ab^4x^{12}+6b^5x^{15}\right)}{18x^{18}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a^5 + 6*a^4*b*x^3 + 15*a^3*b^2*x^6 + 20*a^2*b^3*x^9
+ 15*a*b^4*x^12 + 6*b^5*x^15))/(18*x^18*(a + b*x^3))
```

Maple [B] time = 0.005, size = 78, normalized size = 1.9

$$-\frac{6\,b^{5}x^{15}+15\,ab^{4}x^{12}+20\,a^{2}b^{3}x^{9}+15\,a^{3}b^{2}x^{6}+6\,a^{4}bx^{3}+a^{5}}{18\,x^{18}\left(bx^{3}+a\right)^{5}}\left(\left(bx^{3}+a\right)^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x)
```

```
[Out] -1/18*(6*b^5*x^15+15*a*b^4*x^12+20*a^2*b^3*x^9+15*a^3*b^2*x^6+6*a^4*b*x^3+a
^5)*((b*x^3+a)^2)^(5/2)/x^18/(b*x^3+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.80635, size = 128, normalized size = 3.12

$$-\frac{6\,b^{5}x^{15}+15\,ab^{4}x^{12}+20\,a^{2}b^{3}x^{9}+15\,a^{3}b^{2}x^{6}+6\,a^{4}bx^{3}+a^{5}}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^4
*b*x^3 + a^5)/x^18
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**19,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**19, x)
```

Giac [B] time = 1.12067, size = 143, normalized size = 3.49

$$-\frac{6\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+15\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+20\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+15\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+6\,a^4bx^3\mathrm{sgn}\left(bx^3\right.}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="giac")
```

```
[Out] -1/18*(6*b^5*x^15*sgn(b*x^3 + a) + 15*a*b^4*x^12*sgn(b*x^3 + a) + 20*a^2*b^
3*x^9*sgn(b*x^3 + a) + 15*a^3*b^2*x^6*sgn(b*x^3 + a) + 6*a^4*b*x^3*sgn(b*x^
3 + a) + a^5*sgn(b*x^3 + a))/x^18
```

3.83 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{20}}\,dx$

Optimal. Leaf size=253

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2}}{7x^7}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^2*b^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])/(7*x^7*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a +
b*x^3))
```

Rubi [A] time = 0.0614571, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^2*b^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])/(7*x^7*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a +
b*x^3))
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{20}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{20}}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{20}}+\frac{5a^4b^6}{x^{17}}+\frac{10a^3b^7}{x^{14}}+\frac{10a^2b^8}{x^{11}}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^5}\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)} - \frac{a}{}$$

Mathematica [A] time = 0.0191013, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(27664a^2b^3x^9+21280a^3b^2x^6+8645a^4bx^3+1456a^5+19760ab^4x^{12}+6916b^5x^{15}\right)}{27664x^{19}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(1456*a^5 + 8645*a^4*b*x^3 + 21280*a^3*b^2*x^6 + 2766
4*a^2*b^3*x^9 + 19760*a*b^4*x^12 + 6916*b^5*x^15))/(27664*x^19*(a + b*x^3))
```

Maple [A] time = 0.007, size = 80, normalized size = 0.3

$$-\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x)
```

```
[Out] -1/27664*(6916*b^5*x^15+19760*a*b^4*x^12+27664*a^2*b^3*x^9+21280*a^3*b^2*x^
6+8645*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/x^19/(b*x^3+a)^5
```

Maxima [A] time = 1.02547, size = 80, normalized size = 0.32

$$-\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="maxima")
```

```
[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3*
b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

Fricas [A] time = 1.78799, size = 159, normalized size = 0.63

$$-\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3*
b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{20}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**20,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**20, x)
```

Giac [A] time = 1.11302, size = 144, normalized size = 0.57

$$-\frac{6916\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+19760\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+27664\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+21280\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+864}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="giac")
```

```
[Out] -1/27664*(6916*b^5*x^15*sgn(b*x^3 + a) + 19760*a*b^4*x^12*sgn(b*x^3 + a) +
27664*a^2*b^3*x^9*sgn(b*x^3 + a) + 21280*a^3*b^2*x^6*sgn(b*x^3 + a) + 8645*
a^4*b*x^3*sgn(b*x^3 + a) + 1456*a^5*sgn(b*x^3 + a))/x^19
```

3.84 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{21}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5ab^4}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*x^20*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(8*x^8*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(
a + b*x^3))
```

Rubi [A] time = 0.0563872, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5ab^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*x^20*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(8*x^8*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(
a + b*x^3))
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{21}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{21}}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{21}}+\frac{5a^4b^6}{x^{18}}+\frac{10a^3b^7}{x^{15}}+\frac{10a^2b^8}{x^{12}}+\frac{5ab^9}{x^9}+\frac{b^{10}}{x^6}\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)} - 10a^2b$$

Mathematica [A] time = 0.0178733, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(47600a^2b^3x^9+37400a^3b^2x^6+15400a^4bx^3+2618a^5+32725ab^4x^{12}+10472b^5x^{15}\right)}{52360x^{20}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(2618*a^5 + 15400*a^4*b*x^3 + 37400*a^3*b^2*x^6 + 476
00*a^2*b^3*x^9 + 32725*a*b^4*x^12 + 10472*b^5*x^15))/(52360*x^20*(a + b*x^3
))
```

Maple [A] time = 0.007, size = 80, normalized size = 0.3

$$-\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x)
```

```
[Out] -1/52360*(10472*b^5*x^15+32725*a*b^4*x^12+47600*a^2*b^3*x^9+37400*a^3*b^2*x
^6+15400*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/x^20/(b*x^3+a)^5
```

Maxima [A] time = 1.03713, size = 80, normalized size = 0.31

$$-\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="maxima")
```

```
[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^3
*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

Fricas [A] time = 1.72021, size = 162, normalized size = 0.64

$$-\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="fricas")
```

```
[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^3
*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{21}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**21,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**21, x)
```

Giac [A] time = 1.11415, size = 144, normalized size = 0.56

$$-\frac{10472\,b^5x^{15}\operatorname{sgn}\left(bx^3+a\right)+32725\,ab^4x^{12}\operatorname{sgn}\left(bx^3+a\right)+47600\,a^2b^3x^9\operatorname{sgn}\left(bx^3+a\right)+37400\,a^3b^2x^6\operatorname{sgn}\left(bx^3+a\right)+}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="giac")
```

```
[Out] -1/52360*(10472*b^5*x^15*sgn(b*x^3 + a) + 32725*a*b^4*x^12*sgn(b*x^3 + a) +
 47600*a^2*b^3*x^9*sgn(b*x^3 + a) + 37400*a^3*b^2*x^6*sgn(b*x^3 + a) + 1540
0*a^4*b*x^3*sgn(b*x^3 + a) + 2618*a^5*sgn(b*x^3 + a))/x^20
```

3.85 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx$

Optimal. Leaf size=84

$$\frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}}$$

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*a*x^21) + (b*(a + b*x^
3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(126*a^2*x^18)
```

Rubi [A] time = 0.0400933, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1355, 266, 45, 37}

$$\frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*a*x^21) + (b*(a + b*x^
3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(126*a^2*x^18)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{22}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^8}\,dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^7}\,dx, x, x^3\right)}{21ab^3\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}$$

Mathematica [A] time = 0.0167755, size = 83, normalized size = 0.99

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(105a^2b^3x^9+84a^3b^2x^6+35a^4bx^3+6a^5+70ab^4x^{12}+21b^5x^{15}\right)}{126x^{21}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(6*a^5 + 35*a^4*b*x^3 + 84*a^3*b^2*x^6 + 105*a^2*b^3*
x^9 + 70*a*b^4*x^12 + 21*b^5*x^15))/(126*x^21*(a + b*x^3))
```

Maple [A] time = 0.009, size = 80, normalized size = 1.

$$-\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x)
```

```
[Out] -1/126*(21*b^5*x^15+70*a*b^4*x^12+105*a^2*b^3*x^9+84*a^3*b^2*x^6+35*a^4*b*x
^3+6*a^5)*((b*x^3+a)^2)^(5/2)/x^21/(b*x^3+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73008, size = 136, normalized size = 1.62

$$-\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="fricas")
```

```
[Out] -1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 35
*a^4*b*x^3 + 6*a^5)/x^21
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**22,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**22, x)
```

Giac [A] time = 1.11332, size = 144, normalized size = 1.71

$$-\frac{21\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+70\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+105\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+84\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+35\,a^4bx^3\mathrm{sgn}\left(bx\right.}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="giac")
```

```
[Out] -1/126*(21*b^5*x^15*sgn(b*x^3 + a) + 70*a*b^4*x^12*sgn(b*x^3 + a) + 105*a^2
*b^3*x^9*sgn(b*x^3 + a) + 84*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(
b*x^3 + a) + 6*a^5*sgn(b*x^3 + a))/x^21
```

3.86 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{ab^4\sqrt{}}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*x^22*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(8*x^16*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6])/(2*x^10*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a
 + b*x^3))
```

Rubi [A] time = 0.0580603, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{ab^4\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*x^22*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(8*x^16*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6])/(2*x^10*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a
 + b*x^3))
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{23}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{23}}+\frac{5a^4b^6}{x^{20}}+\frac{10a^3b^7}{x^{17}}+\frac{10a^2b^8}{x^{14}}+\frac{5ab^9}{x^{11}}+\frac{b^{10}}{x^8}\right)dx}{b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}\left(a+bx^3\right)} - \frac{10a^2b}{}$$

Mathematica [A] time = 0.0171454, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(117040a^2b^3x^9+95095a^3b^2x^6+40040a^4bx^3+6916a^5+76076ab^4x^{12}+21736b^5x^{15}\right)}{152152x^{22}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(6916*a^5 + 40040*a^4*b*x^3 + 95095*a^3*b^2*x^6 + 117
040*a^2*b^3*x^9 + 76076*a*b^4*x^12 + 21736*b^5*x^15))/(152152*x^22*(a + b*x
^3))
```

Maple [A] time = 0.007, size = 80, normalized size = 0.3

$$-\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x)
```

```
[Out] -1/152152*(21736*b^5*x^15+76076*a*b^4*x^12+117040*a^2*b^3*x^9+95095*a^3*b^2
*x^6+40040*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/x^22/(b*x^3+a)^5
```

Maxima [A] time = 1.00452, size = 80, normalized size = 0.31

$$-\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="maxima")
```

```
[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*a
^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

Fricas [A] time = 1.65463, size = 165, normalized size = 0.65

$$-\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="fricas")
```

```
[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*a
^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{23}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**23,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**23, x)
```

Giac [A] time = 1.12912, size = 144, normalized size = 0.56

$$-\frac{21736\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+76076\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+117040\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+95095\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="giac")
```

```
[Out] -1/152152*(21736*b^5*x^15*sgn(b*x^3 + a) + 76076*a*b^4*x^12*sgn(b*x^3 + a)
+ 117040*a^2*b^3*x^9*sgn(b*x^3 + a) + 95095*a^3*b^2*x^6*sgn(b*x^3 + a) + 40
040*a^4*b*x^3*sgn(b*x^3 + a) + 6916*a^5*sgn(b*x^3 + a))/x^22
```

3.87 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{24}} dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2}}{11x^1}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*x^23*(a + b*x^3)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^20*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(11*x^11*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a
 + b*x^3))
```

Rubi [A] time = 0.0564672, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2}}{11x^1}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*x^23*(a + b*x^3)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^20*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(11*x^11*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a
 + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{24}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{24}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{24}}+\frac{5a^4b^6}{x^{21}}+\frac{10a^3b^7}{x^{18}}+\frac{10a^2b^8}{x^{15}}+\frac{5ab^9}{x^{12}}+\frac{b^{10}}{x^9}\right)dx}{b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}\left(a+bx^3\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)} - 5a$$

Mathematica [A] time = 0.017519, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(172040a^2b^3x^9+141680a^3b^2x^6+60214a^4bx^3+10472a^5+109480ab^4x^{12}+30107b^5x^{15}\right)}{240856x^{23}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(10472*a^5 + 60214*a^4*b*x^3 + 141680*a^3*b^2*x^6 + 1
72040*a^2*b^3*x^9 + 109480*a*b^4*x^12 + 30107*b^5*x^15))/(240856*x^23*(a +
b*x^3))
```

Maple [A] time = 0.007, size = 80, normalized size = 0.3

$$-\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x)
```

```
[Out] -1/240856*(30107*b^5*x^15+109480*a*b^4*x^12+172040*a^2*b^3*x^9+141680*a^3*b
^2*x^6+60214*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/x^23/(b*x^3+a)^5
```

Maxima [A] time = 1.03738, size = 80, normalized size = 0.31

$$-\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 141680
*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

Fricas [A] time = 1.69174, size = 169, normalized size = 0.66

$$-\frac{30107\, b^{5} x^{15} + 109480\, a b^{4} x^{12} + 172040\, a^{2} b^{3} x^{9} + 141680\, a^{3} b^{2} x^{6} + 60214\, a^{4} b x^{3} + 10472\, a^{5}}{240856\, x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 141680
*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{24}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**24,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**24, x)
```

Giac [A] time = 1.12606, size = 144, normalized size = 0.56

$$-\frac{30107\, b^{5} x^{15} \mathrm{sgn}\left(b x^{3} + a\right) + 109480\, a b^{4} x^{12} \mathrm{sgn}\left(b x^{3} + a\right) + 172040\, a^{2} b^{3} x^{9} \mathrm{sgn}\left(b x^{3} + a\right) + 141680\, a^{3} b^{2} x^{6} \mathrm{sgn}\left(b x^{3} + a\right) +}{240856\, x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="giac")
```

```
[Out] -1/240856*(30107*b^5*x^15*sgn(b*x^3 + a) + 109480*a*b^4*x^12*sgn(b*x^3 + a)
 + 172040*a^2*b^3*x^9*sgn(b*x^3 + a) + 141680*a^3*b^2*x^6*sgn(b*x^3 + a) +
60214*a^4*b*x^3*sgn(b*x^3 + a) + 10472*a^5*sgn(b*x^3 + a))/x^23
```

3.88 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}\,dx$

Optimal. Leaf size=128

$$-\frac{b^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{504a^3x^{18}}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{84a^2x^{21}}-\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{24ax^{24}}$$

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(24*a*x^24) + (b*(a + b*x^
3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(84*a^2*x^21) - (b^2*(a + b*x^3)^5*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(504*a^3*x^18)
```

Rubi [A] time = 0.0566844, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1355, 266, 45, 37}

$$-\frac{b^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{504a^3x^{18}}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{84a^2x^{21}}-\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{24ax^{24}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(24*a*x^24) + (b*(a + b*x^
3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(84*a^2*x^21) - (b^2*(a + b*x^3)^5*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(504*a^3*x^18)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{25}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^9}\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^8}\,dx,x,x^3\right)}{12ab^3\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}} + \frac{\sqrt{a^2+2abx^3+b^2}}{84}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}} - \frac{b^2\left(a+bx^3\right)^5\sqrt{a^2}}{504a^3}$$

Mathematica [A] time = 0.0196098, size = 83, normalized size = 0.65

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(336a^2b^3x^9+280a^3b^2x^6+120a^4bx^3+21a^5+210ab^4x^{12}+56b^5x^{15}\right)}{504x^{24}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(21*a^5 + 120*a^4*b*x^3 + 280*a^3*b^2*x^6 + 336*a^2*b
^3*x^9 + 210*a*b^4*x^12 + 56*b^5*x^15))/(504*x^24*(a + b*x^3))
```

Maple [A] time = 0.007, size = 80, normalized size = 0.6

$$-\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x)
```

```
[Out] -1/504*(56*b^5*x^15+210*a*b^4*x^12+336*a^2*b^3*x^9+280*a^3*b^2*x^6+120*a^4*
b*x^3+21*a^5)*((b*x^3+a)^2)^(5/2)/x^24/(b*x^3+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73209, size = 142, normalized size = 1.11

$$-\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="fricas")
```

```
[Out] -1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
120*a^4*b*x^3 + 21*a^5)/x^24
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{25}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**25,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**25, x)
```

Giac [A] time = 1.11714, size = 144, normalized size = 1.12

$$-\frac{56\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+210\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+336\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+280\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+120\,a^4bx^3\mathrm{s}}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="giac")
```

```
[Out] -1/504*(56*b^5*x^15*sgn(b*x^3 + a) + 210*a*b^4*x^12*sgn(b*x^3 + a) + 336*a^
2*b^3*x^9*sgn(b*x^3 + a) + 280*a^3*b^2*x^6*sgn(b*x^3 + a) + 120*a^4*b*x^3*s
gn(b*x^3 + a) + 21*a^5*sgn(b*x^3 + a))/x^24
```

3.89 $\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=240

$$\frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\tan}{\sqrt{3}b^{5/3}\sqrt{a^2+2}}$$

```
[Out] (x^2*(a + b*x^3))/(2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x
^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a^(2/3)*(a + b*x^3)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])
```

Rubi [A] time = 0.123902, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1355, 321, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\tan}{\sqrt{3}b^{5/3}\sqrt{a^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x^2*(a + b*x^3))/(2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x
^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a^(2/3)*(a + b*x^3)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{\left(ab+b^2x^3\right)\int \frac{x^4}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a\left(ab+b^2x^3\right)\right)\int \frac{x}{ab+b^2x^3}\,dx}{b\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a^{2/3}\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a^{2/3}\left(ab+b^2x^3\right)\right)\int \frac{\sqrt[3]{\ldots}}{a^{2/3}b^{2/3}\ldots}}{3b^2\sqrt{a^2+2abx^3\ldots}}\\
&= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a^{2/3}\left(ab+b^2x^3\right)\right)\int \frac{-\sqrt[3]{a}b+\ldots}{a^{2/3}b^{2/3}-\sqrt[3]{a}\ldots}}{6b^{8/3}\sqrt{a^2+2abx^3+b\ldots}}\\
&= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{\ldots}\right.}{6b^{5/3}\sqrt{a^2+2abx^3+\ldots}}\\
&= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\ldots\right.}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.0464957, size = 131, normalized size = 0.55

$$\frac{\left(a+bx^3\right)\left(-a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+3b^{2/3}x^2\right)}{6b^{5/3}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x] - a^(2/3)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(5/3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.009, size = 113, normalized size = 0.5

$$\frac{bx^3+a}{6\,b^2}\left(3\,x^2b\sqrt[3]{\frac{a}{b}}+2\,\arctan\left(1/3\,\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\sqrt{3}a+2\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)a-\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)a\right)\frac{1}{\sqrt{(bx^3+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(3*x^2*b*(a/b)^(1/3)+2*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/
(a/b)^(1/3))*3^(1/2)*a+2*ln(x+(a/b)^(1/3))*a-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*a)/((b*x^3+a)^2)^(1/2)/b^2/(a/b)^(1/3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73455, size = 298, normalized size = 1.24

$$\frac{3\,x^2-2\,\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)-\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^
(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a
*(a^2/b^2)^(1/3)) + 2*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

Sympy [A] time = 0.359269, size = 32, normalized size = 0.13

$$\text{RootSum}\left(27t^3b^5 - a^2, \left(t \mapsto t\log\left(\frac{9t^2b^3}{a} + x\right)\right)\right) + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(9*_t**2*b**3/a + x))) + x**
2/(2*b)
```

Giac [A] time = 1.15128, size = 197, normalized size = 0.82

$$\frac{x^2\text{sgn}\left(bx^3 + a\right)}{2\,b} + \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\text{sgn}\left(bx^3 + a\right)}{3\,b} + \frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\text{sgn}\left(bx^3 + a\right)}{3\,b^3} - \frac{\left(-ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*sgn(b*x^3 + a)/b + 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn(
b*x^3 + a)/b + 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^3 - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^3
```

3.90 $\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=235

$$\frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\tan^{-1}}{\sqrt{3}b^{4/3}\sqrt{a^2+2ab}}$$

```
[Out] (x*(a + b*x^3))/(b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(4/3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) - (a^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/
(3*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])
```

Rubi [A] time = 0.113682, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1355, 321, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\tan^{-1}}{\sqrt{3}b^{4/3}\sqrt{a^2+2ab}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(a + b*x^3))/(b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(4/3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) - (a^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/
(3*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{x^3}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a\left(ab+b^2x^3\right)\right)\int \frac{1}{ab+b^2x^3}\,dx}{b\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{a}\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{a}\left(ab+b^2x^3\right)\right)\int \frac{2\sqrt[3]{a}\cdots}{a^{2/3}b^{2/3}-\cdots}}{3b^{5/3}\sqrt{a^2+2abx^3+\cdots}}$$

$$= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(\sqrt[3]{a}\left(ab+b^2x^3\right)\right)\int \frac{-\sqrt[3]{a}b+2b^{4\cdots}}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx\cdots}}{6b^{7/3}\sqrt{a^2+2abx^3+b^2x\cdots}}$$

$$= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\cdots\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2\cdots}}$$

$$= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] time = 0.0312101, size = 128, normalized size = 0.54

$$\frac{\left(a+bx^3\right)\left(\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{b}x\right)}{6b^{4/3}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(4/3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.007, size = 110, normalized size = 0.5

$$\frac{bx^3+a}{6b^2}\left(6xb\left(\frac{a}{b}\right)^{2/3}+2\arctan\left(1/3\sqrt{3}\left(-2x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\sqrt{3}a-2\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)a+\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)a\right)\frac{1}{\sqrt{\left(bx^3\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(6*x*b*(a/b)^(2/3)+2*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a
/b)^(1/3))*3^(1/2)*a-2*ln(x+(a/b)^(1/3))*a+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)*a)/((b*x^3+a)^2)^(1/2)/b^2/(a/b)^(2/3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79736, size = 254, normalized size = 1.08

$$\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+6x}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3
)*a)/a) - (-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b)^
(1/3)*log(x - (-a/b)^(1/3)) + 6*x)/b
```

Sympy [A] time = 0.354967, size = 22, normalized size = 0.09

$$\mathrm{RootSum}\left(27t^3b^4 + a, \left(t \mapsto t\log\left(-3tb + x\right)\right)\right) + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(-3*_t*b + x))) + x/b
```

Giac [A] time = 1.10604, size = 193, normalized size = 0.82

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\mathrm{sgn}\left(bx^3 + a\right)}{3\,b} + \frac{x\mathrm{sgn}\left(bx^3 + a\right)}{b} - \frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\mathrm{sgn}\left(bx^3 + a\right)}{3\,b^2} - \frac{\left(-ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*x^3 + a)/b + x*sgn(b*x^3
+ a)/b - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^2 - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^2
```

3.91 $\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx$

Optimal. Leaf size=44

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.034893, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1352, 608, 31}

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}} \, dx, x, x^3\right)\\
&= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{ab+b^2x} \, dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.0083088, size = 35, normalized size = 0.8

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.007, size = 32, normalized size = 0.7

$$\frac{(bx^3+a)\ln(bx^3+a)}{3b}\frac{1}{\sqrt{(bx^3+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^3+a)*ln(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)
```

Maxima [A] time = 1.07148, size = 23, normalized size = 0.52

$$\frac{1}{3}\sqrt{\frac{1}{b^2}}\log\left(x^3+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b^(-2))*log(x^3 + a/b)
```

Fricas [A] time = 1.59779, size = 30, normalized size = 0.68

$$\frac{\log(bx^3+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(b*x^3 + a)/b
```

Sympy [A] time = 0.168491, size = 10, normalized size = 0.23

$$\frac{\log(a+bx^3)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] log(a + b*x**3)/(3*b)
```

Giac [A] time = 1.12937, size = 30, normalized size = 0.68

$$\frac{\log\left(\left|bx^3 + a\right|\right)\operatorname{sgn}\left(bx^3 + a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(b*x^3 + a))*sgn(b*x^3 + a)/b
```

3.92 $\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=202

$$-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) - ((a + b*x^3)*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a +
 b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.0849865, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1355, 292, 31, 634, 617, 204, 628}

$$-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```


```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) - ((a + b*x^3)*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a +
 b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{x}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{\left(ab+b^2x^3\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3\sqrt[3]{a}b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{3\sqrt[3]{a}b\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{6\sqrt[3]{a}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{}{a^{2/3}b^{2/3}-}}{2b^{2/3}\sqrt{a^2+2abx}}$$

$$= -\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\text{Subs}}{\sqrt[3]{a}b^{5/3}\sqrt{}}$$

$$= -\frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{}\right.}{6\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3}}$$

Mathematica [A] time = 0.024666, size = 109, normalized size = 0.54

$$\frac{\left(a+bx^3\right)\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Lo
g[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(
6*a^(1/3)*b^(2/3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.004, size = 97, normalized size = 0.5

$$-\frac{bx^3+a}{6b}\left(2\sqrt{3}\arctan\left(1/3\sqrt{3}\left(-2x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)+2\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)-\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)\frac{1}{\sqrt{\left(bx^3+a\right)^2}}\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x^3+a)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3)
)+2*ln(x+(a/b)^(1/3))-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))/((b*x^3+a)^2)^(1/2
)/b/(a/b)^(1/3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91323, size = 744, normalized size = 3.68

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2+\left(-ab^2\right)^{\frac{1}{3}}bx+\left(-\right.\right.}{6ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(
1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/
a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^
2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3))
)/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2
*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*log(b^2*
x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a
*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A] time = 0.174381, size = 24, normalized size = 0.12

$$\operatorname{RootSum}\left(27t^3ab^2 + 1, \left(t \mapsto t\log\left(9t^2ab + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

Giac [A] time = 1.12462, size = 184, normalized size = 0.91

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}\left(bx^3+a\right)}{3a}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}\left(bx^3+a\right)}{3ab^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)\right.}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*x^3 + a)/a - 1/3*sqrt(3)
*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))*sgn(b
*x^3 + a)/(a*b^2) + 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))*sgn(b*x^3 + a)/(a*b^2)
```

3.93 $\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=202

$$\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + ((a + b*x^3)*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a +
 b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.117491, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1343, 200, 31, 634, 617, 204, 628}

$$\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + ((a + b*x^3)*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a +
 b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{\left(2ab+2b^2x^3\right)\int \frac{1}{2ab+2b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{\left(2ab+2b^2x^3\right)\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}+\sqrt[3]{2}b^{2/3}x}\,dx}{3\ 2^{2/3}a^{2/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(2ab+2b^2x^3\right)\int \frac{2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}-\sqrt[3]{2}b^{2/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/3}x^2}\,dx}{3\ 2^{2/3}a^{2/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(2ab+2b^2x^3\right)\int \frac{-2^{2/3}\sqrt[3]{a}b+2\ 2^{2/3}b^{4/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/3}x^2}\,dx}{12a^{2/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(2ab+2b^2\right.}{2\sqrt[3]{2}}\\
&= \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(2ab+2b^2x^3\right)\text{Sub}}{2a^{2/3}b^{4/3}\sqrt{}}\\
&= -\frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{}\right.}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3}}
\end{aligned}
$$

Mathematica [A] time = 0.0229603, size = 109, normalized size = 0.54

$$
-\frac{\left(a+bx^3\right)\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{\left(a+bx^3\right)^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -((a + b*x^3)*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Lo
g[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(
6*a^(2/3)*b^(1/3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.004, size = 95, normalized size = 0.5

$$-\frac{bx^3+a}{6b}\left(2\sqrt{3}\arctan\left(1/3\sqrt{3}\left(-2x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)-2\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)+\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)\frac{1}{\sqrt{\left(bx^3+a\right)^2}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x^3+a)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3)
)-2*ln(x+(a/b)^(1/3))+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))/((b*x^3+a)^2)^(1/2
)/b/(a/b)^(2/3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74454, size = 749, normalized size = 3.71

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(a^2b\right)^{\frac{1}{3}}a\right.\cdots}{6\,a^2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^
(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a
^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*lo
g(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x
+ (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [A] time = 0.182363, size = 20, normalized size = 0.1

$$\operatorname{RootSum}\left(27t^3a^2b - 1, \left(t \mapsto t\log\left(3ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

Giac [A] time = 1.12051, size = 165, normalized size = 0.82

$$-\frac{1}{6}\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}\right)\operatorname{sgn}\left(bx^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b^2)^(1
/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b))*sgn(b*x^3 + a)
```

3.94 $\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=80

$$\frac{\log(x)\left(a+bx^3\right)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] ((a + b*x^3)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log
[a + b*x^3])/(3*a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.0339088, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1355, 266, 36, 29, 31}

$$\frac{\log(x)\left(a+bx^3\right)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] ((a + b*x^3)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log
[a + b*x^3])/(3*a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{1}{x\left(ab+b^2x^3\right)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(ab+b^2x^3\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(ab+b^2x^3\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)}{3ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(b\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(a+bx^3\right)\log(x)}{a\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] time = 0.011428, size = 42, normalized size = 0.52

$$\frac{\left(a+bx^3\right)\left(3\log(x)-\log\left(a+bx^3\right)\right)}{3a\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] ((a + b*x^3)*(3*Log[x] - Log[a + b*x^3]))/(3*a*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.007, size = 39, normalized size = 0.5

$$\frac{\left(bx^3+a\right)\left(3\ln(x)-\ln\left(bx^3+a\right)\right)}{3a}\frac{1}{\sqrt{\left(bx^3+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^3+a)*(3*ln(x)-ln(b*x^3+a))/((b*x^3+a)^2)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88243, size = 49, normalized size = 0.61

$$-\frac{\log\left(bx^3+a\right)-3\log(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(log(b*x^3 + a) - 3*log(x))/a
```

Sympy [A] time = 0.262047, size = 15, normalized size = 0.19

$$\frac{\log\left(x\right)}{a} - \frac{\log\left(\frac{a}{b} + x^{3}\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] log(x)/a - log(a/b + x**3)/(3*a)
```

Giac [A] time = 1.1237, size = 43, normalized size = 0.54

$$-\frac{1}{3}\left(\frac{\log\left(\left|bx^{3} + a\right|\right)}{a} - \frac{3\log\left(\left|x\right|\right)}{a}\right)\mathrm{sgn}\left(bx^{3} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(log(abs(b*x^3 + a))/a - 3*log(abs(x))/a)*sgn(b*x^3 + a)
```

3.95 $\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=238

$$-\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\tan}{\sqrt{3}a^{4/3}\sqrt{a^2+2}}$$

```
[Out] -((a + b*x^3)/(a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + (b^(1/3)*(a + b*x^3)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])
/(3*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(1/3)*(a + b*x^3)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])
```

Rubi [A] time = 0.107594, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1355, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\tan}{\sqrt{3}a^{4/3}\sqrt{a^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -((a + b*x^3)/(a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + (b^(1/3)*(a + b*x^3)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])
/(3*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(1/3)*(a + b*x^3)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{\left(ab+b^2x^3\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(b\left(ab+b^2x^3\right)\right)\int \frac{x}{ab+b^2x^3}\,dx}{a\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(ab+b^2x^3\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(ab+b^2x^3\right)\int \frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4}\cdots}}{3a^{4/3}\sqrt{a^2+2abx^3+b^{2}\cdots}}\\
&= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(ab+b^2x^3\right)\int \frac{-\sqrt[3]{a}b+2b^{4}\cdots}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+\cdots}}{6a^{4/3}b^{2/3}\sqrt{a^2+2abx^3+\cdots}}\\
&= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{\cdots}\right.}{6a^{4/3}\sqrt{a^2+2abx^3\cdots}}\\
&= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{3a^{4/3}\sqrt{a^2+2abx^3+b^{2}\cdots}}
\end{aligned}
$$

Mathematica [A] time = 0.0312185, size = 133, normalized size = 0.56

$$-\frac{\left(a+bx^3\right)\left(\sqrt[3]{b}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{b}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{b}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{a}\right)}{6a^{4/3}x\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -((a + b*x^3)*(6*a^(1/3) - 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] - 2*b^(1/3)*x*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*x*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(4/3)*x*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.007, size = 111, normalized size = 0.5

$$-\frac{bx^3+a}{6ax}\left(-2\sqrt{3}\arctan\left(1/3\sqrt{3}\left(-2x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)x-2\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x+\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x+6\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt{\left(bx^3+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x^3+a)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3
))*x-2*ln(x+(a/b)^(1/3))*x+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x+6*(a/b)^(1/3
))/((b*x^3+a)^2)^(1/2)/(a/b)^(1/3)/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83057, size = 262, normalized size = 1.1

$$-\frac{2\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)+6}{6ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3
)) + x*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*x*(b/a)
```

```
^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6)/(a*x)
```

Sympy [A] time = 0.384223, size = 29, normalized size = 0.12

$$\mathrm{RootSum}\left(27t^3a^4 - b, \left(t \mapsto t\log\left(\frac{9t^2a^3}{b} + x\right)\right)\right) - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(9*_t**2*a**3/b + x))) - 1/(a*x
)
```

Giac [A] time = 1.11426, size = 177, normalized size = 0.74

$$\frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2} + \frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b} - \frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b} - \frac{6}{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b^2)^(
2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - (-a*b^
2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 6/(a*x))*sgn(b*
x^3 + a)
```

3.96 $\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=240

$$-\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)}{\sqrt{3}a^{5/3}\sqrt{a^2}}$$

```
[Out] -(a + b*x^3)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*x^
3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x
])/(3*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*x^3)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])
```

Rubi [A] time = 0.107053, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1355, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)}{\sqrt{3}a^{5/3}\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -(a + b*x^3)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*x^
3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x
])/(3*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*x^3)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(b\left(ab+b^2x^3\right)\right)\int \frac{1}{ab+b^2x^3}\,dx}{a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{b}\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{b}\left(ab+b^2x^3\right)\right)\int \frac{}{}}{3a^{5/3}\sqrt{a^2+2}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{-\sqrt[3]{a}b+}{a^{2/3}b^{2/3}-\sqrt[3]{a}}}{6a^{5/3}\sqrt[3]{b}\sqrt{a^2+2abx^3}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}\right.}{6a^{5/3}\sqrt{a^2+2a}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}\right.}{3a^{5/3}\sqrt{a^2+2abx^3+}}$$

Mathematica [A] time = 0.0331999, size = 140, normalized size = 0.58

$$-\frac{(a+bx^3)\left(-b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+3a^{2/3}+2b^{2/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}b^{2/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6a^{5/3}x^2\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -((a + b*x^3)*(3*a^(2/3) - 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]] + 2*b^(2/3)*x^2*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*x^2*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(5/3)*x^2*Sqrt[(a + b*x
^3)^2])
```

Maple [A] time = 0.01, size = 118, normalized size = 0.5

$$-\frac{bx^3+a}{6\,ax^2}\left(-2\sqrt{3}\arctan\left(1/3\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)x^2+2\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x^2-\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x^2+3\left(\frac{a}{b}\right)^{2/3}\right)\frac{1}{\sqrt{(b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x^3+a)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3
))*x^2+2*ln(x+(a/b)^(1/3))*x^2-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^2+3*(a/b
)^(2/3))/((b*x^3+a)^2)^(1/2)/(a/b)^(2/3)/a/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83936, size = 335, normalized size = 1.4

$$\frac{2\sqrt{3}x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2\,x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b}{a}\right.\right.}{6\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2
/3) - sqrt(3)*b)/b) - x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(
1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x^2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2
)^(1/3)) - 3)/(a*x^2)
```

Sympy [A] time = 0.415722, size = 32, normalized size = 0.13

$$\operatorname{RootSum}\left(27t^3a^5 + b^2, \left(t \mapsto t\log\left(-\frac{3ta^2}{b} + x\right)\right)\right) - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**5 + b**2, Lambda(_t, _t*log(-3*_t*a**2/b + x))) - 1/(2*
a*x**2)
```

Giac [A] time = 1.10542, size = 169, normalized size = 0.7

$$\frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2} - \frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2} - \frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2} - \frac{3}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 2*sqrt(3)*(-a*b^2)^(
1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - (-a*b^2)^(
1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/(a*x^2))*sgn(b*x^3 +
a)
```

3.97 $\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=122

$$-\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b\log(x)\left(a+bx^3\right)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -(a + b*x^3)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b*(a + b*x^3)*Log
[x])/(a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b*(a + b*x^3)*Log[a + b*x^3])
/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.0505751, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 44}

$$-\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b\log(x)\left(a+bx^3\right)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -(a + b*x^3)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b*(a + b*x^3)*Log
[x])/(a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b*(a + b*x^3)*Log[a + b*x^3])
/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(ab+b^2x^3)\int \frac{1}{x^4(ab+b^2x^3)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{x^2(ab+b^2x)}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \left(\frac{1}{abx^2}-\frac{1}{a^2x}+\frac{b}{a^2(a+bx)}\right)dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= -\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b(a+bx^3)\log(x)}{a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b(a+bx^3)\log(a+bx^3)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] time = 0.0157451, size = 54, normalized size = 0.44

$$-\frac{(a+bx^3)\left(-bx^3\log(a+bx^3)+a+3bx^3\log(x)\right)}{3a^2x^3\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -((a + b*x^3)*(a + 3*b*x^3*Log[x] - b*x^3*Log[a + b*x^3]))/(3*a^2*x^3*Sqrt[
(a + b*x^3)^2])
```

Maple [A] time = 0.01, size = 51, normalized size = 0.4

$$-\frac{(bx^3+a)\left(3\,b\ln(x)\,x^3-b\ln(bx^3+a)\,x^3+a\right)}{3\,a^2x^3}\frac{1}{\sqrt{(bx^3+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/3*(b*x^3+a)*(3*b*ln(x)*x^3-b*ln(b*x^3+a)*x^3+a)/((b*x^3+a)^2)^(1/2)/a^2/
x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73329, size = 80, normalized size = 0.66

$$\frac{bx^3\log\left(bx^3+a\right)-3\,bx^3\log\left(x\right)-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^3*log(b*x^3 + a) - 3*b*x^3*log(x) - a)/(a^2*x^3)
```

Sympy [A] time = 0.531288, size = 31, normalized size = 0.25

$$-\frac{1}{3ax^3}-\frac{b\log\left(x\right)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] -1/(3*a*x**3) - b*log(x)/a**2 + b*log(a/b + x**3)/(3*a**2)
```

Giac [A] time = 1.1288, size = 68, normalized size = 0.56

$$\frac{1}{3}\left(\frac{b\log\left(\left|bx^3+a\right|\right)}{a^2}-\frac{3\,b\log\left(\left|x\right|\right)}{a^2}+\frac{bx^3-a}{a^2x^3}\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*log(abs(b*x^3 + a))/a^2 - 3*b*log(abs(x))/a^2 + (b*x^3 - a)/(a^2*x^3
))*sgn(b*x^3 + a)
```

3.98 $\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=280

$$\frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{\cdots}\right.}{54a^{4/3}b^{5/3}\sqrt{a^2+2\cdots}}$$

```
[Out] x^2/(9*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(6*b*(a + b*x^3)*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 - ((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(54*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.135253, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 290, 292, 31, 634, 617, 204, 628}

$$\frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{\cdots}\right.}{54a^{4/3}b^{5/3}\sqrt{a^2+2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] x^2/(9*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(6*b*(a + b*x^3)*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 - ((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(54*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^4}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{x}{\left(ab+b^2x^3\right)^2}\,dx}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{x}{ab+b^2x^3}\,d\cdots}{9ab\sqrt{a^2+2abx^3+b^2x^{\cdots}}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(ab+b^2x^3\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/\cdots}}}{27a^{4/3}b^2\sqrt{a^2+2abx^3+\cdots}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\cdots\right.}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3\cdots}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\cdots\right.}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3\cdots}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-\cdots}{\sqrt{3}\cdots}\right.}{9\sqrt{3}a^{4/3}b^{5/3}\sqrt{a^2+2abx\cdots}}$$

Mathematica [A] time = 0.0752993, size = 235, normalized size = 0.84

$$\frac{-3a^{4/3}b^{2/3}x^2 + b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) + 2abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) + a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^{2}\cdots\right.}{54a^{4/3}b^{5/3}\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (-3*a^(4/3)*b^(2/3)*x^2 + 6*a^(1/3)*b^(5/3)*x^5 - 2*Sqrt[3]*(a + b*x^3)^2*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(a + b*x^3)^2*Log[a^(1/3) +
b^(1/3)*x] + a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*b*x^3
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + b^2*x^6*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(5/3)*(a + b*x^3)*Sqrt[(a + b*
x^3)^2])
```

Maple [A] time = 0.016, size = 299, normalized size = 1.1

$$\frac{bx^3+a}{54\,ab^2}\left(-2\sqrt{3}\arctan\left(1/3\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)x^6b^2 - 2\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x^6b^2 + \ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x^6b^2 + 6\sqrt[3]{\frac{a}{b}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*b^2
-2*ln(x+(a/b)^(1/3))*x^6*b^2+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*b^2+6*(a
/b)^(1/3)*x^5*b^2-4*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/
3))*x^3*a*b-4*ln(x+(a/b)^(1/3))*x^3*a*b+2*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
*x^3*a*b-3*(a/b)^(1/3)*x^2*a*b-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/
3))/(a/b)^(1/3))*a^2-2*ln(x+(a/b)^(1/3))*a^2+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*a^2)*(b*x^3+a)/(a/b)^(1/3)/b^2/a/((b*x^3+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82331, size = 1152, normalized size = 4.11

$$\left[\frac{6\,ab^3x^5 - 3\,a^2b^2x^2 + 3\,\sqrt{\frac{1}{3}}\left(ab^3x^6 + 2\,a^2b^2x^3 + a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right.}{54\left(a^2b^5x^6+2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3
 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
+ 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b
^2)^(2/3)*x)/(b*x^3 + a)) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(
b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 + 2*a*b*x^3 + a
^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3
+ a^4*b^3), 1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*
a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*
b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2
*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.99 $\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=276

$$\frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{54a^{5/3}b^{4/3}\sqrt{a^2+2ab}}$$

```
[Out] x/(18*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(6*b*(a + b*x^3)*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) +
((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(54*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.133187, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 199, 200, 31, 634, 617, 204, 628}

$$\frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{54a^{5/3}b^{4/3}\sqrt{a^2+2ab}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] x/(18*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(6*b*(a + b*x^3)*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) +
((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(54*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{1}{\left(ab+b^2x^3\right)^2}\,dx}{6\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{1}{ab+b^2x^3}\,dx}{9ab\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}}{27a^{5/3}b^{5/3}\sqrt{a^2+2abx^3+}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{}}{\sqrt{3}\sqrt[3]{}}\right.}{9\sqrt{3}a^{5/3}b^{4/3}\sqrt{a^2+2abx^3}}$$

Mathematica [A] time = 0.0639138, size = 235, normalized size = 0.85

$$\frac{3a^{2/3}b^{4/3}x^4 - b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - 2abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - 6}{54a^{5/3}b^{4/3}\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (-6*a^(5/3)*b^(1/3)*x + 3*a^(2/3)*b^(4/3)*x^4 - 2*Sqrt[3]*(a + b*x^3)^2*Arc
Tan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(a + b*x^3)^2*Log[a^(1/3) + b^
(1/3)*x] - a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*b*x^3*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - b^2*x^6*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(4/3)*(a + b*x^3)*Sqrt[(a + b*x^
3)^2])
```

Maple [A] time = 0.016, size = 297, normalized size = 1.1

$$-\frac{bx^3+a}{54\,ab^2}\left(2\sqrt{3}\arctan\left(1/3\sqrt{3}\left(-2x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)x^6b^2 - 2\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x^6b^2 + \ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x^6b^2 - 3\left(\frac{a}{b}\right)^{2/3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/54*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*b^2
-2*ln(x+(a/b)^(1/3))*x^6*b^2+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*b^2-3*(a
/b)^(2/3)*x^4*b^2+4*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/
3))*x^3*a*b-4*ln(x+(a/b)^(1/3))*x^3*a*b+2*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
*x^3*a*b+6*(a/b)^(2/3)*x*a*b+2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3)
)/(a/b)^(1/3))*a^2-2*ln(x+(a/b)^(1/3))*a^2+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)*a^2)*(b*x^3+a)/(a/b)^(2/3)/b^2/a/((b*x^3+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88762, size = 1152, normalized size = 4.17

$$\left[\frac{3\,a^2b^2x^4 - 6\,a^3bx + 3\,\sqrt{\frac{1}{3}}\left(ab^3x^6 + 2\,a^2b^2x^3 + a^3b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3 - 3\left(a^2b\right)^{\frac{1}{3}}ax - a^2 + 3\,\sqrt{\frac{1}{3}}\left(2\,abx^2 + \left(a^2b\right)^{\frac{2}{3}}x - \left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3 + a}\right.}{54\left(a^3b^4x^6 + 2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 +
 a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^
(1/3)/b))/(b*x^3 + a)) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*
x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a
^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*
b^2), 1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*
x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a
^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^
2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2
*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.100 $\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -1/(6*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.0293781, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 607}

$$-\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -1/(6*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, x^3\right)\\ &= -\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

Mathematica [A] time = 0.010395, size = 27, normalized size = 0.71

$$-\frac{a+bx^3}{6b\left(\left(a+bx^3\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -(a + b*x^3)/(6*b*((a + b*x^3)^2)^(3/2))
```

Maple [A] time = 0.006, size = 24, normalized size = 0.6

$$-\frac{bx^3 + a}{6b}\left(\left(bx^3 + a\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/6*(b*x^3+a)/b/((b*x^3+a)^2)^(3/2)
```

Maxima [A] time = 1.14124, size = 24, normalized size = 0.63

$$-\frac{1}{6\left(x^3 + \frac{a}{b}\right)^2 \left(b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6/((x^3 + a/b)^2*(b^2)^(3/2))
```

Fricas [A] time = 1.7349, size = 51, normalized size = 1.34

$$-\frac{1}{6\left(b^3x^6 + 2\,ab^2x^3 + a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.101 $\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{\ldots}\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx\ldots}}$$

```
[Out] (2*x^2)/(9*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(6*a*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (2*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(2/3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(27*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.14049, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1355, 290, 292, 31, 634, 617, 204, 628}

$$\frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{\ldots}\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (2*x^2)/(9*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(6*a*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (2*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(2/3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(27*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(2b\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^2}\,dx}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(2\left(ab+b^2x^3\right)\right)\int \frac{x}{ab+b^2x^3}\,d}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(2\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/}}}{27a^{7/3}b\sqrt{a^2+2abx^3+b^{}}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{}}{\sqrt{3}\sqrt[3]{}}\right.}{9\sqrt{3}a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+}}$$

Mathematica [A] time = 0.0711672, size = 237, normalized size = 0.86

$$\frac{21a^{4/3}b^{2/3}x^2+2b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+4abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{2/3}\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (21*a^(4/3)*b^(2/3)*x^2 + 12*a^(1/3)*b^(5/3)*x^5 - 4*Sqrt[3]*(a + b*x^3)^2*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*(a + b*x^3)^2*Log[a^(1/3) +
 b^(1/3)*x] + 2*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*a*b*
x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^2*x^6*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(2/3)*(a + b*x^3)*Sqrt[(a
 + b*x^3)^2])
```

Maple [A] time = 0.009, size = 301, normalized size = 1.1

$$\frac{bx^3+a}{54\,ba^2}\left(-4\sqrt{3}\arctan\left(1/3\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)x^6b^2-4\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x^6b^2+2\,\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}\right)x^6b^2+12\sqrt[3]{\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-4*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*b^2
-4*ln(x+(a/b)^(1/3))*x^6*b^2+2*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*b^2+12
*(a/b)^(1/3)*x^5*b^2-8*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^
(1/3))*x^3*a*b-8*ln(x+(a/b)^(1/3))*x^3*a*b+4*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*x^3*a*b+21*(a/b)^(1/3)*x^2*a*b-4*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)
^(1/3))/(a/b)^(1/3))*a^2-4*ln(x+(a/b)^(1/3))*a^2+2*ln(x^2-(a/b)^(1/3)*x+(a/
b)^(2/3))*a^2)*(b*x^3+a)/(a/b)^(1/3)/b/a^2/((b*x^3+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90036, size = 1164, normalized size = 4.2

$$\left[\frac{12\,ab^3x^5 + 21\,a^2b^2x^2 + 6\,\sqrt{\frac{1}{3}}\left(ab^3x^6 + 2\,a^2b^2x^3 + a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab + 3\,\sqrt{\frac{1}{3}}\left(abx + 2\left(-ab^2\right)^{\frac{2}{3}}x^2 + \left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}} - 3\,(-\cdots}{bx^3+a}\cdots\right.}{54\,\left(a^3b^4\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x
^3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*
x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a
*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*
log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b^2*x^6 + 2*a*b*x^3
 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*
x^3 + a^5*b^2), 1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 12*sqrt(1/3)*(a*b^3*x
^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x
 + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^6 + 2*a*b*x^3 + a^
2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b
^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^
4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.102 $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=286

$$\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5\left(a+bx^3\right)^3\log\left(a^{2/3}-\right.}{54a^{8/3}\sqrt[3]{b}\left(a^2+2al\right.}$$

```
[Out] (x*(a + b*x^3))/(6*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*x*(a + b*x^3)^
2)/(18*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(1/3)*(a^2 +
2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*(a + b*x^3)^3*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a^2 + 2*a
*b*x^3 + b^2*x^6)^(3/2))
```

Rubi [A] time = 0.146766, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1343, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5\left(a+bx^3\right)^3\log\left(a^{2/3}-\right.}{54a^{8/3}\sqrt[3]{b}\left(a^2+2al\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

```
[Out] (x*(a + b*x^3))/(6*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*x*(a + b*x^3)^
2)/(18*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(1/3)*(a^2 +
2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*(a + b*x^3)^3*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a^2 + 2*a
*b*x^3 + b^2*x^6)^(3/2))
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(2ab+2b^2x^3\right)^3\int \frac{1}{\left(2ab+2b^2x^3\right)^3}\,dx}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

$$= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{\left(5\left(2ab+2b^2x^3\right)^3\right)\int \frac{1}{\left(2ab+2b^2x^3\right)^2}\,dx}{12ab\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

$$= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{\left(5\left(2ab+2b^2x^3\right)^3\right)\int \frac{1}{2ab+2\cdots}}{36a^2b^2\left(a^2+2abx^3+b^2\cdots\right.}$$

$$= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{\left(5\left(2ab+2b^2x^3\right)^3\right)\int \frac{\cdots}{\sqrt[3]{2}\sqrt[3]{\cdots}}}{108\ 2^{2/3}a^{8/3}b^{8/3}\left(a^2+2ab\cdots\right.}$$

$$= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a}+\cdots\right.}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2\cdots\right.}$$

$$= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a}+\cdots\right.}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2\cdots\right.}$$

$$= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} - \frac{5\left(a+bx^3\right)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-\cdots}{\sqrt{3}\cdots}\right.}{9\sqrt{3}a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+\cdots\right.}$$

Mathematica [A] time = 0.0693797, size = 235, normalized size = 0.82

$$\frac{15a^{2/3}b^{4/3}x^4-5b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-10abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-5a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\cdots\right.}{54a^{8/3}\sqrt[3]{b}\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

```
[Out] (24*a^(5/3)*b^(1/3)*x + 15*a^(2/3)*b^(4/3)*x^4 - 10*Sqrt[3]*(a + b*x^3)^2*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(a + b*x^3)^2*Log[a^(1/3) +
 b^(1/3)*x] - 5*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 10*a*b
*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 5*b^2*x^6*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a + b*x^3)*Sqrt[(
a + b*x^3)^2])
```

Maple [A] time = 0.007, size = 299, normalized size = 1.1

$$\frac{bx^3+a}{54\,ba^2}\left(-10\sqrt{3}\arctan\left(1/3\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)x^6b^2+10\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x^6b^2-5\,\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}\right)x^6b^2+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-10*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*b^
2+10*ln(x+(a/b)^(1/3))*x^6*b^2-5*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*b^2+
15*(a/b)^(2/3)*x^4*b^2-20*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/
b)^(1/3))*x^3*a*b+20*ln(x+(a/b)^(1/3))*x^3*a*b-10*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))*x^3*a*b+24*(a/b)^(2/3)*x*a*b-10*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(
a/b)^(1/3))/(a/b)^(1/3))*a^2+10*ln(x+(a/b)^(1/3))*a^2-5*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))*a^2)*(b*x^3+a)/(a/b)^(2/3)/b/a^2/((b*x^3+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87535, size = 1162, normalized size = 4.06

$$\left[\frac{15\,a^{2}b^{2}x^{4}+24\,a^{3}bx+15\,\sqrt{\frac{1}{3}}\left(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}-3\,\left(a^{2}b\right)^{\frac{1}{3}}ax-a^{2}+3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+\left(a^{2}b\right)^{\frac{2}{3}}x-\left(a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right.}{54\,\left(a^{4}b^{3}x^{6}+2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 15*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^
3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log
(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(b^2*x^6 + 2*a*b*x^3 + a
^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^3*x^6 + 2*a^5*b^2*x^3
+ a^6*b), 1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 30*sqrt(1/3)*(a*b^3*x^6 + 2*a
^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)
*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^6 + 2*a*b*x^3 +
 a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(
b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b
^3*x^6 + 2*a^5*b^2*x^3 + a^6*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.103 $\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*(a + b*x^3)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])
```

Rubi [A] time = 0.0827271, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 44}

$$\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 1/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*(a + b*x^3)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^3}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log(x)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] time = 0.0279135, size = 74, normalized size = 0.5

$$\frac{a\left(3a+2bx^3\right)+6\log(x)\left(a+bx^3\right)^2-2\left(a+bx^3\right)^2\log\left(a+bx^3\right)}{6a^3\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (a*(3*a + 2*b*x^3) + 6*(a + b*x^3)^2*Log[x] - 2*(a + b*x^3)^2*Log[a + b*x^3
])/(6*a^3*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.02, size = 107, normalized size = 0.7

$$\frac{\left(6\ln(x)x^6b^2-2\ln\left(bx^3+a\right)x^6b^2+12\ln(x)x^3ab-4\ln\left(bx^3+a\right)x^3ab+2abx^3+6\ln(x)a^2-2\ln\left(bx^3+a\right)a^2+3\right.}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/6*(6*ln(x)*x^6*b^2-2*ln(b*x^3+a)*x^6*b^2+12*ln(x)*x^3*a*b-4*ln(b*x^3+a)*x
^3*a*b+2*a*b*x^3+6*ln(x)*a^2-2*ln(b*x^3+a)*a^2+3*a^2)*(b*x^3+a)/a^3/((b*x^3
+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82563, size = 196, normalized size = 1.33

$$\frac{2\,abx^3+3\,a^2-2\left(b^2x^6+2\,abx^3+a^2\right)\log\left(bx^3+a\right)+6\left(b^2x^6+2\,abx^3+a^2\right)\log\left(x\right)}{6\left(a^3b^2x^6+2\,a^4bx^3+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*b*x^3 + 3*a^2 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a) + 6*(
b^2*x^6 + 2*a*b*x^3 + a^2)*log(x))/(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**3)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.104 $\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$-\frac{14\left(a+bx^3\right)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}+\frac{14\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{\cdots}\right.}{27a^{10/3}\sqrt{a^2+2abx^3\cdots}}$$

```
[Out] 7/(18*a^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x*(a + b*x^3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) - (14*(a + b*x^3))/(9*a^3*x*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) + (14*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (14*b^
(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]) - (7*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.160381, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{14\left(a+bx^3\right)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}+\frac{14\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{\cdots}\right.}{27a^{10/3}\sqrt{a^2+2abx^3\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```


```
[Out] 7/(18*a^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x*(a + b*x^3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) - (14*(a + b*x^3))/(9*a^3*x*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) + (14*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (14*b^
(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]) - (7*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7b\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^2}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(14\left(ab+b^2x^3\right)\right)}{9a^2\sqrt{a^2+2a}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a+bx^3\right.}{9a^3x\sqrt{a^2+2abx^3}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a+bx^3\right.}{9a^3x\sqrt{a^2+2abx^3}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a+bx^3\right.}{9a^3x\sqrt{a^2+2abx^3}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a+bx^3\right.}{9a^3x\sqrt{a^2+2abx^3}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a+bx^3\right.}{9a^3x\sqrt{a^2+2abx^3}}$$

Mathematica [A] time = 0.0845271, size = 260, normalized size = 0.82

$$\frac{-14b^{7/3}x^7\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-28ab^{4/3}x^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-14a^2\sqrt[3]{b}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right.}{54a^{10/3}x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-54*a^(7/3) - 147*a^(4/3)*b*x^3 - 84*a^(1/3)*b^2*x^6 + 28*Sqrt[3]*b^(1/3)*
x*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 28*b^(1/3)*x*
(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] - 14*a^2*b^(1/3)*x*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2] - 28*a*b^(4/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2] - 14*b^(7/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(54*a^(10/3)*x*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.016, size = 316, normalized size = 1.

$$-\frac{bx^3+a}{54\,xa^3}\left(-28\sqrt{3}\arctan\left(1/3\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)x^7b^2-28\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x^7b^2+14\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}\right)x^7b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/54*(-28*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^7*b
^2-28*ln(x+(a/b)^(1/3))*x^7*b^2+14*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^7*b^
2+84*(a/b)^(1/3)*x^6*b^2-56*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(
a/b)^(1/3))*x^4*a*b-56*ln(x+(a/b)^(1/3))*x^4*a*b+28*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))*x^4*a*b+147*(a/b)^(1/3)*x^3*a*b-28*3^(1/2)*arctan(1/3*3^(1/2)*(-
2*x+(a/b)^(1/3))/(a/b)^(1/3))*x*a^2-28*ln(x+(a/b)^(1/3))*x*a^2+14*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))*x*a^2+54*(a/b)^(1/3)*a^2)*(b*x^3+a)/(a/b)^(1/3)/x/
a^3/((b*x^3+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74604, size = 467, normalized size = 1.48

$$-\frac{84\,b^2x^6+147\,abx^3+28\,\sqrt{3}\left(b^2x^7+2\,abx^4+a^2x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+14\left(b^2x^7+2\,abx^4+a^2x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{lo}}{54\left(a^3b^2x^7+2\,a^4bx^4+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/54*(84*b^2*x^6 + 147*a*b*x^3 + 28*sqrt(3)*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*
(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^7 +
 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)
) - 28*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) +
 54*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**3)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.105 $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$-\frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}-\frac{20b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}\right.}{27a^{11/3}\sqrt{a^2+2abx^3}}$$

```
[Out] 4/(9*a^2*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x^2*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (10*(a + b*x^3))/(9*a^3*x^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + (20*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (
20*b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + (10*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(27*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.158661, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}-\frac{20b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}\right.}{27a^{11/3}\sqrt{a^2+2abx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```


```
[Out] 4/(9*a^2*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x^2*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (10*(a + b*x^3))/(9*a^3*x^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + (20*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (
20*b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + (10*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(27*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(4b\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^2}\,dx}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(20\left(ab+b^2x^3\right)\right)\int}{9a^2\sqrt{a^2+2abx}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3}}$$

Mathematica [A] time = 0.0877373, size = 266, normalized size = 0.84

$$\frac{-60a^{2/3}b^2x^6 + 20b^{8/3}x^8\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) + 40ab^{5/3}x^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) + 20a^2b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\right.}{54a^{11/3}x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-27*a^(8/3) - 96*a^(5/3)*b*x^3 - 60*a^(2/3)*b^2*x^6 + 40*Sqrt[3]*b^(2/3)*x
^2*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 40*b^(2/3)*x
^2*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] + 20*a^2*b^(2/3)*x^2*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 40*a*b^(5/3)*x^5*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(8/3)*x^8*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(11/3)*x^2*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.015, size = 322, normalized size = 1.

$$-\frac{bx^3+a}{54\,x^2a^3}\left(-40\,\sqrt{3}\arctan\left(1/3\,\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)x^8b^2 + 40\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x^8b^2 - 20\,\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}\right)x^8b^2 + 6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/54*(-40*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^8*b
^2+40*ln(x+(a/b)^(1/3))*x^8*b^2-20*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^8*b^
2+60*(a/b)^(2/3)*x^6*b^2-80*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(
a/b)^(1/3))*x^5*a*b+80*ln(x+(a/b)^(1/3))*x^5*a*b-40*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))*x^5*a*b+96*(a/b)^(2/3)*x^3*a*b-40*3^(1/2)*arctan(1/3*3^(1/2)*(-2
*x+(a/b)^(1/3))/(a/b)^(1/3))*x^2*a^2+40*ln(x+(a/b)^(1/3))*x^2*a^2-20*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))*x^2*a^2+27*(a/b)^(2/3)*a^2)*(b*x^3+a)/(a/b)^(2/
3)/x^2/a^3/((b*x^3+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77107, size = 540, normalized size = 1.71

$$-\frac{60\,b^2x^6+96\,abx^3-40\,\sqrt{3}\left(b^2x^8+2\,abx^5+a^2x^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)+20\,\left(b^2x^8+2\,abx^5+a^2x^2\right)\left(-\frac{b^2}{a^2}\right.}{54\,\left(a^3b^2x^8+2\,a^4bx^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/54*(60*b^2*x^6 + 96*a*b*x^3 - 40*sqrt(3)*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)
*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b
) + 20*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x
*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 40*(b^2*x^8 + 2*a*b*x^5 + a^2*x
^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 27*a^2)/(a^3*b^2*x^8 +
 2*a^4*b*x^5 + a^5*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**3)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.106 $\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b\log(x)\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+$$

```
[Out] (-2*b)/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(6*a^2*(a + b*x^3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) - (a + b*x^3)/(3*a^3*x^3*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]) - (3*b*(a + b*x^3)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 + (b*(a + b*x^3)*Log[a + b*x^3])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.0972588, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 44}

$$-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b\log(x)\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```


```
[Out] (-2*b)/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(6*a^2*(a + b*x^3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) - (a + b*x^3)/(3*a^3*x^3*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]) - (3*b*(a + b*x^3)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 + (b*(a + b*x^3)*Log[a + b*x^3])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x^4\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^3}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int\left(\frac{1}{a^3b^3x^2}-\frac{3}{a^4b^2x}+\frac{1}{a^2b(a+bx)^3}+\frac{2}{a^3b(a+bx)^2}+\frac{3}{a^4b(a+bx)}\right)dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+}}$$

Mathematica [A] time = 0.0339763, size = 97, normalized size = 0.52

$$\frac{-a\left(2a^2+9abx^3+6b^2x^6\right)-18bx^3\log(x)\left(a+bx^3\right)^2+6bx^3\left(a+bx^3\right)^2\log\left(a+bx^3\right)}{6a^4x^3\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-(a*(2*a^2 + 9*a*b*x^3 + 6*b^2*x^6)) - 18*b*x^3*(a + b*x^3)^2*Log[x] + 6*b
*x^3*(a + b*x^3)^2*Log[a + b*x^3])/(6*a^4*x^3*(a + b*x^3)*Sqrt[(a + b*x^3)^
2])
```

Maple [A] time = 0.02, size = 133, normalized size = 0.7

$$-\frac{\left(18\,b^3\ln(x)\,x^9-6\,\ln\left(bx^3+a\right)x^9b^3+36\,ab^2\ln(x)\,x^6-12\,\ln\left(bx^3+a\right)x^6ab^2+6\,ab^2x^6+18\,a^2b\ln(x)\,x^3-6\,\ln\left(bx^3+\right.\right.}{6\,x^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/6*(18*b^3*ln(x)*x^9-6*ln(b*x^3+a)*x^9*b^3+36*a*b^2*ln(x)*x^6-12*ln(b*x^3
+a)*x^6*a*b^2+6*a*b^2*x^6+18*a^2*b*ln(x)*x^3-6*ln(b*x^3+a)*x^3*a^2*b+9*a^2*
b*x^3+2*a^3)*(b*x^3+a)/x^3/a^4/((b*x^3+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79788, size = 247, normalized size = 1.31

$$-\frac{6\,ab^{2}x^{6}+9\,a^{2}bx^{3}+2\,a^{3}-6\left(b^{3}x^{9}+2\,ab^{2}x^{6}+a^{2}bx^{3}\right)\log\left(bx^{3}+a\right)+18\left(b^{3}x^{9}+2\,ab^{2}x^{6}+a^{2}bx^{3}\right)\log\left(x\right)}{6\left(a^{4}b^{2}x^{9}+2\,a^{5}bx^{6}+a^{6}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3 - 6*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*
x^3)*log(b*x^3 + a) + 18*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*x^3)*log(x))/(a^4*b
^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^{4}\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**3)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.107 $\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=359

$$-\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{27b^2\left(a+\right.}$$

```
[Out] (5*x)/(486*a^2*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^4/(12*b*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(27*b^2*(a + b*x^3)^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + x/(162*a*b^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6]) - (5*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2
43*Sqrt[3]*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*(a + b*x^3
)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(14
58*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.185065, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{27b^2\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```


```
[Out] (5*x)/(486*a^2*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^4/(12*b*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(27*b^2*(a + b*x^3)^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + x/(162*a*b^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6]) - (5*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2
43*Sqrt[3]*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*(a + b*x^3
)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(14
58*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^6}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^4}\,dx}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3 \ldots}{27\sqrt{a^2+\ldots}}$$

$$= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\ldots}{162ab^2\left(a \ldots\right.}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2 \ldots}}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2 \ldots}}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2 \ldots}}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2 \ldots}}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2 \ldots}}$$

Mathematica [A] time = 0.125536, size = 218, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(-\frac{10\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{8/3}} + \frac{30\sqrt[3]{b}x\left(a+bx^3\right)^3}{a^2} + \frac{20\left(a+bx^3\right)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}} + \frac{20\sqrt{3}\left(a+bx^3\right)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{8/3}} + \frac{18\sqrt[3]{b}x\left(a+b \ldots\right.}{a} \ldots\right.}{2916b^{7/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a*b^(1/3)*x - 351*b^(1/3)*x*(a + b*x^3) + (18*b^(1/3)*x*(
a + b*x^3)^2)/a + (30*b^(1/3)*x*(a + b*x^3)^3)/a^2 + (20*Sqrt[3]*(a + b*x^3
)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(8/3) + (20*(a +
b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - (10*(a + b*x^3)^4*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(8/3)))/(2916*b^(7/3)*((a + b*x^3)^2)
^(5/2))
```

Maple [B] time = 0.017, size = 519, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-20*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^12
*b^4+20*ln(x+(a/b)^(1/3))*x^12*b^4-10*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^1
2*b^4+30*(a/b)^(2/3)*x^10*b^4-80*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/
3))/(a/b)^(1/3))*x^9*a*b^3+80*ln(x+(a/b)^(1/3))*x^9*a*b^3-40*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))*x^9*a*b^3+108*(a/b)^(2/3)*x^7*a*b^3-120*3^(1/2)*arctan(
1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*a^2*b^2+120*ln(x+(a/b)^(1/3
))*x^6*a^2*b^2-60*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*a^2*b^2-225*(a/b)^(
2/3)*x^4*a^2*b^2-80*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/
3))*x^3*a^3*b+80*ln(x+(a/b)^(1/3))*x^3*a^3*b-40*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))*x^3*a^3*b-60*(a/b)^(2/3)*x*a^3*b-20*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x
+(a/b)^(1/3))/(a/b)^(1/3))*a^4+20*ln(x+(a/b)^(1/3))*a^4-10*ln(x^2-(a/b)^(1/
3)*x+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(2/3)/b^3/a^2/((b*x^3+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93184, size = 1632, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7 - 225*a^4*b^2*x^4 - 60*a^5*b*x +
 30*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 +
 a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^
(1/3)/b))/(b*x^3 + a)) - 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3
*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a
) + 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)
^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^
5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7
 - 225*a^4*b^2*x^4 - 60*a^5*b*x + 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9
+ 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(
1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 10*
(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*
log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 20*(b^4*x^12 + 4*a*b^3*x
^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(
2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*
b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**6/((a + b*x**3)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.108 $\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{a}{12b^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] a/(12*b^2*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - 1/(9*b^2*(a + b*
x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.0526029, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{a}{12b^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] a/(12*b^2*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - 1/(9*b^2*(a + b*
x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^5}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^5}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \left(-\frac{a}{b^6(a+bx)^5}+\frac{1}{b^6(a+bx)^4}\right)dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{a}{12b^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] time = 0.016172, size = 39, normalized size = 0.5

$$\frac{-a-4bx^3}{36b^2\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (-a - 4*b*x^3)/(36*b^2*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.007, size = 32, normalized size = 0.4

$$-\frac{\left(bx^3+a\right)\left(4\,bx^3+a\right)}{36\,b^2}\left(\left(bx^3+a\right)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/36*(b*x^3+a)*(4*b*x^3+a)/b^2/((b*x^3+a)^2)^(5/2)
```

Maxima [A] time = 1.02845, size = 65, normalized size = 0.83

$$-\frac{1}{9\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^2}+\frac{a}{12\left(x^3+\frac{a}{b}\right)^4\left(b^2\right)^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2) + 1/12*a/((x^3 + a/b)^4*(b^2)^
(5/2)*b)
```

Fricas [A] time = 1.68398, size = 119, normalized size = 1.53

$$-\frac{4\,bx^3+a}{36\,\left(b^6x^{12}+4\,ab^5x^9+6\,a^2b^4x^6+4\,a^3b^3x^3+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/36*(4*b*x^3 + a)/(b^6*x^12 + 4*a*b^5*x^9 + 6*a^2*b^4*x^6 + 4*a^3*b^3*x^3
 + a^4*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.109 $\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=368

$$\frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{12b\left(a+bx\right.}$$

```
[Out] (7*x^2)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(12*b*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(54*a*b*(a + b*x^3)^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) + (7*x^2)/(324*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6]) - (7*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*(
a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + (7*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(1458*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.190896, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 290, 292, 31, 634, 617, 204, 628}

$$\frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{12b\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (7*x^2)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(12*b*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(54*a*b*(a + b*x^3)^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) + (7*x^2)/(324*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6]) - (7*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*(
a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + (7*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(1458*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^4}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^4}\,dx}{6\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7b\left(ab+\right.\right.}{54a\sqrt{a}}$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{324a^2b\left(a\right.}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2}}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2}}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2}}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2}}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2}}$$

Mathematica [A] time = 0.141174, size = 229, normalized size = 0.62

$$\frac{\left(a+bx^3\right)\left(63a^{4/3}b^{2/3}x^2\left(a+bx^3\right)^2+54a^{7/3}b^{2/3}x^2\left(a+bx^3\right)-243a^{10/3}b^{2/3}x^2+14\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\right.}{2916a^{10/3}b^{5/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(10/3)*b^(2/3)*x^2 + 54*a^(7/3)*b^(2/3)*x^2*(a + b*x^3
) + 63*a^(4/3)*b^(2/3)*x^2*(a + b*x^3)^2 + 84*a^(1/3)*b^(2/3)*x^2*(a + b*x^
3)^3 + 28*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))] - 28*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] + 14*(a + b*x^3)^4*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(10/3)*b^(5/3)*((a + b
*x^3)^2)^(5/2))
```

Maple [B] time = 0.017, size = 521, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-28*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^12
*b^4-28*ln(x+(a/b)^(1/3))*x^12*b^4+14*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^1
2*b^4+84*(a/b)^(1/3)*x^11*b^4-112*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1
/3))/(a/b)^(1/3))*x^9*a*b^3-112*ln(x+(a/b)^(1/3))*x^9*a*b^3+56*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))*x^9*a*b^3+315*(a/b)^(1/3)*x^8*a*b^3-168*3^(1/2)*arcta
n(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*a^2*b^2-168*ln(x+(a/b)^(1
/3))*x^6*a^2*b^2+84*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*a^2*b^2+432*(a/b)
^(1/3)*x^5*a^2*b^2-112*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^
(1/3))*x^3*a^3*b-112*ln(x+(a/b)^(1/3))*x^3*a^3*b+56*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))*x^3*a^3*b-42*(a/b)^(1/3)*x^2*a^3*b-28*3^(1/2)*arctan(1/3*3^(1/2)
*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^4-28*ln(x+(a/b)^(1/3))*a^4+14*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(1/3)/b^2/a^3/((b*x^3+a)^2)^(5
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84126, size = 1632, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(84*a*b^5*x^11 + 315*a^2*b^4*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2
 + 42*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3
 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
+ 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b
^2)^(2/3)*x)/(b*x^3 + a)) + 14*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*
a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)
^(2/3)) - 28*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(
-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*
a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(84*a*b^5*x^11 + 315*a^2*b^4
*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2 + 84*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*
b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2)^(1/3)/a)*ar
ctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*(b^
4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^4*x^12 + 4*a*b^3*x
^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(
1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*
b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.110 $\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=360

$$\frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{12b\left(a+\right.}$$

```
[Out] (5*x)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(12*b*(a + b*x^3)^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(108*a*b*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + x/(81*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 - (10*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*
Sqrt[3]*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (10*(a + b*x^3)
*Log[a^(1/3) + b^(1/3)*x])/(729*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(72
9*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.179909, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{12b\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```


```
[Out] (5*x)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(12*b*(a + b*x^3)^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(108*a*b*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + x/(81*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 - (10*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*
Sqrt[3]*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (10*(a + b*x^3)
*Log[a^(1/3) + b^(1/3)*x])/(729*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(72
9*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{\left(ab+b^2x^3\right)^4}\,dx}{12\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(2b\left(a\right.\right.}{27}$$

$$= -\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{81a^2b}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108ab\left(a+bx^3\right)^2}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108ab\left(a+bx^3\right)^2}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108ab\left(a+bx^3\right)^2}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108ab\left(a+bx^3\right)^2}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108ab\left(a+bx^3\right)^2}$$

Mathematica [A] time = 0.140106, size = 221, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(-20\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+60a^{2/3}\sqrt[3]{b}x\left(a+bx^3\right)^3+36a^{5/3}\sqrt[3]{b}x\left(a+bx^3\right)^2+27a^{8/3}\sqrt[3]{b}x\left(a\right.\right.}{2916a^{11/3}b^{4/3}\left(\left(a+bx^3\right)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(11/3)*b^(1/3)*x + 27*a^(8/3)*b^(1/3)*x*(a + b*x^3) +
36*a^(5/3)*b^(1/3)*x*(a + b*x^3)^2 + 60*a^(2/3)*b^(1/3)*x*(a + b*x^3)^3 + 4
0*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
+ 40*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] - 20*(a + b*x^3)^4*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(11/3)*b^(4/3)*((a + b*x^3)^2)
^(5/2))
```

Maple [B] time = 0.016, size = 519, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-40*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^12
*b^4+40*ln(x+(a/b)^(1/3))*x^12*b^4-20*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^1
2*b^4+60*(a/b)^(2/3)*x^10*b^4-160*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1
/3))/(a/b)^(1/3))*x^9*a*b^3+160*ln(x+(a/b)^(1/3))*x^9*a*b^3-80*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))*x^9*a*b^3+216*(a/b)^(2/3)*x^7*a*b^3-240*3^(1/2)*arcta
n(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*a^2*b^2+240*ln(x+(a/b)^(1
/3))*x^6*a^2*b^2-120*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*a^2*b^2+279*(a/b
)^(2/3)*x^4*a^2*b^2-160*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)
^(1/3))*x^3*a^3*b+160*ln(x+(a/b)^(1/3))*x^3*a^3*b-80*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))*x^3*a^3*b-120*(a/b)^(2/3)*x*a^3*b-40*3^(1/2)*arctan(1/3*3^(1/2)
*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^4+40*ln(x+(a/b)^(1/3))*a^4-20*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(2/3)/b^2/a^3/((b*x^3+a)^2)^(5
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85608, size = 1636, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^7 + 279*a^4*b^2*x^4 - 120*a^5*b*x
+ 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3
+ a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2
+ 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)
^(1/3)/b))/(b*x^3 + a)) - 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^
3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*
a) + 40*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b
)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b
^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^
7 + 279*a^4*b^2*x^4 - 120*a^5*b*x + 120*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x
^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sq
rt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) -
20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/
3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^4*x^12 + 4*a*b^
3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b
)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a
^9*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.111 $\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}} \, dx$

Optimal. Leaf size=38

$$-\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

```
[Out] -1/(12*b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))
```

Rubi [A] time = 0.0295249, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 607}

$$-\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/(12*b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}} \, dx, x, x^3\right)\\ &= -\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0096304, size = 27, normalized size = 0.71

$$-\frac{a+bx^3}{12b\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -(a + b*x^3)/(12*b*((a + b*x^3)^2)^(5/2))
```

Maple [A] time = 0.006, size = 24, normalized size = 0.6

$$-\frac{bx^3+a}{12b}\left(\left(bx^3+a\right)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/12*(b*x^3+a)/b/((b*x^3+a)^2)^(5/2)
```

Maxima [A] time = 1.05502, size = 24, normalized size = 0.63

$$-\frac{1}{12\left(x^3+\frac{a}{b}\right)^4\left(b^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12/((x^3 + a/b)^4*(b^2)^(5/2))
```

Fricas [A] time = 1.74862, size = 97, normalized size = 2.55

$$-\frac{1}{12\left(b^5x^{12}+4\,ab^4x^9+6\,a^2b^3x^6+4\,a^3b^2x^3+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12/(b^5*x^12 + 4*a*b^4*x^9 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^3 + a^4*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.112 $\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\ldots$$

```
[Out] (35*x^2)/(243*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(12*a*(a + b*x^3)^
3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*x^2)/(54*a^2*(a + b*x^3)^2*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]) + (35*x^2)/(324*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]) - (35*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(243*Sqrt[3]*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (
35*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]) + (35*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(1458*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.192751, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1355, 290, 292, 31, 634, 617, 204, 628}

$$\frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```


```
[Out] (35*x^2)/(243*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(12*a*(a + b*x^3)^
3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*x^2)/(54*a^2*(a + b*x^3)^2*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]) + (35*x^2)/(324*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]) - (35*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(243*Sqrt[3]*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (
35*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]) + (35*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(1458*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(5b^3\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^4}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(35b^2\left(a\right.\right.}{54a^2}$$

$$= \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{324a^3\left(a\right.}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x}{54a^2\left(a+bx^3\right)^2\sqrt{a}}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x}{54a^2\left(a+bx^3\right)^2\sqrt{a}}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x}{54a^2\left(a+bx^3\right)^2\sqrt{a}}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x}{54a^2\left(a+bx^3\right)^2\sqrt{a}}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x}{54a^2\left(a+bx^3\right)^2\sqrt{a}}$$

Mathematica [A] time = 0.120359, size = 219, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(\frac{70\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + 315a^{4/3}x^2\left(a+bx^3\right)^2 + 270a^{7/3}x^2\left(a+bx^3\right) + 243a^{10/3}x^2 - \frac{140\left(a+bx^3\right)^4\log\left(\sqrt[3]{a}\right.}{b^{2/3}}\right.}{2916a^{13/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a^(10/3)*x^2 + 270*a^(7/3)*x^2*(a + b*x^3) + 315*a^(4/3)*
x^2*(a + b*x^3)^2 + 420*a^(1/3)*x^2*(a + b*x^3)^3 + (140*Sqrt[3]*(a + b*x^3
)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(2/3) - (140*(a +
 b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (70*(a + b*x^3)^4*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3)))/(2916*a^(13/3)*((a + b*x^3)^
2)^(5/2))
```

Maple [B] time = 0.008, size = 521, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-140*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^1
2*b^4-140*ln(x+(a/b)^(1/3))*x^12*b^4+70*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x
^12*b^4+420*(a/b)^(1/3)*x^11*b^4-560*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)
^(1/3))/(a/b)^(1/3))*x^9*a*b^3-560*ln(x+(a/b)^(1/3))*x^9*a*b^3+280*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))*x^9*a*b^3+1575*(a/b)^(1/3)*x^8*a*b^3-840*3^(1/2)*
arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*a^2*b^2-840*ln(x+(a/
b)^(1/3))*x^6*a^2*b^2+420*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*a^2*b^2+216
0*(a/b)^(1/3)*x^5*a^2*b^2-560*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))
/(a/b)^(1/3))*x^3*a^3*b-560*ln(x+(a/b)^(1/3))*x^3*a^3*b+280*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))*x^3*a^3*b+1248*(a/b)^(1/3)*x^2*a^3*b-140*3^(1/2)*arctan(
1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^4-140*ln(x+(a/b)^(1/3))*a^4+7
0*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(1/3)/b/a^4/((b*x^
3+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8499, size = 1651, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(420*a*b^5*x^11 + 1575*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^
2*x^2 + 210*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b
^2*x^3 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(
a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3
*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^
6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-
a*b^2)^(2/3)) - 140*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 +
 a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x
^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(420*a*b^5*x^11 + 157
5*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^2*x^2 + 420*sqrt(1/3)*(a*b^5*
x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2
)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a
)/b) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 140*(b^4*x^
12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*
b^3*x^3 + a^9*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.113 $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=364

$$\frac{55x\left(a+bx^3\right)^4}{243a^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x\left(a+bx^3\right)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

```
[Out] (x*(a + b*x^3))/(12*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3
)^2)/(108*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3)^3)/(81
*a^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (55*x*(a + b*x^3)^4)/(243*a^4*(a^
2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (110*(a + b*x^3)^5*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(14/3)*b^(1/3)*(a^2 + 2*a*b*x^3
 + b^2*x^6)^(5/2)) + (110*(a + b*x^3)^5*Log[a^(1/3) + b^(1/3)*x])/(729*a^(1
4/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (55*(a + b*x^3)^5*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(14/3)*b^(1/3)*(a^2 + 2*a*b
*x^3 + b^2*x^6)^(5/2))
```

Rubi [A] time = 0.199121, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1343, 199, 200, 31, 634, 617, 204, 628}

$$\frac{55x\left(a+bx^3\right)^4}{243a^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x\left(a+bx^3\right)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

```
[Out] (x*(a + b*x^3))/(12*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3
)^2)/(108*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3)^3)/(81
*a^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (55*x*(a + b*x^3)^4)/(243*a^4*(a^
2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (110*(a + b*x^3)^5*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(14/3)*b^(1/3)*(a^2 + 2*a*b*x^3
 + b^2*x^6)^(5/2)) + (110*(a + b*x^3)^5*Log[a^(1/3) + b^(1/3)*x])/(729*a^(1
4/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (55*(a + b*x^3)^5*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(14/3)*b^(1/3)*(a^2 + 2*a*b
*x^3 + b^2*x^6)^(5/2))
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(2ab+2b^2x^3\right)^5 \int \frac{1}{\left(2ab+2b^2x^3\right)^5}\,dx}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(11\left(2ab+2b^2x^3\right)^5\right)\int \frac{1}{\left(2ab+2b^2x^3\right)^4}\,dx}{24ab\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(11\left(2ab+2b^2x^3\right)^5\right)\int \frac{}{\left(2ab+\right.}}{54a^2b^2\left(a^2+2abx^3+b^2\right.}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

Mathematica [A] time = 0.11405, size = 211, normalized size = 0.58

$$\frac{\left(a+bx^3\right)\left(-\frac{220\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}} + 660a^{2/3}x\left(a+bx^3\right)^3 + 396a^{5/3}x\left(a+bx^3\right)^2 + 297a^{8/3}x\left(a+bx^3\right) + 243a^{11/3}x\right.}{2916a^{14/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a^(11/3)*x + 297*a^(8/3)*x*(a + b*x^3) + 396*a^(5/3)*x*(a
 + b*x^3)^2 + 660*a^(2/3)*x*(a + b*x^3)^3 + (440*Sqrt[3]*(a + b*x^3)^4*ArcT
an[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(1/3) + (440*(a + b*x^3)^
4*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (220*(a + b*x^3)^4*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3)))/(2916*a^(14/3)*((a + b*x^3)^2)^(5/2
))
```

Maple [A] time = 0.01, size = 519, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-440*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^1
2*b^4+440*ln(x+(a/b)^(1/3))*x^12*b^4-220*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*
x^12*b^4+660*(a/b)^(2/3)*x^10*b^4-1760*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/
b)^(1/3))/(a/b)^(1/3))*x^9*a*b^3+1760*ln(x+(a/b)^(1/3))*x^9*a*b^3-880*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^9*a*b^3+2376*(a/b)^(2/3)*x^7*a*b^3-2640*3^(1
/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*a^2*b^2+2640*ln(
x+(a/b)^(1/3))*x^6*a^2*b^2-1320*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*a^2*b
^2+3069*(a/b)^(2/3)*x^4*a^2*b^2-1760*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)
^(1/3))/(a/b)^(1/3))*x^3*a^3*b+1760*ln(x+(a/b)^(1/3))*x^3*a^3*b-880*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))*x^3*a^3*b+1596*(a/b)^(2/3)*x*a^3*b-440*3^(1/2)*a
rctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^4+440*ln(x+(a/b)^(1/3))
*a^4-220*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(2/3)/b/a^4
/((b*x^3+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87585, size = 1652, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(660*a^2*b^4*x^10 + 2376*a^3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*
b*x + 660*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2
*x^3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x -
 a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a
^2*b)^(1/3)/b))/(b*x^3 + a)) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6
+ 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 440*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)
*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 +
6*a^8*b^3*x^6 + 4*a^9*b^2*x^3 + a^10*b), 1/2916*(660*a^2*b^4*x^10 + 2376*a^
3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*b*x + 1320*sqrt(1/3)*(a*b^5*x^12 +
4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b
)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)
/b)/a^2) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 440*(b^4*
```

```
x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a
*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 + 6*a^8*b^3*x^6 + 4*a^
9*b^2*x^3 + a^10*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2abx^3 + b^2x^6\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.114 $\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{1}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{}{3a^4\sqrt{}}$$

```
[Out] 1/(3*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*(a + b*x^3)^3*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]) + 1/(9*a^2*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) + 1/(6*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^
3)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x
^3])/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.123905, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 44}

$$\frac{1}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{}{3a^4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 1/(3*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*(a + b*x^3)^3*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]) + 1/(9*a^2*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) + 1/(6*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^
3)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x
^3])/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^5}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^5b^5x}-\frac{1}{ab^4(a+bx)^5}-\frac{1}{a^2b^4(a+bx)^4}-\frac{1}{a^3b^4(a+bx)^3}-\frac{1}{a^4b^4(a+bx)^2}-\frac{1}{a^5b^4}\right.\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2}}$$

Mathematica [A] time = 0.0417819, size = 96, normalized size = 0.43

$$\frac{a\left(52a^2bx^3+25a^3+42ab^2x^6+12b^3x^9\right)+36\log(x)\left(a+bx^3\right)^4-12\left(a+bx^3\right)^4\log\left(a+bx^3\right)}{36a^5\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (a*(25*a^3 + 52*a^2*b*x^3 + 42*a*b^2*x^6 + 12*b^3*x^9) + 36*(a + b*x^3)^4*L
og[x] - 12*(a + b*x^3)^4*Log[a + b*x^3])/(36*a^5*(a + b*x^3)^3*Sqrt[(a + b*
x^3)^2])
```

Maple [A] time = 0.019, size = 193, normalized size = 0.9

$$\frac{\left(36\ln(x)x^{12}b^4-12\ln\left(bx^3+a\right)x^{12}b^4+144\ln(x)x^9ab^3-48\ln\left(bx^3+a\right)x^9ab^3+12x^9ab^3+216\ln(x)x^6a^2b^2-72\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/36*(36*ln(x)*x^12*b^4-12*ln(b*x^3+a)*x^12*b^4+144*ln(x)*x^9*a*b^3-48*ln(b
*x^3+a)*x^9*a*b^3+12*x^9*a*b^3+216*ln(x)*x^6*a^2*b^2-72*ln(b*x^3+a)*x^6*a^2
*b^2+42*x^6*a^2*b^2+144*ln(x)*x^3*a^3*b-48*ln(b*x^3+a)*x^3*a^3*b+52*x^3*a^3
*b+36*ln(x)*a^4-12*ln(b*x^3+a)*a^4+25*a^4)*(b*x^3+a)/a^5/((b*x^3+a)^2)^(5/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55085, size = 382, normalized size = 1.71

$$\frac{12\,ab^3x^9 + 42\,a^2b^2x^6 + 52\,a^3bx^3 + 25\,a^4 - 12\left(b^4x^{12} + 4\,ab^3x^9 + 6\,a^2b^2x^6 + 4\,a^3bx^3 + a^4\right)\log\left(bx^3 + a\right) + 36\left(b^4x^{12} + \cdots\right.}{36\left(a^5b^4x^{12} + 4\,a^6b^3x^9 + 6\,a^7b^2x^6 + 4\,a^8bx^3 + a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*b^3*x^9 + 42*a^2*b^2*x^6 + 52*a^3*b*x^3 + 25*a^4 - 12*(b^4*x^12
+ 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(b*x^3 + a) + 36*(b^4
*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(x))/(a^5*b^4*x
^12 + 4*a^6*b^3*x^9 + 6*a^7*b^2*x^6 + 4*a^8*b*x^3 + a^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**3)**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.115 $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}} dx$

Optimal. Leaf size=398

$$-\frac{455(a+bx^3)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{65}{324a^3x\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}+\frac{1}{108a^2x\sqrt{a^2+2ab}}$$

```
[Out] 455/(972*a^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x*(a + b*x^3)^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 13/(108*a^2*x*(a + b*x^3)^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + 65/(324*a^3*x*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6]) - (455*(a + b*x^3))/(243*a^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (45
5*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2
43*Sqrt[3]*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (455*b^(1/3)*(a + b*
x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) - (455*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(1458*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.216311, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{455(a+bx^3)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{65}{324a^3x\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}+\frac{1}{108a^2x\sqrt{a^2+2ab}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```


```
[Out] 455/(972*a^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x*(a + b*x^3)^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 13/(108*a^2*x*(a + b*x^3)^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + 65/(324*a^3*x*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6]) - (455*(a + b*x^3))/(243*a^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (45
5*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2
43*Sqrt[3]*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (455*b^(1/3)*(a + b*
x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) - (455*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(1458*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(13b^3\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^4}\,dx}{12a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(65b^2 \cdots\right.}{\cdots}$$

$$= \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{324a^{\cdots}}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{108a^2x\left(a+bx^3\right)\cdots}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{108a^2x\left(a+bx^3\right)\cdots}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{108a^2x\left(a+bx^3\right)\cdots}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{108a^2x\left(a+bx^3\right)\cdots}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{108a^2x\left(a+bx^3\right)\cdots}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{108a^2x\left(a+bx^3\right)\cdots}$$

Mathematica [A] time = 0.130211, size = 242, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(-910\sqrt[3]{b}\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-1179a^{4/3}bx^2\left(a+bx^3\right)^2-594a^{7/3}bx^2\left(a+bx^3\right)-243a^{10/3}bx^{2}\cdots\right.}{2916a^{16/3}\left(\left(\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(10/3)*b*x^2 - 594*a^(7/3)*b*x^2*(a + b*x^3) - 1179*a^
(4/3)*b*x^2*(a + b*x^3)^2 - 2544*a^(1/3)*b*x^2*(a + b*x^3)^3 - (2916*a^(1/3
)*(a + b*x^3)^4)/x - 1820*Sqrt[3]*b^(1/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) +
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 1820*b^(1/3)*(a + b*x^3)^4*Log[a^(1/3) +
b^(1/3)*x] - 910*b^(1/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2]))/(2916*a^(16/3)*((a + b*x^3)^2)^(5/2))
```

Maple [B] time = 0.02, size = 536, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/2916*(-1820*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x
^13*b^4-1820*ln(x+(a/b)^(1/3))*x^13*b^4+910*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))*x^13*b^4+5460*(a/b)^(1/3)*x^12*b^4-7280*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x
+(a/b)^(1/3))/(a/b)^(1/3))*x^10*a*b^3-7280*ln(x+(a/b)^(1/3))*x^10*a*b^3+364
0*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^10*a*b^3+20475*(a/b)^(1/3)*x^9*a*b^3-
10920*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^7*a^2*b^
2-10920*ln(x+(a/b)^(1/3))*x^7*a^2*b^2+5460*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)*x^7*a^2*b^2+28080*(a/b)^(1/3)*x^6*a^2*b^2-7280*3^(1/2)*arctan(1/3*3^(1/2)
*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^4*a^3*b-7280*ln(x+(a/b)^(1/3))*x^4*a^3*b
+3640*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^4*a^3*b+16224*(a/b)^(1/3)*x^3*a^3
*b-1820*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x*a^4-18
20*ln(x+(a/b)^(1/3))*x*a^4+910*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x*a^4+2916
*(a/b)^(1/3)*a^4)*(b*x^3+a)/(a/b)^(1/3)/x/a^5/((b*x^3+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5945, size = 724, normalized size = 1.82

$$-\frac{5460\,b^4x^{12} + 20475\,ab^3x^9 + 28080\,a^2b^2x^6 + 16224\,a^3bx^3 + 2916\,a^4 + 1820\,\sqrt{3}\left(b^4x^{13} + 4\,ab^3x^{10} + 6\,a^2b^2x^7 + 4\,a^3bx \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2916*(5460*b^4*x^12 + 20475*a*b^3*x^9 + 28080*a^2*b^2*x^6 + 16224*a^3*b*
x^3 + 2916*a^4 + 1820*sqrt(3)*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*
a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(
3)) + 910*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(
b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 1820*(b^4*x^13 +
4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*log(b*x + a
*(b/a)^(2/3)))/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^8*b*x^4
 + a^9*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**3)**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.116 $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=398

$$-\frac{385\left(a+bx^3\right)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{77}{324a^3x^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}+\frac{}{54a^2x^2\sqrt{a^2}}$$

```
[Out] 154/(243*a^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x^2*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 7/(54*a^2*x^2*(a + b*x^3)^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]) + 77/(324*a^3*x^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) - (385*(a + b*x^3))/(243*a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) + (770*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (770*b^(2/3)
*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x]])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]) + (385*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2]])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.210628, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{385\left(a+bx^3\right)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{77}{324a^3x^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}+\frac{}{54a^2x^2\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```


```
[Out] 154/(243*a^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x^2*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 7/(54*a^2*x^2*(a + b*x^3)^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]) + 77/(324*a^3*x^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) - (385*(a + b*x^3))/(243*a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) + (770*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (770*b^(2/3)
*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x]])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]) + (385*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2]])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7b^3\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^4}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{54a^2x^2\left(a+\cdots\right.}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{54a^2x^2\left(a+\cdots\right.}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{54a^2x^2\left(a+\cdots\right.}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{54a^2x^2\left(a+\cdots\right.}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{54a^2x^2\left(a+\cdots\right.}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{54a^2x^2\left(a+\cdots\right.}$$

Mathematica [A] time = 0.135487, size = 234, normalized size = 0.59

$$\frac{\left(a+bx^3\right)\left(1540b^{2/3}\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{1458a^{2/3}\left(a+bx^3\right)^4}{x^2}-3162a^{2/3}bx\left(a+bx^3\right)^3-1314a^{5/3}bx\left(a+\cdots\right.\right.}{2916a^{17/3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(11/3)*b*x - 621*a^(8/3)*b*x*(a + b*x^3) - 1314*a^(5/3
)*b*x*(a + b*x^3)^2 - 3162*a^(2/3)*b*x*(a + b*x^3)^3 - (1458*a^(2/3)*(a + b
*x^3)^4)/x^2 - 3080*Sqrt[3]*b^(2/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))] - 3080*b^(2/3)*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3
)*x] + 1540*b^(2/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2]))/(2916*a^(17/3)*((a + b*x^3)^2)^(5/2))
```

Maple [B] time = 0.022, size = 542, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/2916*(-3080*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x
^14*b^4+3080*ln(x+(a/b)^(1/3))*x^14*b^4-1540*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*x^14*b^4+4620*(a/b)^(2/3)*x^12*b^4-12320*3^(1/2)*arctan(1/3*3^(1/2)*(-2
*x+(a/b)^(1/3))/(a/b)^(1/3))*x^11*a*b^3+12320*ln(x+(a/b)^(1/3))*x^11*a*b^3-
6160*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^11*a*b^3+16632*(a/b)^(2/3)*x^9*a*b
^3-18480*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^8*a^2
*b^2+18480*ln(x+(a/b)^(1/3))*x^8*a^2*b^2-9240*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))*x^8*a^2*b^2+21483*(a/b)^(2/3)*x^6*a^2*b^2-12320*3^(1/2)*arctan(1/3*3^(
1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^5*a^3*b+12320*ln(x+(a/b)^(1/3))*x^5*
a^3*b-6160*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^5*a^3*b+11172*(a/b)^(2/3)*x^
3*a^3*b-3080*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^2
*a^4+3080*ln(x+(a/b)^(1/3))*x^2*a^4-1540*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*
x^2*a^4+1458*(a/b)^(2/3)*a^4)*(b*x^3+a)/(a/b)^(2/3)/x^2/a^5/((b*x^3+a)^2)^(
5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57481, size = 799, normalized size = 2.01

$$-\frac{4620\,b^4x^{12}+16632\,ab^3x^9+21483\,a^2b^2x^6+11172\,a^3bx^3+1458\,a^4-3080\,\sqrt{3}\left(b^4x^{14}+4\,ab^3x^{11}+6\,a^2b^2x^8+4\,a^3bx^5+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2916*(4620*b^4*x^12 + 16632*a*b^3*x^9 + 21483*a^2*b^2*x^6 + 11172*a^3*b*
x^3 + 1458*a^4 - 3080*sqrt(3)*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*
a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^
(2/3) - sqrt(3)*b)/b) + 1540*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*a
^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) +
 a^2*(-b^2/a^2)^(2/3)) - 3080*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*
a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^5*b
^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a^9*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**3)**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.117 $\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{b}{2a^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{9a^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{3a^5\sqrt{}}$$

```
[Out] (-4*b)/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(12*a^2*(a + b*x^3)^3*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*b)/(9*a^3*(a + b*x^3)^2*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6]) - b/(2*a^4*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) -
 (a + b*x^3)/(3*a^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (5*b*(a + b*x^3)
*Log[x])/(a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*b*(a + b*x^3)*Log[a + b
*x^3])/(3*a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.142528, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 44}

$$-\frac{b}{2a^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{9a^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{3a^5\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```



```
[Out] (-4*b)/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(12*a^2*(a + b*x^3)^3*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*b)/(9*a^3*(a + b*x^3)^2*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6]) - b/(2*a^4*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) -
 (a + b*x^3)/(3*a^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (5*b*(a + b*x^3)
*Log[x])/(a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*b*(a + b*x^3)*Log[a + b
*x^3])/(3*a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x^4\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^5}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\operatorname{Subst}\left(\int \left(\frac{1}{a^5b^5x^2}-\frac{5}{a^6b^4x}+\frac{1}{a^2b^3(a+bx)^5}+\frac{2}{a^3b^3(a+bx)^4}+\frac{3}{a^4b^3(a+bx)^3}+\right.\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{9a^3\left(a+bx^3\right)^2}$$

Mathematica [A] time = 0.0522613, size = 119, normalized size = 0.44

$$\frac{-a\left(260a^2b^2x^6+125a^3bx^3+12a^4+210ab^3x^9+60b^4x^{12}\right)-180bx^3\log(x)\left(a+bx^3\right)^4+60bx^3\left(a+bx^3\right)^4\log\left(a+bx^3\right)}{36a^6x^3\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-(a*(12*a^4 + 125*a^3*b*x^3 + 260*a^2*b^2*x^6 + 210*a*b^3*x^9 + 60*b^4*x^1
2)) - 180*b*x^3*(a + b*x^3)^4*Log[x] + 60*b*x^3*(a + b*x^3)^4*Log[a + b*x^3
])/(36*a^6*x^3*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.02, size = 219, normalized size = 0.8

$$-\frac{\left(180\,b^5\ln(x)x^{15}-60\,\ln\left(bx^3+a\right)x^{15}b^5+720\,ab^4\ln(x)x^{12}-240\,\ln\left(bx^3+a\right)x^{12}ab^4+60\,ab^4x^{12}+1080\,a^2b^3\ln(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/36*(180*b^5*ln(x)*x^15-60*ln(b*x^3+a)*x^15*b^5+720*a*b^4*ln(x)*x^12-240*
ln(b*x^3+a)*x^12*a*b^4+60*a*b^4*x^12+1080*a^2*b^3*ln(x)*x^9-360*ln(b*x^3+a)
*x^9*a^2*b^3+210*a^2*b^3*x^9+720*a^3*b^2*ln(x)*x^6-240*ln(b*x^3+a)*x^6*a^3*
b^2+260*a^3*b^2*x^6+180*a^4*b*ln(x)*x^3-60*ln(b*x^3+a)*x^3*a^4*b+125*a^4*b*
x^3+12*a^5)*(b*x^3+a)/x^3/a^6/((b*x^3+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61363, size = 446, normalized size = 1.66

$$-\frac{60\,ab^4x^{12}+210\,a^2b^3x^9+260\,a^3b^2x^6+125\,a^4bx^3+12\,a^5-60\left(b^5x^{15}+4\,ab^4x^{12}+6\,a^2b^3x^9+4\,a^3b^2x^6+a^4bx^3\right)\log\left(bx^3\right.}{36\left(a^6b^4x^{15}+4\,a^7b^3x^{12}+6\,a^8b^2x^9+4\,a^9bx^6+a^{10}x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/36*(60*a*b^4*x^12 + 210*a^2*b^3*x^9 + 260*a^3*b^2*x^6 + 125*a^4*b*x^3 +
12*a^5 - 60*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4*a^3*b^2*x^6 + a^4*
b*x^3)*log(b*x^3 + a) + 180*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4*a^
3*b^2*x^6 + a^4*b*x^3)*log(x))/(a^6*b^4*x^15 + 4*a^7*b^3*x^12 + 6*a^8*b^2*x
^9 + 4*a^9*b*x^6 + a^10*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**3)**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.118 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=313

$$\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+7}}{d^7(m+7)\left(a+bx^3\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+\frac{5ab^4}{}$$

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3))
 + (5*a^4*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a
+ b*x^3)) + (10*a^3*b^2*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7
*(7 + m)*(a + b*x^3)) + (10*a^2*b^3*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(d^10*(10 + m)*(a + b*x^3)) + (5*a*b^4*(d*x)^(13 + m)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(d^13*(13 + m)*(a + b*x^3)) + (b^5*(d*x)^(16 + m)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^16*(16 + m)*(a + b*x^3))
```

Rubi [A] time = 0.137861, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 270}

$$\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+7}}{d^7(m+7)\left(a+bx^3\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+\frac{5ab^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3))
 + (5*a^4*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a
+ b*x^3)) + (10*a^3*b^2*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7
*(7 + m)*(a + b*x^3)) + (10*a^2*b^3*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(d^10*(10 + m)*(a + b*x^3)) + (5*a*b^4*(d*x)^(13 + m)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(d^13*(13 + m)*(a + b*x^3)) + (b^5*(d*x)^(16 + m)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^16*(16 + m)*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m \left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^5b^5(dx)^m + \frac{5a^4b^6(dx)^{3+m}}{d^3} + \frac{10a^3b^7(dx)^{6+m}}{d^6} + \frac{10a^2b^8(dx)^{9+m}}{d^9} + \frac{5ab^9(}{}\right.}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)} + \frac{5a^4b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)} + \frac{10a^3b^2(dx)^{7+m}}{d^7(7+}$$

Mathematica [A] time = 0.0998909, size = 111, normalized size = 0.35

$$\frac{x\left(\left(a+bx^3\right)^2\right)^{5/2}(dx)^m\left(\frac{10a^2b^3x^9}{m+10}+\frac{10a^3b^2x^6}{m+7}+\frac{5a^4bx^3}{m+4}+\frac{a^5}{m+1}+\frac{5ab^4x^{12}}{m+13}+\frac{b^5x^{15}}{m+16}\right)}{\left(a+bx^3\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^3)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^3)/(4 + m) + (10
*a^3*b^2*x^6)/(7 + m) + (10*a^2*b^3*x^9)/(10 + m) + (5*a*b^4*x^12)/(13 + m)
 + (b^5*x^15)/(16 + m)))/(a + b*x^3)^5
```

Maple [A] time = 0.007, size = 453, normalized size = 1.5

$$\frac{\left(b^5m^5x^{15} + 35\,b^5m^4x^{15} + 445\,b^5m^3x^{15} + 5\,ab^4m^5x^{12} + 2485\,b^5m^2x^{15} + 190\,ab^4m^4x^{12} + 5714\,b^5mx^{15} + 2555\,ab^4m^3x^{12} + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] x*(b^5*m^5*x^15+35*b^5*m^4*x^15+445*b^5*m^3*x^15+5*a*b^4*m^5*x^12+2485*b^5*
m^2*x^15+190*a*b^4*m^4*x^12+5714*b^5*m*x^15+2555*a*b^4*m^3*x^12+3640*b^5*x^
15+10*a^2*b^3*m^5*x^9+14810*a*b^4*m^2*x^12+410*a^2*b^3*m^4*x^9+34840*a*b^4*
m*x^12+5950*a^2*b^3*m^3*x^9+22400*a*b^4*x^12+10*a^3*b^2*m^5*x^6+36550*a^2*b
^3*m^2*x^9+440*a^3*b^2*m^4*x^6+89240*a^2*b^3*m*x^9+6970*a^3*b^2*m^3*x^6+582
40*a^2*b^3*x^9+5*a^4*b*m^5*x^3+47260*a^3*b^2*m^2*x^6+235*a^4*b*m^4*x^3+1239
20*a^3*b^2*m*x^6+4085*a^4*b*m^3*x^3+83200*a^3*b^2*x^6+a^5*m^5+31685*a^4*b*m
^2*x^3+50*a^5*m^4+100630*a^4*b*m*x^3+955*a^5*m^3+72800*a^4*b*x^3+8650*a^5*m
^2+36824*a^5*m+58240*a^5)*(d*x)^m*((b*x^3+a)^2)^(5/2)/(1+m)/(4+m)/(7+m)/(10
+m)/(13+m)/(16+m)/(b*x^3+a)^5
```

Maxima [A] time = 1.03573, size = 328, normalized size = 1.05

$$\frac{\left(\left(m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640\right)b^5d^mx^{16} + 5\left(m^5 + 38\,m^4 + 511\,m^3 + 2962\,m^2 + 6968\,m + 4480\right)ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)*b^5*d^m*x^16 + 5*(m^5
+ 38*m^4 + 511*m^3 + 2962*m^2 + 6968*m + 4480)*a*b^4*d^m*x^13 + 10*(m^5 + 4
1*m^4 + 595*m^3 + 3655*m^2 + 8924*m + 5824)*a^2*b^3*d^m*x^10 + 10*(m^5 + 44
*m^4 + 697*m^3 + 4726*m^2 + 12392*m + 8320)*a^3*b^2*d^m*x^7 + 5*(m^5 + 47*m
^4 + 817*m^3 + 6337*m^2 + 20126*m + 14560)*a^4*b*d^m*x^4 + (m^5 + 50*m^4 +
955*m^3 + 8650*m^2 + 36824*m + 58240)*a^5*d^m*x)*x^m/(m^6 + 51*m^5 + 1005*m
^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

Fricas [A] time = 1.59833, size = 886, normalized size = 2.83

$$\frac{\left(\left(b^5m^5 + 35\,b^5m^4 + 445\,b^5m^3 + 2485\,b^5m^2 + 5714\,b^5m + 3640\,b^5\right)x^{16} + 5\left(ab^4m^5 + 38\,ab^4m^4 + 511\,ab^4m^3 + 2962\,ab^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 35*b^5*m^4 + 445*b^5*m^3 + 2485*b^5*m^2 + 5714*b^5*m + 3640*b^5
)*x^16 + 5*(a*b^4*m^5 + 38*a*b^4*m^4 + 511*a*b^4*m^3 + 2962*a*b^4*m^2 + 696
8*a*b^4*m + 4480*a*b^4)*x^13 + 10*(a^2*b^3*m^5 + 41*a^2*b^3*m^4 + 595*a^2*b
^3*m^3 + 3655*a^2*b^3*m^2 + 8924*a^2*b^3*m + 5824*a^2*b^3)*x^10 + 10*(a^3*b
^2*m^5 + 44*a^3*b^2*m^4 + 697*a^3*b^2*m^3 + 4726*a^3*b^2*m^2 + 12392*a^3*b^
2*m + 8320*a^3*b^2)*x^7 + 5*(a^4*b*m^5 + 47*a^4*b*m^4 + 817*a^4*b*m^3 + 633
7*a^4*b*m^2 + 20126*a^4*b*m + 14560*a^4*b)*x^4 + (a^5*m^5 + 50*a^5*m^4 + 95
5*a^5*m^3 + 8650*a^5*m^2 + 36824*a^5*m + 58240*a^5)*x)*(d*x)^m/(m^6 + 51*m^
5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20121, size = 1215, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^5*m^5*x^16*sgn(b*x^3 + a) + 35*(d*x)^m*b^5*m^4*x^16*sgn(b*x^3 +
a) + 445*(d*x)^m*b^5*m^3*x^16*sgn(b*x^3 + a) + 5*(d*x)^m*a*b^4*m^5*x^13*sgn
(b*x^3 + a) + 2485*(d*x)^m*b^5*m^2*x^16*sgn(b*x^3 + a) + 190*(d*x)^m*a*b^4*
m^4*x^13*sgn(b*x^3 + a) + 5714*(d*x)^m*b^5*m*x^16*sgn(b*x^3 + a) + 2555*(d*
```

```
x)^m*a*b^4*m^3*x^13*sgn(b*x^3 + a) + 3640*(d*x)^m*b^5*x^16*sgn(b*x^3 + a) +
 10*(d*x)^m*a^2*b^3*m^5*x^10*sgn(b*x^3 + a) + 14810*(d*x)^m*a*b^4*m^2*x^13*
sgn(b*x^3 + a) + 410*(d*x)^m*a^2*b^3*m^4*x^10*sgn(b*x^3 + a) + 34840*(d*x)^
m*a*b^4*m*x^13*sgn(b*x^3 + a) + 5950*(d*x)^m*a^2*b^3*m^3*x^10*sgn(b*x^3 + a
) + 22400*(d*x)^m*a*b^4*x^13*sgn(b*x^3 + a) + 10*(d*x)^m*a^3*b^2*m^5*x^7*sg
n(b*x^3 + a) + 36550*(d*x)^m*a^2*b^3*m^2*x^10*sgn(b*x^3 + a) + 440*(d*x)^m*
a^3*b^2*m^4*x^7*sgn(b*x^3 + a) + 89240*(d*x)^m*a^2*b^3*m*x^10*sgn(b*x^3 + a
) + 6970*(d*x)^m*a^3*b^2*m^3*x^7*sgn(b*x^3 + a) + 58240*(d*x)^m*a^2*b^3*x^1
0*sgn(b*x^3 + a) + 5*(d*x)^m*a^4*b*m^5*x^4*sgn(b*x^3 + a) + 47260*(d*x)^m*a
^3*b^2*m^2*x^7*sgn(b*x^3 + a) + 235*(d*x)^m*a^4*b*m^4*x^4*sgn(b*x^3 + a) +
123920*(d*x)^m*a^3*b^2*m*x^7*sgn(b*x^3 + a) + 4085*(d*x)^m*a^4*b*m^3*x^4*sg
n(b*x^3 + a) + 83200*(d*x)^m*a^3*b^2*x^7*sgn(b*x^3 + a) + (d*x)^m*a^5*m^5*x
*sgn(b*x^3 + a) + 31685*(d*x)^m*a^4*b*m^2*x^4*sgn(b*x^3 + a) + 50*(d*x)^m*a
^5*m^4*x*sgn(b*x^3 + a) + 100630*(d*x)^m*a^4*b*m*x^4*sgn(b*x^3 + a) + 955*(
d*x)^m*a^5*m^3*x*sgn(b*x^3 + a) + 72800*(d*x)^m*a^4*b*x^4*sgn(b*x^3 + a) +
8650*(d*x)^m*a^5*m^2*x*sgn(b*x^3 + a) + 36824*(d*x)^m*a^5*m*x*sgn(b*x^3 + a
) + 58240*(d*x)^m*a^5*x*sgn(b*x^3 + a))/(m^6 + 51*m^5 + 1005*m^4 + 9605*m^3
 + 45474*m^2 + 95064*m + 58240)
```

3.119 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=205

$$\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+7}}{d^7(m+7)\left(a+bx^3\right)}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+\frac{a^3\sqrt{a^2+2ab \cdots}}{d(m+\cdots}$$

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3))
 + (3*a^2*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a
+ b*x^3)) + (3*a*b^2*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7*(7
 + m)*(a + b*x^3)) + (b^3*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
d^10*(10 + m)*(a + b*x^3))
```

Rubi [A] time = 0.0851968, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 270}

$$\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+7}}{d^7(m+7)\left(a+bx^3\right)}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+\frac{a^3\sqrt{a^2+2ab \cdots}}{d(m+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3))
 + (3*a^2*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a
+ b*x^3)) + (3*a*b^2*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7*(7
 + m)*(a + b*x^3)) + (b^3*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
d^10*(10 + m)*(a + b*x^3))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m \left(ab+b^2x^3\right)^3 dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^3b^3(dx)^m+\frac{3a^2b^4(dx)^{3+m}}{d^3}+\frac{3ab^5(dx)^{6+m}}{d^6}+\frac{b^6(dx)^{9+m}}{d^9}\right) dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{3a^2b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}+\frac{3ab^2(dx)^{7+\cdots}}{d^7(7\cdots}\end{aligned}$$

Mathematica [A] time = 0.0709297, size = 131, normalized size = 0.64

$$\frac{x\sqrt{\left(a+bx^3\right)^2}(dx)^m\left(3a^2b\left(m^3+18m^2+87m+70\right)x^3+a^3\left(m^3+21m^2+138m+280\right)+3ab^2\left(m^3+15m^2+54m+40\right)\right.}{(m+1)(m+4)(m+7)(m+10)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a^3*(280 + 138*m + 21*m^2 + m^3) + 3*a^2*b*
(70 + 87*m + 18*m^2 + m^3)*x^3 + 3*a*b^2*(40 + 54*m + 15*m^2 + m^3)*x^6 + b
^3*(28 + 39*m + 12*m^2 + m^3)*x^9))/((1 + m)*(4 + m)*(7 + m)*(10 + m)*(a +
b*x^3))
```

Maple [A] time = 0.006, size = 199, normalized size = 1.

$$\frac{\left(b^3m^3x^9+12\,b^3m^2x^9+39\,b^3mx^9+3\,ab^2m^3x^6+28\,b^3x^9+45\,ab^2m^2x^6+162\,ab^2mx^6+3\,a^2bm^3x^3+120\,ab^2x^6+54\,a^2bm\right.}{(10+m)\,(7+m)\,(4+m)\,(1+m)\left(bx^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] x*(b^3*m^3*x^9+12*b^3*m^2*x^9+39*b^3*m*x^9+3*a*b^2*m^3*x^6+28*b^3*x^9+45*a*
b^2*m^2*x^6+162*a*b^2*m*x^6+3*a^2*b*m^3*x^3+120*a*b^2*x^6+54*a^2*b*m^2*x^3+
261*a^2*b*m*x^3+a^3*m^3+210*a^2*b*x^3+21*a^3*m^2+138*a^3*m+280*a^3)*(d*x)^m
*((b*x^3+a)^2)^(3/2)/(10+m)/(7+m)/(4+m)/(1+m)/(b*x^3+a)^3
```

Maxima [A] time = 1.09974, size = 161, normalized size = 0.79

$$\frac{\left(\left(m^3+12\,m^2+39\,m+28\right)b^3d^mx^{10}+3\left(m^3+15\,m^2+54\,m+40\right)ab^2d^mx^7+3\left(m^3+18\,m^2+87\,m+70\right)a^2bd^mx^4+\left(m^3\right.\right.}{m^4+22\,m^3+159\,m^2+418\,m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 12*m^2 + 39*m + 28)*b^3*d^m*x^10 + 3*(m^3 + 15*m^2 + 54*m + 40)*a*b
^2*d^m*x^7 + 3*(m^3 + 18*m^2 + 87*m + 70)*a^2*b*d^m*x^4 + (m^3 + 21*m^2 + 1
38*m + 280)*a^3*d^m*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Fricas [A] time = 1.58441, size = 358, normalized size = 1.75

$$\frac{\left(\left(b^3m^3+12\,b^3m^2+39\,b^3m+28\,b^3\right)x^{10}+3\left(ab^2m^3+15\,ab^2m^2+54\,ab^2m+40\,ab^2\right)x^7+3\left(a^2bm^3+18\,a^2bm^2+87\,a^2bm\right.\right.}{m^4+22\,m^3+159\,m^2+418\,m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 12*b^3*m^2 + 39*b^3*m + 28*b^3)*x^10 + 3*(a*b^2*m^3 + 15*a*b^2*
m^2 + 54*a*b^2*m + 40*a*b^2)*x^7 + 3*(a^2*b*m^3 + 18*a^2*b*m^2 + 87*a^2*b*m
 + 70*a^2*b)*x^4 + (a^3*m^3 + 21*a^3*m^2 + 138*a^3*m + 280*a^3)*x)*(d*x)^m/
(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**3)**2)**(3/2), x)
```

Giac [B] time = 1.12653, size = 518, normalized size = 2.53

$$\frac{(dx)^m b^3m^3x^{10}\mathrm{sgn}\left(bx^3+a\right)+12\,(dx)^m b^3m^2x^{10}\mathrm{sgn}\left(bx^3+a\right)+39\,(dx)^m b^3mx^{10}\mathrm{sgn}\left(bx^3+a\right)+3\,(dx)^m ab^2m^3x^7\mathrm{sg}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^3*m^3*x^10*sgn(b*x^3 + a) + 12*(d*x)^m*b^3*m^2*x^10*sgn(b*x^3 +
a) + 39*(d*x)^m*b^3*m*x^10*sgn(b*x^3 + a) + 3*(d*x)^m*a*b^2*m^3*x^7*sgn(b*x
^3 + a) + 28*(d*x)^m*b^3*x^10*sgn(b*x^3 + a) + 45*(d*x)^m*a*b^2*m^2*x^7*sgn
(b*x^3 + a) + 162*(d*x)^m*a*b^2*m*x^7*sgn(b*x^3 + a) + 3*(d*x)^m*a^2*b*m^3*
x^4*sgn(b*x^3 + a) + 120*(d*x)^m*a*b^2*x^7*sgn(b*x^3 + a) + 54*(d*x)^m*a^2*
b*m^2*x^4*sgn(b*x^3 + a) + 261*(d*x)^m*a^2*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m
*a^3*m^3*x*sgn(b*x^3 + a) + 210*(d*x)^m*a^2*b*x^4*sgn(b*x^3 + a) + 21*(d*x)
^m*a^3*m^2*x*sgn(b*x^3 + a) + 138*(d*x)^m*a^3*m*x*sgn(b*x^3 + a) + 280*(d*x
)^m*a^3*x*sgn(b*x^3 + a))/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

3.120 $\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=97

$$\frac{b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+1}}{d(m+1)\left(a+bx^3\right)}$$

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3)) +
 (b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a + b*x^3)
)
```

Rubi [A] time = 0.0378413, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$\frac{b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+1}}{d(m+1)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3)) +
 (b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a + b*x^3)
)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(ab(dx)^m+\frac{b^2(dx)^{3+m}}{d^3}\right)dx}{ab+b^2x^3}\\ &= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.0232985, size = 53, normalized size = 0.55

$$\frac{x\sqrt{\left(a+bx^3\right)^2}(dx)^m\left(a(m+4)+b(m+1)x^3\right)}{(m+1)(m+4)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a*(4 + m) + b*(1 + m)*x^3))/((1 + m)*(4 + m
)*(a + b*x^3))
```

Maple [A] time = 0.005, size = 56, normalized size = 0.6

$$\frac{\left(bmx^3 + bx^3 + am + 4\,a\right)x\,(dx)^m}{(4+m)\,(1+m)\left(bx^3+a\right)}\sqrt{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

```
[Out] x*(b*m*x^3+b*x^3+a*m+4*a)*(d*x)^m*((b*x^3+a)^2)^(1/2)/(4+m)/(1+m)/(b*x^3+a)
```

Maxima [A] time = 1.01596, size = 47, normalized size = 0.48

$$\frac{\left(bd^m(m+1)x^4 + ad^m(m+4)x\right)x^m}{m^2+5\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*d^m*(m + 1)*x^4 + a*d^m*(m + 4)*x)*x^m/(m^2 + 5*m + 4)
```

Fricas [A] time = 1.51837, size = 77, normalized size = 0.79

$$\frac{\left((bm+b)x^4 + (am+4\,a)x\right)(dx)^m}{m^2+5\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^4 + (a*m + 4*a)*x)*(d*x)^m/(m^2 + 5*m + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m\sqrt{\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**3)**2), x)
```

Giac [A] time = 1.12445, size = 112, normalized size = 1.15

$$\frac{(dx)^m bmx^4 \text{sgn}\left(bx^3+a\right)+(dx)^m bx^4 \text{sgn}\left(bx^3+a\right)+(dx)^m amx \text{sgn}\left(bx^3+a\right)+4\,(dx)^m ax \text{sgn}\left(bx^3+a\right)}{m^2+5\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m*b*x^4*sgn(b*x^3 + a) + (d*x)^m*a*
m*x*sgn(b*x^3 + a) + 4*(d*x)^m*a*x*sgn(b*x^3 + a))/(m^2 + 5*m + 4)
```

3.121 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx$

Optimal. Leaf size=73

$$\frac{(a+bx^3)(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.033872, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$\frac{(a+bx^3)(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx &= \frac{(ab+b^2x^3)\int \frac{(dx)^m}{ab+b^2x^3} \, dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(dx)^{1+m}(a+bx^3)\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.0184156, size = 62, normalized size = 0.85

$$\frac{x(a+bx^3)(dx)^m\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{a(m+1)\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, 1 + (1 + m)/3, -((b*
x^3)/a)])/(a*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{b^2x^6 + 2\,abx^3 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{b^2x^6 + 2\,abx^3 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{b^2x^6 + 2\,abx^3 + a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{\left(a + bx^3\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt((a + b*x**3)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{b^2x^6 + 2\,abx^3 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

3.122 $\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^3\right)(dx)^{m+1}\,{}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.0345446, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$\frac{\left(a+bx^3\right)(dx)^{m+1}\,{}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{(dx)^m}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\ &= \frac{(dx)^{1+m}\left(a+bx^3\right)\,{}_2F_1\left(3,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

Mathematica [A] time = 0.0199385, size = 60, normalized size = 0.82

$$\frac{x\left(a+bx^3\right)(dx)^m\,{}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^3(m+1)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3)
/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int (dx)^m \left(b^2x^6 + 2\,abx^3 + a^2\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,(dx)^m}{b^4x^{12} + 4\,ab^3x^9 + 6\,a^2b^2x^6 + 4\,a^3bx^3 + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m/(b^4*x^12 + 4*a*b^3*x^9 +
6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**3)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

3.123 $\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^3\right)(dx)^{m+1}\,{}_2F_1\left(5,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.0349544, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$\frac{\left(a+bx^3\right)(dx)^{m+1}\,{}_2F_1\left(5,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{(dx)^m}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\ &= \frac{(dx)^{1+m}\left(a+bx^3\right)\,{}_2F_1\left(5,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

Mathematica [A] time = 0.0185104, size = 60, normalized size = 0.82

$$\frac{x\left(a+bx^3\right)(dx)^m\,{}_2F_1\left(5,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^5(m+1)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b*x^3)
/a)])/(a^5*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int (dx)^m \left(b^2x^6 + 2\,abx^3 + a^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,(dx)^m}{b^6x^{18} + 6\,ab^5x^{15} + 15\,a^2b^4x^{12} + 20\,a^3b^3x^9 + 15\,a^4b^2x^6 + 6\,a^5bx^3 + a^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m/(b^6*x^18 + 6*a*b^5*x^15 +
 15*a^2*b^4*x^12 + 20*a^3*b^3*x^9 + 15*a^4*b^2*x^6 + 6*a^5*b*x^3 + a^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**3)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

3.124 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=77

$$\frac{(dx)^{m+1}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{m+1}{3},-2p;\frac{m+4}{3};-\frac{bx^3}{a}\right)}{d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[(1 + m)/3, -
2*p, (4 + m)/3, -((b*x^3)/a)])/(d*(1 + m)*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] time = 0.0253913, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1356, 364}

$$\frac{(dx)^{m+1}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{m+1}{3},-2p;\frac{m+4}{3};-\frac{bx^3}{a}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[(1 + m)/3, -
2*p, (4 + m)/3, -((b*x^3)/a)])/(d*(1 + m)*(1 + (b*x^3)/a)^(2*p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int (dx)^m \left(1+\frac{bx^3}{a}\right)^{2p} dx\\ &= \frac{(dx)^{1+m}\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{1+m}{3},-2p;\frac{4+m}{3};-\frac{bx^3}{a}\right)}{d(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0172367, size = 66, normalized size = 0.86

$$\frac{x(dx)^m\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p} {}_2F_1\left(\frac{m+1}{3},-2p;\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x*(d*x)^m*((a + b*x^3)^2)^p*Hypergeometric2F1[(1 + m)/3, -2*p, 1 + (1 + m)
/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(2*p))
```

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int (dx)^m \left(b^2x^6 + 2\,abx^3 + a^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^6 + 2\,abx^3 + a^2\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p (dx)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

3.125 $\int x^{11} \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=172

$$\frac{\left(a+bx^3\right)^4\left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(p+2)}-\frac{a\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{b^4(2p+3)}+\frac{a^2\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(p+1)}-\frac{a^3\left(a+b\right.}{}$$

```
[Out] -(a^3*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^4*(1 + 2*p)) + (a^2*(
a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(2*b^4*(1 + p)) - (a*(a + b*x^3
)^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^4*(3 + 2*p)) + ((a + b*x^3)^4*(a^2 +
2*a*b*x^3 + b^2*x^6)^p)/(6*b^4*(2 + p))
```

Rubi [A] time = 0.112672, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1356, 266, 43}

$$\frac{\left(a+bx^3\right)^4\left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(p+2)}-\frac{a\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{b^4(2p+3)}+\frac{a^2\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(p+1)}-\frac{a^3\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] -(a^3*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^4*(1 + 2*p)) + (a^2*(
a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(2*b^4*(1 + p)) - (a*(a + b*x^3
)^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^4*(3 + 2*p)) + ((a + b*x^3)^4*(a^2 +
2*a*b*x^3 + b^2*x^6)^p)/(6*b^4*(2 + p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p dx = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^{11}\left(1+\frac{bx^3}{a}\right)^{2p} dx$$
$$= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x^3\left(1+\frac{bx}{a}\right)^{2p} dx, x, x^3\right)$$
$$= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^3\left(1+\frac{bx}{a}\right)^{2p}}{b^3}+\frac{3a^3\left(1+\frac{bx}{a}\right)^{1+2p}}{b^3}-\right.\right.$$
$$= -\frac{a^3\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^4(1+2p)}+\frac{a^2\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(1+p)}-\frac{a\left(a+bx^3\right)}{}$$

Mathematica [A] time = 0.0558852, size = 110, normalized size = 0.64

$$\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p\left(3a^2b(2p+1)x^3-3a^3-3ab^2\left(2p^2+3p+1\right)x^6+b^3\left(4p^3+12p^2+11p+3\right)x^9\right)}{6b^4(p+1)(p+2)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^3 - 3*a*b^2*(1
 + 3*p + 2*p^2)*x^6 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^9))/(6*b^4*(1 + p)*
(2 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A] time = 0.008, size = 150, normalized size = 0.9

$$-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p\left(-4\,b^3p^3x^9-12\,b^3p^2x^9-11\,b^3px^9-3\,b^3x^9+6\,ab^2p^2x^6+9\,ab^2px^6+3\,ab^2x^6-6\,a^2bpx^3-3\,a^2bx^3\right.}{6\,b^4\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] -1/6*(b^2*x^6+2*a*b*x^3+a^2)^p*(-4*b^3*p^3*x^9-12*b^3*p^2*x^9-11*b^3*p*x^9-
3*b^3*x^9+6*a*b^2*p^2*x^6+9*a*b^2*p*x^6+3*a*b^2*x^6-6*a^2*b*p*x^3-3*a^2*b*x
^3+3*a^3)*(b*x^3+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

Maxima [A] time = 1.15209, size = 155, normalized size = 0.9

$$\frac{\left(\left(4\,p^3+12\,p^2+11\,p+3\right)b^4x^{12}+2\left(2\,p^3+3\,p^2+p\right)ab^3x^9-3\left(2\,p^2+p\right)a^2b^2x^6+6\,a^3bpx^3-3\,a^4\right)\left(bx^3+a\right)^{2\,p}}{6\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/6*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^12 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^9
 - 3*(2*p^2 + p)*a^2*b^2*x^6 + 6*a^3*b*p*x^3 - 3*a^4)*(b*x^3 + a)^(2*p)/((4
```

```
*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

Fricas [A] time = 1.58433, size = 336, normalized size = 1.95

$$\frac{\left(\left(4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4\right)x^{12}+2\left(2\,ab^3p^3+3\,ab^3p^2+ab^3p\right)x^9+6\,a^3bpx^3-3\left(2\,a^2b^2p^2+a^2b^2p\right)x^6-3\,a^4\right)\left(b^2x^6+2\,abx^3+a^2\right)^p}{6\left(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/6*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^12 + 2*(2*a*b^3*p^3 + 3*
a*b^3*p^2 + a*b^3*p)*x^9 + 6*a^3*b*p*x^3 - 3*(2*a^2*b^2*p^2 + a^2*b^2*p)*x^
6 - 3*a^4)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35*b^4*p
^2 + 25*b^4*p + 6*b^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.13232, size = 506, normalized size = 2.94

$$\frac{4\left(b^2x^6+2\,abx^3+a^2\right)^pb^4p^3x^{12}+12\left(b^2x^6+2\,abx^3+a^2\right)^pb^4p^2x^{12}+11\left(b^2x^6+2\,abx^3+a^2\right)^pb^4px^{12}+4\left(b^2x^6+2\,abx^3 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/6*(4*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p^3*x^12 + 12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^p*b^4*p^2*x^12 + 11*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p*x^12 + 4*(b
^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^3*x^9 + 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p
*b^4*x^12 + 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^2*x^9 + 2*(b^2*x^6 + 2*
a*b*x^3 + a^2)^p*a*b^3*p*x^9 - 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2*p^2*
x^6 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2*p*x^6 + 6*(b^2*x^6 + 2*a*b*x^
3 + a^2)^p*a^3*b*p*x^3 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^4)/(4*b^4*p^4 +
20*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

3.126 $\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p\,dx$

Optimal. Leaf size=130

$$\frac{\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+3)}-\frac{a\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(p+1)}+\frac{a^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+1)}$$

```
[Out] (a^2*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + 2*p)) - (a*(a +
 b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + p)) + ((a + b*x^3)^3*(
a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(3 + 2*p))
```

Rubi [A] time = 0.0788709, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1356, 266, 43}

$$\frac{\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+3)}-\frac{a\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(p+1)}+\frac{a^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (a^2*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + 2*p)) - (a*(a +
 b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + p)) + ((a + b*x^3)^3*(
a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(3 + 2*p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^8\left(1+\frac{bx^3}{a}\right)^{2p}dx$$
$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\mathrm{Subst}\left(\int x^2\left(1+\frac{bx}{a}\right)^{2p}dx,x,x^3\right)$$
$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\mathrm{Subst}\left(\int\left(\frac{a^2\left(1+\frac{bx}{a}\right)^{2p}}{b^2}-\frac{2a^2\left(1+\frac{bx}{a}\right)^{1+2p}}{b^2}\right.\right.$$
$$=\frac{a^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(1+2p)}-\frac{a\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(1+p)}+\frac{\left(a+bx^3\right)^3}{3}$$

Mathematica [A] time = 0.0329366, size = 77, normalized size = 0.59

$$\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p\left(a^2-ab(2p+1)x^3+b^2\left(2p^2+3p+1\right)x^6\right)}{3b^3(p+1)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(a^2 - a*b*(1 + 2*p)*x^3 + b^2*(1 + 3*p + 2*
p^2)*x^6))/(3*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A] time = 0.007, size = 96, normalized size = 0.7

$$\frac{\left(2b^2p^2x^6+3b^2px^6+b^2x^6-2abpx^3-abx^3+a^2\right)\left(bx^3+a\right)\left(b^2x^6+2abx^3+a^2\right)^p}{3b^3\left(4p^3+12p^2+11p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] 1/3*(b*x^3+a)*(2*b^2*p^2*x^6+3*b^2*p*x^6+b^2*x^6-2*a*b*p*x^3-a*b*x^3+a^2)*(
b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

Maxima [A] time = 1.02904, size = 107, normalized size = 0.82

$$\frac{\left(\left(2p^2+3p+1\right)b^3x^9+\left(2p^2+p\right)ab^2x^6-2a^2bpx^3+a^3\right)\left(bx^3+a\right)^{2p}}{3\left(4p^3+12p^2+11p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/3*((2*p^2 + 3*p + 1)*b^3*x^9 + (2*p^2 + p)*a*b^2*x^6 - 2*a^2*b*p*x^3 + a^
3)*(b*x^3 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Fricas [A] time = 1.62213, size = 223, normalized size = 1.72

$$\frac{\left(\left(2\,b^{3}p^{2}+3\,b^{3}p+b^{3}\right)x^{9}-2\,a^{2}bpx^{3}+\left(2\,ab^{2}p^{2}+ab^{2}p\right)x^{6}+a^{3}\right)\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}}{3\left(4\,b^{3}p^{3}+12\,b^{3}p^{2}+11\,b^{3}p+3\,b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/3*((2*b^3*p^2 + 3*b^3*p + b^3)*x^9 - 2*a^2*b*p*x^3 + (2*a*b^2*p^2 + a*b^2
*p)*x^6 + a^3)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b
^3*p + 3*b^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.12567, size = 317, normalized size = 2.44

$$\frac{2\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}b^{3}p^{2}x^{9}+3\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}b^{3}px^{9}+\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}b^{3}x^{9}+2\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}ab^{2}p}{3\left(4\,b^{3}p^{3}+12\,b^{3}p^{2}+11\,b^{3}p+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/3*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*p^2*x^9 + 3*(b^2*x^6 + 2*a*b*x^3 +
 a^2)^p*b^3*p*x^9 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*x^9 + 2*(b^2*x^6 + 2*
a*b*x^3 + a^2)^p*a*b^2*p^2*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^2*p*x^6
- 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b*p*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)
^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

3.127 $\int x^5 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=84

$$\frac{\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(p+1)}-\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(2p+1)}$$

```
[Out] -(a*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^2*(1 + 2*p)) + ((a + b*
x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(6*b^2*(1 + p))
```

Rubi [A] time = 0.055235, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1356, 266, 43}

$$\frac{\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(p+1)}-\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] -(a*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^2*(1 + 2*p)) + ((a + b*
x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(6*b^2*(1 + p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^5\left(1+\frac{bx^3}{a}\right)^{2p}dx$$
$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{2p}dx,x,x^3\right)$$
$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b}+\frac{a\left(1+\frac{bx}{a}\right)^{1+2p}}{b}\right)dx,$$
$$=-\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(1+2p)}+\frac{\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(1+p)}$$

Mathematica [A] time = 0.0200858, size = 51, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p\left(b(2p+1)x^3-a\right)}{6b^2(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-a + b*(1 + 2*p)*x^3))/(6*b^2*(1 + p)*(1 +
2*p))
```

Maple [A] time = 0.007, size = 60, normalized size = 0.7

$$-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p\left(-2\,x^3pb-bx^3+a\right)\left(bx^3+a\right)}{6\,b^2\left(2\,p^2+3\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] -1/6*(b^2*x^6+2*a*b*x^3+a^2)^p*(-2*b*p*x^3-b*x^3+a)*(b*x^3+a)/b^2/(2*p^2+3*
p+1)
```

Maxima [A] time = 1.07303, size = 73, normalized size = 0.87

$$\frac{\left(b^2(2\,p+1)x^6+2\,abpx^3-a^2\right)\left(bx^3+a\right)^{2\,p}}{6\left(2\,p^2+3\,p+1\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/6*(b^2*(2*p + 1)*x^6 + 2*a*b*p*x^3 - a^2)*(b*x^3 + a)^(2*p)/((2*p^2 + 3*p
 + 1)*b^2)
```

Fricas [A] time = 1.55878, size = 142, normalized size = 1.69

$$\frac{\left(\left(2\, b^{2} p + b^{2}\right) x^{6} + 2\, a b p x^{3} - a^{2}\right) \left(b^{2} x^{6} + 2\, a b x^{3} + a^{2}\right)^{p}}{6 \left(2\, b^{2} p^{2} + 3\, b^{2} p + b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/6*((2*b^2*p + b^2)*x^6 + 2*a*b*p*x^3 - a^2)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p
/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.12001, size = 178, normalized size = 2.12

$$\frac{2 \left(b^{2} x^{6} + 2\, a b x^{3} + a^{2}\right)^{p} b^{2} p x^{6} + \left(b^{2} x^{6} + 2\, a b x^{3} + a^{2}\right)^{p} b^{2} x^{6} + 2 \left(b^{2} x^{6} + 2\, a b x^{3} + a^{2}\right)^{p} a b p x^{3} - \left(b^{2} x^{6} + 2\, a b x^{3} + a^{2}\right)^{p} a^{2}}{6 \left(2\, b^{2} p^{2} + 3\, b^{2} p + b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/6*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^2*p*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2
)^p*b^2*x^6 + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b*p*x^3 - (b^2*x^6 + 2*a*b*
x^3 + a^2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

3.128 $\int x^4 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{1}{5}x^5\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

```
[Out] (x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*x
^3)/a)])/(5*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] time = 0.0192857, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1356, 364}

$$\frac{1}{5}x^5\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*x
^3)/a)])/(5*(1 + (b*x^3)/a)^(2*p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^4\left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= \frac{1}{5}x^5\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.007131, size = 51, normalized size = 0.85

$$\frac{1}{5}x^5\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p} {}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^5*((a + b*x^3)^2)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*x^3)/a)])/(5*
(1 + (b*x^3)/a)^(2*p))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int x^{4} \left(b^{2} x^{6} + 2\,ab x^{3} + a^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^{2} x^{6} + 2\,ab x^{3} + a^{2}\right)^{p} x^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{2} x^{6} + 2\,ab x^{3} + a^{2}\right)^{p} x^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{4} \left(\left(a + b x^{3}\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Integral(x**4*((a + b*x**3)**2)**p, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

3.129 $\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{1}{4}x^4\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

```
[Out] (x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*x
^3)/a)])/(4*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] time = 0.0193485, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1356, 364}

$$\frac{1}{4}x^4\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*x
^3)/a)])/(4*(1 + (b*x^3)/a)^(2*p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^3\left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= \frac{1}{4}x^4\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.006243, size = 51, normalized size = 0.85

$$\frac{1}{4}x^4\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p} {}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^4*((a + b*x^3)^2)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*x^3)/a)])/(4*
(1 + (b*x^3)/a)^(2*p))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int x^3 \left(b^2x^6 + 2\,abx^3 + a^2\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\left(a + bx^3\right)^2\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Integral(x**3*((a + b*x**3)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

3.130 $\int x^2 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=41

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(2p+1)}$$

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b*(1 + 2*p))
```

Rubi [A] time = 0.0276851, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1352, 609}

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b*(1 + 2*p))
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \frac{1}{3}\text{Subst}\left(\int\left(a^2+2abx+b^2x^2\right)^p\,dx, x, x^3\right)\\ &= \frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.0062623, size = 32, normalized size = 0.78

$$\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p}{3b(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p)/(3*b*(1 + 2*p))
```

Maple [A] time = 0.004, size = 40, normalized size = 1.

$$\frac{\left(bx^{3}+a\right)\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}}{3\,b\left(1+2\,p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] 1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b/(1+2*p)
```

Maxima [A] time = 1.04373, size = 41, normalized size = 1.

$$\frac{\left(bx^{3}+a\right)\left(bx^{3}+a\right)^{2\,p}}{3\,b\left(2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(b*x^3 + a)*(b*x^3 + a)^(2*p)/(b*(2*p + 1))
```

Fricas [A] time = 1.55186, size = 80, normalized size = 1.95

$$\frac{\left(bx^{3}+a\right)\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}}{3\left(2\,bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^3 + a)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(2*b*p + b)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.11458, size = 78, normalized size = 1.9

$$\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}bx^{3}+\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}a}{3\left(2\,bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/3*((b^2*x^6 + 2*a*b*x^3 + a^2)^p*b*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a)
/(2*b*p + b)
```

3.131 $\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx$

Optimal. Leaf size=58

$$\frac{x^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(1,2p+\frac{5}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}$$

```
[Out] (x^2*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1, 5/3 + 2
*p, 5/3, -((b*x^3)/a)])/(2*a)
```

Rubi [A] time = 0.0151618, antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1356, 364}

$$\frac{1}{2}x^2\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{2}{3},-2p;\frac{5}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*x
^3)/a)])/(2*(1 + (b*x^3)/a)^(2*p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x\left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= \frac{1}{2}x^2\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{2}{3},-2p;\frac{5}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0061913, size = 51, normalized size = 0.88

$$\frac{1}{2}x^2\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p} {}_2F_1\left(\frac{2}{3},-2p;\frac{5}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^2*((a + b*x^3)^2)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*x^3)/a)])/(2*
(1 + (b*x^3)/a)^(2*p))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x\left(b^2x^6 + 2\,abx^3 + a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^6 + 2\,abx^3 + a^2\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\left(a + bx^3\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

3.132 $\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx$

Optimal. Leaf size=53

$$\frac{x\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(1,2p+\frac{4}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{a}$$

```
[Out] (x*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1, 4/3 + 2*p
, 4/3, -((b*x^3)/a)])/a
```

Rubi [A] time = 0.019247, antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1343, 246, 245}

$$x\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{1}{3},-2p;\frac{4}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1/3, -2*p, 4/3, -((b*x^3
)/a)])/(1 + (b*x^3)/a)^(2*p)
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(2ab+2b^2x^3\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \left(2ab+2b^2x^3\right)^{2p}\,dx\\ &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= x\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{1}{3},-2p;\frac{4}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [C] time = 0.166889, size = 204, normalized size = 3.85

$$\frac{4^{-p}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-2p}\left(\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}\right)^{-2p}\left(\left(a+bx^3\right)^2\right)^pF_1\left(2p+1;-2p,-2p;2(p+1);-\frac{i\left(\sqrt[3]{b}x+(-1)^{2/3}\sqrt[3]{a}\right)}{\sqrt{3}\sqrt[3]{a}},\right.}{\sqrt[3]{b}(2p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*((a + b*x^3)^2)^p*AppellF1[1 + 2*p, -2*p,
 -2*p, 2*(1 + p), ((-I)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(Sqrt[3]*a^(1/3))
, (I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(4^p*b^(1/3)*
(1 + 2*p)*((a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^(2*
p)*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^(2*p))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \left(b^2x^6+2\,abx^3+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6+2\,abx^3+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2x^6+2\,abx^3+a^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

3.133 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x}\,dx$

Optimal. Leaf size=63

$$-\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(1,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a(2p+1)}$$

```
[Out] -((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1, 1 + 2*p, 2
*(1 + p), 1 + (b*x^3)/a])/(3*a*(1 + 2*p))
```

Rubi [A] time = 0.0373573, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1356, 266, 65}

$$-\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(1,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
```

```
[Out] -((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1, 1 + 2*p, 2
*(1 + p), 1 + (b*x^3)/a])/(3*a*(1 + 2*p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x}\,dx = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x}\,dx$$

$$= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\mathrm{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(1,1+2p;2(1+p);1+\frac{bx^3}{a}\right)}{3a(1+2p)}$$

Mathematica [A] time = 0.0108053, size = 54, normalized size = 0.86

$$-\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p\,{}_2F_1\left(1,2p+1;2p+2;\frac{bx^3}{a}+1\right)}{3a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
```

```
[Out] -((a + b*x^3)*((a + b*x^3)^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 + 2*p, 1 +
(b*x^3)/a])/(3*a*(1 + 2*p))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x,x)
```

```
[Out] Integral(((a + b*x**3)**2)**p/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

3.134 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

```
[Out] -(((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x^
3)/a)])/(x*(1 + (b*x^3)/a)^(2*p)))
```

Rubi [A] time = 0.0183031, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1356, 364}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

```
[Out] -(((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x^
3)/a)])/(x*(1 + (b*x^3)/a)^(2*p)))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^2}\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^2}\,dx\\ &= -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.0064322, size = 49, normalized size = 0.84

$$-\frac{\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p} {}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

```
[Out] -((((a + b*x^3)^2)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x^3)/a)])/(x*(
1 + (b*x^3)/a)^(2*p)))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^{3}\right)^{2}\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**2,x)
```

```
[Out] Integral(((a + b*x**3)**2)**p/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

3.135 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^3}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

```
[Out] -((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((b*x^3
)/a)])/(2*x^2*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] time = 0.0188288, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1356, 364}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
```

```
[Out] -((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((b*x^3
)/a)])/(2*x^2*(1 + (b*x^3)/a)^(2*p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^3}\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^3}\,dx\\ &= -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0060824, size = 51, normalized size = 0.85

$$-\frac{\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p}\,{}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
```

```
[Out] -(((a + b*x^3)^2)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((b*x^3)/a)])/(2*x^
2*(1 + (b*x^3)/a)^(2*p))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**3,x)
```

```
[Out] Integral(((a + b*x**3)**2)**p/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

3.136 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx$

Optimal. Leaf size=64

$$\frac{b(a+bx^3)(a^2+2abx^3+b^2x^6)^p\,{}_2F_1\left(2,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a^2(2p+1)}$$

```
[Out] (b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 + (b*x^3)/a])/(3*a^2*(1 + 2*p))
```

Rubi [A] time = 0.0411508, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1356, 266, 65}

$$\frac{b(a+bx^3)(a^2+2abx^3+b^2x^6)^p\,{}_2F_1\left(2,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
```

```
[Out] (b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 + (b*x^3)/a])/(3*a^2*(1 + 2*p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^4}\,dx = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^4}\,dx$$

$$= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\mathrm{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x^2}\,dx, x, x^3\right)$$

$$= \frac{b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(2,1+2p;2(1+p);1+\frac{bx^3}{a}\right)}{3a^2(1+2p)}$$

Mathematica [A] time = 0.0114877, size = 55, normalized size = 0.86

$$\frac{b\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p\,{}_2F_1\left(2,2p+1;2p+2;\frac{bx^3}{a}+1\right)}{3a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
```

```
[Out] (b*(a + b*x^3)*((a + b*x^3)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, 1 +
 (b*x^3)/a])/(3*a^2*(1 + 2*p))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**p/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

3.137 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^5} dx$

Optimal. Leaf size=60

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

```
[Out] -((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -((b*x^
3)/a)])/(4*x^4*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] time = 0.0197642, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1356, 364}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
```

```
[Out] -((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -((b*x^
3)/a)])/(4*x^4*(1 + (b*x^3)/a)^(2*p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^5} dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^5} dx\\
&= -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.0063592, size = 51, normalized size = 0.85

$$-\frac{\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p} {}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
```

```
[Out] -(((a + b*x^3)^2)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -((b*x^3)/a)])/(4*x
^4*(1 + (b*x^3)/a)^(2*p))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{\left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)^{p}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^{2} x^{6}+2 a b x^{3}+a^{2}\right)^{p}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^{2} x^{6}+2 a b x^{3}+a^{2}\right)^{p}}{x^{5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+b x^{3}\right)^{2}\right)^{p}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**5,x)
```

```
[Out] Integral(((a + b*x**3)**2)**p/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

3.138 $\int \frac{x^8}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=81

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{x^3}{3c}$$

```
[Out] x^3/(3*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

Rubi [A] time = 0.0827972, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1357, 703, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3 + c*x^6),x]
```

```
[Out] x^3/(3*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^3\right)\\
&= \frac{x^3}{3c} + \frac{\operatorname{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^3\right)}{3c}\\
&= \frac{x^3}{3c} - \frac{b\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2} + \frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2}\\
&= \frac{x^3}{3c} - \frac{b\log\left(a+bx^3+cx^6\right)}{6c^2} - \frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3c^2}\\
&= \frac{x^3}{3c} - \frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} - \frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}
\end{aligned}$$

Mathematica [A] time = 0.0539784, size = 78, normalized size = 0.96

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - b\log\left(a+bx^3+cx^6\right) + 2cx^3}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*x^3 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

Maple [A] time = 0.004, size = 111, normalized size = 1.4

$$\frac{x^3}{3c} - \frac{b\ln\left(cx^6+bx^3+a\right)}{6c^2} - \frac{2a}{3c}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} + \frac{b^2}{3c^2}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*x^3/c-1/6*b*ln(c*x^6+b*x^3+a)/c^2-2/3/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^3+b)/(4*a*c-b^2)^(1/2))*a+1/3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*
a*c-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59755, size = 556, normalized size = 6.86

$$\left[\frac{2\left(b^2c-4\,ac^2\right)x^3-\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\left(b^3-4\,abc\right)\log\left(cx^6+bx^3+a\right)}{6\left(b^2c^2-4\,ac^3\right)}, \frac{2\left(b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c - 4*a*c^2)*x^3 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b
*x^3 + a)) - (b^3 - 4*a*b*c)*log(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1
/6*(2*(b^2*c - 4*a*c^2)*x^3 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^6 +
 b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 2.30463, size = 316, normalized size = 3.9

$$\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6c^2\left(4ac-b^2\right)}\right)\log\left(x^3+\frac{-ab-12ac^2\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6c^2\left(4ac-b^2\right)}\right)+3b^2c\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6c^2\left(4ac-b^2\right)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))*
log(x**3 + (-a*b - 12*a*c**2*(-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(6*c**2*(4*a*c - b**2))) + 3*b**2*c*(-b/(6*c**2) - sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(6*c**2) + s
qrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))*log(x**3 + (-a*b
 - 12*a*c**2*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a
*c - b**2))) + 3*b**2*c*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**3/(3*c)
```

Giac [A] time = 1.4113, size = 101, normalized size = 1.25

$$\frac{x^3}{3\,c} - \frac{b\log\left(cx^6 + bx^3 + a\right)}{6\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3/c - 1/6*b*log(c*x^6 + b*x^3 + a)/c^2 + 1/3*(b^2 - 2*a*c)*arctan((2*
c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.139 $\int \frac{x^5}{a+bx^3+cx^6} \, dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^3 + c*x^6]/(6*c)
```

Rubi [A] time = 0.0578286, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1357, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3 + c*x^6),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^3 + c*x^6]/(6*c)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,x^3\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6c}$$
$$= \frac{\log\left(a+bx^3+cx^6\right)}{6c} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^3\right)}{3c}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

Mathematica [A] time = 0.0230189, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^3+cx^6\right) - \frac{2b\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^3 + c*x^6])/(6*c)
```

Maple [A] time = 0.003, size = 60, normalized size = 1.

$$\frac{\ln\left(cx^6+bx^3+a\right)}{6c} - \frac{b}{3c}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/6*ln(c*x^6+b*x^3+a)/c-1/3*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c
-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51339, size = 443, normalized size = 7.03

$$\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^6+bx^3+a\right)}{6\left(b^2c-4\,ac^2\right)},\frac{2\sqrt{-b^2+4\,ac}b\arctan\left(-\frac{\left(2\,cx^3+b\right)\sqrt{\ldots}}{b^2-\ldots}\right)\ldots}{6\left(\ldots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x
^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) + (b^2 - 4*a*c)*log(c*x^6 +
 b*x^3 + a))/(b^2*c - 4*a*c^2), 1/6*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^6 + b*x^3
+ a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] time = 1.28991, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{6c\left(4ac-b^2\right)}+\frac{1}{6c}\right)\log\left(x^3+\frac{-12ac\left(-\frac{b\sqrt{-4ac+b^2}}{6c\left(4ac-b^2\right)}+\frac{1}{6c}\right)+2a+3b^2\left(-\frac{b\sqrt{-4ac+b^2}}{6c\left(4ac-b^2\right)}+\frac{1}{6c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{6c\left(4ac-b^2\right)}+\frac{1}{6c}\right)\log\left(x^3\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c
*(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(-b
*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b) + (b*sqrt(-4*a*c +
 b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c*(b*sqrt(-4*a*c +
 b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(b*sqrt(-4*a*c + b**2
)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b)
```

Giac [A] time = 1.42903, size = 80, normalized size = 1.27

$$-\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\sqrt{-b^2+4\,ac}c}+\frac{\log\left(cx^6+bx^3+a\right)}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
6*log(c*x^6 + b*x^3 + a)/c
```

3.140 $\int \frac{x^2}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=38

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

```
[Out] (-2*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.0357311, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3 + c*x^6),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*Sqrt[b^2 - 4*a*c])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)\\
&= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)\right)\\
&= -\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [A] time = 0.0093453, size = 42, normalized size = 1.11

$$\frac{2\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/(3*Sqrt[-b^2 + 4*a*c])
```

Maple [A] time = 0.002, size = 37, normalized size = 1.

$$\frac{2}{3}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^6+b*x^3+a),x)
```

```
[Out] 2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47343, size = 296, normalized size = 7.79

$$\left[\frac{\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac-(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{3\,\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{3\,(b^2-4\,ac)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*
a*c))/(c*x^6 + b*x^3 + a))/sqrt(b^2 - 4*a*c), -2/3*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] time = 0.632454, size = 131, normalized size = 3.45

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**3 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3 + sqrt(-1/(4*a*c - b**2))*log(x**3 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3
```

Giac [A] time = 1.39525, size = 49, normalized size = 1.29

$$\frac{2\arctan\left(\frac{2cx^3+b}{\sqrt{-b^2+4ac}}\right)}{3\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.141 $\int \frac{1}{x(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^3 + c*x^6]/(6*a)
```

Rubi [A] time = 0.0695275, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1357, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^3 + c*x^6]/(6*a)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx,x,x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)}{3a}+\frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,x^3\right)}{3a}\\
&= \frac{\log(x)}{a}-\frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^3\right)}{3a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}
\end{aligned}
$$

Mathematica [C] time = 0.0234794, size = 66, normalized size = 0.96

$$
\frac{\log(x)}{a}-\frac{\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\#1^3c\log(x-\#1)+b\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] Log[x]/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]*#
1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

Maple [A] time = 0.006, size = 66, normalized size = 1.

$$
\frac{\ln(x)}{a}-\frac{\ln\left(cx^6+bx^3+a\right)}{6a}-\frac{b}{3a}\arctan\left(\left(2cx^3+b\right)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^6+b*x^3+a),x)
```

```
[Out] ln(x)/a-1/6*ln(c*x^6+b*x^3+a)/a-1/3/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b
)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61825, size = 510, normalized size = 7.39

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^6+bx^3+a\right)+6\left(b^2-4\,ac\right)\log\left(x\right)}{6\left(ab^2-4\,a^2c\right)},\ \frac{2\sqrt{-b^2+4\,a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x
^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^2 - 4*a*c)*log(c*x^6 +
 b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/6*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^6 + b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] time = 3.6923, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)}-\frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)}-\frac{1}{6a}\right)+3ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)}-\frac{1}{6a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-12*a**
2*c*(-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(-b*
sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c))
+ (b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-12*a*
*2*c*(b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(b*s
qrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c)) +
 log(x)/a
```

Giac [A] time = 1.43558, size = 89, normalized size = 1.29

$$-\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a}-\frac{\log\left(cx^6+bx^3+a\right)}{6\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
6*log(c*x^6 + b*x^3 + a)/a + log(abs(x))/a
```

3.142 $\int \frac{1}{x^4(a+bx^3+cx^6)} dx$

Optimal. Leaf size=89

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

```
[Out] -1/(3*a*x^3) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^3 + c*x^6])/(6*a^2
)
```

Rubi [A] time = 0.125024, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/(3*a*x^3) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^3 + c*x^6])/(6*a^2
)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{3ax^3}+\frac{\operatorname{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx,x,x^3\right)}{3a}\\
&= -\frac{1}{3ax^3}+\frac{\operatorname{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^3\right)}{3a}\\
&= -\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{\operatorname{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,x^3\right)}{3a^2}\\
&= -\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{b\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6a^2}+\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6a^2}\\
&= -\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^3\right)}{3a^2}\\
&= -\frac{1}{3ax^3}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}
\end{aligned}$$

Mathematica [C] time = 0.029396, size = 92, normalized size = 1.03

$$\frac{\operatorname{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\#1^3bc\log(x-\#1)-ac\log(x-\#1)+b^2\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/(3*a*x^3) - (b*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6 & , (b^2*Log[x
- #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a^2)
```

Maple [A] time = 0.008, size = 119, normalized size = 1.3

$$-\frac{1}{3\,a x^{3}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(c x^{6}+b x^{3}+a\right)}{6\,a^{2}}-\frac{2\,c}{3\,a}\arctan\left(\left(2\,c x^{3}+b\right)\frac{1}{\sqrt{4\,ac-b^{2}}}\right)\frac{1}{\sqrt{4\,ac-b^{2}}}+\frac{b^{2}}{3\,a^{2}}\arctan\left(\left(2\,c x^{3}+b\right)\frac{1}{\sqrt{4\,a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/3/a/x^3-b*ln(x)/a^2+1/6*b*ln(c*x^6+b*x^3+a)/a^2-2/3/a/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*c+1/3/a^2/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87747, size = 664, normalized size = 7.46

$$\left[-\frac{\left(b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}x^{3}\log\left(\frac{2\,c^{2}x^{6}+2\,bcx^{3}+b^{2}-2\,ac+\left(2\,cx^{3}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{6}+bx^{3}+a}\right)-\left(b^{3}-4\,abc\right)x^{3}\log\left(cx^{6}+bx^{3}+a\right)+6\left(b^{3}-4\,abc\right)x^{3}}{6\left(a^{2}b^{2}-4\,a^{3}c\right)x^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2
 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^3 - 4
*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3), -1/6*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3)]
```

Sympy [B] time = 80.3891, size = 345, normalized size = 3.88

$$\left(\frac{b}{6a^{2}}-\frac{\sqrt{-4ac+b^{2}}\left(2ac-b^{2}\right)}{6a^{2}\left(4ac-b^{2}\right)}\right)\log\left(x^{3}+\frac{-12a^{3}c\left(\frac{b}{6a^{2}}-\frac{\sqrt{-4ac+b^{2}}\left(2ac-b^{2}\right)}{6a^{2}\left(4ac-b^{2}\right)}\right)+3a^{2}b^{2}\left(\frac{b}{6a^{2}}-\frac{\sqrt{-4ac+b^{2}}\left(2ac-b^{2}\right)}{6a^{2}\left(4ac-b^{2}\right)}\right)+3abc-b^{3}}{2ac^{2}-b^{2}c}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] (b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a**2*(4*a*c - b**2)))*l
og(x**3 + (-12*a**3*c*(b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a
**2*(4*a*c - b**2))) + 3*a**2*b**2*(b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(6*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) +
(b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a**2*(4*a*c - b**2)))*l
og(x**3 + (-12*a**3*c*(b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a
**2*(4*a*c - b**2))) + 3*a**2*b**2*(b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(6*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) -
1/(3*a*x**3) - b*log(x)/a**2
```

Giac [A] time = 1.41367, size = 126, normalized size = 1.42

$$\frac{b\log\left(cx^6+bx^3+a\right)}{6\,a^2}-\frac{b\log\left(|x|\right)}{a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a^2}+\frac{bx^3-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*b*log(c*x^6 + b*x^3 + a)/a^2 - b*log(abs(x))/a^2 + 1/3*(b^2 - 2*a*c)*ar
ctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/3*(b*x^
3 - a)/(a^2*x^3)
```

3.143 $\int \frac{x^7}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=636

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4a\cdots}}\right.}{6\ 2^{2/3}c^{5/3}\sqrt[3]{\cdots}}$$

```
[Out] x^2/(2*c) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)
*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2
^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((b - (b^2 - 2*a*c)
/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*c)/Sq
rt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3
*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqr
t[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2
^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rubi [A] time = 1.24899, antiderivative size = 636, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1367, 1510, 292, 31, 634, 617, 204, 628}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4a\cdots}}\right.}{6\ 2^{2/3}c^{5/3}\sqrt[3]{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] x^2/(2*c) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)
*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2
^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((b - (b^2 - 2*a*c)
/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*c)/Sq
rt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3
*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqr
t[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2
^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \frac{x^2}{2c} - \frac{\int \frac{x(2a+2bx^3)}{a+bx^3+cx^6}\,dx}{2c}$$

$$= \frac{x^2}{2c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\,2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\,2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\cdots\right.}{\cdots}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{\cdots}\right.}{3\,2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}\cdots$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{\cdots}\right.}{3\,2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}\cdots$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\cdots}{3\,2^{2/3}c^{5\cdots}}$$

Mathematica [C] time = 0.0301584, size = 70, normalized size = 0.11

$$\frac{3x^2 - 2\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\#1^3b\log(x-\#1)+a\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] (3*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#
1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

Maple [C] time = 0.089, size = 61, normalized size = 0.1

$$\frac{x^2}{2c} - \frac{1}{3c}\sum_{\_R=\text{RootOf}(c\_Z^6+b\_Z^3+a)}\frac{\left(\_R^4b+\_R\,a\right)\ln(x-\_R)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/2*x^2/c-1/3/c*sum((_R^4*b+_R*a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6
*c+_Z^3*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 7.78997, size = 12034, normalized size = 18.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(3)*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*
c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2
*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5
 - 4*a*c^6))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^
6 + 32*a^2*b^2*c^7 - 32*a^3*c^8)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 -
 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^
12 - 64*a^3*c^13)) - sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2
*c^3))*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*
b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*
b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*sqrt
((2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 + (1/2)^(2/3)*((b^8*c^5 - 13*
a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b^10 - 1
0*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 1
2*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) - (b^10 - 12*a*b^8*c + 52*a^
2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2
 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^
4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3*b^5*c +
 25*a^4*b^3*c^2 - 20*a^5*b*c^3 - (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^
7)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*
a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^
6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3))/(a^3*b^5 - 5*a^4*b^3
*c + 5*a^5*b*c^2)) - (1/2)^(1/3)*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^6 + 32*a^2*
b^2*c^7 - 32*a^3*c^8)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b
^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a
^3*c^13)) - sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3)*x)*
((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c +
 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3) + sqrt(3)*(
a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2))/(a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)
) - 4*sqrt(3)*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^
6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 -
 4*a*c^6))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^6
+ 32*a^2*b^2*c^7 - 32*a^3*c^8)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 5
```

0*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c ^3))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^ 8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^ 4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*sqrt((2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 - (1/2)^(2/3)*((b^8*c^5 - 13*a* b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b^10 - 10* a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12* a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^10 - 12*a*b^8*c + 52*a^2* b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4* c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3* c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3*b^5*c + 2 5*a^4*b^3*c^2 - 20*a^5*b*c^3 + (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7) *sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a* b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6* c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b ^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3))/(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)) - (1/2)^(1/3)*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^6 + 32*a^2*b^ 2*c^7 - 32*a^3*c^8)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4 *c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3 *c^13)) + sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 3 5*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3) - sqrt(3)*(a^ 2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2))/(a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)) + (1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^1 0 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^1 0 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*log(2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 + (1/2)^(2/3)*((b^8*c^ 5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b ^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c ^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) - (b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50 *a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3* b^5*c + 25*a^4*b^3*c^2 - 20*a^5*b*c^3 - (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a ^4*b*c^7)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^ 4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b ^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35 *a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)) + (1/2)^(1/3) *c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8* c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4* c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*log(2*(a ^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 - (1/2)^(2/3)*((b^8*c^5 - 13*a*b^6* c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b^10 - 10*a*b^ 8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^ 4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^10 - 12*a*b^8*c + 52*a^2*b^6* c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^ 2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3*b^5*c + 25*a^ 4*b^3*c^2 - 20*a^5*b*c^3 + (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7)*sqr t((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b ^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2* c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2

```
- 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c
^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)) - 2*(1/2)^(1/3)*c*((b^4 -
3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*
b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*log((1/2)^(2/3)*(b^
10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4 - (b^8*c
^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*sqrt((b^
10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^
10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a
^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*
a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) + 2*(a^3*b^5 - 5*a^4*b^3*c + 5*a
^5*b*c^2)*x) - 2*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a
*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^
2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^
5 - 4*a*c^6))^(1/3)*log((1/2)^(2/3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 9
5*a^3*b^4*c^3 + 60*a^4*b^2*c^4 + (b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 -
 112*a^3*b^2*c^8 + 64*a^4*c^9)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 5
0*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b
^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c
^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))
^(2/3) + 2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x) - 3*x^2)/c
```

Sympy [A] time = 3.9408, size = 279, normalized size = 0.44

$$\operatorname{RootSum}\left(t^6\left(46656a^3c^8-34992a^2b^2c^7+8748ab^4c^6-729b^6c^5\right)+t^3\left(432a^4c^4-1512a^3b^2c^3+1107a^2b^4c^2-297ab^6c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**8 - 34992*a**2*b**2*c**7 + 8748*a*b**4*c**6 -
729*b**6*c**5) + _t**3*(432*a**4*c**4 - 1512*a**3*b**2*c**3 + 1107*a**2*b**
4*c**2 - 297*a*b**6*c + 27*b**8) + a**5, Lambda(_t, _t*log(x + (-15552*_t**
5*a**4*c**9 + 27216*_t**5*a**3*b**2*c**8 - 14580*_t**5*a**2*b**4*c**7 + 315
9*_t**5*a*b**6*c**6 - 243*_t**5*b**8*c**5 - 72*_t**2*a**5*c**5 + 594*_t**2*
a**4*b**2*c**4 - 864*_t**2*a**3*b**4*c**3 + 468*_t**2*a**2*b**6*c**2 - 108*
_t**2*a*b**8*c + 9*_t**2*b**10)/(5*a**5*b*c**2 - 5*a**4*b**3*c + a**3*b**5)
))) + x**2/(2*c)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^7/(c*x^6 + b*x^3 + a), x)
```

3.144 $\int \frac{x^6}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=631

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}}\right.}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2}\right.}$$

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 -
 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi [A] time = 1.02409, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}}\right.}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^3 + c*x^6),x]
```


```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 -
 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```


Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx = \frac{x}{c} - \frac{\int \frac{a+bx^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\right.}{3}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right.}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right.}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\right.}{3\sqrt[3]{2}c^{4/3}\left(\right.}$$

Mathematica [C] time = 0.0296814, size = 70, normalized size = 0.11

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\#1^3b\log(x-\#1)+a\log(x-\#1)}{\#1^2b+2\#1^5c}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^3 + c*x^6),x]
```

```
[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/
(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] time = 0.003, size = 59, normalized size = 0.1

$$\frac{x}{c} + \frac{1}{3c}\sum_{\_R=\text{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(-\_R^3b-a\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^6+b*x^3+a),x)
```

```
[Out] x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*
b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 5.37546, size = 11169, normalized size = 17.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*ar
ctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6
- 112*a^3*b^2*c^7 + 64*a^4*c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*
a^3*c^11)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 +
24*a^4*b*c^4)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 +
 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*
(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^
4*c^8)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(b^9 -
 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a
*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^1
1)))/(b^2*c^4 - 4*a*c^5))^(2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)
*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16
*a^4*c^4 - (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 -
 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c
^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4
*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c
^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c
+ 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8
- 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3
))/(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)) + 2*sqrt(3)*(a^3*b^4 - 4*a^4*b^2*c
+ 2*a^5*c^2))/(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)) - 4*sqrt(3)*(1/2)^(1/3)*
c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(s
qrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*
c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^9 -
```

```
11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*(-(b^3 - 2
*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^
3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c
^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(b^8*c^4 - 13*a*b^
6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8)*sqrt((b^8 - 8*a*b^6*
c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 +
48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^
2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c
^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(
2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 1
0*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 + (b^7*c^4 - 12*a*
b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9
 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/3
)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*
b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^
3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2
*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2
*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3))/(a^2*b^4 - 4*a^3*b^2*c +
 2*a^4*c^2)) - 2*sqrt(3)*(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2))/(a^3*b^4 - 4*
a^4*b^2*c + 2*a^5*c^2)) - (1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5
))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 -
 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12*
a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c
^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c
^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1
/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^
2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*
c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b
^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/3)*c*(-(b^3
 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a
^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*
c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3
+ 16*a^4*c^4 + (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8
- 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c
^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a
^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4
 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6
)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^
4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)
*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*
c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^
(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 -
 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b
^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*
(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2
```

```
*b^2*c^2 - 8*a^3*c^3 - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4
*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5
))^(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*
a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log
(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*
a^2*b^2*c^2 - 8*a^3*c^3 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8
- 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*
b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*
a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*
c^5))^(1/3)) + 6*x)/c
```

Sympy [A] time = 2.66067, size = 196, normalized size = 0.31

$$\mathrm{RootSum}\left(t^6\left(46656a^3c^7 - 34992a^2b^2c^6 + 8748ab^4c^5 - 729b^6c^4\right) + t^3\left(864a^3bc^3 - 864a^2b^3c^2 + 270ab^5c - 27b^7\right) + a^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5*c
 - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*_t*
*4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**
2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4))) +
 x/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^6/(c*x^6 + b*x^3 + a), x)
```

3.145 $\int \frac{x^4}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\right.}{}$$

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[
b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c])
- ((b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c]
) + ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*
a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2
/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sq
rt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x +
2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^
2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[
b^2 - 4*a*c])
```

Rubi [A] time = 0.519725, antiderivative size = 558, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1374, 292, 31, 634, 617, 204, 628}

$$-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[
b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c])
- ((b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c]
) + ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*
a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2
/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sq
rt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x +
2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^
2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[
b^2 - 4*a*c])
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
```

```
n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+c\ldots}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\,2^{2/3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\,2^{2/3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \ldots$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\ldots\right.}{3\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\ldots\right.}{3\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}} + \frac{\left(b-\sqrt{b^2-4\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.0171399, size = 44, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^3 b+\#1^6 c+a\&,\frac{\#1^2\log(x-\#1)}{2\#1^3 c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1^2)/(b + 2*c*#1^3) & ]/3
```

Maple [C] time = 0.003, size = 43, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\_R^4\ln(x-\_R)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

Fricas [B] time = 2.9361, size = 8007, normalized size = 14.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*(1/2)^(1/3)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^
2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^
2 - 4*a*c^3))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(b^5*c^2 - 8*a*b^3*c^
3 + 16*a^2*b*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^
5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2))*
sqrt((2*(a*b^2 - 2*a^2*c)*x^2 + (1/2)^(2/3)*((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12
*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2
)*x)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 1
2*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3)
 - 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sqrt((b^4 - 4*a*b
^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(
-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^
4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3))/(a*b
^2 - 2*a^2*c))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b
^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c
^3))^(1/3) - (1/2)^(1/3)*(sqrt(3)*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x*
sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6
 - 64*a^3*c^7)) - sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)*(-((b^2*c^2 - 4*
a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*
b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3) - sqrt(3)*(a*b^2 - 2
*a^2*c))/(a*b^2 - 2*a^2*c)) - 2/3*sqrt(3)*(1/2)^(1/3)*(((b^2*c^2 - 4*a*c^3)
*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^
6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(
sqrt(3)*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^
2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + sqrt(3)*(b
^4 - 6*a*b^2*c + 8*a^2*c^2))*sqrt((2*(a*b^2 - 2*a^2*c)*x^2 - (1/2)^(2/3)*((
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + (
b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b
)/(b^2*c^2 - 4*a*c^3))^(2/3) + 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 1
6*a^3*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*
a^2*b^2*c^6 - 64*a^3*c^7))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*
c^2 - 4*a*c^3))^(1/3))/(a*b^2 - 2*a^2*c))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 -
 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c
^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3) - (1/2)^(1/3)*(sqrt(3)*(b^5*c^2 - 8*a*
b^3*c^3 + 16*a^2*b*c^4)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*
a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^
2*c^2)*x)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4
 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(
1/3) + sqrt(3)*(a*b^2 - 2*a^2*c))/(a*b^2 - 2*a^2*c)) - 1/6*(1/2)^(1/3)*(-((
b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c
^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3)*log(-2*(
```

```
a*b^2 - 2*a^2*c)*x^2 - (1/2)^(2/3)*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^
5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(-((
b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c
^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) + 2*(1/2
)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(-((b^2*c^
2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 4
8*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3)) - 1/6*(1/2)^(
1/3)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12
*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)*
log(-2*(a*b^2 - 2*a^2*c)*x^2 + (1/2)^(2/3)*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*
a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)
*x)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*
a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) -
 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(((
b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c
^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)) + 1/3*(
1/2)^(1/3)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c
^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))
^(1/3)*log(-(1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 - (b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^
6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(-((b^2*c^2 - 4*a*c^3
)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c
^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 2*(a*b^2 - 2*a^2*c)*x)
+ 1/3*(1/2)^(1/3)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/
(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a
*c^3))^(1/3)*log(-(1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 + (b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^
2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(((b^2*c^2 - 4*
a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*
b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 2*(a*b^2 - 2*a^2*c
)*x)
```

Sympy [A] time = 1.75708, size = 175, normalized size = 0.31

$$\operatorname{RootSum}\left(t^6\left(46656a^3c^5-34992a^2b^2c^4+8748ab^4c^3-729b^6c^2\right)+t^3\left(-432a^2bc^2+216ab^3c-27b^5\right)+a^2,\left(t\mapsto t\log\left(x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**5 - 34992*a**2*b**2*c**4 + 8748*a*b**4*c**3 -
729*b**6*c**2) + _t**3*(-432*a**2*b*c**2 + 216*a*b**3*c - 27*b**5) + a**2,
Lambda(_t, _t*log(x + (15552*_t**5*a**3*c**5 - 11664*_t**5*a**2*b**2*c**4 +
 2916*_t**5*a*b**4*c**3 - 243*_t**5*b**6*c**2 - 108*_t**2*a**2*b*c**2 + 63*
_t**2*a*b**3*c - 9*_t**2*b**5)/(2*a**2*c - a*b**2))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

3.146 $\int \frac{x^3}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2}}\right.}{6\sqrt[3]{2}}$$

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[
b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c])
- ((b + Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c]
) - ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^2 - 4*
a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1
/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sq
rt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x +
2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^
2 - 4*a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[
b^2 - 4*a*c])
```

Rubi [A] time = 0.573605, antiderivative size = 558, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1374, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2}}\right.}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[
b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c])
- ((b + Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c]
) - ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^2 - 4*
a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1
/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sq
rt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x +
2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^
2 - 4*a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[
b^2 - 4*a*c])
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
```

```
n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+c\ldots}$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}\sqrt{b^2-4ac}}+\ldots$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\ldots\right.}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}\ldots$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\ldots\right.}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}\ldots$$

$$= \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\,\mathrm{l}\ldots}{3\ldots}$$

Mathematica [C] time = 0.0160134, size = 42, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^3 b+\#1^6 c+a\&,\frac{\#1\log(x-\#1)}{2\#1^3 c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1)/(b + 2*c*#1^3) & ]/3
```

Maple [C] time = 0.003, size = 43, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\_R^3\ln(x-\_R)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

Fricas [B] time = 1.99071, size = 5501, normalized size = 9.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*arctan(-1/
3*((1/2)^(2/3)*(sqrt(3)*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4
)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - sqrt(3
)*(b^4 - 4*a*b^2*c))*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 +
 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(2/3)*sqrt(-((1/2)^(
1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3) - b
*x^2 - (1/2)^(2/3)*(b^3 - 4*a*b*c)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 -
12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(2/3))
/b) - sqrt(3)*a*b - (1/2)^(2/3)*(sqrt(3)*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3 - 64*a^3*c^4)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))*x - sqrt(3)*(b^4 - 4*a*b^2*c)*x)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))
^(2/3))/(a*b)) + 2/3*sqrt(3)*(1/2)^(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))
^(1/3)*arctan(-1/3*((1/2)^(2/3)*(sqrt(3)*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3 - 64*a^3*c^4)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)) + sqrt(3)*(b^4 - 4*a*b^2*c))*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(2
/3)*sqrt(((1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(-((b^2*c - 4*a*c^2)*sqrt(
b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4
*a*c^2))^(1/3) + b*x^2 + (1/2)^(2/3)*(b^3 - 4*a*b*c)*(-((b^2*c - 4*a*c^2)*s
qrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c
 - 4*a*c^2))^(2/3))/b) + sqrt(3)*a*b - (1/2)^(2/3)*(sqrt(3)*(b^6*c - 12*a*b
^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5))*x + sqrt(3)*(b^4 - 4*a*b^2*c)*x)*(-((b^2*c - 4*
a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1
)/(b^2*c - 4*a*c^2))^(2/3))/(a*b)) - 1/6*(1/2)^(1/3)*(((b^2*c - 4*a*c^2)*sq
rt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c
- 4*a*c^2))^(1/3)*log(-(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(
b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(((b^2*c - 4*
a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1
)/(b^2*c - 4*a*c^2))^(1/3) + b*x^2 + (1/2)^(2/3)*(b^3 - 4*a*b*c)*(((b^2*c -
 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
+ 1)/(b^2*c - 4*a*c^2))^(2/3)) - 1/6*(1/2)^(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(
b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4
*a*c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(-((b^2*c - 4*a*c
^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(
b^2*c - 4*a*c^2))^(1/3) + b*x^2 + (1/2)^(2/3)*(b^3 - 4*a*b*c)*(-((b^2*c - 4
*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) -
1)/(b^2*c - 4*a*c^2))^(2/3)) + 1/3*(1/2)^(1/3)*(((b^2*c - 4*a*c^2)*sqrt(b^2
```

```
/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*
c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(((b^2*c - 4*a*c^2)*sq
rt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c
- 4*a*c^2))^(1/3) + b*x) + 1/3*(1/2)^(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2
))^(1/3)*log(-(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(-((b^2*c - 4*a*c^2)*sqr
t(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c -
 4*a*c^2))^(1/3) + b*x)
```

Sympy [A] time = 1.8966, size = 122, normalized size = 0.22

$$\operatorname{RootSum}\left(t^6\left(46656a^3c^4 - 34992a^2b^2c^3 + 8748ab^4c^2 - 729b^6c\right) + t^3\left(432a^2c^2 - 216ab^2c + 27b^4\right) + a, \left(t \mapsto t\log\left(x + \frac{25\ldots}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**4 - 34992*a**2*b**2*c**3 + 8748*a*b**4*c**2 -
729*b**6*c) + _t**3*(432*a**2*c**2 - 216*a*b**2*c + 27*b**4) + a, Lambda(_t
, _t*log(x + (2592*_t**4*a**2*c**3 - 1296*_t**4*a*b**2*c**2 + 162*_t**4*b**
4*c + 12*_t*a*c - 3*_t*b**2)/b)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

3.147 $\int \frac{x}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4a\right.}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}}$$

```
[Out] -((2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)
)) + (2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a
*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1
/3)) - (2^(1/3)*c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)
*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (2^(1/3)*c^(1/3)
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2
/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2
])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^
(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])^(1/3))
```

Rubi [A] time = 0.471834, antiderivative size = 558, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1375, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4a\right.}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3 + c*x^6),x]
```


```
[Out] -((2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)
)) + (2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a
*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1
/3)) - (2^(1/3)*c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)
*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (2^(1/3)*c^(1/3)
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2
/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2
])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^
(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])^(1/3))
```


Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
```

```
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{a+bx^3+cx^6}\,dx = \frac{c\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{\left(\sqrt[3]{2}c^{2/3}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(\sqrt[3]{2}c^{2/3}\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(\sqrt[3]{2}c^{2/3}\right)\int}{3\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{c^{2/3}\int}{}$$

$$= -\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\log\left(\right.}{}$$

$$= -\frac{\sqrt[3]{2}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}\right.}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2}}}$$

Mathematica [C] time = 0.0175309, size = 43, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\log(x-\#1)}{2\#1^4c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1 + 2*c*#1^4) & ]/3
```

Maple [C] time = 0.002, size = 41, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\_R\ln(x-\_R)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(_R/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a), x)
```

Fricas [B] time = 2.12046, size = 6175, normalized size = 11.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*arctan(-1
/3*(2*sqrt(3)*(1/2)^(1/3)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt(b^2/(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x*(-((a*b^2 - 4*a^2*c)*s
qrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2
 - 4*a^2*c))^(1/3) - 2*sqrt(3)*(1/2)^(5/6)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*(-((a*b
^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^
3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*sqrt((2*b*c*x^2 + (1/2)^(2/3)*((a*b^6 - 1
2*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x - (b^4 - 4*a*b^2*c)*x)*(-((a*b^2 - 4*a^2
*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(
a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c - (a*b^5 - 8*a^2*b^3*c
 + 16*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5
*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3))/(b*c)) + sqrt(3)*b)/b) +
2/3*sqrt(3)*(1/2)^(1/3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)*arctan(-1/3
*(2*sqrt(3)*(1/2)^(1/3)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt(b^2/(a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x*(((a*b^2 - 4*a^2*c)*sqrt
(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 -
4*a^2*c))^(1/3) - 2*sqrt(3)*sqrt(1/2)*(1/2)^(1/3)*(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*
(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)*sqrt((2*b*c*x^2 - (1/2)^(2/3)*((a*b
^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3
*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x + (b^4 - 4*a*b^2*c)*x)*(((a*b^2 -
4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) -
 1)/(a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c + (a*b^5 - 8*a^2*
b^3*c + 16*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*
b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3))/(b*c)) - sqrt(3)*b)/b
) - 1/6*(1/2)^(1/3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x^2
+ (1/2)^(2/3)*((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^
2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x - (b^4 - 4*a*b^
2*c)*x)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b
*c - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3))
- 1/6*(1/2)^(1/3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x^2 - (
```

```
1/2)^(2/3)*((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(
a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x + (b^4 - 4*a*b^2*c
)*x)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
- 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3
*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)) + 1/3
*(1/2)^(1/3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4
*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x + (1/2)^(
2/3)*(b^4 - 4*a*b^2*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3
)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b
^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^
3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3)) + 1/3*(1/2)^(1/3)*(((a*b^2 - 4*a^2*c)*sq
rt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2
- 4*a^2*c))^(1/3)*log(2*b*c*x + (1/2)^(2/3)*(b^4 - 4*a*b^2*c + (a*b^6 - 12*
a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(2/3))
```

Sympy [A] time = 1.52902, size = 158, normalized size = 0.28

$$\operatorname{RootSum}\left(t^6\left(46656a^4c^3 - 34992a^3b^2c^2 + 8748a^2b^4c - 729ab^6\right) + t^3\left(-432a^2c^2 + 216ab^2c - 27b^4\right) + c, \left(t \mapsto t\log\left(x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**4*c**3 - 34992*a**3*b**2*c**2 + 8748*a**2*b**4*c -
729*a*b**6) + _t**3*(-432*a**2*c**2 + 216*a*b**2*c - 27*b**4) + c, Lambda(_
t, _t*log(x + (-15552*_t**5*a**4*c**3 + 11664*_t**5*a**3*b**2*c**2 - 2916*_
t**5*a**2*b**4*c + 243*_t**5*a*b**6 + 72*_t**2*a**2*c**2 - 54*_t**2*a*b**2*
c + 9*_t**2*b**4)/(b*c))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a), x)
```

3.148 $\int \frac{1}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}\right.\right.}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right.}$$

```
[Out] -((2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)
)) + (2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a
*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2
/3)) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)
*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (2^(2/3)*c^(2/3)
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2
/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2
])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^
(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])^(2/3))
```

Rubi [A] time = 0.595439, antiderivative size = 558, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1347, 200, 31, 634, 617, 204, 628}

$$-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}\right.\right.}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(-1),x]
```

```
[Out] -((2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)
)) + (2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a
*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2
/3)) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)
*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (2^(2/3)*c^(2/3)
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2
/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2
])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^
(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])^(2/3))
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
```

```
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{a+bx^3+cx^6}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2^{2/3}c\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(2^{2/3}c\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(2^{2/3}c\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-\cdots}}}{\sqrt[3]{2}}\cdots}}{3\sqrt{b^2-4ac}\left(b+\sqrt{\cdots}\right.}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\int \frac{\cdots}{\frac{\left(b-\sqrt{b^2\cdots}\right.}{2^{\cdots}}\cdots}}{3\sqrt[3]{2}\sqrt{b^2\cdots}}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\log\left(\left(b\cdots\right.\right.}{\cdots}$$

$$= -\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-\cdots}}\right.}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2\cdots}\right.}$$

Mathematica [C] time = 0.0177215, size = 45, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\log(x-\#1)}{\#1^2b+2\#1^5c}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(-1),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1^2 + 2*c*#1^5) & ]/3
```

Maple [C] time = 0.002, size = 40, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\ln(x-\_R)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(1/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^6 + b*x^3 + a), x)
```

Fricas [B] time = 2.94829, size = 8374, normalized size = 15.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2
 - 4*a^3*c))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^
4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - sqrt(3)*(b^5 - 6*a*b
^3*c + 8*a^2*b*c^2)*x)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*
c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2
- 4*a^3*c))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b
^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c
^2))*sqrt((2*(b^2*c^2 - 2*a*c^3)*x^2 + (1/2)^(2/3)*(b^6 - 8*a*b^4*c + 20*a^
2*b^2*c^2 - 16*a^3*c^3 - (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*
b*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2
 - 4*a^3*c))^(2/3) - (1/2)^(1/3)*((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)) - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*x)*(((a^2*b^2 - 4*a
^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b
^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3))/(b^2*c^2 - 2*a*c^3))
*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5
*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(2/3) + 2*
sqrt(3)*(b^2*c - 2*a*c^2))/(b^2*c - 2*a*c^2)) - 2/3*sqrt(3)*(1/2)^(1/3)*(-(
(a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^
4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3)*arctan(
-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*
a^6*b^2*c^2 - 64*a^7*c^3)) + sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(-(
(a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^
4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) - (1/2)
^(1/6)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*sqrt
((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 6
4*a^7*c^3)) + sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2))*sqrt((2*(b^2*c^2 - 2
*a*c^3)*x^2 + (1/2)^(2/3)*(b^6 - 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^3*c^3 +
(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*sqrt((b^4 - 4*a*b^
2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(
-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*
b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) + (1/
2)^(1/3)*((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x*sqrt((b^4 - 4*a*b^2*
c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + (b
^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*x)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b
^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) -
 b)/(a^2*b^2 - 4*a^3*c))^(1/3))/(b^2*c^2 - 2*a*c^3))*(-((a^2*b^2 - 4*a^3*c)
```

```
*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^
2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) - 2*sqrt(3)*(b^2*c - 2*a*c
^2))/(b^2*c - 2*a*c^2)) - 1/6*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 -
 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c
^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)*log(-2*(b^2*c^2 - 2*a*c^3)*x^2 - (1/2)
^(2/3)*(b^6 - 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^3*c^3 - (a^2*b^7 - 12*a^3*b
^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a
^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c
)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c
^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(2/3) + (1/2)^(1/3)*((a^2*b^5*c
 - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*
b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - (b^4*c - 6*a*b^2*c^2 +
 8*a^2*c^3)*x)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^
4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*
c))^(1/3)) - 1/6*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c +
 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a
^2*b^2 - 4*a^3*c))^(1/3)*log(-2*(b^2*c^2 - 2*a*c^3)*x^2 - (1/2)^(2/3)*(b^6
- 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^3*c^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^
4*b^3*c^2 - 64*a^5*b*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*
a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4
 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7
*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) - (1/2)^(1/3)*((a^2*b^5*c - 8*a^3*b^
3*c^2 + 16*a^4*b*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^
5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)
*x)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12
*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3))
 + 1/3*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)
/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*
a^3*c))^(1/3)*log(-2*(b^2*c - 2*a*c^2)*x + (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8
*a^2*c^2 - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqrt((b^4 - 4*a*b^2*c + 4
*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b
^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c +
48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)) + 1/3*(1/2)^
(1/3)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 -
12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3
)*log(-2*(b^2*c - 2*a*c^2)*x + (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2 + (
a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a
^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4*a^3*
c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*
c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3))
```

Sympy [A] time = 2.82539, size = 155, normalized size = 0.28

$$\operatorname{RootSum}\left(t^6\left(46656a^5c^3 - 34992a^4b^2c^2 + 8748a^3b^4c - 729a^2b^6\right) + t^3\left(432a^2bc^2 - 216ab^3c + 27b^5\right) + c^2, \left(t \mapsto t\log\left(x +\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**5*c**3 - 34992*a**4*b**2*c**2 + 8748*a**3*b**4*c -
729*a**2*b**6) + _t**3*(432*a**2*b*c**2 - 216*a*b**3*c + 27*b**5) + c**2, L
ambda(_t, _t*log(x + (-1296*_t**4*a**4*b*c**2 + 648*_t**4*a**3*b**3*c - 81*
_t**4*a**2*b**5 + 12*_t*a**2*c**2 - 15*_t*a*b**2*c + 3*_t*b**4)/(2*a*c**2 -
 b**2*c))))
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^6 + b*x^3 + a), x)
```

3.149 $\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=610

$$-\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\cdots}\right.}{6\ 2^{2/3}\cdots}$$

```
[Out] -(1/(a*x)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1
/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a*(b - Sqr
t[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3
]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log
[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt
[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^
2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(1/3)) - (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(
2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^
2])/(6*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(1 - b/Sqrt[b^2
- 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2
 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a
*c])^(1/3))
```

Rubi [A] time = 0.818118, antiderivative size = 610, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1368, 1510, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\cdots}\right.}{6\ 2^{2/3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(1/(a*x)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1
/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a*(b - Sqr
t[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3
]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log
[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt
[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^
2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(1/3)) - (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(
2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^
2])/(6*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(1 - b/Sqrt[b^2
- 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2
 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a
*c])^(1/3))
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx = -\frac{1}{ax}+\frac{\int \frac{x\left(-b-cx^3\right)}{a+bx^3+cx^6}\,dx}{a}$$

$$= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}$$

$$= -\frac{1}{ax}+\frac{\left(c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{cx}}\,dx}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\cdots}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4}\cdots}}{\sqrt[3]{2}}\cdots}}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots$$

$$= -\frac{1}{ax}+\frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{cx}\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-}}\cdots\right.}{3\,2^{2/3}a\sqrt[3]{b+\sqrt{b^2-}\cdots}}\cdots$$

$$= -\frac{1}{ax}+\frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{cx}\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-}}\cdots\right.}{3\,2^{2/3}a\sqrt[3]{b+\sqrt{b^2-}\cdots}}\cdots$$

$$= -\frac{1}{ax}+\frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{cx}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{cx}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.0314018, size = 71, normalized size = 0.12

$$-\frac{\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\#1^3c\log(x-\#1)+b\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{3a}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(1/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]
*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

Maple [C] time = 0.006, size = 61, normalized size = 0.1

$$-\frac{1}{ax}-\frac{1}{3\,a}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(\_R^4c+\_R\,b\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/a/x-1/3/a*sum((_R^4*c+_R*b)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+
_Z^3*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 6.21081, size = 11187, normalized size = 18.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((
b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 1
2*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*a
rctan(1/3*((1/2)^(5/6)*(sqrt(3)*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*sqrt
((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 -
 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) - sqrt(3)*(b^6 - 8*a*b^4*c
+ 18*a^2*b^2*c^2 - 8*a^3*c^3))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((
b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 1
2*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*s
qrt((2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x^2 + (1/2)^(2/3)*((a^4*b^8 - 13
*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b
^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) - (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2
 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b
^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(
2/3) - (1/2)^(1/3)*(b^7*c - 8*a*b^5*c^2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 - (a
^4*b^6*c - 10*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^
6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c
+ 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sq
rt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6
 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/
3))/(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)) - (1/2)^(1/3)*(sqrt(3)*(a^4*b^5 -
8*a^5*b^3*c + 16*a^6*b*c^2)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11
*c^3)) - sqrt(3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*x)*((b^3 -
2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11
*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3) + sqrt(3)*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c
^3))/(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)) - 4*sqrt(3)*(1/2)^(1/3)*a*x*((b^3 -
 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*
a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^1
1*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*arctan(1/3*((1/2)^(5/6)*(sqrt(3)*(a^4*b
^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a
^11*c^3)) + sqrt(3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3))*((b^3 -
 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*
```

```
a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^1
1*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*sqrt((2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*
c^5)*x^2 - (1/2)^(2/3)*((a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*
b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2
*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3))
 + (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*(
(b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - (1/2)^(1/3)*(b^7*c - 8*a*b^5*c^
2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 + (a^4*b^6*c - 10*a^5*b^4*c^2 + 32*a^6*b^2
*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
+ 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b
^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64
*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3))/(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5
)) - (1/2)^(1/3)*(sqrt(3)*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*
a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) + sqrt(3)*(b^6 - 8*a*b^4*c + 18
*a^2*b^2*c^2 - 8*a^3*c^3)*x)*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*
a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3) - s
qrt(3)*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3))/(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)
) - (1/2)^(1/3)*a*x*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c
 + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log(2*(b^4*c
^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x^2 + (1/2)^(2/3)*((a^4*b^8 - 13*a^5*b^6*c +
60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)) - (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3
*c^3 + 24*a^4*b*c^4)*x)*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b
^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - (1/2)^
(1/3)*(b^7*c - 8*a*b^5*c^2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 - (a^4*b^6*c - 10
*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*
b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2
*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4
*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)) - (1/2)^(
1/3)*a*x*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*
b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log(2*(b^4*c^3 - 4*a*b^
2*c^4 + 2*a^2*c^5)*x^2 - (1/2)^(2/3)*((a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*
c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^
2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
 64*a^11*c^3)) + (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a
^4*b*c^4)*x)*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a
^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - (1/2)^(1/3)*(b^7*
c - 8*a*b^5*c^2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 + (a^4*b^6*c - 10*a^5*b^4*c^
2 + 32*a^6*b^2*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a
^11*c^3)))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)) + 2*(1/2)^(1/3)*a*x*
((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^
2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log((1/2)^(2/3)*(b^9 - 11*a*b^7*
c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 - (a^4*b^8 - 13*a^5*b^6*
c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*sqrt((b^8 - 8*a*b^6*c +
```

```
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a
^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*
a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) + 2
*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x) + 2*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c
 - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*
c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))
/(a^4*b^2 - 4*a^5*c))^(1/3)*log((1/2)^(2/3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*
c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 + (a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*
c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
- 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 6
4*a^11*c^3)))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c +
 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*
a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) + 2*(b^4*c^3 - 4*a
*b^2*c^4 + 2*a^2*c^5)*x) - 6)/(a*x)
```

Sympy [A] time = 2.54423, size = 252, normalized size = 0.41

$$\text{RootSum}\left(t^6\left(46656a^7c^3-34992a^6b^2c^2+8748a^5b^4c-729a^4b^6\right)+t^3\left(-864a^3bc^3+864a^2b^3c^2-270ab^5c+27b^7\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**7*c**3 - 34992*a**6*b**2*c**2 + 8748*a**5*b**4*c -
729*a**4*b**6) + _t**3*(-864*a**3*b*c**3 + 864*a**2*b**3*c**2 - 270*a*b**5*
c + 27*b**7) + c**4, Lambda(_t, _t*log(x + (-15552*_t**5*a**8*c**4 + 27216*
_t**5*a**7*b**2*c**3 - 14580*_t**5*a**6*b**4*c**2 + 3159*_t**5*a**5*b**6*c
- 243*_t**5*a**4*b**8 + 252*_t**2*a**4*b*c**4 - 567*_t**2*a**3*b**3*c**3 +
378*_t**2*a**2*b**5*c**2 - 99*_t**2*a*b**7*c + 9*_t**2*b**9)/(2*a**2*c**5 -
 4*a*b**2*c**4 + b**4*c**3)))) - 1/(a*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6+bx^3+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^2), x)
```

3.150 $\int \frac{1}{x^3\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=612

$$\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{}}\right.}{6\sqrt[3]{2}a\left(\right.}$$

```
[Out] -1/(2*a*x^2) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^
(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b - S
qrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (
2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt
[3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*L
og[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b - Sq
rt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[
b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*
c])^(2/3)) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])
^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*
x^2])/(6*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[b^
2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b
^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 - 4
*a*c])^(2/3))
```

Rubi [A] time = 0.815099, antiderivative size = 612, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1368, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{}}\right.}{6\sqrt[3]{2}a\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/(2*a*x^2) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^
(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b - S
qrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (
2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt
[3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*L
og[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b - Sq
rt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[
b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*
c])^(2/3)) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])
^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*
x^2])/(6*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[b^
2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b
^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 - 4
*a*c])^(2/3))
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)}\,dx = -\frac{1}{2ax^2}+\frac{\int \frac{-2b-2cx^3}{a+bx^3+cx^6}\,dx}{2a}$$

$$= -\frac{1}{2ax^2}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}$$

$$= -\frac{1}{2ax^2}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}}}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= -\frac{1}{2ax^2}-\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b\ldots}}\right.}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-\ldots}\right.}$$

$$= -\frac{1}{2ax^2}-\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b\ldots}}\right.}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-\ldots}\right.}$$

$$= -\frac{1}{2ax^2}+\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}-\ldots$$

Mathematica [C] time = 0.0325425, size = 75, normalized size = 0.12

$$-\frac{\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\#1^3c\log(x-\#1)+b\log(x-\#1)}{\#1^2b+2\#1^5c}\&\right]}{3a}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/(2*a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #
1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

Maple [C] time = 0.006, size = 62, normalized size = 0.1

$$\frac{1}{3\,a}\sum_{\_R=\text{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(-\_R^3c-b\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3/a*sum((-_R^3*c-b)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a)
)-1/2/a/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 5.72078, size = 12411, normalized size = 20.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 -
4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4
*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5
*b^2 - 4*a^6*c))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^5*b^8 - 13*a^
6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*x*sqrt((b^10 - 10*
a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*
a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - sqrt(3)*(b^10 - 12*a*b^8*c +
 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*(-(b^4 - 3*a*b^2*c +
a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50
*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
- 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(a^5*b^8
 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^10
 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - sqrt(3)*(b^10 - 12*a*b
^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4))*(-(b^4 - 3*a*b^2*
c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
- 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*
c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3)*sqrt((2*(b^5*c^4 - 5*a*b^3*
c^5 + 5*a^2*b*c^6)*x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 -
138*a^3*b^5*c^3 + 130*a^4*b^3*c^4 - 40*a^5*b*c^5 - (a^5*b^9 - 14*a^6*b^7*c
+ 72*a^7*b^5*c^2 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b^10 - 10*a*b^8*c
 + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*
b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 +
25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))
)/(a^5*b^2 - 4*a^6*c))^(2/3) + (1/2)^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*c^3 +
 32*a^7*b^2*c^4 - 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 -
50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^
2 - 64*a^13*c^3)) - (b^8*c^2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4 - 20*a^3*b^2*c^
5)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*
b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3))/(b^
5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6)) + 2*sqrt(3)*(b^5*c^3 - 5*a*b^3*c^4 + 5*
a^2*b*c^5))/(b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)) - 4*sqrt(3)*(1/2)^(1/3)*
a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*
b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*arct
```

```
an(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 -
112*a^8*b^2*c^3 + 64*a^9*c^4)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 -
50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^
2 - 64*a^13*c^3)) + sqrt(3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^
4*c^3 + 60*a^4*b^2*c^4)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*
c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c
^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 -
 4*a^6*c))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^
4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*
c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)) + sqrt(3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*
a^3*b^4*c^3 + 60*a^4*b^2*c^4))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*
a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b
^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^2 - 4*a^6*c))^(2/3)*sqrt((2*(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6)*x^2 + (1
/2)^(2/3)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 130*a^4*b
^3*c^4 - 40*a^5*b*c^5 + (a^5*b^9 - 14*a^6*b^7*c + 72*a^7*b^5*c^2 - 160*a^8*
b^3*c^3 + 128*a^9*b*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*
b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 -
 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3
) - (1/2)^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*c^3 + 32*a^7*b^2*c^4 - 32*a^8*c^
5)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2
*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) + (b^8*c^
2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3*a*b^2*c +
a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50
*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
- 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3))/(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2
*b*c^6)) - 2*sqrt(3)*(b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5))/(b^5*c^3 - 5*a*
b^3*c^4 + 5*a^2*b*c^5)) - (1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 +
(a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c
^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*
c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^
6)*x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3
+ 130*a^4*b^3*c^4 - 40*a^5*b*c^5 - (a^5*b^9 - 14*a^6*b^7*c + 72*a^7*b^5*c^2
 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^
2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^
2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*s
qrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/
(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a
^6*c))^(2/3) + (1/2)^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*c^3 + 32*a^7*b^2*c^4
- 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 +
 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)
) - (b^8*c^2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3
*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b
^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)) - (1/2)^(1/3)*a*x^2
*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c
 + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^
5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6)*x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 6
3*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 130*a^4*b^3*c^4 - 40*a^5*b*c^5 + (a^5*b^9
 - 14*a^6*b^7*c + 72*a^7*b^5*c^2 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b
^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*
b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c +
 a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 5
0*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3) - (1/2)^(1/3)*((a^5*b^6*c^2 -
```

```
10*a^6*b^4*c^3 + 32*a^7*b^2*c^4 - 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 3
5*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)) + (b^8*c^2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4
 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sq
rt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(
a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^
6*c))^(1/3)) + 2*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2
- 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a
^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a
^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*x + (1
/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 - (a^5*b^6 - 1
0*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqrt((b^10 - 10*a*b^8*c + 35*a^2
*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a
^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^
2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^
2 - 4*a^6*c))^(1/3)) + 2*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (
a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^
3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4
)*x + (1/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 + (a^5
*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqrt((b^10 - 10*a*b^8*c
+ 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b
^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 2
5*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
/(a^5*b^2 - 4*a^6*c))^(1/3)) - 3)/(a*x^2)
```

Sympy [A] time = 5.78583, size = 241, normalized size = 0.39

$$\operatorname{RootSum}\left(t^6\left(46656a^8c^3 - 34992a^7b^2c^2 + 8748a^6b^4c - 729a^5b^6\right) + t^3\left(-432a^4c^4 + 1512a^3b^2c^3 - 1107a^2b^4c^2 + 297ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**8*c**3 - 34992*a**7*b**2*c**2 + 8748*a**6*b**4*c -
729*a**5*b**6) + _t**3*(-432*a**4*c**4 + 1512*a**3*b**2*c**3 - 1107*a**2*b*
*4*c**2 + 297*a*b**6*c - 27*b**8) + c**5, Lambda(_t, _t*log(x + (-2592*_t**
4*a**8*c**3 + 2592*_t**4*a**7*b**2*c**2 - 810*_t**4*a**6*b**4*c + 81*_t**4*
a**5*b**6 + 12*_t*a**4*c**4 - 75*_t*a**3*b**2*c**3 + 78*_t*a**2*b**4*c**2 -
 27*_t*a*b**6*c + 3*_t*b**8)/(5*a**2*b*c**4 - 5*a*b**3*c**3 + b**5*c**2))))
 - 1/(2*a*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^3), x)
```

3.151 $\int \frac{x^{11}}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=35

$$\frac{x^6}{6} - \frac{4x^3}{3} - \frac{1}{6}\log\left(x^3+1\right) + \frac{9}{2}\log\left(x^3+3\right)$$

```
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Rubi [A] time = 0.024081, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 701, 632, 31}

$$\frac{x^6}{6} - \frac{4x^3}{3} - \frac{1}{6}\log\left(x^3+1\right) + \frac{9}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{3+4x+x^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-4+x+\frac{12+13x}{3+4x+x^2}\right)dx, x, x^3\right)$$
$$= -\frac{4x^3}{3}+\frac{x^6}{6}+\frac{1}{3}\,\text{Subst}\left(\int \frac{12+13x}{3+4x+x^2}\,dx, x, x^3\right)$$
$$= -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right)+\frac{9}{2}\,\text{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right)$$
$$= -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)$$

Mathematica [A] time = 0.0061133, size = 35, normalized size = 1.

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{9}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Maple [A] time = 0.005, size = 28, normalized size = 0.8

$$-\frac{4\,x^3}{3}+\frac{x^6}{6}-\frac{\ln\left(x^3+1\right)}{6}+\frac{9\,\ln\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^6+4*x^3+3),x)
```

```
[Out] -4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

Maxima [A] time = 1.2259, size = 36, normalized size = 1.03

$$\frac{1}{6}\,x^6-\frac{4}{3}\,x^3+\frac{9}{2}\,\log\left(x^3+3\right)-\frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Fricas [A] time = 1.41851, size = 77, normalized size = 2.2

$$\frac{1}{6}\,x^6-\frac{4}{3}\,x^3+\frac{9}{2}\,\log\left(x^3+3\right)-\frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Sympy [A] time = 0.120889, size = 29, normalized size = 0.83

$$\frac{x^6}{6} - \frac{4x^3}{3} - \frac{\log\left(x^3+1\right)}{6} + \frac{9\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**6+4*x**3+3),x)
```

```
[Out] x**6/6 - 4*x**3/3 - log(x**3 + 1)/6 + 9*log(x**3 + 3)/2
```

Giac [A] time = 1.09476, size = 39, normalized size = 1.11

$$\frac{1}{6}x^6 - \frac{4}{3}x^3 + \frac{9}{2}\log\left(\left|x^3+3\right|\right) - \frac{1}{6}\log\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

3.152 $\int \frac{x^8}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=28

$$\frac{x^3}{3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{3}{2}\log\left(x^3+3\right)$$

```
[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Rubi [A] time = 0.0187006, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 703, 632, 31}

$$\frac{x^3}{3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{3}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(3 + 4*x^3 + x^6),x]
```

```
[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{3+4x+x^2}\,dx, x, x^3\right)$$
$$= \frac{x^3}{3} + \frac{1}{3}\operatorname{Subst}\left(\int \frac{-3-4x}{3+4x+x^2}\,dx, x, x^3\right)$$
$$= \frac{x^3}{3} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right) - \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right)$$
$$= \frac{x^3}{3} + \frac{1}{6}\log\left(1+x^3\right) - \frac{3}{2}\log\left(3+x^3\right)$$

Mathematica [A] time = 0.0050508, size = 28, normalized size = 1.

$$\frac{x^3}{3} + \frac{1}{6}\log\left(x^3+1\right) - \frac{3}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(3 + 4*x^3 + x^6),x]

[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Maple [A] time = 0.003, size = 23, normalized size = 0.8

$$\frac{x^3}{3} + \frac{\ln\left(x^3+1\right)}{6} - \frac{3\ln\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6+4*x^3+3),x)

[Out] 1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

Maxima [A] time = 1.184, size = 30, normalized size = 1.07

$$\frac{1}{3}x^3 - \frac{3}{2}\log\left(x^3+3\right) + \frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Fricas [A] time = 1.47735, size = 63, normalized size = 2.25

$$\frac{1}{3}x^3 - \frac{3}{2}\log\left(x^3+3\right) + \frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Sympy [A] time = 0.127806, size = 22, normalized size = 0.79

$$\frac{x^3}{3} + \frac{\log\left(x^3 + 1\right)}{6} - \frac{3\log\left(x^3 + 3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6+4*x**3+3),x)
```

```
[Out] x**3/3 + log(x**3 + 1)/6 - 3*log(x**3 + 3)/2
```

Giac [A] time = 1.11344, size = 32, normalized size = 1.14

$$\frac{1}{3}\,x^3 - \frac{3}{2}\,\log\left(\left|x^3 + 3\right|\right) + \frac{1}{6}\,\log\left(\left|x^3 + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 3/2*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

3.153 $\int \frac{x^5}{3+4x^3+x^6} dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

```
[Out] -Log[1 + x^3]/6 + Log[3 + x^3]/2
```

Rubi [A] time = 0.0141699, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 632, 31}

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(3 + 4*x^3 + x^6),x]
```

```
[Out] -Log[1 + x^3]/6 + Log[3 + x^3]/2
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{3+4x^3+x^6}\,dx&=\frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{3+4x+x^2}\,dx,x,x^3\right)\\&=-\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^3\right)\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{3+x}\,dx,x,x^3\right)\\&=-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{2}\log\left(3+x^3\right)\end{aligned}$$

Mathematica [A] time = 0.0041518, size = 21, normalized size = 1.

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(3 + 4*x^3 + x^6),x]
```

```
[Out] -Log[1 + x^3]/6 + Log[3 + x^3]/2
```

Maple [A] time = 0.004, size = 18, normalized size = 0.9

$$-\frac{\ln\left(x^3+1\right)}{6}+\frac{\ln\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+4*x^3+3),x)
```

```
[Out] -1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

Maxima [A] time = 1.30684, size = 23, normalized size = 1.1

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Fricas [A] time = 1.48159, size = 50, normalized size = 2.38

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Sympy [A] time = 0.116274, size = 15, normalized size = 0.71

$$-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+4*x**3+3),x)
```

```
[Out] -log(x**3 + 1)/6 + log(x**3 + 3)/2
```

Giac [A] time = 1.09246, size = 26, normalized size = 1.24

$$\frac{1}{2}\,\log\left(\left|x^3+3\right|\right)-\frac{1}{6}\,\log\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

3.154 $\int \frac{x^2}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=10

$$-\frac{1}{3}\tanh^{-1}\left(x^3+2\right)$$

```
[Out] -ArcTanh[2 + x^3]/3
```

Rubi [B] time = 0.0139787, antiderivative size = 21, normalized size of antiderivative = 2.1, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 616, 31}

$$\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(3 + 4*x^3 + x^6),x]
```

```
[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{3+4x^3+x^6} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{3+4x+x^2} \, dx, x, x^3\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, x^3\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{3+x} \, dx, x, x^3\right)\\
&= \frac{1}{6}\log\left(1+x^3\right)-\frac{1}{6}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [B] time = 0.0035071, size = 21, normalized size = 2.1

$$\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(3 + 4*x^3 + x^6),x]
```

```
[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6
```

Maple [B] time = 0.004, size = 18, normalized size = 1.8

$$\frac{\ln\left(x^3+1\right)}{6}-\frac{\ln\left(x^3+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+4*x^3+3),x)
```

```
[Out] 1/6*ln(x^3+1)-1/6*ln(x^3+3)
```

Maxima [B] time = 1.38805, size = 23, normalized size = 2.3

$$-\frac{1}{6}\log\left(x^3+3\right)+\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Fricas [B] time = 1.4608, size = 51, normalized size = 5.1

$$-\frac{1}{6}\log\left(x^3+3\right)+\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Sympy [A] time = 0.111578, size = 15, normalized size = 1.5

$$\frac{\log\left(x^3+1\right)}{6}-\frac{\log\left(x^3+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+4*x**3+3),x)
```

```
[Out] log(x**3 + 1)/6 - log(x**3 + 3)/6
```

Giac [B] time = 1.09131, size = 26, normalized size = 2.6

$$-\frac{1}{6}\log\left(\left|x^3+3\right|\right)+\frac{1}{6}\log\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/6*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

3.155 $\int \frac{1}{x(3+4x^3+x^6)} dx$

Optimal. Leaf size=27

$$-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^3+3\right)+\frac{\log(x)}{3}$$

```
[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

Rubi [A] time = 0.0189696, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 705, 29, 632, 31}

$$-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^3+3\right)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(3 + 4*x^3 + x^6)),x]

[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(3+4x^3+x^6\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\left(3+4x+x^2\right)}\,dx,x,x^3\right)\\ &= \frac{1}{9}\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)+\frac{1}{9}\,\text{Subst}\left(\int \frac{-4-x}{3+4x+x^2}\,dx,x,x^3\right)\\ &= \frac{\log(x)}{3}+\frac{1}{18}\,\text{Subst}\left(\int \frac{1}{3+x}\,dx,x,x^3\right)-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^3\right)\\ &= \frac{\log(x)}{3}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{18}\log\left(3+x^3\right)\end{aligned}$$

Mathematica [A] time = 0.0055941, size = 27, normalized size = 1.

$$-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^3+3\right)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + 4*x^3 + x^6)),x]
```

```
[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

Maple [A] time = 0.013, size = 31, normalized size = 1.2

$$\frac{\ln(x)}{3}-\frac{\ln\left(x^2-x+1\right)}{6}+\frac{\ln\left(x^3+3\right)}{18}-\frac{\ln(1+x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+4*x^3+3),x)
```

```
[Out] 1/3*ln(x)-1/6*ln(x^2-x+1)+1/18*ln(x^3+3)-1/6*ln(1+x)
```

Maxima [A] time = 1.19491, size = 31, normalized size = 1.15

$$\frac{1}{18}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{9}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/9*log(x^3)
```

Fricas [A] time = 1.43902, size = 69, normalized size = 2.56

$$\frac{1}{18}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/3*log(x)
```

Sympy [A] time = 0.143223, size = 20, normalized size = 0.74

$$\frac{\log(x)}{3} - \frac{\log(x^3+1)}{6} + \frac{\log(x^3+3)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+4*x**3+3),x)
```

```
[Out] log(x)/3 - log(x**3 + 1)/6 + log(x**3 + 3)/18
```

Giac [A] time = 1.15731, size = 32, normalized size = 1.19

$$\frac{1}{18}\log\left(|x^3+3|\right) - \frac{1}{6}\log\left(|x^3+1|\right) + \frac{1}{3}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/18*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1)) + 1/3*log(abs(x))
```

3.156 $\int \frac{1}{x^4(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

```
[Out] -1/(9*x^3) - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Rubi [A] time = 0.0318425, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 709, 800}

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(9*x^3) - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^4\left(3+4x^3+x^6\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\left(3+4x+x^2\right)}\,dx, x, x^3\right)$$
$$= -\frac{1}{9x^3}+\frac{1}{9}\,\text{Subst}\left(\int \frac{-4-x}{x\left(3+4x+x^2\right)}\,dx, x, x^3\right)$$
$$= -\frac{1}{9x^3}+\frac{1}{9}\,\text{Subst}\left(\int \left(-\frac{4}{3x}+\frac{3}{2(1+x)}-\frac{1}{6(3+x)}\right)dx, x, x^3\right)$$
$$= -\frac{1}{9x^3}-\frac{4\log(x)}{9}+\frac{1}{6}\log\left(1+x^3\right)-\frac{1}{54}\log\left(3+x^3\right)$$

Mathematica [A] time = 0.0072617, size = 34, normalized size = 1.

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(9*x^3) - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Maple [A] time = 0.008, size = 36, normalized size = 1.1

$$-\frac{1}{9\,x^3}-\frac{4\,\ln(x)}{9}+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\ln\left(x^3+3\right)}{54}+\frac{\ln(1+x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6+4*x^3+3),x)
```

```
[Out] -1/9/x^3-4/9*ln(x)+1/6*ln(x^2-x+1)-1/54*ln(x^3+3)+1/6*ln(1+x)
```

Maxima [A] time = 1.16239, size = 38, normalized size = 1.12

$$-\frac{1}{9\,x^3}-\frac{1}{54}\log\left(x^3+3\right)+\frac{1}{6}\log\left(x^3+1\right)-\frac{4}{27}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/9/x^3 - 1/54*log(x^3 + 3) + 1/6*log(x^3 + 1) - 4/27*log(x^3)
```

Fricas [A] time = 1.42205, size = 96, normalized size = 2.82

$$-\frac{x^3\log\left(x^3+3\right)-9\,x^3\log\left(x^3+1\right)+24\,x^3\log(x)+6}{54\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/54*(x^3*log(x^3 + 3) - 9*x^3*log(x^3 + 1) + 24*x^3*log(x) + 6)/x^3
```

Sympy [A] time = 0.17, size = 29, normalized size = 0.85

$$-\frac{4\log(x)}{9}+\frac{\log\left(x^3+1\right)}{6}-\frac{\log\left(x^3+3\right)}{54}-\frac{1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6+4*x**3+3),x)
```

```
[Out] -4*log(x)/9 + log(x**3 + 1)/6 - log(x**3 + 3)/54 - 1/(9*x**3)
```

Giac [A] time = 1.10187, size = 49, normalized size = 1.44

$$\frac{4\,x^3-3}{27\,x^3}-\frac{1}{54}\,\log\left(\left|x^3+3\right|\right)+\frac{1}{6}\,\log\left(\left|x^3+1\right|\right)-\frac{4}{9}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/27*(4*x^3 - 3)/x^3 - 1/54*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1)) - 4/9
*log(abs(x))
```

3.157 $\int \frac{1}{x^7(3+4x^3+x^6)} dx$

Optimal. Leaf size=41

$$\frac{4}{27x^3} - \frac{1}{18x^6} - \frac{1}{6}\log\left(x^3+1\right) + \frac{1}{162}\log\left(x^3+3\right) + \frac{13\log(x)}{27}$$

```
[Out] -1/(18*x^6) + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/1
62
```

Rubi [A] time = 0.03602, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 709, 800}

$$\frac{4}{27x^3} - \frac{1}{18x^6} - \frac{1}{6}\log\left(x^3+1\right) + \frac{1}{162}\log\left(x^3+3\right) + \frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(18*x^6) + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/1
62
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^7\left(3+4x^3+x^6\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{x^3\left(3+4x+x^2\right)}\,dx, x, x^3\right)$$
$$= -\frac{1}{18x^6} + \frac{1}{9}\,\mathrm{Subst}\left(\int \frac{-4-x}{x^2\left(3+4x+x^2\right)}\,dx, x, x^3\right)$$
$$= -\frac{1}{18x^6} + \frac{1}{9}\,\mathrm{Subst}\left(\int \left(-\frac{4}{3x^2} + \frac{13}{9x} - \frac{3}{2(1+x)} + \frac{1}{18(3+x)}\right) dx, x, x^3\right)$$
$$= -\frac{1}{18x^6} + \frac{4}{27x^3} + \frac{13\log(x)}{27} - \frac{1}{6}\log\left(1+x^3\right) + \frac{1}{162}\log\left(3+x^3\right)$$

Mathematica [A] time = 0.0057354, size = 41, normalized size = 1.

$$\frac{4}{27x^3} - \frac{1}{18x^6} - \frac{1}{6}\log\left(x^3+1\right) + \frac{1}{162}\log\left(x^3+3\right) + \frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(18*x^6) + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/1
62
```

Maple [A] time = 0.009, size = 41, normalized size = 1.

$$-\frac{1}{18\,x^6} + \frac{4}{27\,x^3} + \frac{13\,\ln(x)}{27} - \frac{\ln\left(x^2-x+1\right)}{6} + \frac{\ln\left(x^3+3\right)}{162} - \frac{\ln(1+x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^6+4*x^3+3),x)
```

```
[Out] -1/18/x^6+4/27/x^3+13/27*ln(x)-1/6*ln(x^2-x+1)+1/162*ln(x^3+3)-1/6*ln(1+x)
```

Maxima [A] time = 1.05047, size = 47, normalized size = 1.15

$$\frac{8\,x^3-3}{54\,x^6} + \frac{1}{162}\log\left(x^3+3\right) - \frac{1}{6}\log\left(x^3+1\right) + \frac{13}{81}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/54*(8*x^3 - 3)/x^6 + 1/162*log(x^3 + 3) - 1/6*log(x^3 + 1) + 13/81*log(x^
3)
```

Fricas [A] time = 1.48597, size = 109, normalized size = 2.66

$$\frac{x^6\log\left(x^3+3\right) - 27\,x^6\log\left(x^3+1\right) + 78\,x^6\log(x) + 24\,x^3 - 9}{162\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/162*(x^6*log(x^3 + 3) - 27*x^6*log(x^3 + 1) + 78*x^6*log(x) + 24*x^3 - 9)
/x^6
```

Sympy [A] time = 0.20182, size = 34, normalized size = 0.83

$$\frac{13\log(x)}{27} - \frac{\log\left(x^3+1\right)}{6} + \frac{\log\left(x^3+3\right)}{162} + \frac{8x^3-3}{54x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**6+4*x**3+3),x)
```

```
[Out] 13*log(x)/27 - log(x**3 + 1)/6 + log(x**3 + 3)/162 + (8*x**3 - 3)/(54*x**6)
```

Giac [A] time = 1.0856, size = 55, normalized size = 1.34

$$-\frac{13\,x^6-8\,x^3+3}{54\,x^6} + \frac{1}{162}\log\left(\left|x^3+3\right|\right) - \frac{1}{6}\log\left(\left|x^3+1\right|\right) + \frac{13}{27}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/54*(13*x^6 - 8*x^3 + 3)/x^6 + 1/162*log(abs(x^3 + 3)) - 1/6*log(abs(x^3
+ 1)) + 13/27*log(abs(x))
```

3.158 $\int \frac{x^{10}}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=124

$$\frac{x^5}{5}-2x^2-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{3}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+\frac{1}{6}\log(x+1)-\frac{3}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}$$

```
[Out] -2*x^2 + x^5/5 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (9*3^(1/6)*ArcTan[
(3^(1/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3*3^(2/3)*Log[3^(1/3) + x])/2
 - Log[1 - x + x^2]/12 + (3*3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rubi [A] time = 0.114143, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1367, 1502, 1510, 292, 31, 634, 618, 204, 628, 617}

$$\frac{x^5}{5}-2x^2-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{3}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+\frac{1}{6}\log(x+1)-\frac{3}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] -2*x^2 + x^5/5 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (9*3^(1/6)*ArcTan[
(3^(1/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3*3^(2/3)*Log[3^(1/3) + x])/2
 - Log[1 - x + x^2]/12 + (3*3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{x^5}{5} - \frac{1}{5}\int \frac{x^4\left(15+20x^3\right)}{3+4x^3+x^6}\,dx$$
$$= -2x^2 + \frac{x^5}{5} + \frac{1}{10}\int \frac{x\left(120+130x^3\right)}{3+4x^3+x^6}\,dx$$
$$= -2x^2 + \frac{x^5}{5} - \frac{1}{2}\int \frac{x}{1+x^3}\,dx + \frac{27}{2}\int \frac{x}{3+x^3}\,dx$$
$$= -2x^2 + \frac{x^5}{5} + \frac{1}{6}\int \frac{1}{1+x}\,dx - \frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx - \frac{1}{2}\left(3\ 3^{2/3}\right)\int \frac{1}{\sqrt[3]{3}+x}\,dx + \frac{1}{2}\left(3\ 3^{2/3}\right)\int \frac{}{3^{2/3}\ldots}$$
$$= -2x^2 + \frac{x^5}{5} + \frac{1}{6}\log(1+x) - \frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right) - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{27}{4}\ldots$$
$$= -2x^2 + \frac{x^5}{5} + \frac{1}{6}\log(1+x) - \frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{3}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+\ldots\right.$$
$$= -2x^2 + \frac{x^5}{5} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{9}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + \frac{1}{6}\log(1+x) - \frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right) - \frac{1}{12}\ldots$$

Mathematica [A] time = 0.0535701, size = 118, normalized size = 0.95

$$\frac{1}{60}\left(12x^5 - 120x^2 - 5\log\left(x^2-x+1\right) + 45\ 3^{2/3}\log\left(\sqrt[3]{3}x^2 - 3^{2/3}x + 3\right) + 10\log(x+1) - 90\ 3^{2/3}\log\left(3^{2/3}x+3\right) - 270\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-120*x^2 + 12*x^5 - 270*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 10*Sqrt[
3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Log[1 + x] - 90*3^(2/3)*Log[3 + 3^(2/3)*
x] - 5*Log[1 - x + x^2] + 45*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/60
```

Maple [A] time = 0.009, size = 94, normalized size = 0.8

$$\frac{x^5}{5} - 2x^2 - \frac{\ln\left(x^2-x+1\right)}{12} - \frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) - \frac{3\,3^{2/3}\ln\left(\sqrt[3]{3}+x\right)}{2} + \frac{3\,3^{2/3}\ln\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4} + \frac{9\sqrt[6]{3}}{2}\mathrm{arc}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^6+4*x^3+3),x)
```

```
[Out] 1/5*x^5-2*x^2-1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-3/2*
3^(2/3)*ln(3^(1/3)+x)+3/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+9/2*3^(1/6)*arc
tan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)
```

Maxima [A] time = 1.63335, size = 127, normalized size = 1.02

$$\frac{1}{5}x^5 - 2x^2 + \frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{3}{2}\cdot 3^{\frac{2}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{9}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)*
log(x + 3^(1/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3^(1/6)*
arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1
)
```

Fricas [A] time = 1.52044, size = 358, normalized size = 2.89

$$\frac{1}{5}x^5 - 2x^2 + \frac{3}{2}\sqrt{3}(-9)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\sqrt{3}\left(2(-9)^{\frac{1}{3}}x + 3\right)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{4}(-9)^{\frac{1}{3}}\log\left(3x^2 - (-9)^{\frac{2}{3}}x - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/2*sqrt(3)*(-9)^(1/3)*arctan(1/9*sqrt(3)*(2*(-9)^(1/3)*x
 + 3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*(-9)^(1/3)*log(3*x
^2 - (-9)^(2/3)*x - 3*(-9)^(1/3)) + 3/2*(-9)^(1/3)*log(3*x + (-9)^(2/3)) -
1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] time = 0.636538, size = 144, normalized size = 1.16

$$\frac{x^5}{5} - 2x^2 + \frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{3872\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{3281} + \frac{3188648\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{88587}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(x**6+4*x**3+3),x)
```

```
[Out] x**5/5 - 2*x**2 + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 3872*(-1/12
 - sqrt(3)*I/12)**5/3281 + 3188648*(-1/12 - sqrt(3)*I/12)**2/88587) + (-1/1
2 + sqrt(3)*I/12)*log(x + 3188648*(-1/12 + sqrt(3)*I/12)**2/88587 + 3872*(-
1/12 + sqrt(3)*I/12)**5/3281) + RootSum(8*_t**3 + 243, Lambda(_t, _t*log(38
72*_t**5/3281 + 3188648*_t**2/88587 + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.159 $\int \frac{x^9}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=122

$$\frac{x^4}{4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{3}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-4x-\frac{1}{6}\log(x+1)+\frac{3}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5}$$

```
[Out] -4*x + x^4/4 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3*3^(5/6)*ArcTan[(3
^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3*3^(1/3)*Log[3^(1/3) + x])/2 +
 Log[1 - x + x^2]/12 - (3*3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rubi [A] time = 0.0936454, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1367, 1502, 1422, 200, 31, 634, 618, 204, 628, 617}

$$\frac{x^4}{4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{3}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-4x-\frac{1}{6}\log(x+1)+\frac{3}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] -4*x + x^4/4 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3*3^(5/6)*ArcTan[(3
^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3*3^(1/3)*Log[3^(1/3) + x])/2 +
 Log[1 - x + x^2]/12 - (3*3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
```

```
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{3+4x^3+x^6}\,dx &= \frac{x^4}{4}-\frac{1}{4}\int \frac{x^3\left(12+16x^3\right)}{3+4x^3+x^6}\,dx\\ &= -4x+\frac{x^4}{4}+\frac{1}{4}\int \frac{48+52x^3}{3+4x^3+x^6}\,dx\\ &= -4x+\frac{x^4}{4}-\frac{1}{2}\int \frac{1}{1+x^3}\,dx+\frac{27}{2}\int \frac{1}{3+x^3}\,dx\\ &= -4x+\frac{x^4}{4}-\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx+\frac{1}{2}\left(3\sqrt[3]{3}\right)\int \frac{1}{\sqrt[3]{3}+x}\,dx+\frac{1}{2}\left(3\sqrt[3]{3}\right)\int \frac{2\sqrt[3]{3}}{3^{2/3}-\sqrt[3]{}}\\ &= -4x+\frac{x^4}{4}-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{4}(\\ &= -4x+\frac{x^4}{4}-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{3}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right.\\ &= -4x+\frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\text{lo}\end{aligned}$$

Mathematica [A] time = 0.0295371, size = 114, normalized size = 0.93

$$\frac{1}{12}\left(3x^4+\log\left(x^2-x+1\right)-9\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-48x-2\log(x+1)+18\sqrt[3]{3}\log\left(3^{2/3}x+3\right)-18\ 3^{5/6}\tan^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-48*x + 3*x^4 - 18*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*Arc
Tan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 18*3^(1/3)*Log[3 + 3^(2/3)*x] + Lo
g[1 - x + x^2] - 9*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [A] time = 0.007, size = 92, normalized size = 0.8

$$\frac{x^4}{4}-4x+\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{3\sqrt[3]{3}\ln\left(\sqrt[3]{3}+x\right)}{2}-\frac{3\sqrt[3]{3}\ln\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4}+\frac{3\,3^{5/6}}{2}\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^6+4*x^3+3),x)
```

```
[Out] 1/4*x^4-4*x+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+3/2*3^
(1/3)*ln(3^(1/3)+x)-3/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+3/2*3^(5/6)*arcta
n(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)
```

Maxima [A] time = 1.67359, size = 124, normalized size = 1.02

$$\frac{1}{4}x^4+\frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Fricas [A] time = 1.55849, size = 305, normalized size = 2.5

$$\frac{1}{4}x^4+\frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{2}{3}\cdot 3^{\frac{1}{6}}x-\frac{1}{3}\sqrt{3}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(2/3*3^(1/6)*x - 1/3*sqrt(3)) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Sympy [C] time = 0.644529, size = 129, normalized size = 1.06

$$\frac{x^4}{4}-4x-\frac{\log(x+1)}{6}+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{9841}{19692}-\frac{9841\sqrt{3}i}{19692}+\frac{360\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{547}\right)+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{9841}{19692}+\frac{360}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**6+4*x**3+3),x)
```

```
[Out] x**4/4 - 4*x - log(x + 1)/6 + (1/12 + sqrt(3)*I/12)*log(x - 9841/19692 - 98
41*sqrt(3)*I/19692 + 360*(1/12 + sqrt(3)*I/12)**4/547) + (1/12 - sqrt(3)*I/
12)*log(x - 9841/19692 + 360*(1/12 - sqrt(3)*I/12)**4/547 + 9841*sqrt(3)*I/
19692) + RootSum(8*_t**3 - 81, Lambda(_t, _t*log(360*_t**4/547 - 9841*_t/16
41 + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.160 $\int \frac{x^7}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=119

$$\frac{x^2}{2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\frac{1}{6}\log(x+1)+\frac{1}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\,\text{ta}$$

```
[Out] x^2/2 - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3*3^(1/6)*ArcTan[(3^(1/3)
- 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3^(2/3)*Log[3^(1/3) + x])/2 + Log[1 -
x + x^2]/12 - (3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rubi [A] time = 0.081783, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1367, 1510, 292, 31, 634, 618, 204, 628, 617}

$$\frac{x^2}{2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\frac{1}{6}\log(x+1)+\frac{1}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\,\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(3 + 4*x^3 + x^6),x]
```

```
[Out] x^2/2 - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3*3^(1/6)*ArcTan[(3^(1/3)
- 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3^(2/3)*Log[3^(1/3) + x])/2 + Log[1 -
x + x^2]/12 - (3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{3+4x^3+x^6}\,dx &= \frac{x^2}{2}-\frac{1}{2}\int \frac{x\left(6+8x^3\right)}{3+4x^3+x^6}\,dx\\
&= \frac{x^2}{2}+\frac{1}{2}\int \frac{x}{1+x^3}\,dx-\frac{9}{2}\int \frac{x}{3+x^3}\,dx\\
&= \frac{x^2}{2}-\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx+\frac{1}{2}3^{2/3}\int \frac{1}{\sqrt[3]{3}+x}\,dx-\frac{1}{2}3^{2/3}\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx\\
&= \frac{x^2}{2}-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{9}{4}\int \frac{1}{3^{2/3}-\sqrt[3]{}\ldots}\\
&= \frac{x^2}{2}-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)-\frac{1}{2}\mathrm{Su}\ldots\\
&= \frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+\ldots\right.
\end{aligned}$$

Mathematica [A] time = 0.0273695, size = 111, normalized size = 0.93

$$\frac{1}{12}\left(6x^2+\log\left(x^2-x+1\right)-3\ 3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-2\log(x+1)+6\ 3^{2/3}\log\left(3^{2/3}x+3\right)+18\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\ldots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(3 + 4*x^3 + x^6),x]
```

```
[Out] (6*x^2 + 18*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1
+ 2*x)/Sqrt[3]] - 2*Log[1 + x] + 6*3^(2/3)*Log[3 + 3^(2/3)*x] + Log[1 - x +
 x^2] - 3*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [A] time = 0.006, size = 89, normalized size = 0.8

$$\frac{x^2}{2}+\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{3^{\frac{2}{3}}\ln\left(\sqrt[3]{3}+x\right)}{2}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{4}-\frac{3\sqrt[6]{3}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2}{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6+4*x^3+3),x)
```

```
[Out] 1/2*x^2+1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/2*3^(2/3
)*ln(3^(1/3)+x)-1/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-3/2*3^(1/6)*arctan(1/
3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)
```

Maxima [A] time = 1.70047, size = 120, normalized size = 1.01

$$\frac{1}{2}x^2-\frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(x +
3^(1/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*arctan(1
/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Fricas [A] time = 1.52441, size = 327, normalized size = 2.75

$$\frac{1}{2}x^2-\frac{1}{2}\cdot 9^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{2}{9}\cdot 9^{\frac{1}{3}}\sqrt{3}x-\frac{1}{3}\sqrt{3}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{4}\cdot 9^{\frac{1}{3}}\log\left(3x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+\frac{1}{2}\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*9^(1/3)*sqrt(3)*arctan(2/9*9^(1/3)*sqrt(3)*x - 1/3*sqrt(3)) +
 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*9^(1/3)*log(3*x^2 - 9^(2/3
)*x + 3*9^(1/3)) + 1/2*9^(1/3)*log(3*x + 9^(2/3)) + 1/12*log(x^2 - x + 1) -
 1/6*log(x + 1)
```

Sympy [C] time = 0.66824, size = 134, normalized size = 1.13

$$\frac{x^2}{2} - \frac{\log(x+1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{6562\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{183} - \frac{1872\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{61}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{1872\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{61}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6+4*x**3+3),x)
```

```
[Out] x**2/2 - log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 6562*(1/12 - sqrt(3)*
I/12)**2/183 - 1872*(1/12 - sqrt(3)*I/12)**5/61) + (1/12 + sqrt(3)*I/12)*lo
g(x - 1872*(1/12 + sqrt(3)*I/12)**5/61 + 6562*(1/12 + sqrt(3)*I/12)**2/183)
 + RootSum(8*_t**3 - 9, Lambda(_t, _t*log(-1872*_t**5/61 + 6562*_t**2/183 +
 x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.161 $\int \frac{x^6}{3+4x^3+x^6} dx$

Optimal. Leaf size=113

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+x+\frac{1}{6}\log(x+1)-\frac{1}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\text{ta}$$

```
[Out] x - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3^(5/6)*ArcTan[(3^(1/3) - 2*x)
/3^(5/6)])/2 + Log[1 + x]/6 - (3^(1/3)*Log[3^(1/3) + x])/2 - Log[1 - x + x^
2]/12 + (3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rubi [A] time = 0.0739025, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1367, 1422, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+x+\frac{1}{6}\log(x+1)-\frac{1}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(3 + 4*x^3 + x^6),x]
```

```
[Out] x - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3^(5/6)*ArcTan[(3^(1/3) - 2*x)
/3^(5/6)])/2 + Log[1 + x]/6 - (3^(1/3)*Log[3^(1/3) + x])/2 - Log[1 - x + x^
2]/12 + (3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{3+4x^3+x^6}\,dx &= x-\int \frac{3+4x^3}{3+4x^3+x^6}\,dx\\ &= x+\frac{1}{2}\int \frac{1}{1+x^3}\,dx-\frac{9}{2}\int \frac{1}{3+x^3}\,dx\\ &= x+\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx-\frac{1}{2}\sqrt[3]{3}\int \frac{1}{\sqrt[3]{3}+x}\,dx-\frac{1}{2}\sqrt[3]{3}\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx\\ &= x+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\sqrt[3]{3}\int \frac{-\sqrt[3]{}}{3^{2/3}-}\\ &= x+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{1}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)-\frac{1}{2}\text{Subs}\\ &= x-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x\right.\end{aligned}$$

Mathematica [A] time = 0.0263998, size = 111, normalized size = 0.98

$$\frac{1}{12}\left(-\log\left(x^2-x+1\right)+3\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+12x+2\log(x+1)-6\sqrt[3]{3}\log\left(3^{2/3}x+3\right)+6\ 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(3 + 4*x^3 + x^6),x]
```

```
[Out] (12*x + 6*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] + 2*Log[1 + x] - 6*3^(1/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x
^2] + 3*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [A] time = 0.005, size = 85, normalized size = 0.8

$$x-\frac{\ln\left(x^{2}-x+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\sqrt[3]{3}\ln\left(\sqrt[3]{3}+x\right)}{2}+\frac{\sqrt[3]{3}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^{2}\right)}{4}-\frac{3^{\frac{5}{6}}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/}}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6+4*x^3+3),x)
```

```
[Out] x-1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/2*3^(1/3)*ln(3
^(1/3)+x)+1/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/2*3^(5/6)*arctan(1/3*3^(1
/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)
```

Maxima [A] time = 1.62974, size = 115, normalized size = 1.02

$$-\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1/3
)*log(x + 3^(1/3)) + x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Fricas [A] time = 1.43372, size = 331, normalized size = 2.93

$$\frac{1}{2}\sqrt{3}(-3)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\sqrt{3}\left(2(-3)^{\frac{2}{3}}x-3\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{4}(-3)^{\frac{1}{3}}\log\left(x^{2}+(-3)^{\frac{1}{3}}x+(-3)^{\frac{2}{3}}\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*(-3)^(1/3)*arctan(1/9*sqrt(3)*(2*(-3)^(2/3)*x - 3)) + 1/6*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*(-3)^(1/3)*log(x^2 + (-3)^(1/3)*x +
(-3)^(2/3)) + 1/2*(-3)^(1/3)*log(x - (-3)^(1/3)) + x - 1/12*log(x^2 - x + 1
) + 1/6*log(x + 1)
```

Sympy [C] time = 0.625421, size = 126, normalized size = 1.12

$$x + \frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{121}{246} - \frac{121\sqrt{3}i}{246} + \frac{864\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^4}{41}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{121}{246} + \frac{864\left(-\frac{1}{12} + \right.}{41}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6+4*x**3+3),x)
```

```
[Out] x + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 121/246 - 121*sqrt(3)*I/2
46 + 864*(-1/12 - sqrt(3)*I/12)**4/41) + (-1/12 + sqrt(3)*I/12)*log(x - 121
/246 + 864*(-1/12 + sqrt(3)*I/12)**4/41 + 121*sqrt(3)*I/246) + RootSum(8*_t
**3 + 3, Lambda(_t, _t*log(864*_t**4/41 + 242*_t/41 + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.162 $\int \frac{x^4}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=112

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\sqrt[3]{3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{2\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(
5/6)])/2 + Log[1 + x]/6 - Log[3^(1/3) + x]/(2*3^(1/3)) - Log[1 - x + x^2]/1
2 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(1/3))
```

Rubi [A] time = 0.0680056, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1374, 292, 31, 634, 617, 204, 628, 618}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\sqrt[3]{3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{2\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(3 + 4*x^3 + x^6),x]
```


```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(
5/6)])/2 + Log[1 + x]/6 - Log[3^(1/3) + x]/(2*3^(1/3)) - Log[1 - x + x^2]/1
2 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(1/3))
```


Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = -\left(\frac{1}{2}\int \frac{x}{1+x^3}\,dx\right)+\frac{3}{2}\int \frac{x}{3+x^3}\,dx$$

$$= \frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{2\sqrt[3]{3}}+\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{2\sqrt[3]{3}}$$

$$= \frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{3}{4}\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx$$

$$= \frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\sqrt[3]{3}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\right.$$

$$= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log(}{}$$

Mathematica [A] time = 0.0377344, size = 107, normalized size = 0.96

$$\frac{1}{12}\left(-\log\left(x^2-x+1\right)+3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+2\log(x+1)-2\,3^{2/3}\log\left(3^{2/3}x+3\right)-6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] + 2*Log[1 + x] - 2*3^(2/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x^2] +
3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [A] time = 0.007, size = 84, normalized size = 0.8

$$-\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{3^{\frac{2}{3}}\ln\left(\sqrt[3]{3}+x\right)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{12}+\frac{\sqrt[6]{3}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+4*x^3+3),x)
```

```
[Out] -1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/6*3^(2/3)*ln(3^
(1/3)+x)+1/12*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/2*3^(1/6)*arctan(1/3*3^(1
/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)
```

Maxima [A] time = 1.62697, size = 113, normalized size = 1.01

$$\frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(x + 3^(1/3))
- 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3*3^(1/6
)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Fricas [A] time = 1.51484, size = 392, normalized size = 3.5

$$-\frac{1}{12}\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-3^{\frac{1}{3}}(-1)^{\frac{2}{3}}x+x^2-3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(3^{\frac{1}{3}}(-1)^{\frac{2}{3}}+x\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/12*3^(2/3)*(-1)^(1/3)*log(-3^(1/3)*(-1)^(2/3)*x + x^2 - 3^(2/3)*(-1)^(1/
3)) + 1/6*3^(2/3)*(-1)^(1/3)*log(3^(1/3)*(-1)^(2/3) + x) - 1/6*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*(-1)^(1/3)*arctan(1/3*3^(1/6)*(2*(-
1)^(1/3)*x + 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] time = 0.619713, size = 134, normalized size = 1.2

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{2592\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{5}+\frac{168\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{5}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{168\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right.}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 2592*(-1/12 - sqrt(3)*I/12)**
5/5 + 168*(-1/12 - sqrt(3)*I/12)**2/5) + (-1/12 + sqrt(3)*I/12)*log(x + 168
*(-1/12 + sqrt(3)*I/12)**2/5 + 2592*(-1/12 + sqrt(3)*I/12)**5/5) + RootSum(
24*_t**3 + 1, Lambda(_t, _t*log(2592*_t**5/5 + 168*_t**2/5 + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.163 $\int \frac{x^3}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{2\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2*
3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(2*3^(2/3)) + Log[1 - x + x^2]/1
2 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(2/3))
```

Rubi [A] time = 0.0693467, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1374, 200, 31, 634, 617, 204, 628, 618}

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{2\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(3 + 4*x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2*
3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(2*3^(2/3)) + Log[1 - x + x^2]/1
2 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(2/3))
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{3+4x^3+x^6}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1+x^3}\,dx\right)+\frac{3}{2}\int \frac{1}{3+x^3}\,dx \\
&= -\left(\frac{1}{6}\int \frac{1}{1+x}\,dx\right)-\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx+\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{2\ 3^{2/3}}+\frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{2\ 3^{2/3}} \\
&= -\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}}+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{4\ 3^{2/3}}+\frac{1}{4}3 \\
&= -\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\ 3^{2/3}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x}\right. \\
&= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\right.}{4\ 3^2}
\end{aligned}$$

Mathematica [A] time = 0.0241865, size = 106, normalized size = 0.95

$$\frac{1}{12}\left(\log\left(x^2-x+1\right)-\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-2\log(x+1)+2\sqrt[3]{3}\log\left(3^{2/3}x+3\right)-2\ 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\,\text{ta}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] - 2*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + Log[1 - x + x^2] -
3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [A] time = 0.007, size = 84, normalized size = 0.8

$$\frac{\ln\left(x^{2}-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt[3]{3}\ln\left(\sqrt[3]{3}+x\right)}{6}-\frac{\sqrt[3]{3}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^{2}\right)}{12}+\frac{3^{\frac{5}{6}}}{6}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+4*x^3+3),x)
```

```
[Out] 1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*3^(1/3)*ln(3^(
1/3)+x)-1/12*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/6*3^(5/6)*arctan(1/3*3^(1/
2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)
```

Maxima [A] time = 1.56599, size = 113, normalized size = 1.01

$$\frac{1}{6}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\cdot 3^{\frac{1}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1/3
)*log(x + 3^(1/3)) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Fricas [A] time = 1.54202, size = 342, normalized size = 3.05

$$\frac{1}{6}\cdot 9^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{36}\cdot 9^{\frac{2}{3}}\log\left(3x^{2}-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+\frac{1}{18}\cdot 9^{\frac{2}{3}}\log\left(3x+9^{\frac{2}{3}}\right)-\frac{1}{6}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/6*9^(1/6)*sqrt(3)*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*9^(1/3)*sq
rt(3))) - 1/36*9^(2/3)*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 1/18*9^(2/3)*lo
g(3*x + 9^(2/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/12*log(x^2
 - x + 1) - 1/6*log(x + 1)
```

Sympy [C] time = 0.65432, size = 110, normalized size = 0.98

$$-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^{4}+\frac{\sqrt{3}i}{4}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^{4}-\frac{\sqrt{3}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 - sqrt(3)*I/1
2)**4 + sqrt(3)*I/4) + (1/12 + sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 + sqrt
(3)*I/12)**4 - sqrt(3)*I/4) + RootSum(72*_t**3 - 1, Lambda(_t, _t*log(648*_
t**4 - 3*_t + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.164 $\int \frac{x}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\sqrt[3]{3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2
*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(6*3^(1/3)) + Log[1 - x + x^2]/
12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(1/3))
```

Rubi [A] time = 0.0690096, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1375, 292, 31, 634, 618, 204, 628, 617}

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\sqrt[3]{3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(3 + 4*x^3 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2
*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(6*3^(1/3)) + Log[1 - x + x^2]/
12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(1/3))
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{3+4x^3+x^6}\,dx = \frac{1}{2}\int \frac{x}{1+x^3}\,dx - \frac{1}{2}\int \frac{x}{3+x^3}\,dx$$

$$= -\left(\frac{1}{6}\int \frac{1}{1+x}\,dx\right) + \frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx + \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{6\sqrt[3]{3}} - \frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{6\sqrt[3]{3}}$$

$$= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}$$

$$= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\sqrt[3]{3}} - \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-3-}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\,3^{5/6}} - \frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\right.}{12}$$

Mathematica [A] time = 0.0378439, size = 108, normalized size = 0.96

$$\frac{1}{36}\left(3\log\left(x^2-x+1\right) - 3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right) - 6\log(x+1) + 2\,3^{2/3}\log\left(3^{2/3}x+3\right) + 6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 6\sqrt{3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(3 + 4*x^3 + x^6),x]
```

```
[Out] (6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] - 6*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] + 3*Log[1 - x + x^2] -
 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

Maple [A] time = 0.007, size = 84, normalized size = 0.8

$$\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{3^{\frac{2}{3}}\ln\left(\sqrt[3]{3}+x\right)}{18}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{36}-\frac{\sqrt[6]{3}}{6}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+4*x^3+3),x)
```

```
[Out] 1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/18*3^(2/3)*ln(3^
(1/3)+x)-1/36*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*3^(1/6)*arctan(1/3*3^(1
/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)
```

Maxima [A] time = 1.65587, size = 113, normalized size = 1.01

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3)
) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1/3*3^(1
/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Fricas [A] time = 1.49911, size = 289, normalized size = 2.58

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(-\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3)
) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*3^(1/6)*arctan(-1/3*3^(
1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Sympy [C] time = 1.30997, size = 119, normalized size = 1.06

$$-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+90\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2+11664\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5\right)+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+11664\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 90*(1/12 - sqrt(3)*I/12)**2 +
 11664*(1/12 - sqrt(3)*I/12)**5) + (1/12 + sqrt(3)*I/12)*log(x + 11664*(1/1
```

```
2 + sqrt(3)*I/12)**5 + 90*(1/12 + sqrt(3)*I/12)**2) + RootSum(648*_t**3 - 1
, Lambda(_t, _t*log(11664*_t**5 + 90*_t**2 + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.165 $\int \frac{1}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{6\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(6
*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(6*3^(2/3)) - Log[1 - x + x^2]/
12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(2/3))
```

Rubi [A] time = 0.0652153, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1347, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{6\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x^3 + x^6)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(6
*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(6*3^(2/3)) - Log[1 - x + x^2]/
12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(2/3))
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{3+4x^3+x^6}\,dx &= \frac{1}{2}\int \frac{1}{1+x^3}\,dx - \frac{1}{2}\int \frac{1}{3+x^3}\,dx \\
&= \frac{1}{6}\int \frac{1}{1+x}\,dx + \frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx - \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{6\ 3^{2/3}} - \frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{6\ 3^{2/3}} \\
&= \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}} - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{12\ 3^{2/3}} - \frac{\int \frac{1}{3^{2/}}
\\
&= \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\ 3^{2/3}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}
\\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}} + \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\log\left(3^{2/3}-\right.}{12}
\end{aligned}$$

Mathematica [A] time = 0.0255537, size = 107, normalized size = 0.96

$$\frac{1}{36}\left(-3\log\left(x^2-x+1\right)+\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+6\log(x+1)-2\sqrt[3]{3}\log\left(3^{2/3}x+3\right)+2\ 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+6\sqrt{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x^3 + x^6)^(-1),x]
```

```
[Out] (2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] + 6*Log[1 + x] - 2*3^(1/3)*Log[3 + 3^(2/3)*x] - 3*Log[1 - x + x^2] +
 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

Maple [A] time = 0.006, size = 84, normalized size = 0.8

$$-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\sqrt[3]{3}\ln\left(\sqrt[3]{3}+x\right)}{18}+\frac{\sqrt[3]{3}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{36}-\frac{3^{\frac{5}{6}}}{18}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+4*x^3+3),x)
```

```
[Out] -1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/18*3^(1/3)*ln(3
^(1/3)+x)+1/36*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/18*3^(5/6)*arctan(1/3*3^
(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)
```

Maxima [A] time = 1.68026, size = 113, normalized size = 1.01

$$-\frac{1}{18}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{36}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{18}\cdot 3^{\frac{1}{3}}\log\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3^(
1/3)*log(x + 3^(1/3)) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Fricas [A] time = 1.51014, size = 450, normalized size = 4.02

$$\frac{1}{18}\cdot 9^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{108}\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(9^{\frac{2}{3}}(-1)^{\frac{1}{3}}x+3x^2+3\cdot 9^{\frac{1}{3}}(-1)^{\frac{2}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/18*9^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*(-1)
^(2/3)*x - 3*9^(1/3)*sqrt(3))) - 1/108*9^(2/3)*(-1)^(1/3)*log(9^(2/3)*(-1)^
(1/3)*x + 3*x^2 + 3*9^(1/3)*(-1)^(2/3)) + 1/54*9^(2/3)*(-1)^(1/3)*log(-9^(2
/3)*(-1)^(1/3) + 3*x) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/12*lo
g(x^2 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] time = 1.36963, size = 124, normalized size = 1.11

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{13}{10}-\frac{13\sqrt{3}i}{10}+\frac{23328\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{5}\right)+\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{13}{10}+\frac{23328\left(-\frac{1}{12}-\right.}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 13/10 - 13*sqrt(3)*I/10 + 233
28*(-1/12 + sqrt(3)*I/12)**4/5) + (-1/12 - sqrt(3)*I/12)*log(x + 13/10 + 23
328*(-1/12 - sqrt(3)*I/12)**4/5 + 13*sqrt(3)*I/10) + RootSum(1944*_t**3 + 1
, Lambda(_t, _t*log(23328*_t**4/5 - 78*_t/5 + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.166 $\int \frac{1}{x^2(3+4x^3+x^6)} \, dx$

Optimal. Leaf size=119

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\sqrt[3]{3}}-\frac{1}{3x}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{18\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\ 3^{5/6}}$$

```
[Out] -1/(3*x) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3
^(5/6)]/(6*3^(5/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(18*3^(1/3)) - Log[1
- x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(1/3))
```

Rubi [A] time = 0.0822884, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1368, 1510, 292, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\sqrt[3]{3}}-\frac{1}{3x}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{18\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\ 3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(3 + 4*x^3 + x^6)),x]
```


```
[Out] -1/(3*x) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3
^(5/6)]/(6*3^(5/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(18*3^(1/3)) - Log[1
- x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(1/3))
```


Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(3+4x^3+x^6\right)}\,dx &= -\frac{1}{3x}+\frac{1}{3}\int \frac{x\left(-4-x^3\right)}{3+4x^3+x^6}\,dx\\
&= -\frac{1}{3x}+\frac{1}{6}\int \frac{x}{3+x^3}\,dx-\frac{1}{2}\int \frac{x}{1+x^3}\,dx\\
&= -\frac{1}{3x}+\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{18\sqrt[3]{3}}+\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{18\sqrt[3]{3}}\\
&= -\frac{1}{3x}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{12}\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx-\frac{1}{4}\int\\
&= -\frac{1}{3x}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\sqrt[3]{3}}+\frac{1}{2}\text{Subst}\\
&= -\frac{1}{3x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\ 3^{5/6}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\end{aligned}$$

Mathematica [A] time = 0.0435589, size = 118, normalized size = 0.99

$$-\frac{9x\log\left(x^2-x+1\right)-3^{2/3}x\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-18x\log(x+1)+2\ 3^{2/3}x\log\left(3^{2/3}x+3\right)+6\sqrt[6]{3}x\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+18}{108x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -(36 + 6*3^(1/6)*x*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 18*Sqrt[3]*x*ArcTan[(-
1 + 2*x)/Sqrt[3]] - 18*x*Log[1 + x] + 2*3^(2/3)*x*Log[3 + 3^(2/3)*x] + 9*x*
Log[1 - x + x^2] - 3^(2/3)*x*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/(108*x)
```

Maple [A] time = 0.008, size = 89, normalized size = 0.8

$$-\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{1}{3x}-\frac{3^{\frac{2}{3}}\ln\left(\sqrt[3]{3}+x\right)}{54}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{108}+\frac{\sqrt[6]{3}}{18}\arctan\left(\frac{\sqrt{3}}{3}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6+4*x^3+3),x)
```

```
[Out] -1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/3/x-1/54*3^(2/3
)*ln(3^(1/3)+x)+1/108*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/18*3^(1/6)*arctan
(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)
```

Maxima [A] time = 1.67379, size = 120, normalized size = 1.01

$$\frac{1}{108}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{18}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(x + 3^(1/3)
) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(1/3*3^(
1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Fricas [A] time = 1.49493, size = 409, normalized size = 3.44

$$-\frac{3^{\frac{2}{3}}(-1)^{\frac{1}{3}}x\log\left(-3^{\frac{1}{3}}(-1)^{\frac{2}{3}}x+x^2-3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\right)-2\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}x\log\left(3^{\frac{1}{3}}(-1)^{\frac{2}{3}}+x\right)+18\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-}{108x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/108*(3^(2/3)*(-1)^(1/3)*x*log(-3^(1/3)*(-1)^(2/3)*x + x^2 - 3^(2/3)*(-1)
^(1/3)) - 2*3^(2/3)*(-1)^(1/3)*x*log(3^(1/3)*(-1)^(2/3) + x) + 18*sqrt(3)*x
*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*(-1)^(1/3)*x*arctan(1/3*3^(1/6)*
(2*(-1)^(1/3)*x + 3^(1/3))) + 9*x*log(x^2 - x + 1) - 18*x*log(x + 1) + 36)/
x
```

Sympy [C] time = 1.28348, size = 139, normalized size = 1.17

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{8188128\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{41}+\frac{39384\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{41}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{39384\left(-\frac{1}{12}\right.}{41}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 8188128*(-1/12 - sqrt(3)*I/12
)**5/41 + 39384*(-1/12 - sqrt(3)*I/12)**2/41) + (-1/12 + sqrt(3)*I/12)*log(
x + 39384*(-1/12 + sqrt(3)*I/12)**2/41 - 8188128*(-1/12 + sqrt(3)*I/12)**5/
41) + RootSum(17496*_t**3 + 1, Lambda(_t, _t*log(-8188128*_t**5/41 + 39384*
_t**2/41 + x))) - 1/(3*x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.167 $\int \frac{1}{x^3(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{6x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{18\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}$$

```
[Out] -1/(6*x^2) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)
/3^(5/6)]/(18*3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(18*3^(2/3)) + Log
[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(2/3))
```

Rubi [A] time = 0.0775538, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1368, 1422, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{6x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{18\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(3 + 4*x^3 + x^6)),x]
```



```
[Out] -1/(6*x^2) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)
/3^(5/6)]/(18*3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(18*3^(2/3)) + Log
[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(2/3))
```


Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(3+4x^3+x^6\right)}\,dx = -\frac{1}{6x^2}+\frac{1}{6}\int \frac{-8-2x^3}{3+4x^3+x^6}\,dx$$

$$= -\frac{1}{6x^2}+\frac{1}{6}\int \frac{1}{3+x^3}\,dx-\frac{1}{2}\int \frac{1}{1+x^3}\,dx$$

$$= -\frac{1}{6x^2}-\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx+\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{18\ 3^{2/3}}+\frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{18\ 3^{2/3}}$$

$$= -\frac{1}{6x^2}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+}}{36\ 3^{2/3}}$$

$$= -\frac{1}{6x^2}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\ 3^{2/3}}+\frac{1}{2}\text{Subst}$$

$$= -\frac{1}{6x^2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-$$

Mathematica [A] time = 0.0561699, size = 113, normalized size = 0.95

$$\frac{1}{108}\left(-\frac{18}{x^2}+9\log\left(x^2-x+1\right)-\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-18\log(x+1)+2\sqrt[3]{3}\log\left(3^{2/3}x+3\right)-2\,3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}\cdot}{3^{5}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-18/x^2 - 2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 18*Sqrt[3]*ArcTan[(-
1 + 2*x)/Sqrt[3]] - 18*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + 9*Log[1
- x + x^2] - 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/108
```

Maple [A] time = 0.007, size = 89, normalized size = 0.8

$$\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt[3]{3}\ln\left(\sqrt[3]{3}+x\right)}{54}-\frac{\sqrt[3]{3}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{108}+\frac{3^{\frac{5}{6}}}{54}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6+4*x^3+3),x)
```

```
[Out] 1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/54*3^(1/3)*ln(3^
(1/3)+x)-1/108*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/54*3^(5/6)*arctan(1/3*3^
(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)-1/6/x^2
```

Maxima [A] time = 1.61766, size = 120, normalized size = 1.01

$$\frac{1}{54}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{108}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{54}\cdot 3^{\frac{1}{3}}\log\left(x+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3^(
1/3)*log(x + 3^(1/3)) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Fricas [A] time = 1.48877, size = 378, normalized size = 3.18

$$\frac{6\cdot 9^{\frac{1}{6}}\sqrt{3}x^2\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-9^{\frac{2}{3}}x^2\log\left(3x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+2\cdot 9^{\frac{2}{3}}x^2\log\left(3x+9^{\frac{2}{3}}\right)-54\sqrt{\cdot}}{324x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/324*(6*9^(1/6)*sqrt(3)*x^2*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*9
^(1/3)*sqrt(3))) - 9^(2/3)*x^2*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 2*9^(2/
```

```
3)*x^2*log(3*x + 9^(2/3)) - 54*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x - 1)) +
27*x^2*log(x^2 - x + 1) - 54*x^2*log(x + 1) - 54)/x^2
```

Sympy [C] time = 1.30238, size = 128, normalized size = 1.08

$$-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1093}{244}-\frac{1093\sqrt{3}i}{244}+\frac{787320\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{61}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1093}{244}+\frac{787320\left(\frac{1}{12}\right.}{61}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 1093/244 - 1093*sqrt(3)*I/244
 + 787320*(1/12 - sqrt(3)*I/12)**4/61) + (1/12 + sqrt(3)*I/12)*log(x + 1093
/244 + 787320*(1/12 + sqrt(3)*I/12)**4/61 + 1093*sqrt(3)*I/244) + RootSum(5
2488*_t**3 - 1, Lambda(_t, _t*log(787320*_t**4/61 + 3279*_t/61 + x))) - 1/(
6*x**2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.168 $\int \frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{12x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\sqrt[3]{3}}+\frac{4}{9x}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{54\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\cdots\right)}{18\,3\cdots}$$

```
[Out] -1/(12*x^4) + 4/(9*x) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(
1/3) - 2*x)/3^(5/6)]/(18*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(54*3^(
1/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(1/3))
```

Rubi [A] time = 0.10298, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1368, 1504, 1510, 292, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\sqrt[3]{3}}+\frac{4}{9x}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{54\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\cdots\right)}{18\,3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(12*x^4) + 4/(9*x) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(
1/3) - 2*x)/3^(5/6)]/(18*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(54*3^(
1/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(1/3))
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx = -\frac{1}{12x^4}+\frac{1}{12}\int \frac{-16-4x^3}{x^2\left(3+4x^3+x^6\right)}\,dx$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{36}\int \frac{x\left(-52-16x^3\right)}{3+4x^3+x^6}\,dx$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{18}\int \frac{x}{3+x^3}\,dx+\frac{1}{2}\int \frac{x}{1+x^3}\,dx$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx+\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{54\sqrt[3]{3}}-\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{3}}$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}-\frac{1}{36}\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{3}}$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\ 3^{5/6}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}+\frac{1}{12}\log\left(1-\right.$$

Mathematica [A] time = 0.0510884, size = 118, normalized size = 0.94

$$\frac{1}{324}\left(-\frac{27}{x^4}+27\log\left(x^2-x+1\right)-3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+\frac{144}{x}-54\log(x+1)+2\ 3^{2/3}\log\left(3^{2/3}x+3\right)+6\sqrt[6]{3}\tan\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-27/x^4 + 144/x + 6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 54*Sqrt[3]*A
rcTan[(-1 + 2*x)/Sqrt[3]] - 54*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] +
27*Log[1 - x + x^2] - 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/324
```

Maple [A] time = 0.008, size = 94, normalized size = 0.8

$$-\frac{1}{12\,x^4}+\frac{4}{9\,x}+\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)+\frac{3^{\frac{2}{3}}\ln\left(\sqrt[3]{3}+x\right)}{162}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{324}-\frac{\sqrt[6]{3}}{54}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^6+4*x^3+3),x)
```

```
[Out] -1/12/x^4+4/9/x+1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/
162*3^(2/3)*ln(3^(1/3)+x)-1/324*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/54*3^(1
/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)
```

Maxima [A] time = 1.60797, size = 130, normalized size = 1.03

$$-\frac{1}{324}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{162}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)-\frac{1}{54}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(x + 3^(1/
3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arctan(1/3*3
^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x + 1) - 1
/6*log(x + 1)
```

Fricas [A] time = 1.47367, size = 339, normalized size = 2.69

$$-\frac{3^{\frac{2}{3}}x^4\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-2\cdot 3^{\frac{2}{3}}x^4\log\left(x+3^{\frac{1}{3}}\right)-54\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6\cdot 3^{\frac{1}{6}}x^4\arctan\left(-\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right.\right.}{324x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/324*(3^(2/3)*x^4*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 2*3^(2/3)*x^4*log(x +
3^(1/3)) - 54*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*x^4*arc
tan(-1/3*3^(1/6)*(2*x - 3^(1/3))) - 27*x^4*log(x^2 - x + 1) + 54*x^4*log(x
+ 1) - 144*x^3 + 27)/x^4
```

Sympy [C] time = 1.3051, size = 141, normalized size = 1.12

$$-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{4782978\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{547}+\frac{1028869776\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{547}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1028869776}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 4782978*(1/12 - sqrt(3)*I/12)
**2/547 + 1028869776*(1/12 - sqrt(3)*I/12)**5/547) + (1/12 + sqrt(3)*I/12)*
log(x + 1028869776*(1/12 + sqrt(3)*I/12)**5/547 + 4782978*(1/12 + sqrt(3)*I
/12)**2/547) + RootSum(472392*_t**3 - 1, Lambda(_t, _t*log(1028869776*_t**5
/547 + 4782978*_t**2/547 + x))) + (16*x**3 - 3)/(36*x**4)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.169 $\int \frac{1}{x^6(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=126

$$\frac{2}{9x^2}-\frac{1}{15x^5}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{54\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}$$

```
[Out] -1/(15*x^5) + 2/(9*x^2) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3
^(1/3) - 2*x)/3^(5/6)]/(54*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(54*3
^(2/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(2/3)
)
```

Rubi [A] time = 0.0985897, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1368, 1504, 1422, 200, 31, 634, 618, 204, 628, 617}

$$\frac{2}{9x^2}-\frac{1}{15x^5}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{54\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(15*x^5) + 2/(9*x^2) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3
^(1/3) - 2*x)/3^(5/6)]/(54*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(54*3
^(2/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(2/3)
)
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
```

```
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^6\left(3+4x^3+x^6\right)}\,dx = -\frac{1}{15x^5}+\frac{1}{15}\int \frac{-20-5x^3}{x^3\left(3+4x^3+x^6\right)}\,dx$$
$$= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{90}\int \frac{-130-40x^3}{3+4x^3+x^6}\,dx$$
$$= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{18}\int \frac{1}{3+x^3}\,dx+\frac{1}{2}\int \frac{1}{1+x^3}\,dx$$
$$= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{54\ 3^{2/3}}-\frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{54\ 3^{2/3}}$$
$$= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+$$
$$= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{108\ 3^{2/3}}$$
$$= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}}-\frac{1}{12}\log\left(1\right.$$

Mathematica [A] time = 0.0612248, size = 118, normalized size = 0.94

$$\frac{\frac{360}{x^2}-\frac{108}{x^5}-135\log\left(x^2-x+1\right)+5\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+270\log(x+1)-10\sqrt[3]{3}\log\left(3^{2/3}x+3\right)+10\ 3^{5/6}\tan^{-}}{1620}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-108/x^5 + 360/x^2 + 10*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 270*Sqrt
[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 270*Log[1 + x] - 10*3^(1/3)*Log[3 + 3^(2/3
)*x] - 135*Log[1 - x + x^2] + 5*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/1
620
```

Maple [A] time = 0.01, size = 94, normalized size = 0.8

$$-\frac{1}{15\,x^5}+\frac{2}{9\,x^2}-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)-\frac{\sqrt[3]{3}\ln\left(\sqrt[3]{3}+x\right)}{162}+\frac{\sqrt[3]{3}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{324}-\frac{3^{\frac{5}{6}}}{162}\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^6+4*x^3+3),x)
```

```
[Out] -1/15/x^5+2/9/x^2-1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-
1/162*3^(1/3)*ln(3^(1/3)+x)+1/324*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/162*3
^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)
```

Maxima [A] time = 1.6985, size = 130, normalized size = 1.03

$$-\frac{1}{162}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{324}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{162}\cdot 3^{\frac{1}{3}}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/162*
3^(1/3)*log(x + 3^(1/3)) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x + 1) +
1/6*log(x + 1)
```

Fricas [A] time = 1.52307, size = 510, normalized size = 4.05

$$\frac{30\cdot 9^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\,x^5\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\,x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-5\cdot 9^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\,x^5\log\left(9^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\,x+3\,x^2+3\cdot 9^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/4860*(30*9^(1/6)*sqrt(3)*(-1)^(1/3)*x^5*arctan(1/27*9^(1/6)*(2*9^(2/3)*sq
rt(3)*(-1)^(2/3)*x - 3*9^(1/3)*sqrt(3))) - 5*9^(2/3)*(-1)^(1/3)*x^5*log(9^(
2/3)*(-1)^(1/3)*x + 3*x^2 + 3*9^(1/3)*(-1)^(2/3)) + 10*9^(2/3)*(-1)^(1/3)*x
^5*log(-9^(2/3)*(-1)^(1/3) + 3*x) + 810*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x
 - 1)) - 405*x^5*log(x^2 - x + 1) + 810*x^5*log(x + 1) + 1080*x^3 - 324)/x^
5
```

Sympy [C] time = 1.28467, size = 136, normalized size = 1.08

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{88573}{6562}-\frac{88573\sqrt{3}i}{6562}+\frac{119042784\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{3281}\right)+\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{88573}{6562}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 88573/6562 - 88573*sqrt(3)*I/
6562 + 119042784*(-1/12 + sqrt(3)*I/12)**4/3281) + (-1/12 - sqrt(3)*I/12)*l
og(x + 88573/6562 + 119042784*(-1/12 - sqrt(3)*I/12)**4/3281 + 88573*sqrt(3
)*I/6562) + RootSum(1417176*_t**3 + 1, Lambda(_t, _t*log(119042784*_t**4/32
81 - 531438*_t/3281 + x))) + (10*x**3 - 3)/(45*x**5)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.170 $\int \frac{x^6}{1-x^3+x^6}\,dx$

Optimal. Leaf size=412

$$-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+x$$

```
[Out] x + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(
3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I
*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*S
qrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^
(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3
)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*
(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3))
 - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x
 + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rubi [A] time = 0.428322, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1367, 1422, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+x$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - x^3 + x^6),x]
```

```
[Out] x + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(
3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I
*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*S
qrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^
(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3
)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*
(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3))
 - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x
 + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x - \int \frac{1-x^3}{1-x^3+x^6}\,dx$$

$$= x - \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx + \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= x + \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\int \cdots}{9\sqrt[3]{2}\left(1+i\cdots\right.}$$

$$= x + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\int \cdots}{18\sqrt[3]{2}\left(1\cdots\right.}$$

$$= x + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\cdots\right.\right.}{18\cdots}$$

$$= x + \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\cdots\right.}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] time = 0.0142571, size = 59, normalized size = 0.14

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]+x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - x^3 + x^6),x]
```

```
[Out] x + RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 +
2*#1^5) & ]/3
```

Maple [C] time = 0.008, size = 44, normalized size = 0.1

$$x+\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(\_R^3-1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6-x^3+1),x)
```

```
[Out] x+1/3*sum((_R^3-1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x+\int \frac{x^3-1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] x + integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 1.87502, size = 3906, normalized size = 9.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(1/6)*s
qrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arcta
n(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18
^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) + 2/27*18^(2/3)*12
^(1/6)*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3)
- 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 108*sqrt(3)*sin(2/3*ar
ctan(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3)
- 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt
(2)*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arct
an(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(
3) - 2))^2))*sin(2/3*arctan(sqrt(3) - 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)
*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) -
2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arcta
n(sqrt(3) - 2))^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) - 2
)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3
*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))
+ 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*
sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)
*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(
sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3)
- 2))^2)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) +
18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/216*(18^(1/3)*12^(
5/6)*sqrt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqr
t(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3
)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*s
qrt(3)*x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))
- 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*
12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) + 1/108*(18^(2/3)*12^(1/6)*sqrt
(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3)
 - 2)))*log(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3
*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin
(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) + x
```

Sympy [A] time = 0.176818, size = 26, normalized size = 0.06

$$x + \text{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 - 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6-x**3+1),x)
```

```
[Out] x + RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t
 + x)))
```

Giac [B] time = 1.19542, size = 861, normalized size = 2.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9
*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6
*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi
)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*pi) + 2*s
in(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*si
n(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*
pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)
*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*arctan(((sqrt(3)*i
+ 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(4*sqrt(3)*co
s(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)
^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi)
- 2*cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) -
1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*p
i)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sq
rt(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi
))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos
(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + si
n(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(1/9*pi))*log((sqrt(3)*i*cos(1/9
*pi) + cos(1/9*pi))*x + x^2 + 1) + x
```

3.171 $\int \frac{x^5}{1-x^3+x^6}\,dx$

Optimal. Leaf size=39

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rubi [A] time = 0.0354415, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - x^3 + x^6),x]

[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{1-x^3+x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}
$$

Mathematica [A] time = 0.0100902, size = 39, normalized size = 1.

$$\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - x^3 + x^6),x]
```

```
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Maple [A] time = 0.003, size = 33, normalized size = 0.9

$$\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}}{9}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6-x^3+1),x)
```

```
[Out] 1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 1.59843, size = 43, normalized size = 1.1

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Fricas [A] time = 1.43572, size = 95, normalized size = 2.44

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Sympy [A] time = 0.129845, size = 37, normalized size = 0.95

$$\frac{\log\left(x^6 - x^3 + 1\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6-x**3+1),x)
```

```
[Out] log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Giac [A] time = 1.09055, size = 43, normalized size = 1.1

$$\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3 - 1\right)\right) + \frac{1}{6}\,\log\left(x^6 - x^3 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

3.172 $\int \frac{x^4}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+$$

```
[Out] ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^
(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqr
t[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[
3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3
)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1
 + I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - (
(3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rubi [A] time = 0.280592, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1374, 292, 31, 634, 617, 204, 628}

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^3 + x^6),x]
```


```
[Out] ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^
(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqr
t[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[
3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3
)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1
 + I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - (
(3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```


Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4}{1-x^3+x^6}\,dx = -\left(\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\left(-\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)+\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\int \cdots}{9\cdots}$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(-3-i\sqrt{3}\right)\int \cdots}{18\ 2^{2/3}\sqrt[3]{1\cdots}}$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)\cdots\right.}{18\ 2^{2/3}\cdots}$$

$$= \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\cdots$$

Mathematica [C] time = 0.0098653, size = 41, normalized size = 0.1

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^3 + x^6),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/3
```

Maple [C] time = 0.004, size = 40, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 2.30185, size = 5936, normalized size = 14.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) +
2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3
*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^
2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/
3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2) + 2/27*18^(2/3)*12^(1/6)*arc
tan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 1
08*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt
(3) + 2))^4 + 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))
^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2
))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arct
an(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arctan(sqrt(3
) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*
12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36
*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)*
12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3*arctan(sq
rt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(sqrt(3) +
2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4))*sin(2/3*arctan(sqrt(3) + 2)) - 1
/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(
1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3
)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2
))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 864*cos(2/3*arctan(sqrt
(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x -
 36*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2
+ 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(
sqrt(3) + 2))^3)*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(
2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))
^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*ar
ctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 +
 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*
x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(
2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(
3) + 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arcta
n(sqrt(3) + 2))))/(3*cos(2/3*arctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqr
t(3) + 2))^2*sin(2/3*arctan(sqrt(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2)
)^4)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(
2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/432*(6*18^(2/3)*12^(2
/3)*x - 216*cos(2/3*arctan(sqrt(3) + 2))^2 + 216*sin(2/3*arctan(sqrt(3) + 2
))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2)
```

```
)^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36
*x^2))/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)))) - 1/108
*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6
)*cos(2/3*arctan(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3
) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*
12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)
) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan
(sqrt(3) + 2))^2 + 36*x^2) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arcta
n(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(18^(2
/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arc
tan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2
 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3
)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2)
```

Sympy [A] time = 0.180416, size = 26, normalized size = 0.06

$$\text{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(6561t^5+54t^2+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t**
2 + x)))
```

Giac [B] time = 1.18076, size = 1112, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10
*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(
4/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)
*sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/
9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(2/9*pi)^5
- 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*sin(2/9*p
i)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)^3 - 2*si
n(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*cos(2/9*pi)
*sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*
sin(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3)*cos(1/9*pi)^3*sin
(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*cos(1/9*pi)^4*sin(1/
9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5 - sqrt(3)*cos(1/9*
pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)
*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(10*sqrt(3
)*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sq
rt(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10
*cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi
)^2 - sin(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1
```

```
) - 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*s
in(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*cos(2/9*pi)^5 - 20*cos(2/9*pi)^3
*sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^4 + 2*sqrt(3)*cos(2/9*pi)*sin(2
/9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2
/9*pi))*x + x^2 + 1) - 1/18*(10*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 20*sqrt
(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*sin(1/9*pi)^5 - 2*cos(1/9*pi)^5
 + 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3
)*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 - sin(1/9*pi)^2)*log((sqrt(3)*i*c
os(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

3.173 $\int \frac{x^3}{1-x^3+x^6} \, dx$

Optimal. Leaf size=411

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+ \cdots$$

```
[Out] -((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2
^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sq
rt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt
[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/
3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(
1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1
- I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) -
((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x +
2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rubi [A] time = 0.277631, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1374, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+ \cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - x^3 + x^6),x]
```


```
[Out] -((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2
^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sq
rt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt
[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/
3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(
1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1
- I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) -
((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x +
2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```


Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{1-x^3+x^6}\,dx = -\left(\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\right.}}}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)}$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1-\right.}}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+}}{18\sqrt[3]{2}\left(1+i\right.}$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)\right.}{18\sqrt[3]{2}}$$

$$= -\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\right.}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] time = 0.008928, size = 39, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - x^3 + x^6),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

Maple [C] time = 0.004, size = 40, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 1.9112, size = 3903, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(2*18^(2/3)*12^(1/6)
*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arcta
n(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18
*x^2) + 2/27*18^(2/3)*12^(1/6)*arctan(1/216*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt
(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^
(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3
*arctan(sqrt(3) + 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*sqrt(3)*x - 216*sin
(2/3*arctan(sqrt(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqr
t(3) + 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) -
 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/108*(6*18^(1/3)*
12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 108*sqrt(3)*cos(2/3*arcta
n(sqrt(3) + 2))^2 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 18*(18^(1/
3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2
)) - sqrt(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^
(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3
*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^
2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2))
 - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arct
an(sqrt(3) + 2))^2 - 3*sin(2/3*arctan(sqrt(3) + 2))^2)) - 1/27*(18^(2/3)*12
^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arc
tan(sqrt(3) + 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arct
an(sqrt(3) + 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 108*sqrt(3)
*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arct
an(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sq
rt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan
(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^
(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*
sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*
sin(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*
arctan(sqrt(3) + 2))^2)) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(
sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/
3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*
cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) +
 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) + 1/1
08*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1
/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*
arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) +
 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*s
in(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)
```

Sympy [A] time = 0.178717, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log\left(-1458t^4 - 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t +
 x)))
```

Giac [B] time = 1.15075, size = 860, normalized size = 2.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2*
sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4/9
*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/
9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(2/9*pi)^4
- 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^4 + 8*cos(
2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos(2/9*pi) +
 sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*
sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3)*cos(1/9*pi)^2*sin
(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*sin(1/9*pi) + 8*cos(
1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*pi))*arctan(((sqrt(3)
*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(8*sqrt(3)
*cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/
9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/
9*pi) - cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1
) - 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2
/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)
^4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2
/9*pi))*x + x^2 + 1) + 1/18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3
)*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi
)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi) - cos(1/9*pi))*log((sqrt(3)*i*c
os(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

3.174 $\int \frac{x^2}{1-x^3+x^6} \, dx$

Optimal. Leaf size=23

$$-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0230573, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 204}

$$-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^3 + x^6),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1-x^3+x^6} \, dx &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{1-x+x^2} \, dx, x, x^3\right)\\
&= -\left(\frac{2}{3} \text{Subst}\left(\int \frac{1}{-3-x^2} \, dx, x, -1+2x^3\right)\right)\\
&= -\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0058599, size = 23, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - x^3 + x^6),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A] time = 0.001, size = 19, normalized size = 0.8

$$\frac{2\sqrt{3}}{9}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6-x^3+1),x)
```

```
[Out] 2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 1.70142, size = 24, normalized size = 1.04

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Fricas [A] time = 1.45039, size = 61, normalized size = 2.65

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Sympy [A] time = 0.123806, size = 27, normalized size = 1.17

$$\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6-x**3+1),x)
```

```
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Giac [A] time = 1.1399, size = 24, normalized size = 1.04

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

3.175 $\int \frac{x}{1-x^3+x^6}\,dx$

Optimal. Leaf size=375

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\cdots}\right.}{3\sqrt{3}\sqrt[3]{\frac{1}{2}}\left(1-i\cdots\right.}$$

```
[Out] ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sqrt[
3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]]
)/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]
)/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) -
2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 - I*Sqrt[3]
)^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 -
 I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))
^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))
```

Rubi [A] time = 0.25039, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1375, 292, 31, 634, 617, 204, 628}

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\cdots}\right.}{3\sqrt{3}\sqrt[3]{\frac{1}{2}}\left(1-i\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^3 + x^6),x]
```


```
[Out] ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sqrt[
3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]]
)/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]
)/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) -
2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 - I*Sqrt[3]
)^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 -
 I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))
^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))
```


Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[b, 3]*Rt[a, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{1-x^3+x^6}\,dx = -\frac{i\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}} + \frac{i\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$= \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right.} \cdots}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}} \cdots}}{3\sqrt{3}\sqrt[3]{\frac{1}{2}}\left(1+ \cdots\right.}$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{2\sqrt{3}} - \frac{i\int \cdots}{\left(\frac{1}{2}\left(1+i\sqrt{\cdots}\right.\right.}$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} - \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}$$

$$= \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

Mathematica [C] time = 0.0090665, size = 40, normalized size = 0.11

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^4-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^3 + x^6),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

Maple [C] time = 0.004, size = 38, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R\ln(x-\_R)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(_R/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 2.35626, size = 5936, normalized size = 15.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) -
2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3
*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^
2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/
3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 2/27*18^(2/3)*12^(1/6)*arc
tan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 1
08*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt
(3) - 2))^4 - 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))
^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2
```

```
))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arct
an(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3
) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*
12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36
*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*
12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sq
rt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) -
2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3) - 2)) - 1
/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(
1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3
)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2
))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 864*cos(2/3*arctan(sqrt
(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x -
 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3*arctan(sqrt(3) - 2))^2
- 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 72*cos(2/3*arctan(
sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(
2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))
^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*ar
ctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 +
 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*
x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(
2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(
3) - 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arcta
n(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4 - 10*cos(2/3*arctan(sqr
t(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2)
)^4)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) - 18^(
2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/432*(6*18^(2/3)*12^(2
/3)*x - 216*cos(2/3*arctan(sqrt(3) - 2))^2 + 216*sin(2/3*arctan(sqrt(3) - 2
))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2)
)^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36
*x^2))/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)))) + 1/108
*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6
)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3
) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*
12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)
) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan
(sqrt(3) - 2))^2 + 36*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arcta
n(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2
/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arc
tan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2
 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3
)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)
```

Sympy [A] time = 0.173484, size = 26, normalized size = 0.07

$$\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(6561t^5 - 27t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 - 27*_t**
2 + x)))
```

Giac [B] time = 1.16175, size = 1096, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sq
rt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*
pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 + sqrt(3)*sin(4
/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*pi)
- 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^5 - 10*sqr
t(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^4 - 5*
cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(2/9*pi)^5
- sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2 + 2*cos(2/9*pi)*sin(2/9*pi)
)*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2/9*pi))
) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 5
*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*sin(1/9*pi) - 10*cos(1
/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 + sqrt(3)*cos(1/9*pi)^2 - sqrt(3)*si
n(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9*pi
) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin
(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 +
 cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)
^4 - 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) - cos(4/9*pi)^2 + sin(4/9*pi)^2)*log
(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(
2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*si
n(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*cos(2/9*pi
)*sin(2/9*pi)^4 - 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) - cos(2/9*pi)^2 + sin(2
/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) - 1/18*(5
*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3
 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi)^3*sin(1/9*pi)^2 -
 5*cos(1/9*pi)*sin(1/9*pi)^4 + 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) - cos(1/9*
pi)^2 + sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x + x^2 +
1)
```

3.176 $\int \frac{1}{1-x^3+x^6}\,dx$

Optimal. Leaf size=186

$$-\frac{(-1)^{5/18}\left(3\log\left(\sqrt[9]{-1}-x\right)+\log(2)\right)}{9\sqrt{3}}+\frac{(-1)^{13/18}\log\left(-\sqrt[3]{2}\left(x+(-1)^{8/9}\right)\right)}{3\sqrt{3}}-\frac{(-1)^{13/18}\log\left(-2^{2/3}\left(\left((-1)^{8/9}-x\right)x+(-1)^{7/9}\right)\cdots\right)}{6\sqrt{3}}\cdots$$

```
[Out] -((-1)^(13/18)*ArcTan[(1 + 2*(-1)^(1/9)*x)/Sqrt[3]])/3 + ((-1)^(5/18)*ArcTa
n[(1 - 2*(-1)^(8/9)*x)/Sqrt[3]])/3 - ((-1)^(5/18)*(Log[2] + 3*Log[(-1)^(1/9
) - x]))/(9*Sqrt[3]) + ((-1)^(13/18)*Log[-(2^(1/3)*((-1)^(8/9) + x))])/(3*S
qrt[3]) - ((-1)^(13/18)*Log[-(2^(2/3)*((-1)^(7/9) + ((-1)^(8/9) - x)*x))])/
(6*Sqrt[3]) + ((-1)^(5/18)*Log[2^(2/3)*((-1)^(2/9) + x*((-1)^(1/9) + x))])/
(6*Sqrt[3])
```

Rubi [C] time = 0.24245, antiderivative size = 375, normalized size of antiderivative = 2.02, number of steps used = 13, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {1347, 200, 31, 634, 617, 204, 628}

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\cdots}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/\cdots}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3 + x^6)^(-1),x]
```

```
[Out] ((-I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sqrt
[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]
])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x
])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) -
 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[3
])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1
- I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(2/3))
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{1-x^3+x^6}\,dx = -\frac{i\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}+\frac{i\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$=\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \frac{-2^{2/\cdots}}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2\cdots}}}{3\sqrt{3}\left(\frac{1}{2}\cdots\right.}$$

$$=\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{i\int \frac{\cdots}{\left(\frac{1}{2}\left(1\cdots\right.\right.}}{\cdots}$$

$$=\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x\cdots\right.}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$=-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{\cdots}\right.}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\cdots}$$

Mathematica [C] time = 0.0100208, size = 42, normalized size = 0.23

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^5-\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3 + x^6)^(-1),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

Maple [C] time = 0.005, size = 37, normalized size = 0.2

$$\frac{1}{3}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(1/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6 - x^3 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 1.89363, size = 3900, normalized size = 20.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(1/6)*s
qrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arcta
n(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18
^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 2/27*18^(2/3)*12
^(1/6)*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arc
tan(sqrt(3) - 2))^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) -
 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2
/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)
) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3
)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(
2)*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arcta
n(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3
) - 2))^2))*sin(2/3*arctan(sqrt(3) - 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*
cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2
```

```
)))*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2
)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 108*sqrt(3)*sin(2/3*arcta
n(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) - 2
)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3
*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))
+ 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*
sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)
*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(
sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3)
- 2))^2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) -
18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/216*(18^(1/3)*12^(
5/6)*sqrt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqr
t(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3
)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*s
qrt(3)*x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))
+ 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*
12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt
(3)*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3)
 - 2)))*log(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3
*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin
(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)
```

Sympy [A] time = 0.180478, size = 20, normalized size = 0.11

$$\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log\left(729t^4 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x)))
```

Giac [B] time = 1.17408, size = 849, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*pi)
 - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*sqr
t(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)^3*
sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9*pi) - sin(2/9*p
i))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2/9*pi
))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 +
sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)*sin(1/9
*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9
*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*
```

```
sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4
/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 - sqrt(3)*sin(4/9*pi) + cos(4/9*pi))
*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*
cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*p
i)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 - sqrt(3)*sin(2/9*pi)
+ cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) + 1/
18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)
^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 + sqrt(3
)*sin(1/9*pi) + cos(1/9*pi))*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x +
x^2 + 1)
```

3.177 $\int \frac{1}{x(1-x^3+x^6)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)$$

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rubi [A] time = 0.0394801, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3 + x^6)),x]
```

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \log(x)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] time = 0.0119664, size = 55, normalized size = 1.34

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3 + x^6)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-1
+ 2*#1^3) & ]/3
```

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}}{9}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6-x^3+1),x)
```

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 1.61205, size = 51, normalized size = 1.24

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3*
log(x^3)
```

Fricas [A] time = 1.50589, size = 107, normalized size = 2.61

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

Sympy [A] time = 0.149243, size = 41, normalized size = 1.

$$\log\left(x\right)-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9
```

Giac [A] time = 1.11291, size = 47, normalized size = 1.15

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

3.178 $\int \frac{1}{x^2(1-x^3+x^6)}\,dx$

Optimal. Leaf size=416

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-$$

```
[Out] -x^(-1) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/(
(1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3
 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqr
t[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*
2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
 + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^
(1/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(
1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rubi [A] time = 0.299715, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1368, 1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^3 + x^6)),x]
```


```
[Out] -x^(-1) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/(
(1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3
 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqr
t[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*
2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
 + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^
(1/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(
1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```


Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2(1-x^3+x^6)}\,dx = -\frac{1}{x} + \int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{x} + \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{1}{x} - \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}+\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\int -}{9\ 2^{2/3}\sqrt[3]{}}$$

$$= -\frac{1}{x} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\int \frac{}{\left(\frac{1}{2}(1-\right.}}{18\ 2}$$

$$= -\frac{1}{x} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left((1\right.}{}$$

$$= -\frac{1}{x} + \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}\right.}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

Mathematica [C] time = 0.0122603, size = 61, normalized size = 0.15

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^4-\#1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#
1 + 2*#1^4) & ]/3
```

Maple [C] time = 0.007, size = 50, normalized size = 0.1

$$-\frac{1}{3}\sum_{\_R=\text{RootOf}(\_Z^6-\_Z^3+1)}\frac{\left(\_R^4-\_R\right)\ln(x-\_R)}{2\_R^5-\_R^2}-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))-1/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x} - \int \frac{x^4 - x}{x^6 - x^3 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^4 - x)/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 2.26592, size = 5952, normalized size = 14.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2))*log(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt
(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18
^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin
(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2) + 8*18^(2/3)*12^(1/6)*x*arctan(-1/432
*(6*18^(2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) + 2))^2 + 216*sin(2/3*
arctan(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*a
rctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 -
12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3
)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sq
rt(3) + 2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(
3) + 2))))*sin(2/3*arctan(sqrt(3) + 2)) + 4*(18^(2/3)*12^(1/6)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(sqrt(3) + 2
)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)
)^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*arct
an(sqrt(3) + 2))^4 + 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3
) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt
(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2
/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arctan
(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18
^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(
3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)
*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*a
rctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))
^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 18
^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 2*18^(2/3)*12^(2/3
)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3*ar
ctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(sqr
t(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + 4*(18^(2/3)*12^(1/6)*sq
rt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(s
qrt(3) + 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sq
rt(3) + 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*si
n(2/3*arctan(sqrt(3) + 2))^4 - 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arc
tan(sqrt(3) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*a
rctan(sqrt(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 + 12*(18^(2/3)*12^(2/
3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(
2/3*arctan(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) +
2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*12^(
```

```
1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) +
6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)
*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqr
t(3) + 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) +
2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 2*18^(2/
3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*
cos(2/3*arctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*
arctan(sqrt(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + (18^(2/3)*12^
(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*x*cos(2/3*
arctan(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4
+ 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*12^(1/3)*s
qrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(
1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) +
 2))^2 + 36*x^2) - (18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)
) + 18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)
*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3)
+ 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2
/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2)
)^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(
1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2) - 108)/x
```

Sympy [A] time = 0.204916, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-27t^2 + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x))) - 1
/x
```

Giac [B] time = 1.16506, size = 1115, normalized size = 2.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sqr
t(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*p
i)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)*si
n(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*p
i) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^5 - 10*
sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^4 -
 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(2/9*pi)
^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 - 4*cos(2/9*pi)*sin(
2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2
/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi
)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*sin(1/9*pi) - 1
0*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 - 2*sqrt(3)*cos(1/9*pi)^2 + 2
*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)
```

```
*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(5*sqrt(3)*cos(4/
9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(
4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*
sin(4/9*pi)^4 + 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + 2*cos(4/9*pi)^2 - 2*sin
(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) + 1/18*
(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)
^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2
 + 5*cos(2/9*pi)*sin(2/9*pi)^4 + 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) + 2*cos(
2/9*pi)^2 - 2*sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x +
 x^2 + 1) + 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*
pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi
)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 - 4*sqrt(3)*cos(1/9*pi)*sin
(1/9*pi) + 2*cos(1/9*pi)^2 - 2*sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) +
cos(1/9*pi))*x + x^2 + 1) - 1/x
```

3.179 $\int \frac{1}{x^3(1-x^3+x^6)}\,dx$

Optimal. Leaf size=418

$$-\frac{1}{2x^2}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -1/(2*x^2) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(1 + (2*x
)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) -
((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*
Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/
(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2
/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3
])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rubi [A] time = 0.342475, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1368, 1422, 200, 31, 634, 617, 204, 628}

$$-\frac{1}{2x^2}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^3 + x^6)),x]
```


```
[Out] -1/(2*x^2) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(1 + (2*x
)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) -
((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*
Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/
(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2
/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3
])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```


Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(1-x^3+x^6\right)}\,dx = -\frac{1}{2x^2}+\frac{1}{2}\int \frac{2-2x^3}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{2x^2}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\int}{9\sqrt[3]{2}\left(\right.}$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\int}{18}$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\right.}{}$$

$$= -\frac{1}{2x^2}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{}}\right.}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2}}$$

Mathematica [C] time = 0.0132168, size = 65, normalized size = 0.16

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(2*x^2) - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/
(-#1^2 + 2*#1^5) & ]/3
```

Maple [C] time = 0.007, size = 50, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(-\_R^3+1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))-1/2/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2x^2}-\int \frac{x^3-1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 1.94167, size = 3947, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) + 2))*log(-18^(2/3)*1
2^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*cos(
2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))
^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 8*18^(2
/3)*12^(1/6)*x^2*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arcta
n(sqrt(3) + 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*
sin(2/3*arctan(sqrt(3) + 2))^2 - 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arcta
n(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sqr
t(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(
sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(
1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*s
qrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*s
in(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*a
rctan(sqrt(3) + 2))^2))*sin(2/3*arctan(sqrt(3) + 2)) + 4*(18^(2/3)*12^(1/6)
*sqrt(3)*x^2*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*x^2*sin(2/3*a
rctan(sqrt(3) + 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*ar
ctan(sqrt(3) + 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 108*sqrt(
3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*ar
ctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*
sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arct
an(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*1
8^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6
)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2
)*sin(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/
3*arctan(sqrt(3) + 2))^2)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arcta
n(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*x^2*sin(2/3*arctan(sqrt(3) + 2)))*arcta
n(1/216*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)
*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3
) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) -
6*18^(1/3)*12^(5/6)*sqrt(3)*x - 216*sin(2/3*arctan(sqrt(3) + 2)))/cos(2/3*a
rctan(sqrt(3) + 2))) + (18^(2/3)*12^(1/6)*sqrt(3)*x^2*sin(2/3*arctan(sqrt(3
) + 2)) - 18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) + 2)))*log(2*18^(2/3
)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos
(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) +
2))^2 + 18*x^2) - (18^(2/3)*12^(1/6)*sqrt(3)*x^2*sin(2/3*arctan(sqrt(3) + 2
)) + 18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/3)*12^(
1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3
*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2
+ 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 54)/x^2
```

Sympy [A] time = 0.207414, size = 31, normalized size = 0.07

$$\mathrm{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 + 9t + x\right)\right)\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + 9*_t + x
))) - 1/(2*x**2)
```

Giac [B] time = 1.19414, size = 867, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(3
)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^3
 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*
pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*
sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)
^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*pi) + 2*si
n(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin
(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*p
i)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)*
sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*arctan(((sqrt(3)*i +
 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(4*sqrt(3)*cos
(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^
4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) -
 2*cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) + 1
/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi
)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqr
t(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi)
)*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(
1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin
(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(1/9*pi))*log((sqrt(3)*i*cos(1/9*
pi) + cos(1/9*pi))*x + x^2 + 1) - 1/2/x^2
```

3.180 $\int \frac{1}{x^4(1-x^3+x^6)} \, dx$

Optimal. Leaf size=48

$$-\frac{1}{3x^3} - \frac{1}{6}\log\left(x^6 - x^3 + 1\right) + \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x)$$

```
[Out] -1/(3*x^3) + ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3
 + x^6]/6
```

Rubi [A] time = 0.0520241, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{3x^3} - \frac{1}{6}\log\left(x^6 - x^3 + 1\right) + \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(3*x^3) + ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3
 + x^6]/6
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\log(x)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\log(x)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= -\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] time = 0.0136032, size = 51, normalized size = 1.06

$$-\frac{1}{3}\operatorname{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{3x^3}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(3*x^3) + Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 +
2*#1^3) & ]/3
```

Maple [A] time = 0.008, size = 40, normalized size = 0.8

$$-\frac{1}{3\,x^3}+\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}}{9}\arctan\left(\frac{\left(2\,x^3-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6-x^3+1),x)
```

```
[Out] -1/3/x^3+ln(x)-1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 1.56136, size = 58, normalized size = 1.21

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3 - 1/6*log(x^6 - x^3
+ 1) + 1/3*log(x^3)
```

Fricas [A] time = 1.44726, size = 143, normalized size = 2.98

$$-\frac{2\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+3x^3\log\left(x^6-x^3+1\right)-18x^3\log\left(x\right)+6}{18x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3*x^3*log(x^6 - x^3
+ 1) - 18*x^3*log(x) + 6)/x^3
```

Sympy [A] time = 0.174397, size = 48, normalized size = 1.

$$\log\left(x\right)-\frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9 - 1/(3*x**3)
```

Giac [A] time = 1.12789, size = 61, normalized size = 1.27

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{x^3+1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3*(x^3 + 1)/x^3 - 1/6*log(
x^6 - x^3 + 1) + log(abs(x))
```

3.181 $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

Optimal. Leaf size=423

$$-\frac{1}{4x^4}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2}\right.}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -1/(4*x^4) - x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^
(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])*ArcTan[
(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^
(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3
)*(1 - I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(
1/3)*x])/(9*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sq
rt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 -
 I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I
*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rubi [A] time = 0.369171, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1368, 1504, 12, 1374, 292, 31, 634, 617, 204, 628}

$$-\frac{1}{4x^4}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2}\right.}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(4*x^4) - x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^
(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])*ArcTan[
(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^
(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3
)*(1 - I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(
1/3)*x])/(9*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sq
rt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 -
 I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I
*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(1-x^3+x^6\right)}\,dx = -\frac{1}{4x^4}+\frac{1}{4}\int \frac{4-4x^3}{x^2\left(1-x^3+x^6\right)}\,dx$$

$$= -\frac{1}{4x^4}-\frac{1}{x}-\frac{1}{4}\int \frac{4x^4}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{4x^4}-\frac{1}{x}-\int \frac{x^4}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{4x^4}-\frac{1}{x}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{1}{4x^4}-\frac{1}{x}+-\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\cdots$$

$$= -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+-\frac{\left(-3-\cdots\right.}{\cdots}$$

$$= -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\cdots\right.}{\cdots}$$

$$= -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\cdots\right)}{9\ 2^{2/3}\cdots}$$

Mathematica [C] time = 0.0148857, size = 54, normalized size = 0.13

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{4x^4}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(4*x^4) - x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 
2*#1^3) & ]/3
```

Maple [C] time = 0.007, size = 51, normalized size = 0.1

$$-\frac{1}{4\,x^4}-x^{-1}-\frac{1}{3}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^6-x^3+1),x)
```

```
[Out] -1/4/x^4-1/x-1/3*sum(_R^4/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\,x^3+1}{4\,x^4}-\int\frac{x^4}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/4*(4*x^3 + 1)/x^4 - integrate(x^4/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 2.29423, size = 5994, normalized size = 14.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x^4*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12
^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sq
rt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))
*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3
) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3
)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) + 8*18^(2/3)*12^(1/6
)*x^4*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*ar
ctan(sqrt(3) - 2))^4 + 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt
(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sq
rt(3) - 2))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos
(2/3*arctan(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arct
an(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 +
18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqr
t(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/
3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3
*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2
))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 -
18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2
/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*
arctan(sqrt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(s
qrt(3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3)
- 2)) - 108*x^3 - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^4*cos(2/3*arctan(sqrt(3) -
 2)) - 18^(2/3)*12^(1/6)*x^4*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*
18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*co
s(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 -
 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2
/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3
*arctan(sqrt(3) - 2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) -
2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqr
t(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(
2/3*arctan(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(
sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*
arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^
```

```
2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/
3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt
(3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqr
t(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4
 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin
(2/3*arctan(sqrt(3) - 2))^4)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^4*cos(2/3*ar
ctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*x^4*sin(2/3*arctan(sqrt(3) - 2)))*ar
ctan(-1/432*(6*18^(2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) - 2))^2 + 2
16*sin(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3
)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3)
 - 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2
/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/
3*arctan(sqrt(3) - 2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*a
rctan(sqrt(3) - 2)))) - (18^(2/3)*12^(1/6)*sqrt(3)*x^4*sin(2/3*arctan(sqrt(
3) - 2)) + 18^(2/3)*12^(1/6)*x^4*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)
*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan
(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqr
t(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(
1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) + (18^(2/3)*12^(1
/6)*sqrt(3)*x^4*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*x^4*cos(2/
3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^
4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)
*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqr
t(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x
^2) - 27)/x^4
```

Sympy [A] time = 0.215088, size = 37, normalized size = 0.09

$$\operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(-6561t^5+54t^2+x\right)\right)\right)-\frac{4x^3+1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**5 + 54*_t*
*2 + x))) - (4*x**3 + 1)/(4*x**4)
```

Giac [B] time = 1.15665, size = 1129, normalized size = 2.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(4
/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)*
sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9
*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2/9*pi)^5 -
 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*sin(2/9*pi
)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)^3 - 2*sin
(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*cos(2/9*pi)*
```

```
sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*s
in(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3)*cos(1/9*pi)^3*sin(
1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*cos(1/9*pi)^4*sin(1/9
*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5 - sqrt(3)*cos(1/9*p
i)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*
i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(10*sqrt(3)
*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqr
t(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi)
^2 - sin(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1)
 + 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*si
n(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*cos(2/9*pi)^5 - 20*cos(2/9*pi)^3*
sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^4 + 2*sqrt(3)*cos(2/9*pi)*sin(2/
9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/
9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 20*sqrt(
3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*sin(1/9*pi)^5 - 2*cos(1/9*pi)^5
+ 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3)
*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 - sin(1/9*pi)^2)*log((sqrt(3)*i*co
s(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/4*(4*x^3 + 1)/x^4
```

3.182 $\int \frac{1}{2+x^3+x^6}\,dx$

Optimal. Leaf size=381

$$\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\cdots}\right.}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2\cdots}}$$

```
[Out] (I*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/(Sqrt[21]*((1 - I
*Sqrt[7])/2)^(2/3)) - (I*ArcTan[(1 - (2*x)/((1 + I*Sqrt[7])/2)^(1/3))/Sqrt[
3]])/(Sqrt[21]*((1 + I*Sqrt[7])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[7])^(1/3
) + 2^(1/3)*x])/(Sqrt[7]*((1 - I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[(1 + I*Sqr
t[7])^(1/3) + 2^(1/3)*x])/(Sqrt[7]*((1 + I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[
(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3
)*Sqrt[7]*(1 - I*Sqrt[7])^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[7])^(2/3) - (2*(1
 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[7]*(1 + I*Sqrt[7])^(2/
3))
```

Rubi [A] time = 0.402723, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {1347, 200, 31, 634, 617, 204, 628}

$$\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\cdots}\right.}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^3 + x^6)^(-1),x]
```

```
[Out] (I*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/(Sqrt[21]*((1 - I
*Sqrt[7])/2)^(2/3)) - (I*ArcTan[(1 - (2*x)/((1 + I*Sqrt[7])/2)^(1/3))/Sqrt[
3]])/(Sqrt[21]*((1 + I*Sqrt[7])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[7])^(1/3
) + 2^(1/3)*x])/(Sqrt[7]*((1 - I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[(1 + I*Sqr
t[7])^(1/3) + 2^(1/3)*x])/(Sqrt[7]*((1 + I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[
(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3
)*Sqrt[7]*(1 - I*Sqrt[7])^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[7])^(2/3) - (2*(1
 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[7]*(1 + I*Sqrt[7])^(2/
3))
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{2+x^3+x^6}\,dx = -\frac{i\int \frac{1}{\frac{1}{2}-\frac{i\sqrt{7}}{2}+x^3}\,dx}{\sqrt{7}} + \frac{i\int \frac{1}{\frac{1}{2}+\frac{i\sqrt{7}}{2}+x^3}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \frac{1}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{7})}+x}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{\left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3}-\sqrt[3]{\frac{1}{2}(1-i\sqrt{7})}x+x^2}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\int \frac{1}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{7})}+x}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\int \frac{2}{\left(\frac{1}{2}(1+i\sqrt{7}\right.}}{3\sqrt{7}\left(\right.}$$

$$= -\frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{7})}+2x}{\left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3}-\sqrt[3]{\frac{1}{2}(1-i\sqrt{7})}x+x^2}\,dx}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}} - \frac{i\int -}{}$$

$$= -\frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+2^{2}\right.}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}}$$

$$= \frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{7})}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{7})}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}\right.}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2}}$$

Mathematica [C] time = 0.0091525, size = 38, normalized size = 0.1

$$\frac{1}{3}\text{RootSum}\left[\#1^6+\#1^3+2\&,\frac{\log(x-\#1)}{2\#1^5+\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^3 + x^6)^(-1),x]
```

```
[Out] RootSum[2 + #1^3 + #1^6 & , Log[x - #1]/(#1^2 + 2*#1^5) & ]/3
```

Maple [C] time = 0.003, size = 33, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6+\_Z^3+2\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5+\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+x^3+2),x)
```

```
[Out] 1/3*sum(1/(2*_R^5+_R^2)*ln(x-_R),_R=RootOf(_Z^6+_Z^3+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6+x^3+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 + x^3 + 2), x)
```

Fricas [B] time = 5.27856, size = 7470, normalized size = 19.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="fricas")
```

```
[Out] 1/294*112^(1/6)*49^(2/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))*log(112^(1/6)*4
9^(2/3)*sqrt(7)*x*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 7*112^(1/6)*49^(2/3)
*x*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + 14*49^(1/3)*14^(1/3)*cos(2/3*arctan
(1/3*sqrt(7) + 4/3))^2 + 14*49^(1/3)*14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) +
4/3))^2 + 98*x^2) - 2/147*112^(1/6)*49^(2/3)*arctan(1/2744*(14*112^(5/6)*49
^(1/3)*sqrt(7)*x*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + 2744*sqrt(7)*cos(2/3*
arctan(1/3*sqrt(7) + 4/3))^2 + 2744*sqrt(7)*sin(2/3*arctan(1/3*sqrt(7) + 4/
3))^2 + 98*(112^(5/6)*49^(1/3)*x + 224*cos(2/3*arctan(1/3*sqrt(7) + 4/3)))*
sin(2/3*arctan(1/3*sqrt(7) + 4/3)) - sqrt(112^(1/6)*49^(2/3)*sqrt(7)*x*sin(
2/3*arctan(1/3*sqrt(7) + 4/3)) + 7*112^(1/6)*49^(2/3)*x*cos(2/3*arctan(1/3*
sqrt(7) + 4/3)) + 14*49^(1/3)*14^(1/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2
 + 14*49^(1/3)*14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 98*x^2)*(112
^(5/6)*49^(1/3)*sqrt(7)*sqrt(2)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + 7*112^
(5/6)*49^(1/3)*sqrt(2)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))))/(cos(2/3*arctan
(1/3*sqrt(7) + 4/3))^2 - 7*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2))*sin(2/3*a
rctan(1/3*sqrt(7) + 4/3)) + 1/147*(112^(1/6)*49^(2/3)*sqrt(3)*cos(2/3*arcta
n(1/3*sqrt(7) + 4/3)) + 112^(1/6)*49^(2/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3
)))*arctan(1/5488*(70*112^(5/6)*49^(1/3)*(sqrt(7)*x + 7*sqrt(3)*x)*cos(2/3*
arctan(1/3*sqrt(7) + 4/3))^3 - 27440*(sqrt(7) + 2*sqrt(3))*cos(2/3*arctan(1
/3*sqrt(7) + 4/3))^4 - 5488*(sqrt(7) - 2*sqrt(3))*sin(2/3*arctan(1/3*sqrt(7
) + 4/3))^4 - 14*(112^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3)*x - 7*x) - 1568*(sqrt
(7)*sqrt(3) - 5)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)))*sin(2/3*arctan(1/3*sqr
t(7) + 4/3))^3 + 14*(112^(5/6)*49^(1/3)*(13*sqrt(7)*x - 21*sqrt(3)*x)*cos(2
/3*arctan(1/3*sqrt(7) + 4/3)) - 784*(3*sqrt(7) + 4*sqrt(3))*cos(2/3*arctan(
1/3*sqrt(7) + 4/3))^2)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 - 14*(112^(5/6)
*49^(1/3)*(9*sqrt(7)*sqrt(3)*x + 49*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2
 - 1568*(sqrt(7)*sqrt(3) + 11)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^3)*sin(2/
3*arctan(1/3*sqrt(7) + 4/3)) - (5*112^(5/6)*49^(1/3)*(sqrt(7) + 7*sqrt(3))*
cos(2/3*arctan(1/3*sqrt(7) + 4/3))^3 - 112^(5/6)*49^(1/3)*(9*sqrt(7)*sqrt(3
) + 49)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2*sin(2/3*arctan(1/3*sqrt(7) + 4
/3)) + 112^(5/6)*49^(1/3)*(13*sqrt(7) - 21*sqrt(3))*cos(2/3*arctan(1/3*sqrt
(7) + 4/3))*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 - 112^(5/6)*49^(1/3)*(sqrt
(7)*sqrt(3) - 7)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^3)*sqrt(-112^(1/6)*49^(
2/3)*(sqrt(7)*sqrt(3)*x + 7*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^(1/
6)*49^(2/3)*(sqrt(7)*x - 7*sqrt(3)*x)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) +
28*49^(1/3)*14^(1/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 28*49^(1/3)*14^
(1/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 196*x^2))/(25*cos(2/3*arctan(1
/3*sqrt(7) + 4/3))^4 - 38*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2*sin(2/3*arct
an(1/3*sqrt(7) + 4/3))^2 + sin(2/3*arctan(1/3*sqrt(7) + 4/3))^4)) + 1/147*(
112^(1/6)*49^(2/3)*sqrt(3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^(1/6)*4
9^(2/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)))*arctan(-1/5488*(70*112^(5/6)*49
^(1/3)*(sqrt(7)*x - 7*sqrt(3)*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^3 - 274
40*(sqrt(7) - 2*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^4 - 5488*(sqrt(
```

```
7) + 2*sqrt(3))*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^4 + 14*(112^(5/6)*49^(1/
3)*(sqrt(7)*sqrt(3)*x + 7*x) - 1568*(sqrt(7)*sqrt(3) + 5)*cos(2/3*arctan(1/
3*sqrt(7) + 4/3)))*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^3 + 14*(112^(5/6)*49^
(1/3)*(13*sqrt(7)*x + 21*sqrt(3)*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 78
4*(3*sqrt(7) - 4*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2)*sin(2/3*arc
tan(1/3*sqrt(7) + 4/3))^2 + 14*(112^(5/6)*49^(1/3)*(9*sqrt(7)*sqrt(3)*x - 4
9*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 - 1568*(sqrt(7)*sqrt(3) - 11)*cos
(2/3*arctan(1/3*sqrt(7) + 4/3))^3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) - (5*
112^(5/6)*49^(1/3)*(sqrt(7) - 7*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))
^3 + 112^(5/6)*49^(1/3)*(9*sqrt(7)*sqrt(3) - 49)*cos(2/3*arctan(1/3*sqrt(7)
 + 4/3))^2*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 112^(5/6)*49^(1/3)*(13*sqrt
(7) + 21*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))*sin(2/3*arctan(1/3*sqr
t(7) + 4/3))^2 + 112^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3) + 7)*sin(2/3*arctan(1/
3*sqrt(7) + 4/3))^3)*sqrt(112^(1/6)*49^(2/3)*(sqrt(7)*sqrt(3)*x - 7*x)*cos(
2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^(1/6)*49^(2/3)*(sqrt(7)*x + 7*sqrt(3)*
x)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 28*49^(1/3)*14^(1/3)*cos(2/3*arctan
(1/3*sqrt(7) + 4/3))^2 + 28*49^(1/3)*14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) +
4/3))^2 + 196*x^2))/(25*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^4 - 38*cos(2/3*a
rctan(1/3*sqrt(7) + 4/3))^2*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + sin(2/3*
arctan(1/3*sqrt(7) + 4/3))^4)) + 1/588*(112^(1/6)*49^(2/3)*sqrt(3)*sin(2/3*
arctan(1/3*sqrt(7) + 4/3)) - 112^(1/6)*49^(2/3)*cos(2/3*arctan(1/3*sqrt(7)
+ 4/3)))*log(-112^(1/6)*49^(2/3)*(sqrt(7)*sqrt(3)*x + 7*x)*cos(2/3*arctan(1
/3*sqrt(7) + 4/3)) - 112^(1/6)*49^(2/3)*(sqrt(7)*x - 7*sqrt(3)*x)*sin(2/3*a
rctan(1/3*sqrt(7) + 4/3)) + 28*49^(1/3)*14^(1/3)*cos(2/3*arctan(1/3*sqrt(7)
 + 4/3))^2 + 28*49^(1/3)*14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 19
6*x^2) - 1/588*(112^(1/6)*49^(2/3)*sqrt(3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3
)) + 112^(1/6)*49^(2/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)))*log(112^(1/6)*4
9^(2/3)*(sqrt(7)*sqrt(3)*x - 7*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^
(1/6)*49^(2/3)*(sqrt(7)*x + 7*sqrt(3)*x)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))
 + 28*49^(1/3)*14^(1/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 28*49^(1/3)*
14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 196*x^2)
```

Sympy [A] time = 0.153661, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(1000188t^6+1323t^3+1,\left(t\mapsto t\log\left(-5292t^4+7t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+x**3+2),x)
```

```
[Out] RootSum(1000188*_t**6 + 1323*_t**3 + 1, Lambda(_t, _t*log(-5292*_t**4 + 7*_
t + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6+x^3+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^6 + x^3 + 2), x)
```

3.183 $\int \frac{x^2}{2+x^3+x^6} \, dx$

Optimal. Leaf size=23

$$\frac{2\tan^{-1}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

```
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Rubi [A] time = 0.0241188, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1352, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + x^3 + x^6),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{2+x^3+x^6} \, dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{2+x+x^2} \, dx, x, x^3\right) \\
&= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-7-x^2} \, dx, x, 1+2x^3\right)\right) \\
&= \frac{2\tan^{-1}\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}
\end{aligned}$$

Mathematica [A] time = 0.0076714, size = 23, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + x^3 + x^6),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Maple [A] time = 0.003, size = 19, normalized size = 0.8

$$\frac{2\sqrt{7}}{21}\arctan\left(\frac{(2x^3+1)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+x^3+2),x)
```

```
[Out] 2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

Maxima [A] time = 1.55674, size = 24, normalized size = 1.04

$$\frac{2}{21}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^3+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="maxima")
```

```
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Fricas [A] time = 1.45253, size = 62, normalized size = 2.7

$$\frac{2}{21}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^3+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="fricas")
```

```
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Sympy [A] time = 0.11846, size = 27, normalized size = 1.17

$$\frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+x**3+2),x)
```

```
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**3/7 + sqrt(7)/7)/21
```

Giac [A] time = 1.20174, size = 24, normalized size = 1.04

$$\frac{2}{21}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^{3}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="giac")
```

```
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

3.184 $\int \frac{x^3}{2+x^3+x^6}\,dx$

Optimal. Leaf size=399

$$-\frac{(7+i\sqrt{7})\log\left(2^{2/3}x^2-\sqrt[3]{2(1-i\sqrt{7})}x+(1-i\sqrt{7})^{2/3}\right)}{42\sqrt[3]{2}(1-i\sqrt{7})^{2/3}}-\frac{(7-i\sqrt{7})\log\left(2^{2/3}x^2-\sqrt[3]{2(1+i\sqrt{7})}x+(1+i\sqrt{7})^{2/3}\right)}{42\sqrt[3]{2}(1+i\sqrt{7})^{2/3}}+\cdots$$

```
[Out] ((-I)*((1 - I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3)
)/Sqrt[3]])/Sqrt[21] + (I*((1 + I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 +
 I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/Sqrt[21] + ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt
[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) + ((7 - I*Sqrt[
7])*Log[(1 + I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 + I*Sqrt[7])^(2/
3)) - ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3
)*x + 2^(2/3)*x^2])/(42*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) - ((7 - I*Sqrt[7])*L
og[(1 + I*Sqrt[7])^(2/3) - (2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(42*
2^(1/3)*(1 + I*Sqrt[7])^(2/3))
```

Rubi [A] time = 0.305094, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1374, 200, 31, 634, 617, 204, 628}

$$-\frac{(7+i\sqrt{7})\log\left(2^{2/3}x^2-\sqrt[3]{2(1-i\sqrt{7})}x+(1-i\sqrt{7})^{2/3}\right)}{42\sqrt[3]{2}(1-i\sqrt{7})^{2/3}}-\frac{(7-i\sqrt{7})\log\left(2^{2/3}x^2-\sqrt[3]{2(1+i\sqrt{7})}x+(1+i\sqrt{7})^{2/3}\right)}{42\sqrt[3]{2}(1+i\sqrt{7})^{2/3}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 + x^3 + x^6),x]
```

```
[Out] ((-I)*((1 - I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3)
)/Sqrt[3]])/Sqrt[21] + (I*((1 + I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 +
 I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/Sqrt[21] + ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt
[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) + ((7 - I*Sqrt[
7])*Log[(1 + I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 + I*Sqrt[7])^(2/
3)) - ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3
)*x + 2^(2/3)*x^2])/(42*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) - ((7 - I*Sqrt[7])*L
og[(1 + I*Sqrt[7])^(2/3) - (2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(42*
2^(1/3)*(1 + I*Sqrt[7])^(2/3))
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \frac{1}{14}\left(7-i\sqrt{7}\right)\int \frac{1}{\frac{1}{2}+\frac{i\sqrt{7}}{2}+x^3}\,dx + \frac{1}{14}\left(7+i\sqrt{7}\right)\int \frac{1}{\frac{1}{2}-\frac{i\sqrt{7}}{2}+x^3}\,dx$$

$$= \frac{\left(7-i\sqrt{7}\right)\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}+x}\,dx}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} + \frac{\left(7-i\sqrt{7}\right)\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+x^2}\,dx}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} + \frac{\left(7+i\sqrt{7}\right)\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right.}}}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2}}$$

$$= \frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} + \frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} - \frac{\left(7-i\sqrt{7}\right)\int \frac{-\sqrt[3]{\frac{1}{2}(}}{\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}}{42\sqrt[3]{2}\left(1+i\right.}$$

$$= \frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} + \frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} - \frac{\left(7+i\sqrt{7}\right)\log\left(\left(1-i\sqrt{7}\right.\right.}{42\sqrt[3]{2}}$$

$$= -\frac{i\sqrt[3]{\frac{1}{2}}\left(1-i\sqrt{7}\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{i\sqrt[3]{\frac{1}{2}}\left(1+i\sqrt{7}\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1\cdots}\right.}{21\sqrt[3]{2}\left(1-i\right.}$$

Mathematica [C] time = 0.0083866, size = 37, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^6+\#1^3+2\&,\frac{\#1\log(x-\#1)}{2\#1^3+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 + x^3 + x^6),x]
```

```
[Out] RootSum[2 + #1^3 + #1^6 & , (Log[x - #1]*#1)/(1 + 2*#1^3) & ]/3
```

Maple [C] time = 0.003, size = 36, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6+\_Z^3+2\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5+\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+x^3+2),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5+_R^2)*ln(x-_R),_R=RootOf(_Z^6+_Z^3+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{x^6+x^3+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="maxima")

[Out] integrate(x^3/(x^6 + x^3 + 2), x)
```

Fricas [B] time = 2.02283, size = 5416, normalized size = 13.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="fricas")

[Out] 1/294*98^(2/3)*56^(1/6)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))*log
(-2*98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(
7))) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^
2 + 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 +
 98*x^2) - 2/147*98^(2/3)*56^(1/6)*arctan(1/5488*(98^(1/3)*56^(5/6)*sqrt(7)
*sqrt(2)*sqrt(-2*98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sq
rt(7) + sqrt(7))) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7)
 + sqrt(7)))^2 + 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) +
sqrt(7)))^2 + 98*x^2) - 14*98^(1/3)*56^(5/6)*sqrt(7)*x + 5488*sin(2/3*arcta
n(2/7*sqrt(14)*sqrt(7) + sqrt(7))))/cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + s
qrt(7))))*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + 1/147*(98^(2/3)
*56^(1/6)*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + 98^(2/3
)*56^(1/6)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*arctan(1/2744*(
14*98^(1/3)*56^(5/6)*sqrt(7)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7
))) + 2744*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 2744
*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 14*(98^(1/3)*5
6^(5/6)*sqrt(7)*sqrt(3)*x + 784*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(
7))))*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) - sqrt(98^(2/3)*56^(1
/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + 98^
(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) +
14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 14*
98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 98*x^2
)*(98^(1/3)*56^(5/6)*sqrt(7)*sqrt(3)*sqrt(2)*sin(2/3*arctan(2/7*sqrt(14)*sq
rt(7) + sqrt(7))) + 98^(1/3)*56^(5/6)*sqrt(7)*sqrt(2)*cos(2/3*arctan(2/7*sq
rt(14)*sqrt(7) + sqrt(7)))))/(cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)
))^2 - 3*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2)) + 1/147*(98^(2
/3)*56^(1/6)*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) - 98^(
2/3)*56^(1/6)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*arctan(-1/27
44*(14*98^(1/3)*56^(5/6)*sqrt(7)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sq
rt(7))) - 2744*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 -
2744*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 - 14*(98^(1/
3)*56^(5/6)*sqrt(7)*sqrt(3)*x - 784*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + s
qrt(7))))*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + sqrt(-98^(2/3)*
56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))
+ 98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)
)) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2
+ 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 9
8*x^2)*(98^(1/3)*56^(5/6)*sqrt(7)*sqrt(3)*sqrt(2)*sin(2/3*arctan(2/7*sqrt(1
4)*sqrt(7) + sqrt(7))) - 98^(1/3)*56^(5/6)*sqrt(7)*sqrt(2)*cos(2/3*arctan(2
/7*sqrt(14)*sqrt(7) + sqrt(7)))))/(cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sq
rt(7)))^2 - 3*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2)) + 1/588*(
98^(2/3)*56^(1/6)*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) -
 98^(2/3)*56^(1/6)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*log(98^
(2/3)*56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt
(7))) + 98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + s
```

```
qrt(7))) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7
)))^2 + 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))
^2 + 98*x^2) - 1/588*(98^(2/3)*56^(1/6)*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)
*sqrt(7) + sqrt(7))) + 98^(2/3)*56^(1/6)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7
) + sqrt(7))))*log(-98^(2/3)*56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*
sqrt(14)*sqrt(7) + sqrt(7))) + 98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2
/7*sqrt(14)*sqrt(7) + sqrt(7))) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sq
rt(14)*sqrt(7) + sqrt(7)))^2 + 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(
14)*sqrt(7) + sqrt(7)))^2 + 98*x^2)
```

Sympy [A] time = 0.145134, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(250047t^6 + 1323t^3 + 2, \left(t \mapsto t\log\left(7938t^4 + 21t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+x**3+2),x)
```

```
[Out] RootSum(250047*_t**6 + 1323*_t**3 + 2, Lambda(_t, _t*log(7938*_t**4 + 21*_t
 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{x^6 + x^3 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + x^3 + 2), x)
```

3.185 $\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=231

$$\frac{\left(16a^2c^2-56ab^2c+21b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5}-\frac{\left(b^2-4ac\right)\left(16a^2c^2-56ab^2c+21b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}}-\frac{(7b}{}$$

```
[Out] ((21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/
(1536*c^5) - (b*x^6*(a + b*x^3 + c*x^6)^(3/2))/(20*c^2) + (x^9*(a + b*x^3 +
 c*x^6)^(3/2))/(18*c) - ((7*b*(15*b^2 - 28*a*c) - 6*c*(21*b^2 - 20*a*c)*x^3
)*(a + b*x^3 + c*x^6)^(3/2))/(2880*c^4) - ((b^2 - 4*a*c)*(21*b^4 - 56*a*b^2
*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]
)/(3072*c^(11/2))
```

Rubi [A] time = 0.299663, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1357, 742, 832, 779, 612, 621, 206}

$$\frac{\left(16a^2c^2-56ab^2c+21b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5}-\frac{\left(b^2-4ac\right)\left(16a^2c^2-56ab^2c+21b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}}-\frac{(7b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```


```
[Out] ((21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/
(1536*c^5) - (b*x^6*(a + b*x^3 + c*x^6)^(3/2))/(20*c^2) + (x^9*(a + b*x^3 +
 c*x^6)^(3/2))/(18*c) - ((7*b*(15*b^2 - 28*a*c) - 6*c*(21*b^2 - 20*a*c)*x^3
)*(a + b*x^3 + c*x^6)^(3/2))/(2880*c^4) - ((b^2 - 4*a*c)*(21*b^4 - 56*a*b^2
*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]
)/(3072*c^(11/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
```

```
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int x^4\sqrt{a+bx+cx^2}\,dx,x,x^3\right)$$

$$= \frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c} + \frac{\operatorname{Subst}\left(\int x^2\left(-3a-\frac{9bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{18c}$$

$$= -\frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c} + \frac{\operatorname{Subst}\left(\int x\left(9ab+\frac{3}{4}\left(21b^2-20ac\right)x\right)\sqrt{\right.}}{90c^2}$$

$$= -\frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c} - \frac{\left(7b\left(15b^2-28ac\right)-6c\left(21b^2-20ac\right)x^3\right.}{2880c^4}$$

$$= \frac{\left(21b^4-56ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a+bx\right.}{1}$$

$$= \frac{\left(21b^4-56ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a+bx\right.}{1}$$

$$= \frac{\left(21b^4-56ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a+bx\right.}{1}$$

Mathematica [A] time = 0.165049, size = 208, normalized size = 0.9

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(16bc^2\left(113a^2-34acx^6+8c^2x^{12}\right)+160c^3x^3\left(-3a^2+2acx^6+8c^2x^{12}\right)+16b^2c^2x^3\left(56a-9cx^6\right)+168b\right.}{46080c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(315*b^5 - 210*b^4*c*x^3 + 16*b^2*c^2*x^
3*(56*a - 9*c*x^6) + 168*b^3*c*(-10*a + c*x^6) + 16*b*c^2*(113*a^2 - 34*a*c
*x^6 + 8*c^2*x^12) + 160*c^3*x^3*(-3*a^2 + 2*a*c*x^6 + 8*c^2*x^12)) - 15*(2
1*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*ArcTanh[(b + 2*c*x^3)/(
2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(46080*c^(11/2))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x^{14}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75189, size = 1071, normalized size = 4.64

$$\left[-\frac{15\left(21\,b^6-140\,ab^4c+240\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)-4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)
*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)
*sqrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*(9*b^2*c^4 - 20*
a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c - 1680*a*b^3*c^2 +
```

```
 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c
*x^6 + b*x^3 + a))/c^6, 1/46080*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2
 - 64*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sq
rt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*
(9*b^2*c^4 - 20*a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c -
1680*a*b^3*c^2 + 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*
c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**14*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6+bx^3+a}x^{14}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^14, x)
```

3.186 $\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=171

$$\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh}{768c^{9/2}}$$

```
[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*c^4) + (x^
6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + ((35*b^2 - 32*a*c - 42*b*c*x^3)*(a +
b*x^3 + c*x^6)^(3/2))/(720*c^3) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(768*c^(9/2))
```

Rubi [A] time = 0.151818, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 742, 779, 612, 621, 206}

$$\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh}{768c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*c^4) + (x^
6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + ((35*b^2 - 32*a*c - 42*b*c*x^3)*(a +
b*x^3 + c*x^6)^(3/2))/(720*c^3) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(768*c^(9/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{3}\,\text{Subst}\left(\int x^3\sqrt{a+bx+cx^2}\,dx, x, x^3\right)$$
$$= \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\text{Subst}\left(\int x\left(-2a-\frac{7bx}{2}\right)\sqrt{a+bx+cx^2}\,dx, x, x^3\right)}{15c}$$
$$= \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\left(35b^2-32ac-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3} - \frac{\left(b\left(7b^2-12ac\right)\right)\text{Subs}}{}$$
$$= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4} + \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\left(35b^2-32ac-42bc\right.}{72}$$
$$= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4} + \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\left(35b^2-32ac-42bc\right.}{72}$$
$$= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4} + \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\left(35b^2-32ac-42bc\right.}{72}$$

Mathematica [A] time = 0.13708, size = 164, normalized size = 0.96

$$\frac{-\frac{\left(32ac-35b^2+42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{5\left(12abc-7b^3\right)\left(2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)}{256c^{7/2}}+x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^(3/2) - ((-35*b^2 + 32*a*c + 42*b*c*x^3)*(a + b*x^
3 + c*x^6)^(3/2))/(48*c^2) + (5*(-7*b^3 + 12*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^3
)*Sqrt[a + b*x^3 + c*x^6] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*
Sqrt[a + b*x^3 + c*x^6])]))/(256*c^(7/2)))/(15*c)
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int x^{11}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70713, size = 863, normalized size = 5.05

$$\left[\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(384c^5x^{12}+48\right.}{23040c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8
*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) +
 4*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a*c^4)*x^6 - 105*b^4*c
+ 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^3)*sqrt(c*x^
6 + b*x^3 + a))/c^5, -1/11520*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(
-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*
c*x^3 + a*c)) - 2*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a*c^4)*x
^6 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*a*b*c^3)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**11*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6+bx^3+a}\,x^{11}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^11, x)
```

3.187 $\int x^8\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=153

$$\frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}-\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2}+\frac{x^3\left(a+\right.}{}$$

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*c^3) - (5*b*(a
 + b*x^3 + c*x^6)^(3/2))/(72*c^2) + (x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c)
- ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6])])/(384*c^(7/2))
```

Rubi [A] time = 0.135841, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 742, 640, 612, 621, 206}

$$\frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}-\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2}+\frac{x^3\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```



```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*c^3) - (5*b*(a
 + b*x^3 + c*x^6)^(3/2))/(72*c^2) + (x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c)
- ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6])])/(384*c^(7/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^8\sqrt{a+bx^3+cx^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int x^2\sqrt{a+bx+cx^2}\,dx,x,x^3\right)\\
&= \frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c}+\frac{\text{Subst}\left(\int\left(-a-\frac{5bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{12c}\\
&= -\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c}+\frac{\left(5b^2-4ac\right)\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x\right.}{48c^2}\\
&= \frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c}-\frac{((}{}\\
&= \frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c}-\frac{((}{}\\
&= \frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c}-\frac{(b}{}
\end{aligned}$$

Mathematica [A] time = 0.0717056, size = 136, normalized size = 0.89

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(b\left(8c^2x^6-52ac\right)+24c^2x^3\left(a+2cx^6\right)-10b^2cx^3+15b^3\right)-3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.}{1152c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(15*b^3 - 10*b^2*c*x^3 + 24*c^2*x^3*(a +
 2*c*x^6) + b*(-52*a*c + 8*c^2*x^6)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*
ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(1152*c^(7/2))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int x^8\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64847, size = 699, normalized size = 4.57

$$\left[\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2+4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(48c^4x^9+8bc^3x^6\right.}{2304c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c
*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(
48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/1152*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*
c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^
2*x^6 + b*c*x^3 + a*c)) + 2*(48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52*a*b*c
^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**8*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^6+bx^3+a}\,x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^8, x)
```

3.188 $\int x^5\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}-\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}$$

```
[Out] -(b*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c^2) + (a + b*x^3 + c*x^6)^(
3/2)/(9*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x
^3 + c*x^6])])/(48*c^(5/2))
```

Rubi [A] time = 0.0853352, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 640, 612, 621, 206}

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}-\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -(b*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c^2) + (a + b*x^3 + c*x^6)^(
3/2)/(9*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x
^3 + c*x^6])])/(48*c^(5/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5\sqrt{a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int x\sqrt{a+bx+cx^2}\,dx,x,x^3\right)\\
&= \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}-\frac{b\,\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{6c}\\
&= -\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}+\frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx,\right.}{48c^2}\\
&= -\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}+\frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{}{\sqrt{}}\right.}{24c^2}\\
&= -\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}+\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0760385, size = 99, normalized size = 0.92

$$\frac{\sqrt{a+bx^3+cx^6}\left(8c\left(a+cx^6\right)-3b^2+2bcx^3\right)}{72c^2}+\frac{\left(b^3-4abc\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 2*b*c*x^3 + 8*c*(a + c*x^6)))/(72*c^2) +
 ((b^3 - 4*a*b*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])
])/(48*c^(5/2))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int x^5\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.64762, size = 554, normalized size = 5.13

$$\left[-\frac{3\left(b^3-4\,abc\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)-4\left(8\,c^3x^6+2\,bc^2x^3-3\,b^2c+8\,a\right.}{288\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqr
t(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 + 2*b*c^
2*x^3 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^3, -1/144*(3*(b^3 - 4
*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/
(c^2*x^6 + b*c*x^3 + a*c)) - 2*(8*c^3*x^6 + 2*b*c^2*x^3 - 3*b^2*c + 8*a*c^2
)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [A] time = 1.16761, size = 132, normalized size = 1.22

$$\frac{1}{72}\sqrt{cx^6+bx^3+a}\left(2\left(4\,x^3+\frac{b}{c}\right)x^3-\frac{3\,b^2-8\,ac}{c^2}\right)-\frac{\left(b^3-4\,abc\right)\log\left(\left|-2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{48\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*sqrt(c*x^6 + b*x^3 + a)*(2*(4*x^3 + b/c)*x^3 - (3*b^2 - 8*a*c)/c^2) -
1/48*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqr
t(c) - b))/c^(5/2)
```

3.189 $\int x^2\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=83

$$\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{12c}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

```
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) - ((b^2 - 4*a*c)*ArcTanh[(b
+ 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(3/2))
```

Rubi [A] time = 0.0601921, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1352, 612, 621, 206}

$$\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{12c}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```


```
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) - ((b^2 - 4*a*c)*ArcTanh[(b
+ 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(3/2))
```


Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^3\right)$$
$$= \frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c} - \frac{(b^2-4ac)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24c}$$
$$= \frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c} - \frac{(b^2-4ac)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12c}$$
$$= \frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c} - \frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

Mathematica [A] time = 0.010546, size = 87, normalized size = 1.05

$$\frac{1}{3}\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c} - \frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b
+ 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2)))/3
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int x^2\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54722, size = 463, normalized size = 5.58

$$\left[-\frac{(b^2-4\,ac)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^6+bx^3+a}\left(2\,c^2x^3+bc\right)}{48\,c^2}, \frac{(b^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x
^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*
(2*c^2*x^3 + b*c))/c^2, 1/24*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x
^6 + b*x^3 + a)*(2*c^2*x^3 + b*c))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [A] time = 1.15953, size = 103, normalized size = 1.24

$$\frac{1}{12}\sqrt{cx^6+bx^3+a}\left(2\,x^3+\frac{b}{c}\right)+\frac{\left(b^2-4\,ac\right)\log\left(\left|-2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{24\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*log(abs(-2*
(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(3/2)
```

3.190 $\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$

Optimal. Leaf size=109

$$\frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

```
[Out] Sqrt[a + b*x^3 + c*x^6]/3 - (Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])/3 + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^
3 + c*x^6])])/(6*Sqrt[c])
```

Rubi [A] time = 0.111379, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 734, 843, 621, 206, 724}

$$\frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/3 - (Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])/3 + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^
3 + c*x^6])])/(6*Sqrt[c])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{6}\,\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\sqrt{a+bx^3+cx^6} + \frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right) + \frac{1}{6}b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,d$$

$$= \frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right) + \frac{1}{3}b\,\text{Subst}\left(\int \frac{1}{4c-x^2}$$

$$= \frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{3}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

Mathematica [A] time = 0.0434411, size = 106, normalized size = 0.97

$$\frac{1}{6}\left(2\sqrt{a+bx^3+cx^6} - 2\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3 + c*x^6] - 2*Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqr
t[a + b*x^3 + c*x^6])] + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
 + c*x^6])])/Sqrt[c])/6
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82982, size = 1335, normalized size = 12.25

$$\left[\frac{b\sqrt{c}\log\left(-8\,c^2x^6 - 8\,bcx^3 - b^2 - 4\,\sqrt{cx^6 + bx^3 + a}\left(2\,cx^3 + b\right)\sqrt{c} - 4\,ac\right) + 2\,\sqrt{a}c\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\ldots}{x^6}\right.}{12\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 +
a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^6 + 8
*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) +
4*sqrt(c*x^6 + b*x^3 + a)*c)/c, -1/6*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*
x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - sqrt(a)*c*log(
-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(a) + 8*a^2)/x^6) - 2*sqrt(c*x^6 + b*x^3 + a)*c)/c, 1/12*(4*sqrt(-a)*c*a
rctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3
 + a^2)) + b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^
3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c, 1/6
*(2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a
*c*x^6 + a*b*x^3 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2
*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)
*c)/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x, x)
```

3.191 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^4} dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*x^3) - (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt
[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt
[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rubi [A] time = 0.114291, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 732, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*x^3) - (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt
[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt
[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} + \frac{1}{6}b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right) + \frac{1}{3}c\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,d\right.\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{1}{3}b\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right) + \frac{1}{3}(2c)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\right.\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0493902, size = 112, normalized size = 1.

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*x^3) - (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt
[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt
[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8756, size = 1411, normalized size = 12.6

$$\left[\frac{2\,a\sqrt{c}x^3\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)+\sqrt{a}bx^3\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(b\right.}{x^6}\right.}{12\,ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(2*a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*
x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)
*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)
/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), -1/12*(4*a*sqrt(-c)*x^3*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a
*c)) - sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b
*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*a
)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8
*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) -
 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(
c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*a*
sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2
*x^6 + b*c*x^3 + a*c)) - 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^4, x)
```

3.192 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx$

Optimal. Leaf size=88

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}$$

```
[Out] -((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) + ((b^2 - 4*a*c)*ArcTan
h[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(3/2))
```

Rubi [A] time = 0.0727628, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1357, 720, 724, 206}

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] -((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) + ((b^2 - 4*a*c)*ArcTan
h[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(3/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^3\right)\\
&= -\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{12ax^6} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{24a}\\
&= -\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{12ax^6} + \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12a}\\
&= -\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{12ax^6} + \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0448208, size = 89, normalized size = 1.01

$$
\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{x^6}}{24a^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] ((-2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/x^6 + (b^2 - 4*a*c)*Arc
Tanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(3/2))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$
\int \frac{1}{x^7}\sqrt{cx^6+bx^3+a}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76622, size = 502, normalized size = 5.7

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{a}x^6\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)+4\,\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right)}{48\,a^2x^6},\,-\frac{\left(b^2-4\,ac\right)\sqrt{-a}x^6}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/48*((b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*s
qrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 +
 b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6), -1/24*((b^2 - 4*a*c)*sqrt(-a)*x^6
*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x
^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^6+bx^3+a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^7, x)
```

3.193 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}} \, dx$

Optimal. Leaf size=116

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}+\frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9}$$

```
[Out] (b*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a^2*x^6) - (a + b*x^3 + c*x^6
)^(3/2)/(9*a*x^9) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])/(48*a^(5/2))
```

Rubi [A] time = 0.0954686, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 730, 720, 724, 206}

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}+\frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] (b*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a^2*x^6) - (a + b*x^3 + c*x^6
)^(3/2)/(9*a*x^9) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])/(48*a^(5/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{6a}$$

$$= \frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{48a^2}$$

$$= \frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} - \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+c}}\right)}{24a^2}$$

$$= \frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} - \frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}$$

Mathematica [A] time = 0.0856186, size = 108, normalized size = 0.93

$$-\frac{\sqrt{a+bx^3+cx^6}\left(8a^2+2ax^3\left(b+4cx^3\right)-3b^2x^6\right)}{72a^2x^9} - \frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*(8*a^2 - 3*b^2*x^6 + 2*a*x^3*(b + 4*c*x^3)))/(72*
a^2*x^9) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3
 + c*x^6])])/(48*a^(5/2))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x^{10}}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91256, size = 595, normalized size = 5.13

$$\left[-\frac{3\left(b^3-4\,abc\right)\sqrt{a}x^9\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)-4\left(\left(3\,ab^2-8\,a^2c\right)x^6-2\,a^2bx^3-8\,a^3\right)\sqrt{cx^6+\ldots}}{288\,a^3x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3
+ 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b
^2 - 8*a^2*c)*x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^9)
, 1/144*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*
(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 - 8*a^2*c)*
x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^9)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**10,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**10, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^6+bx^3+a}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^10, x)
```

3.194 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}} dx$

Optimal. Leaf size=161

$$-\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6}+\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}+\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9}-\frac{\left(a+\right.}{}$$

```
[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*a^3*x^6) - (a
 + b*x^3 + c*x^6)^(3/2)/(12*a*x^12) + (5*b*(a + b*x^3 + c*x^6)^(3/2))/(72*a
^2*x^9) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*S
qrt[a + b*x^3 + c*x^6])])/(384*a^(7/2))
```

Rubi [A] time = 0.147009, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 744, 806, 720, 724, 206}

$$-\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6}+\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}+\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9}-\frac{\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```


```
[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*a^3*x^6) - (a
 + b*x^3 + c*x^6)^(3/2)/(12*a*x^12) + (5*b*(a + b*x^3 + c*x^6)^(3/2))/(72*a
^2*x^9) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*S
qrt[a + b*x^3 + c*x^6])])/(384*a^(7/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^3\right)\\
&= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{5b}{2}+cx\right)\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^3\right)}{12a}\\
&= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}} + \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9} + \frac{\left(5b^2-4ac\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{48a^2}\\
&= -\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}} + \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9} - \frac{\left(\left(b^2 \right.\right.}{}\\
&= -\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}} + \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9} + \frac{\left(\left(b^2 \right.\right.}{}\\
&= -\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}} + \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9} + \frac{\left(b^2 - \right.}{}
\end{aligned}$$

Mathematica [A] time = 0.102086, size = 139, normalized size = 0.86

$$\frac{\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}} - \frac{\sqrt{a+bx^3+cx^6}\left(8a^2x^3\left(b+3cx^3\right)+48a^3-2abx^6\left(5b+26cx^3\right)+1\right.}{576a^3x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*(48*a^3 + 15*b^3*x^9 + 8*a^2*x^3*(b + 3*c*x^3) -
2*a*b*x^6*(5*b + 26*c*x^3)))/(576*a^3*x^12) + ((5*b^4 - 24*a*b^2*c + 16*a^2
*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(384*a^(7
/2))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^{13}}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06822, size = 749, normalized size = 4.65

$$\left[\frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\sqrt{a}x^{12}\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)-4\left(\left(15\,ab^3-52\,a^2bc\right)x^9+8\,a^3bx^3-\right.}{2304\,a^4x^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*
c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^
2)/x^6) - 4*((15*a*b^3 - 52*a^2*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*
a^3*c)*x^6 + 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^12), -1/1152*(3*(5*b^4
 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a
)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3 - 52*a^2
*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^6 + 48*a^4)*sqrt(c*x^6
 + b*x^3 + a))/(a^4*x^12)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**13, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{13}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^13, x)
```

3.195 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$

Optimal. Leaf size=199

$$-\frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6}-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{a}}\right.}{768a^{9/2}}$$

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*a^4*x^6) -
(a + b*x^3 + c*x^6)^(3/2)/(15*a*x^15) + (7*b*(a + b*x^3 + c*x^6)^(3/2))/(12
0*a^2*x^12) - ((35*b^2 - 32*a*c)*(a + b*x^3 + c*x^6)^(3/2))/(720*a^3*x^9) -
 (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a
+ b*x^3 + c*x^6])])/(768*a^(9/2))
```

Rubi [A] time = 0.226768, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1357, 744, 834, 806, 720, 724, 206}

$$-\frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6}-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{a}}\right.}{768a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*a^4*x^6) -
(a + b*x^3 + c*x^6)^(3/2)/(15*a*x^15) + (7*b*(a + b*x^3 + c*x^6)^(3/2))/(12
0*a^2*x^12) - ((35*b^2 - 32*a*c)*(a + b*x^3 + c*x^6)^(3/2))/(720*a^3*x^9) -
 (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a
+ b*x^3 + c*x^6])])/(768*a^(9/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
```

```
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^6}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+2cx\right)\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^3\right)}{15a}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{4}\left(35b^2-32ac\right)+\frac{7bcx}{2}\right)\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^3\right)}{60a^2}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} - \frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9} - \frac{\left(b\left(7b^2-12ac\right.\right.}{}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} - \frac{\left(35b^2\right.}{}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} - \frac{\left(35b^2\right.}{}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} - \frac{\left(35b^2\right.}{}$$

Mathematica [A] time = 0.170417, size = 173, normalized size = 0.87

$$-\frac{\sqrt{a+bx^3+cx^6}\left(-8a^2x^6\left(7b^2+29bcx^3+32c^2x^6\right)+16a^3\left(3bx^3+8cx^6\right)+384a^4+10ab^2x^9\left(7b+46cx^3\right)-105b^4x^{12}\right)}{5760a^4x^{15}} -$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*(384*a^4 - 105*b^4*x^12 + 10*a*b^2*x^9*(7*b + 46*
c*x^3) + 16*a^3*(3*b*x^3 + 8*c*x^6) - 8*a^2*x^6*(7*b^2 + 29*b*c*x^3 + 32*c^
2*x^6)))/(5760*a^4*x^15) - (b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*
a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(768*a^(9/2))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{x^{16}}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.6458, size = 910, normalized size = 4.57

$$\left[\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{a}x^{15}\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)+4\left(\left(105\,ab^4-460\,a^2b^2c+256\right.\right.}{23040\,a^5x^{15}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="fricas")
```

```
[Out] [1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 +
4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) +
8*a^2)/x^6) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^12 - 2*(35*a^2
*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a^3*b^2 - 16*a^4*c)*x^6 - 384
*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15), 1/11520*(15*(7*b^5 - 40*a*b^3*c
+ 48*a^2*b*c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2
*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^2*c + 2
56*a^3*c^2)*x^12 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a
^3*b^2 - 16*a^4*c)*x^6 - 384*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**16,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**16, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6+bx^3+a}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^16, x)
```

3.196 $\int x^3\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=140

$$\frac{x^4\sqrt{a+bx^3+cx^6}F_1\left(\frac{4}{3};-\frac{1}{2},-\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -1/2, -1/2, 7/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.149481, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$\frac{x^4\sqrt{a+bx^3+cx^6}F_1\left(\frac{4}{3};-\frac{1}{2},-\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -1/2, -1/2, 7/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx = \frac{\sqrt{a+bx^3+cx^6}\int x^3\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{x^4\sqrt{a+bx^3+cx^6}F_1\left(\frac{4}{3};-\frac{1}{2},-\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.577437, size = 358, normalized size = 2.56

$$\frac{x\left(3x^3\left(16ac-5b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-24ab\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\cdots}\right.}{448c\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(8*(3*b + 8*c*x^3)*(a + b*x^3 + c*x^6) - 24*a*b*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*(-5*b^2 + 16*a*c
)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/
2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
 4*a*c])]))/(448*c*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int x^3\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6+bx^3+a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^6 + bx^3 + a}x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{a + bx^3 + cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6 + bx^3 + a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

3.197 $\int x\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=140

$$\frac{x^2\sqrt{a+bx^3+cx^6}F_1\left(\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -1/2, -1/2, 5/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.0988441, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$\frac{x^2\sqrt{a+bx^3+cx^6}F_1\left(\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -1/2, -1/2, 5/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x\sqrt{a+bx^3+cx^6}\,dx = \frac{\sqrt{a+bx^3+cx^6}\int x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{x^2\sqrt{a+bx^3+cx^6}F_1\left(\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.422417, size = 337, normalized size = 2.41

$$\frac{x^2\left(3bx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+15a\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\right.}{50\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*(10*(a + b*x^3 + c*x^6) + 15*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(50*Sqrt[a + b*x^3
+ c*x^6])
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6+bx^3+a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^6 + bx^3 + a}x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a + bx^3 + cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6 + bx^3 + a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

3.198 $\int \sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=135

$$\frac{x\sqrt{a+bx^3+cx^6}F_1\left(\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -1/2, -1/2, 4/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.0646869, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1348, 429}

$$\frac{x\sqrt{a+bx^3+cx^6}F_1\left(\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -1/2, -1/2, 4/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sqrt{a+bx^3+cx^6}\,dx = \frac{\sqrt{a+bx^3+cx^6}\int \sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{x\sqrt{a+bx^3+cx^6}F_1\left(\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.342673, size = 335, normalized size = 2.48

$$\frac{x\left(3bx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+24a\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}\right.}{32\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(8*(a + b*x^3 + c*x^6) + 24*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(32*Sqrt[a + b*x^3 + c
*x^6])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^6 + bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx^3 + cx^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

3.199 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^2} dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{a+bx^3+cx^6}F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -((Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -1/2, -1/2, 2/3, (-2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c*x
^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rubi [A] time = 0.123717, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$-\frac{\sqrt{a+bx^3+cx^6}F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -1/2, -1/2, 2/3, (-2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c*x
^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \frac{\sqrt{a+bx^3+cx^6}\int \frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}{x^2}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.38586, size = 340, normalized size = 2.46

$$\frac{12cx^6\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+15bx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1}{20x\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

```
[Out] (-20*(a + b*x^3 + c*x^6) + 15*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 12*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*x*Sqrt[a + b*x^
3 + c*x^6])
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6+bx^3+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^6 + bx^3 + a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

3.200 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^3} dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{a+bx^3+cx^6}F_1\left(-\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -1/2, -1/2, 1/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rubi [A] time = 0.12271, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$-\frac{\sqrt{a+bx^3+cx^6}F_1\left(-\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -1/2, -1/2, 1/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \frac{\sqrt{a+bx^3+cx^6}\int \frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}{x^3}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{\sqrt{a+bx^3+cx^6}F_1\left(-\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.330792, size = 340, normalized size = 2.43

$$\frac{3cx^6\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+6bx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1}{8x^2\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

```
[Out] (-4*(a + b*x^3 + c*x^6) + 6*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(8*x^2*Sqrt[a + b*x^3
+ c*x^6])
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6+bx^3+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^6 + bx^3 + a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

3.201 $\int x^{14} \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=293

$$\frac{\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{6144c^5}-\frac{\left(b^2-4ac\right)\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+}}{16384c^6}$$

```
[Out] -((b^2 - 4*a*c)*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b
*x^3 + c*x^6])/(16384*c^6) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x
^3)*(a + b*x^3 + c*x^6)^(3/2))/(6144*c^5) - (11*b*x^6*(a + b*x^3 + c*x^6)^(
5/2))/(336*c^2) + (x^9*(a + b*x^3 + c*x^6)^(5/2))/(24*c) - ((3*b*(77*b^2 -
124*a*c) - 10*c*(33*b^2 - 28*a*c)*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(13440*c^
4) + ((b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x
^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(32768*c^(13/2))
```

Rubi [A] time = 0.402334, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1357, 742, 832, 779, 612, 621, 206}

$$\frac{\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{6144c^5}-\frac{\left(b^2-4ac\right)\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+}}{16384c^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```



```
[Out] -((b^2 - 4*a*c)*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b
*x^3 + c*x^6])/(16384*c^6) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x
^3)*(a + b*x^3 + c*x^6)^(3/2))/(6144*c^5) - (11*b*x^6*(a + b*x^3 + c*x^6)^(
5/2))/(336*c^2) + (x^9*(a + b*x^3 + c*x^6)^(5/2))/(24*c) - ((3*b*(77*b^2 -
124*a*c) - 10*c*(33*b^2 - 28*a*c)*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(13440*c^
4) + ((b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x
^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(32768*c^(13/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^{14}\left(a+bx^3+cx^6\right)^{3/2}dx=\frac{1}{3}\text{Subst}\left(\int x^4\left(a+bx+cx^2\right)^{3/2}dx,x,x^3\right)$$
$$=\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}+\frac{\text{Subst}\left(\int x^2\left(-3a-\frac{11bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}dx,x,x^3\right)}{24c}$$
$$=-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}+\frac{\text{Subst}\left(\int x\left(11ab+\frac{3}{4}\left(33b^2-28a\right.\right.\right.}{168}$$
$$=-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}-\frac{\left(3b\left(77b^2-124ac\right)-10c\left(33b^2-2\right.\right.}{13440c^4}$$
$$=\frac{\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{6144c^5}-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2}+\frac{x^9}{}$$
$$=-\frac{\left(b^2-4ac\right)\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{\left(33b^4-72ab^2c+1\right.}{}$$
$$=-\frac{\left(b^2-4ac\right)\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{\left(33b^4-72ab^2c+1\right.}{}$$
$$=-\frac{\left(b^2-4ac\right)\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{\left(33b^4-72ab^2c+1\right.}{}$$

Mathematica [A] time = 0.359141, size = 241, normalized size = 0.82

$$\frac{\frac{\left(16a^2c^2-72ab^2c+33b^4\right)\left(2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\left(4c\left(5a+2cx^6\right)-3b^2+8bcx^3\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)}{4096c^{11/2}}+\frac{\left(372abc-280ac^2x^3+330b^2cx^3-231b\right.}{560c^3}}{24c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((-11*b*x^6*(a + b*x^3 + c*x^6)^(5/2))/(14*c) + x^9*(a + b*x^3 + c*x^6)^(5/
2) + ((-231*b^3 + 372*a*b*c + 330*b^2*c*x^3 - 280*a*c^2*x^3)*(a + b*x^3 + c
*x^6)^(5/2))/(560*c^3) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(2*Sqrt[c]*(b
+ 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 8*b*c*x^3 + 4*c*(5*a + 2*c*x^6
)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*
x^6])]))/(4096*c^(11/2)))/(24*c)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x^{14}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89127, size = 1570, normalized size = 5.36

$$\left[\frac{105\left(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6881280*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3
 + 256*a^4*c^4)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b
*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(71680*c^8*x^21 + 87040*b*c^7*
x^18 + 1280*(b^2*c^6 + 84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12
+ 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*
b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3
*c^4 + 2416*a^2*b*c^5)*x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b
^2*c^4 - 6720*a^3*c^5)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7, -1/3440640*(105*(
33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*s
qrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6
+ b*c*x^3 + a*c)) - 2*(71680*c^8*x^21 + 87040*b*c^7*x^18 + 1280*(b^2*c^6 +
84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12 + 16*(99*b^4*c^4 - 568*
a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3
*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*
x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*
x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{14}\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**14*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^{14}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^14, x)
```

3.202 $\int x^{11} \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=223

$$\frac{\left(-16ac+21b^2-30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{840c^3}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384c^4}+\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+\right.}{1024c^5}$$

```
[Out] (b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(10
24*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(384*
c^4) + (x^6*(a + b*x^3 + c*x^6)^(5/2))/(21*c) + ((21*b^2 - 16*a*c - 30*b*c*
x^3)*(a + b*x^3 + c*x^6)^(5/2))/(840*c^3) - (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a
*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2048*c^(11
/2))
```

Rubi [A] time = 0.205978, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 742, 779, 612, 621, 206}

$$\frac{\left(-16ac+21b^2-30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{840c^3}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384c^4}+\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+\right.}{1024c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```


```
[Out] (b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(10
24*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(384*
c^4) + (x^6*(a + b*x^3 + c*x^6)^(5/2))/(21*c) + ((21*b^2 - 16*a*c - 30*b*c*
x^3)*(a + b*x^3 + c*x^6)^(5/2))/(840*c^3) - (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a
*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2048*c^(11
/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
```

```
2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{1}{3}\,\text{Subst}\left(\int x^3\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)$$

$$=\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}+\frac{\text{Subst}\left(\int x\left(-2a-\frac{9bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)}{21c}$$

$$=\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}+\frac{\left(21b^2-16ac-30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{840c^3}-\frac{\left(b\left(3b^2-4ac\right)\right)\text{Su}}{}$$

$$=-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384c^4}+\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}+\frac{\left(21b^2-16ac-\right.}{}$$

$$=\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3\right.}{384c^4}$$

$$=\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3\right.}{384c^4}$$

$$=\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3\right.}{384c^4}$$

Mathematica [A] time = 0.196137, size = 192, normalized size = 0.86

$$\frac{-\frac{\left(16ac-21b^2+30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{40c^2}+\frac{7\left(4abc-3b^3\right)\left(2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\left(4c\left(5a+2cx^6\right)-3b^2+8bcx^3\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)}{2048c^{9/2}}+x^6}{21c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^(5/2) - ((-21*b^2 + 16*a*c + 30*b*c*x^3)*(a + b*x^
3 + c*x^6)^(5/2))/(40*c^2) + (7*(-3*b^3 + 4*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^3)
*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 8*b*c*x^3 + 4*c*(5*a + 2*c*x^6)) + 3*(b^
2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]))/(
2048*c^(9/2)))/(21*c)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^{11} \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07128, size = 1268, normalized size = 5.69

$$\left[-\frac{105 \left(3 b^7 - 28 a b^5 c + 80 a^2 b^3 c^2 - 64 a^3 b c^3\right) \sqrt{c} \log\left(-8 c^2 x^6 - 8 b c x^3 - b^2 - 4 \sqrt{c x^6 + b x^3 + a}\left(2 c x^3 + b\right) \sqrt{c} - 4 a c\right) - 4 (\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128*(b^2*c^5 + 64
*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c - 2520*a*b^4*c^2
 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^
2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^3)*sqrt(c*x
^6 + b*x^3 + a))/c^6, 1/215040*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 -
64*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128
*(b^2*c^5 + 64*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c -
2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^
2*c^4 + 128*a^2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{11}\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**11*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^{11}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^11, x)
```

3.203 $\int x^8 \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=204

$$\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3}-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\text{ta}}{3072c}$$

```
[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(153
6*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*c^3
) - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(180*c^2) + (x^3*(a + b*x^3 + c*x^6)^(5
/2))/(18*c) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqr
t[c]*Sqrt[a + b*x^3 + c*x^6])])/(3072*c^(9/2))
```

Rubi [A] time = 0.187612, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 742, 640, 612, 621, 206}

$$\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3}-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\text{ta}}{3072c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```



```
[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(153
6*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*c^3
) - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(180*c^2) + (x^3*(a + b*x^3 + c*x^6)^(5
/2))/(18*c) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqr
t[c]*Sqrt[a + b*x^3 + c*x^6])])/(3072*c^(9/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^8\left(a+bx^3+cx^6\right)^{3/2}dx=\frac{1}{3}\text{Subst}\left(\int x^2\left(a+bx+cx^2\right)^{3/2}dx,x,x^3\right)$$
$$=\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{18c}+\frac{\text{Subst}\left(\int\left(-a-\frac{7bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}dx,x,x^3\right)}{18c}$$
$$=-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{18c}+\frac{\left(7b^2-4ac\right)\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}\cdots\right.}{72c^2}$$
$$=\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3}-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{18c}\cdots$$
$$=-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+\cdots\right.}{576c^3}$$
$$=-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+\cdots\right.}{576c^3}$$
$$=-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+\cdots\right.}{576c^3}$$

Mathematica [A] time = 0.169545, size = 175, normalized size = 0.86

$$\frac{\frac{\left(7b^2-4ac\right)\left(2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\left(4c\left(5a+2cx^6\right)-3b^2+8bcx^3\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)}{512c^{7/2}}+x^3\left(a+bx^3+cx^6\right)^{5/2}-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{10c}\cdots}{18c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((-7*b*(a + b*x^3 + c*x^6)^(5/2))/(10*c) + x^3*(a + b*x^3 + c*x^6)^(5/2) +
((7*b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 +
 8*b*c*x^3 + 4*c*(5*a + 2*c*x^6)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)
/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]))/(512*c^(7/2)))/(18*c)
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int x^{8}\left(cx^{6}+bx^{3}+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76723, size = 1064, normalized size = 5.22

$$\left[-\frac{15\left(7b^{6}-60ab^{4}c+144a^{2}b^{2}c^{2}-64a^{3}c^{3}\right)\sqrt{c}\log\left(-8c^{2}x^{6}-8bcx^{3}-b^{2}+4\sqrt{cx^{6}+bx^{3}+a}\left(2cx^{3}+b\right)\sqrt{c}-4ac\right)-4(1}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*l
og(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*s
qrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*b^2*c^4 + 140*
a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1
296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c*x^
6 + b*x^3 + a))/c^5, -1/46080*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 6
4*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-
c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*
b^2*c^4 + 140*a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760
*a*b^3*c^2 - 1296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*
x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{8}\left(a+bx^{3}+cx^{6}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**8*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^8, x)
```

3.204 $\int x^5 \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=150

$$\frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{(a+b}{}$$

```
[Out] (b*(b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*c^3) - (b*(b +
 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*c^2) + (a + b*x^3 + c*x^6)^(5/2)/(
15*c) - (b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
+ c*x^6])])/(256*c^(7/2))
```

Rubi [A] time = 0.116251, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 640, 612, 621, 206}

$$\frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```


```
[Out] (b*(b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*c^3) - (b*(b +
 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*c^2) + (a + b*x^3 + c*x^6)^(5/2)/(
15*c) - (b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
+ c*x^6])])/(256*c^(7/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5 \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{1}{3}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^{3/2} dx, x, x^3\right)$$
$$= \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c} - \frac{b\,\text{Subst}\left(\int \left(a+bx+cx^2\right)^{3/2} dx, x, x^3\right)}{6c}$$
$$= -\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2} + \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c} + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \sqrt{a+b}\right.}{32c^2}$$
$$= \frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3} - \frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2} + \frac{\left(a+bx^3+c\right.}{15c}$$
$$= \frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3} - \frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2} + \frac{\left(a+bx^3+c\right.}{15c}$$
$$= \frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3} - \frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2} + \frac{\left(a+bx^3+c\right.}{15c}$$

Mathematica [A] time = 0.152161, size = 149, normalized size = 0.99

$$-\frac{b\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\right)}{256c^{7/2}} - \frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -(b*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*c^2) + (a + b*x^3 + c*x^6)
^(5/2)/(15*c) - (b*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x^3)*Sqrt[a + b*x^3 +
 c*x^6] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c
*x^6])]))/(256*c^(7/2))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int x^5 \left(cx^6+bx^3+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65705, size = 840, normalized size = 5.6

$$\left[\frac{15\left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)\sqrt{c}\log\left(-8\,c^2x^6 - 8\,bcx^3 - b^2 + 4\,\sqrt{cx^6 + bx^3 + a}\left(2\,cx^3 + b\right)\sqrt{c} - 4\,ac\right) + 4\left(128\,c^5x^{12} + 176\,bc\cdots\right.}{7680\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c
*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(
128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c - 100*
a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6 + b*x^
3 + a))/c^4, 1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arctan(1/
2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c))
 + 2*(128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c
- 100*a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6
+ b*x^3 + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [A] time = 1.15868, size = 232, normalized size = 1.55

$$\frac{1}{1920}\,\sqrt{cx^6 + bx^3 + a}\left(2\left(4\left(2\left(8\,cx^3 + 11\,b\right)x^3 + \frac{b^2c^3 + 32\,ac^4}{c^4}\right)x^3 - \frac{5\,b^3c^2 - 28\,abc^3}{c^4}\right)x^3 + \frac{15\,b^4c - 100\,ab^2c^2 + 128\,a^2c^3}{c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*(8*c*x^3 + 11*b)*x^3 + (b^2*c^3 + 3
2*a*c^4)/c^4)*x^3 - (5*b^3*c^2 - 28*a*b*c^3)/c^4)*x^3 + (15*b^4*c - 100*a*b
^2*c^2 + 128*a^2*c^3)/c^4) + 1/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*log(abs
(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(7/2)
```

3.205 $\int x^2 \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=124

$$-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}$$

```
[Out] -((b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*c^2) + ((b + 2*c
*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*c) + ((b^2 - 4*a*c)^2*ArcTanh[(b + 2*c
*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(128*c^(5/2))
```

Rubi [A] time = 0.086428, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1352, 612, 621, 206}

$$-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```


```
[Out] -((b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*c^2) + ((b + 2*c
*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*c) + ((b^2 - 4*a*c)^2*ArcTanh[(b + 2*c
*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(128*c^(5/2))
```


Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2 \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{1}{3}\operatorname{Subst}\left(\int \left(a+bx+cx^2\right)^{3/2} dx, x, x^3\right)$$

$$= \frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c} - \frac{\left(b^2-4ac\right)\operatorname{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^3\right)}{16c}$$

$$= -\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2} + \frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c} + \frac{\left(b^2-4ac\right)^2 \operatorname{Subs}}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2} + \frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c} + \frac{\left(b^2-4ac\right)^2 \operatorname{Subs}}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2} + \frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c} + \frac{\left(b^2-4ac\right)^2 \tanh}{128}$$

Mathematica [A] time = 0.088241, size = 126, normalized size = 1.02

$$\frac{\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\right)}{8c^{3/2}}+2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2) + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(
b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/
(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]))/(8*c^(3/2)))/(48*c)
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int x^2 \left(cx^6+bx^3+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63293, size = 684, normalized size = 5.52

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)+4\left(16\,c^4x^9+24\,bc^3x\right.}{768\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3
 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(16*c
^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^3)*
sqrt(c*x^6 + b*x^3 + a))/c^3, -1/384*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b
*c*x^3 + a*c)) - 2*(16*c^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2 + 2*(b
^2*c^2 + 20*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [A] time = 1.14813, size = 182, normalized size = 1.47

$$\frac{1}{192}\sqrt{cx^6+bx^3+a}\left(2\left(4\left(2\,cx^3+3\,b\right)x^3+\frac{b^2c^2+20\,ac^3}{c^3}\right)x^3-\frac{3\,b^3c-20\,abc^2}{c^3}\right)-\frac{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\log\left(\left|-2\left(\sqrt{c}\right.\right.\right.}{128\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*c*x^3 + 3*b)*x^3 + (b^2*c^2 + 20*a*c
^3)/c^3)*x^3 - (3*b^3*c - 20*a*b*c^2)/c^3) - 1/128*(b^4 - 8*a*b^2*c + 16*a^
2*c^2)*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(
5/2)
```

3.206 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x} dx$

Optimal. Leaf size=155

$$-\frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}+\frac{(8ac+b^2+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c}+\frac{1}{9}($$

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c) + (a + b*x^3 + c
*x^6)^(3/2)/9 - (a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
c*x^6])])/3 - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b
*x^3 + c*x^6])])/(48*c^(3/2))
```

Rubi [A] time = 0.178907, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1357, 734, 814, 843, 621, 206, 724}

$$-\frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}+\frac{(8ac+b^2+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c}+\frac{1}{9}($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```


```
[Out] ((b^2 + 8*a*c + 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c) + (a + b*x^3 + c
*x^6)^(3/2)/9 - (a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
c*x^6])])/3 - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b
*x^3 + c*x^6])])/(48*c^(3/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
```

```
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx, x, x^3\right)$$

$$= \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} - \frac{1}{6}\,\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^3\right)$$

$$= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} + \frac{\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b\left(b^2-12ac\right)x}{x\sqrt{a+bx+cx^2}}\,dx, \ldots\right)}{24c}$$

$$= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} + \frac{1}{3}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}} \ldots\right)$$

$$= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} - \frac{1}{3}\left(2a^2\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx \ldots\right)$$

$$= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} - \frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3 \ldots}}\right)$$

Mathematica [A] time = 0.149561, size = 143, normalized size = 0.92

$$\frac{1}{144}\left(-48a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{c^{3/2}}+\frac{2\sqrt{a+bx^3+cx^6}\left(8c\left(4a+cx^6\right)+\right.}{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

```
[Out] ((2*Sqrt[a + b*x^3 + c*x^6]*(3*b^2 + 14*b*c*x^3 + 8*c*(4*a + c*x^6)))/c - 4
8*a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])] - (3*b
*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])
/c^(3/2))/144
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.49825, size = 1717, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/288*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 +
 b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^3 - 12*a*b*c)*sqrt(c
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt
(c*x^6 + b*x^3 + a))/c^2, 1/144*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8
*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) +
3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
```

```
)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2
*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/288*(96*sqrt(-a)*a*c^2*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a
^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt
(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^
2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/144*(48*sqrt(-a
)*a*c^2*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6
+ a*b*x^3 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x
^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 +
14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x, x)
```

3.207 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=150

$$\frac{\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}+\frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{1}{2}\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+\cdots}}\right.$$

```
[Out] ((3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/4 - (a + b*x^3 + c*x^6)^(3/2)/(3*
x^3) - (Sqrt[a]*b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])
])/2 + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x
^6])])/(8*Sqrt[c])
```

Rubi [A] time = 0.169818, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1357, 732, 814, 843, 621, 206, 724}

$$\frac{\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}+\frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{1}{2}\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

```
[Out] ((3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/4 - (a + b*x^3 + c*x^6)^(3/2)/(3*
x^3) - (Sqrt[a]*b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])
])/2 + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x
^6])])/(8*Sqrt[c])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
```

```
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^4}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx,x,x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3} + \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx,x,x^3\right)$$

$$= \frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3} - \frac{\text{Subst}\left(\int \frac{-4abc-c\left(b^2+4ac\right)x}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{8c}$$

$$= \frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3} + \frac{1}{2}(ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,\right.$$

$$= \frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3} - (ab)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+}{\sqrt{a+b}}\right.$$

$$= \frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3} - \frac{1}{2}\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)$$

Mathematica [A] time = 0.110244, size = 134, normalized size = 0.89

$$\frac{1}{24}\left(\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}+\frac{2\sqrt{a+bx^3+cx^6}\left(-4a+5bx^3+2cx^6\right)}{x^3}-12\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

```
[Out] ((2*Sqrt[a + b*x^3 + c*x^6]*(-4*a + 5*b*x^3 + 2*c*x^6))/x^3 - 12*Sqrt[a]*b*
ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])] + (3*(b^2 + 4*a*
c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/Sqrt[c])/24
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.25671, size = 1667, normalized size = 11.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(12*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^
6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 3*(b^2 + 4*a*c)*sqrt(c
)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c*x^6 + b*x
^3 + a))/(c*x^3), 1/24*(6*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x
^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c
```

```
*x^6 + b*x^3 + a))/(c*x^3), 1/48*(24*sqrt(-a)*b*c*x^3*arctan(1/2*sqrt(c*x^6
 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 3*(b^2 +
4*a*c)*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqr
t(c*x^6 + b*x^3 + a))/(c*x^3), 1/24*(12*sqrt(-a)*b*c*x^3*arctan(1/2*sqrt(c*
x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c
*x^6 + b*x^3 + a))/(c*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**4,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^4, x)
```

3.208 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=151

$$-\frac{(4ac+b^2)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}}-\frac{(a+bx^3+cx^6)^{3/2}}{6x^6}-\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3}+\frac{1}{2}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+\cdots}}\right.$$

```
[Out] -((b - 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*x^3) - (a + b*x^3 + c*x^6)^(3/2
)/(6*x^6) - ((b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3
+ c*x^6])])/(8*Sqrt[a]) + (b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[
a + b*x^3 + c*x^6])])/2
```

Rubi [A] time = 0.162014, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1357, 732, 812, 843, 621, 206, 724}

$$-\frac{(4ac+b^2)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}}-\frac{(a+bx^3+cx^6)^{3/2}}{6x^6}-\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3}+\frac{1}{2}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] -((b - 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*x^3) - (a + b*x^3 + c*x^6)^(3/2
)/(6*x^6) - ((b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3
+ c*x^6])])/(8*Sqrt[a]) + (b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[
a + b*x^3 + c*x^6])])/2
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^7}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6} + \frac{1}{4}\operatorname{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^3\right)$$

$$= -\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6} - \frac{1}{8}\operatorname{Subst}\left(\int \frac{-b^2-4ac-4bcx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right.$$

$$= -\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6} + \frac{1}{2}(bc)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \right.$$

$$= -\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6} + (bc)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c}{\sqrt{a+bx^3}}\right.$$

$$= -\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6} - \frac{\left(b^2+4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}} + \frac{1}{2}$$

Mathematica [A] time = 0.176054, size = 134, normalized size = 0.89

$$\frac{1}{24}\left(-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{a}}-\frac{2\sqrt{a+bx^3+cx^6}\left(2a+5bx^3-4cx^6\right)}{x^6}+12b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] ((-2*(2*a + 5*b*x^3 - 4*c*x^6)*Sqrt[a + b*x^3 + c*x^6])/x^6 - (3*(b^2 + 4*a
*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/Sqrt[a] + 1
2*b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/24
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x^7}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20497, size = 1670, normalized size = 11.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/48*(12*a*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 +
 b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*(b^2 + 4*a*c)*sqrt(a)*x^6*lo
g(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)
*sqrt(a) + 8*a^2)/x^6) + 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt(c*x^6 + b*x
^3 + a))/(a*x^6), -1/48*(24*a*b*sqrt(-c)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 3*(b^2 + 4*a*c)*sq
rt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(
b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt
```

```
(c*x^6 + b*x^3 + a))/(a*x^6), 1/24*(6*a*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*
c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*
(b^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)
*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sq
rt(c*x^6 + b*x^3 + a))/(a*x^6), -1/24*(12*a*b*sqrt(-c)*x^6*arctan(1/2*sqrt(
c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 3*(b
^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt
(c*x^6 + b*x^3 + a))/(a*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**7,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^7, x)
```

3.209 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=163

$$\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}}-\frac{\left(x^3\left(8ac+b^2\right)+2ab\right)\sqrt{a+bx^3+cx^6}}{24ax^6}+\frac{1}{3}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-\frac{(a$$

```
[Out] -((2*a*b + (b^2 + 8*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a*x^6) - (a + b*
x^3 + c*x^6)^(3/2)/(9*x^9) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqr
t[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(3/2)) + (c^(3/2)*ArcTanh[(b + 2*c*x^
3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rubi [A] time = 0.17553, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1357, 732, 810, 843, 621, 206, 724}

$$\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}}-\frac{\left(x^3\left(8ac+b^2\right)+2ab\right)\sqrt{a+bx^3+cx^6}}{24ax^6}+\frac{1}{3}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-\frac{(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] -((2*a*b + (b^2 + 8*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a*x^6) - (a + b*
x^3 + c*x^6)^(3/2)/(9*x^9) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqr
t[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(3/2)) + (c^(3/2)*ArcTanh[(b + 2*c*x^
3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
```

```
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{10}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}b\left(b^2-12ac\right)-8ac^2x}{x\sqrt{a+bx+cx^2}}\,d\right.}{24a}$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9} + \frac{1}{3}c^2\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+c}}\right.$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9} + \frac{1}{3}\left(2c^2\right)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\right.$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9} + \frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2}{2\sqrt{a}\sqrt{}}\right.}{48a^{3/2}}$$

Mathematica [A] time = 0.213963, size = 149, normalized size = 0.91

$$\frac{1}{144}\left(-\frac{2\sqrt{a+bx^3+cx^6}\left(8a^2+14abx^3+32acx^6+3b^2x^6\right)}{ax^9}+\frac{3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{a^{3/2}}+48c^{3/2}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] ((-2*Sqrt[a + b*x^3 + c*x^6]*(8*a^2 + 14*a*b*x^3 + 3*b^2*x^6 + 32*a*c*x^6))
/(a*x^9) + (3*b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*
x^3 + c*x^6])])/a^(3/2) + 48*c^(3/2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[
a + b*x^3 + c*x^6])])/144
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^{10}}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.47602, size = 1806, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/288*(48*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*x^
9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8
*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/288*(96*a^2*sqrt(-c)*c*x^9*arc
tan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 +
 a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3
```

```
 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((3*a*
b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x
^9), 1/144*(24*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*
x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(-
a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 +
 a*b*x^3 + a^2)) - 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt
(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/144*(48*a^2*sqrt(-c)*c*x^9*arctan(1/2*sq
rt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 3
*(b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14
*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**10,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**10, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^10, x)
```

3.210 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=133

$$\frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}}$$

```
[Out] ((b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*a^2*x^6) - ((2*a
+ b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*a*x^12) - ((b^2 - 4*a*c)^2*ArcTanh[
(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(128*a^(5/2))
```

Rubi [A] time = 0.1076, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1357, 720, 724, 206}

$$\frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] ((b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*a^2*x^6) - ((2*a
+ b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*a*x^12) - ((b^2 - 4*a*c)^2*ArcTanh[
(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(128*a^(5/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{13}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx, x, x^3\right)$$

$$= -\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{16a}$$

$$= \frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}} + \frac{\left(b^2-4ac\right)^2\text{Subst}}{}$$

$$= \frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}} - \frac{\left(b^2-4ac\right)^2\text{Subst}}{}$$

$$= \frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}} - \frac{\left(b^2-4ac\right)^2\tanh^{-}}{128}$$

Mathematica [A] time = 0.175532, size = 138, normalized size = 1.04

$$-\frac{\frac{3\left(b^2-4ac\right)\left(x^6\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}\right)}{8a^{3/2}x^6}+\frac{2\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{x^{12}}}{48a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] -((2*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/x^12 + (3*(b^2 - 4*a*c)*(-2*S
qrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6] + (b^2 - 4*a*c)*x^6*ArcTanh[(2
*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]))/(8*a^(3/2)*x^6))/(48*a)
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{x^{13}}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02661, size = 732, normalized size = 5.5

$$\left[\frac{3\left(b^4-8ab^2c+16a^2c^2\right)\sqrt{a}x^{12}\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)+4\left(\left(3ab^3-20a^2bc\right)x^9-24a^3bx^3-2\right.}{768a^3x^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*c)*x
^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x
^6) + 4*((3*a*b^3 - 20*a^2*b*c)*x^9 - 24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)
*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^12), 1/384*(3*(b^4 - 8*a*b^2
*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*x^9 -
24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a)
)/(a^3*x^12)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**13,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**13, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^13, x)
```

3.211 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=162

$$-\frac{b\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}+\frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}}-($$

```
[Out] -(b*(b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*a^3*x^6) + (b
*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*a^2*x^12) - (a + b*x^3 + c*x^
6)^(5/2)/(15*a*x^15) + (b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*
Sqrt[a + b*x^3 + c*x^6])])/(256*a^(7/2))
```

Rubi [A] time = 0.145584, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 730, 720, 724, 206}

$$-\frac{b\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}+\frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}}-($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] -(b*(b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*a^3*x^6) + (b
*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*a^2*x^12) - (a + b*x^3 + c*x^
6)^(5/2)/(15*a*x^15) + (b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*
Sqrt[a + b*x^3 + c*x^6])])/(256*a^(7/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{16}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15ax^{15}} - \frac{b\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx, x, x^3\right)}{6a}$$

$$= \frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}} - \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15ax^{15}} + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x\right.}{32a^2}$$

$$= -\frac{b\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}} - \frac{\left(a+bx^3+cx^6\right)^{5}}{15ax^{15}}$$

$$= -\frac{b\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}} - \frac{\left(a+bx^3+cx^6\right)^{5}}{15ax^{15}}$$

$$= -\frac{b\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}} - \frac{\left(a+bx^3+cx^6\right)^{5}}{15ax^{15}}$$

Mathematica [A] time = 0.143209, size = 167, normalized size = 1.03

$$\frac{b\left(16a^{3/2}\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}-3x^6\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}-x^6\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx}}\right.\right.\right.}{768a^{7/2}x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] -(a + b*x^3 + c*x^6)^(5/2)/(15*a*x^15) + (b*(16*a^(3/2)*(2*a + b*x^3)*(a +
b*x^3 + c*x^6)^(3/2) - 3*(b^2 - 4*a*c)*x^6*(2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a
+ b*x^3 + c*x^6] - (b^2 - 4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])))/(768*a^(7/2)*x^12)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^{16}}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.73701, size = 890, normalized size = 5.49

$$\left[\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\sqrt{a}x^{15}\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)-4\left(\left(15\,ab^4-100\,a^2b^2c+128\,a^3c\right.\right.}{7680\,a^4x^{15}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 + 4*a*
c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^
2)/x^6) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^12 - 2*(5*a^2*b^3 -
 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^6 + 128*a^5)*sq
rt(c*x^6 + b*x^3 + a))/(a^4*x^15), -1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a
)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*
x^12 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4
*c)*x^6 + 128*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^15)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**16,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**16, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^16, x)
```

3.212 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}} dx$

Optimal. Leaf size=216

$$-\frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}}+\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6}-\frac{(b^2-4ac)^2(7b^2-4$$

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1536
*a^4*x^6) - ((7*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*
a^3*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(18*a*x^18) + (7*b*(a + b*x^3 + c*x^6
)^(5/2))/(180*a^2*x^15) - ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b
*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(3072*a^(9/2))
```

Rubi [A] time = 0.207242, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 744, 806, 720, 724, 206}

$$-\frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}}+\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6}-\frac{(b^2-4ac)^2(7b^2-4$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1536
*a^4*x^6) - ((7*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*
a^3*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(18*a*x^18) + (7*b*(a + b*x^3 + c*x^6
)^(5/2))/(180*a^2*x^15) - ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b
*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(3072*a^(9/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
```

```
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{19}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{18ax^{18}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+cx\right)\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx, x, x^3\right)}{18a}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{18ax^{18}} + \frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180a^2x^{15}} + \frac{\left(7b^2-4ac\right)\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx, x, x^3\right)}{72a^2}$$

$$= -\frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576a^3x^{12}} - \frac{\left(a+bx^3+cx^6\right)^{5/2}}{18ax^{18}} + \frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180a^2x^{15}} - \frac{((b}{}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576a^3x^{12}}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576a^3x^{12}}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576a^3x^{12}}$$

Mathematica [A] time = 0.212798, size = 206, normalized size = 0.95

$$-\frac{\frac{\left(\frac{7b^2}{2}-2ac\right)\left(16a^{3/2}\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}-3x^6\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}-x^6\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)\right)\right)}{256a^{7/2}x^{12}} - \frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{10ax^{15}} + \frac{(a+bx^3}{}}{18a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^(5/2)/x^18 - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(10*a*x^
15) + (((7*b^2)/2 - 2*a*c)*(16*a^(3/2)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3
/2) - 3*(b^2 - 4*a*c)*x^6*(2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6]
- (b^2 - 4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6
])])))/(256*a^(7/2)*x^12))/(18*a)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x^{19}} \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.90839, size = 1111, normalized size = 5.14

$$\left[-\frac{15 \left(7 b^6 - 60 ab^4c + 144 a^2b^2c^2 - 64 a^3c^3\right)\sqrt{a}x^{18} \log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right) - 4\left(\left(105ab^5 - 76\right.\right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*x
^18*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
+ 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c
^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^12 + 8*(7*a^3*b^3
 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*b^2 + 140*a^5*c)*x^6 - 1280
*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18), 1/46080*(15*(7*b^6 - 60*a*b^4*c
+ 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^18*arctan(1/2*sqrt(c*x^6 + b*x^3
 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^5 - 7
60*a^2*b^3*c + 1296*a^3*b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a
```

```
^4*c^2)*x^12 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*
b^2 + 140*a^5*c)*x^6 - 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**19,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**19, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^19, x)
```

3.213 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx$

Optimal. Leaf size=255

$$-\frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{840a^3x^{15}}+\frac{b(3b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{384a^4x^{12}}-\frac{b(b^2-4ac)(3b^2-4ac)(2a+bx^3)}{1024a^5x^6}$$

```
[Out] -(b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1
024*a^5*x^6) + (b*(3*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/
(384*a^4*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(21*a*x^21) + (b*(a + b*x^3 + c*
x^6)^(5/2))/(28*a^2*x^18) - ((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(
840*a^3*x^15) + (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2
*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2048*a^(11/2))
```

Rubi [A] time = 0.311733, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1357, 744, 834, 806, 720, 724, 206}

$$-\frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{840a^3x^{15}}+\frac{b(3b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{384a^4x^{12}}-\frac{b(b^2-4ac)(3b^2-4ac)(2a+bx^3)}{1024a^5x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```


```
[Out] -(b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1
024*a^5*x^6) + (b*(3*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/
(384*a^4*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(21*a*x^21) + (b*(a + b*x^3 + c*
x^6)^(5/2))/(28*a^2*x^18) - ((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(
840*a^3*x^15) + (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2
*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2048*a^(11/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
```

```
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{22}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^8}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{9b}{2}+2cx\right)\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx, x, x^3\right)}{21a}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}} + \frac{b\left(a+bx^3+cx^6\right)^{5/2}}{28a^2x^{18}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{3}{4}\left(21b^2-16ac\right)+\frac{9bcx}{2}\right)\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx, x,\right.}{126a^2}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}} + \frac{b\left(a+bx^3+cx^6\right)^{5/2}}{28a^2x^{18}} - \frac{\left(21b^2-16ac\right)\left(a+bx^3+cx^6\right)^{5/2}}{840a^3x^{15}} - \frac{\left(b\left(3b^2-\right.\right.}{}$$

$$= \frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384a^4x^{12}} - \frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}} + \frac{b\left(a+bx^3+cx^6\right)^{5/2}}{28a^2x^{18}} - \frac{(}{}$$

$$= -\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+\right.}{384a^4x^{12}}$$

$$= -\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+\right.}{384a^4x^{12}}$$

$$= -\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+\right.}{384a^4x^{12}}$$

Mathematica [A] time = 0.506127, size = 243, normalized size = 0.95

$$-\frac{\frac{\left(21b^2-16ac\right)\left(a+bx^3+cx^6\right)^{5/2}}{40a^2x^{15}} + \frac{\left(21abc-\frac{63b^3}{4}\right)\left(16a^{3/2}\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}-3x^6\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}-x^6\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+}}\right.\right.\right.}{1536a^{9/2}x^{12}}}{21a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^(5/2)/x^21 - (3*b*(a + b*x^3 + c*x^6)^(5/2))/(4*a*x^1
8) + ((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(40*a^2*x^15) + (((-63*b
^3)/4 + 21*a*b*c)*(16*a^(3/2)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2) - 3*(
b^2 - 4*a*c)*x^6*(2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6] - (b^2 -
4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])))/(1
536*a^(9/2)*x^12))/(21*a)
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{1}{x^{22}}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.65438, size = 1318, normalized size = 5.17

$$\left[-\frac{105\left(3\,b^7 - 28\,ab^5c + 80\,a^2b^3c^2 - 64\,a^3bc^3\right)\sqrt{a}x^{21}\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right) + 4\left(\left(315\,ab^6 - 252\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(a
)*x^21*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x
^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((315*a*b^6 - 2520*a^2*b^4*c + 5488*a^3
*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5 - 728*a^3*b^3*c + 1168*a^4*b
*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 128*a^5*c^2)*x^12 + 6400*a^6*b
*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*a^7 + 128*(a^5*b^2 + 64*a^6*c
)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(a^6*x^21), -1/215040*(105*(3*b^7 - 28*a*b^
5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-a)*x^21*arctan(1/2*sqrt(c*x^6 +
b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((315*a*b^
6 - 2520*a^2*b^4*c + 5488*a^3*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5
 - 728*a^3*b^3*c + 1168*a^4*b*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 1
28*a^5*c^2)*x^12 + 6400*a^6*b*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*
a^7 + 128*(a^5*b^2 + 64*a^6*c)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(a^6*x^21)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**22,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**22, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^22, x)
```

3.214 $\int x^3 \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=141

$$\frac{ax^4\sqrt{a+bx^3+cx^6}F_1\left(\frac{4}{3};-\frac{3}{2},-\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -3/2, -3/2, 7/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rubi [A] time = 0.129406, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$\frac{ax^4\sqrt{a+bx^3+cx^6}F_1\left(\frac{4}{3};-\frac{3}{2},-\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -3/2, -3/2, 7/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int x^3\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2} dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{ax^4\sqrt{a+bx^3+cx^6}F_1\left(\frac{4}{3};-\frac{3}{2},-\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.903106, size = 453, normalized size = 3.21

$$\frac{x\left(27x^3\left(640a^2c^2-404ab^2c+55b^4\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+8\left(4a^2c\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(8*(-297*b^4*x^3 - 81*b^3*c*x^6 + 3464*b^2*c^2*x^9 + 5488*b*c^3*x^12 + 2
240*c^4*x^15 + 4*a^2*c*(459*b + 1280*c*x^3) + a*(-297*b^3 + 2052*b^2*c*x^3
+ 10204*b*c^2*x^6 + 7360*c^3*x^9)) + 216*a*b*(11*b^2 - 68*a*c)*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 27*(55
*b^4 - 404*a*b^2*c + 640*a^2*c^2)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(232960*c^2*Sqrt[a + b*x^3 + c*
x^6])
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int x^3\left(cx^6+bx^3+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^9 + bx^6 + ax^3\right)\sqrt{cx^6 + bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^9 + b*x^6 + a*x^3)*sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

3.215 $\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx$

Optimal. Leaf size=141

$$\frac{ax^2\sqrt{a+bx^3+cx^6}F_1\left(\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -3/2, -3/2, 5/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rubi [A] time = 0.0962197, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$\frac{ax^2\sqrt{a+bx^3+cx^6}F_1\left(\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -3/2, -3/2, 5/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x\left(a+bx^3+cx^6\right)^{3/2}dx=\frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$=\frac{ax^2\sqrt{a+bx^3+cx^6}F_1\left(\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.731032, size = 410, normalized size = 2.91

$$x^2\left(10\left(448a^2c+27ab^2+698abcx^3+608ac^2x^6+277b^2cx^6+27b^3x^3+410bc^2x^9+160c^3x^{12}\right)-27bx^3\left(7b^2-52ac\right)\sqrt{\frac{-\sqrt{}}{}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*(10*(27*a*b^2 + 448*a^2*c + 27*b^3*x^3 + 698*a*b*c*x^3 + 277*b^2*c*x^6
 + 608*a*c^2*x^6 + 410*b*c^2*x^9 + 160*c^3*x^12) - 270*a*(b^2 - 16*a*c)*Sqr
t[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]
 - 27*b*(7*b^2 - 52*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(17600*c*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int x\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^7 + bx^4 + ax\right)\sqrt{cx^6 + bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^7 + b*x^4 + a*x)*sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

3.216 $\int \left(a+bx^3+cx^6\right)^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{ax\sqrt{a+bx^3+cx^6}F_1\left(\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -3/2, -3/2, 4/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.0664819, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1348, 429}

$$\frac{ax\sqrt{a+bx^3+cx^6}F_1\left(\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -3/2, -3/2, 4/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int \left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2} dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{ax\sqrt{a+bx^3+cx^6}F_1\left(\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.661665, size = 408, normalized size = 3.

$$\frac{x\left(8\left(364a^2c+27ab^2+548abcx^3+476ac^2x^6+211b^2cx^6+27b^3x^3+296bc^2x^9+112c^3x^{12}\right)-27bx^3\left(5b^2-44ac\right)\sqrt{-\cdots}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(8*(27*a*b^2 + 364*a^2*c + 27*b^3*x^3 + 548*a*b*c*x^3 + 211*b^2*c*x^6 +
476*a*c^2*x^6 + 296*b*c^2*x^9 + 112*c^3*x^12) - 216*a*(b^2 - 28*a*c)*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] -
27*b*(5*b^2 - 44*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(8960*c*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2), x)
```

3.217 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=139

$$-\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -((a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -3/2, -3/2, 2/3, (-2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
)
```

Rubi [A] time = 0.128221, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$-\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -3/2, -3/2, 2/3, (-2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^2}dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{a\sqrt{a+bx^3+cx^6}F_1\left(-\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.513515, size = 379, normalized size = 2.73

$$\frac{10\left(-80a^2-61abx^3-70acx^6+19b^2x^6+29bcx^9+10c^2x^{12}\right)+27x^6\left(20ac+b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{3};\right.}{800x\sqrt{a+bx^3+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

```
[Out] (10*(-80*a^2 - 61*a*b*x^3 + 19*b^2*x^6 - 70*a*c*x^6 + 29*b*c*x^9 + 10*c^2*x
^12) + 810*a*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*App
ellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
 + Sqrt[b^2 - 4*a*c])] + 27*(b^2 + 20*a*c)*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(800*x*Sqrt[a + b*x^3 +
 c*x^6])
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**2,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

3.218 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=141

$$-\frac{a\sqrt{a+bx^3+cx^6}F_1\left(-\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -(a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -3/2, -3/2, 1/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*Sqrt[1 + (
2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
)])
```

Rubi [A] time = 0.127185, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$-\frac{a\sqrt{a+bx^3+cx^6}F_1\left(-\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

```
[Out] -(a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -3/2, -3/2, 1/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*Sqrt[1 + (
2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
)])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^3}\,dx = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^3}dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{a\sqrt{a+bx^3+cx^6}F_1\left(-\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.498844, size = 379, normalized size = 2.69

$$\frac{8\left(-28a^2-11abx^3-20acx^6+17b^2x^6+25bcx^9+8c^2x^{12}\right)+27x^6\left(8ac+b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2}\right.}{448x^2\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

```
[Out] (8*(-28*a^2 - 11*a*b*x^3 + 17*b^2*x^6 - 20*a*c*x^6 + 25*b*c*x^9 + 8*c^2*x^1
2) + 648*a*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
 Sqrt[b^2 - 4*a*c])] + 27*(b^2 + 8*a*c)*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(448*x^2*Sqrt[a + b*x^3 +
c*x^6])
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

3.219 $\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=171

$$\frac{(48a^2c^2-120ab^2c+35b^4)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}}-\frac{(5b(21b^2-44ac)-2cx^3(35b^2-36ac))\sqrt{a+bx^3+cx^6}}{576c^4}-\frac{7bx^6}{}$$

```
[Out] (-7*b*x^6*Sqrt[a + b*x^3 + c*x^6])/(72*c^2) + (x^9*Sqrt[a + b*x^3 + c*x^6])
/(12*c) - ((5*b*(21*b^2 - 44*a*c) - 2*c*(35*b^2 - 36*a*c)*x^3)*Sqrt[a + b*x
^3 + c*x^6])/(576*c^4) + ((35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(b +
2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(384*c^(9/2))
```

Rubi [A] time = 0.216241, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 742, 832, 779, 621, 206}

$$\frac{(48a^2c^2-120ab^2c+35b^4)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}}-\frac{(5b(21b^2-44ac)-2cx^3(35b^2-36ac))\sqrt{a+bx^3+cx^6}}{576c^4}-\frac{7bx^6}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```


```
[Out] (-7*b*x^6*Sqrt[a + b*x^3 + c*x^6])/(72*c^2) + (x^9*Sqrt[a + b*x^3 + c*x^6])
/(12*c) - ((5*b*(21*b^2 - 44*a*c) - 2*c*(35*b^2 - 36*a*c)*x^3)*Sqrt[a + b*x
^3 + c*x^6])/(576*c^4) + ((35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(b +
2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(384*c^(9/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
```

```
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} + \frac{\text{Subst}\left(\int \frac{x^2\left(-3a-\frac{7bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{12c}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} + \frac{\text{Subst}\left(\int \frac{x\left(7ab+\frac{1}{4}\left(35b^2-36ac\right)x\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{36c^2}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(5b\left(21b^2-44ac\right)-2c\left(35b^2-36ac\right)x^3\right)\sqrt{a+bx^3}}{576c^4}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(5b\left(21b^2-44ac\right)-2c\left(35b^2-36ac\right)x^3\right)\sqrt{a+bx^3}}{576c^4}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(5b\left(21b^2-44ac\right)-2c\left(35b^2-36ac\right)x^3\right)\sqrt{a+bx^3}}{576c^4}$$

Mathematica [A] time = 0.113647, size = 137, normalized size = 0.8

$$\frac{3\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)+2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(4bc\left(55a-14cx^6\right)+24c^2x^3\left(2cx^6-3a\right)+70b\right.}{1152c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-105*b^3 + 70*b^2*c*x^3 + 4*b*c*(55*a -
 14*c*x^6) + 24*c^2*x^3*(-3*a + 2*c*x^6)) + 3*(35*b^4 - 120*a*b^2*c + 48*a^
```

```
2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(1152*c^
(9/2))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x^{14}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.642, size = 717, normalized size = 4.19

$$\left[\frac{3\left(35\,b^4-120\,ab^2c+48\,a^2c^2\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)+4\left(48\,c^4x^9-5\right.}{2304\,c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b
*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4
*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*
a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5, -1/1152*(3*(35*b^4 - 120*a*b^2*c
+ 48*a^2*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c
 + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^
5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**14/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\sqrt{cx^6 + bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/sqrt(c*x^6 + b*x^3 + a), x)
```

3.220 $\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=121

$$\frac{\left(-16ac+15b^2-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3}-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}+\frac{x^6\sqrt{a+bx^3+cx^6}}{9c}$$

```
[Out] (x^6*Sqrt[a + b*x^3 + c*x^6])/(9*c) + ((15*b^2 - 16*a*c - 10*b*c*x^3)*Sqrt[
a + b*x^3 + c*x^6])/(72*c^3) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(7/2))
```

Rubi [A] time = 0.105398, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 742, 779, 621, 206}

$$\frac{\left(-16ac+15b^2-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3}-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}+\frac{x^6\sqrt{a+bx^3+cx^6}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^6*Sqrt[a + b*x^3 + c*x^6])/(9*c) + ((15*b^2 - 16*a*c - 10*b*c*x^3)*Sqrt[
a + b*x^3 + c*x^6])/(72*c^3) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(7/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{9c}$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\left(15b^2-16ac-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{\left(b\left(5b^2-12ac\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+}}\right.}{48c^3}$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\left(15b^2-16ac-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{\left(b\left(5b^2-12ac\right)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,d\right.}{24c^3}$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\left(15b^2-16ac-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3}}\right.}{48c^{7/2}}$$

Mathematica [A] time = 0.053277, size = 104, normalized size = 0.86

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(8c\left(cx^6-2a\right)+15b^2-10bcx^3\right)+\left(36abc-15b^3\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{144c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(15*b^2 - 10*b*c*x^3 + 8*c*(-2*a + c*x^6
)) + (-15*b^3 + 36*a*b*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 +
 c*x^6])])/(144*c^(7/2))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^{11}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58784, size = 568, normalized size = 4.69

$$\left[-\frac{3\left(5\,b^3-12\,abc\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)-4\left(8\,c^3x^6-10\,bc^2x^3+15\,b\right.}{288\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*
sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 - 10*
b*c^2*x^3 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/144*(3*(5*
b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*s
qrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 - 10*b*c^2*x^3 + 15*b^2*c
 - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**11/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^11/sqrt(c*x^6 + b*x^3 + a), x)
```

3.221 $\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=104

$$\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}-\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}$$

```
[Out] -(b*Sqrt[a + b*x^3 + c*x^6])/(4*c^2) + (x^3*Sqrt[a + b*x^3 + c*x^6])/(6*c)
+ ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]
)])/(24*c^(5/2))
```

Rubi [A] time = 0.0886356, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 742, 640, 621, 206}

$$\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}-\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -(b*Sqrt[a + b*x^3 + c*x^6])/(4*c^2) + (x^3*Sqrt[a + b*x^3 + c*x^6])/(6*c)
+ ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]
)])/(24*c^(5/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\operatorname{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6c}\\
&= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\left(3b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24c^2}\\
&= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\left(3b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12c^2}\\
&= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0333565, size = 88, normalized size = 0.85

$$\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)+2\sqrt{c}\left(2cx^3-3b\right)\sqrt{a+bx^3+cx^6}}{24c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*(-3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6] + (3*b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(5/2))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int x^8\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61118, size = 475, normalized size = 4.57

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^6+bx^3+a}\left(2\,c^2x^3-3\,bc\right)}{48\,c^3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c
*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a
)*(2*c^2*x^3 - 3*b*c))/c^3, -1/24*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt
(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*s
qrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 - 3*b*c))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**8/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/sqrt(c*x^6 + b*x^3 + a), x)
```

3.222 $\int \frac{x^5}{\sqrt{a+bx^3+cx^6}} \, dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a+bx^3+cx^6}}{3c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6])])/(6*c^(3/2))
```

Rubi [A] time = 0.0565937, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1357, 640, 621, 206}

$$\frac{\sqrt{a+bx^3+cx^6}}{3c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6])])/(6*c^(3/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6c}$$
$$= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3c}$$
$$= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Mathematica [A] time = 0.0161302, size = 68, normalized size = 1.

$$\frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6])])/(6*c^(3/2))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int x^5 \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57334, size = 385, normalized size = 5.66

$$\left[\frac{b\sqrt{c}\log\left(-8\,c^2x^6 - 8\,bcx^3 - b^2 + 4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c} - 4\,ac\right) + 4\,\sqrt{cx^6+bx^3+a}\,c}{12\,c^2}, \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2}{2\left(c^2x^6+b\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 +
a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c^2, 1/6*(
b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x
^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*c)/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [A] time = 1.15987, size = 82, normalized size = 1.21

$$\frac{b \log\left(\left|-2\left(\sqrt{c}x^3 - \sqrt{cx^6 + bx^3 + a}\right)\sqrt{c} - b\right|\right)}{6\, c^{\frac{3}{2}}} + \frac{\sqrt{cx^6 + bx^3 + a}}{3\, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*b*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(3
/2) + 1/3*sqrt(c*x^6 + b*x^3 + a)/c
```

3.223 $\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

```
[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])
```

Rubi [A] time = 0.0341713, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1352, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0057731, size = 43, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{cx^6 + bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52184, size = 285, normalized size = 6.63

$$\left[\frac{\log\left(-8\,c^2x^6 - 8\,bcx^3 - b^2 - 4\,\sqrt{cx^6 + bx^3 + a}\left(2\,cx^3 + b\right)\sqrt{c} - 4\,ac\right)}{6\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)}{3\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(c) - 4*a*c)/sqrt(c), -1/3*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [A] time = 1.18438, size = 54, normalized size = 1.26

$$-\frac{\log\left(\left|-2\left(\sqrt{c}x^3 - \sqrt{cx^6 + bx^3 + a}\right)\sqrt{c} - b\right|\right)}{3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/sqrt(
c)
```

3.224 $\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

```
[Out] -ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[a])
```

Rubi [A] time = 0.0401252, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1357, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[a])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\ &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0110785, size = 44, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[a])
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58869, size = 294, normalized size = 6.68

$$\left[\frac{\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)}{6\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)}{3\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(a) + 8*a^2)/x^6)/sqrt(a), 1/3*sqrt(-a)*arctan(1/2*sqrt(c*x^6 +
 b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x), x)
```

3.225 $\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=72

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])])/(6*a^(3/2))
```

Rubi [A] time = 0.0600472, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1357, 730, 724, 206}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])])/(6*a^(3/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} - \frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6a}$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} + \frac{b\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3a}$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} + \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}$$

Mathematica [A] time = 0.0237053, size = 72, normalized size = 1.

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}} - \frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])])/(6*a^(3/2))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63848, size = 424, normalized size = 5.89

$$\left[\frac{\sqrt{a}bx^3\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)-4\,\sqrt{cx^6+bx^3+a}a}{12\,a^2x^3}, -\frac{\sqrt{-a}bx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{-}}{2\left(acx^6+abx^3+a^2\right)}\right.}{6\,a^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b
*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a
)/(a^2*x^3), -1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*a)/
(a^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^4), x)
```

3.226 $\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=108

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}$$

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(6*a*x^6) + (b*Sqrt[a + b*x^3 + c*x^6])/(4*a^2*x^3
) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^
6])])/(24*a^(5/2))
```

Rubi [A] time = 0.100508, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 744, 806, 724, 206}

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(6*a*x^6) + (b*Sqrt[a + b*x^3 + c*x^6])/(4*a^2*x^3
) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^
6])])/(24*a^(5/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}-\frac{\text{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{6a}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}+\frac{\left(3b^2-4ac\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{24a^2}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{\left(3b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12a^2}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0616632, size = 92, normalized size = 0.85

$$\frac{\left(4ac-3b^2\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{2\sqrt{a}\left(3bx^3-2a\right)\sqrt{a+bx^3+cx^6}}{x^6}}{24a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] ((2*Sqrt[a]*(-2*a + 3*b*x^3)*Sqrt[a + b*x^3 + c*x^6])/x^6 + (-3*b^2 + 4*a*c
)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(5/2))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{1}{x^7}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74093, size = 512, normalized size = 4.74

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{a}x^6\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)-4\,\sqrt{cx^6+bx^3+a}\left(3\,abx^3-2\,a^2\right)}{48\,a^3x^6},\ \frac{\left(3\,b^2-4\,ac\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4
*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6
 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6), 1/24*((3*b^2 - 4*a*c)*sqrt(-a
)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 +
a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^7), x)
```

3.227 $\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=145

$$-\frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}$$

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(9*a*x^9) + (5*b*Sqrt[a + b*x^3 + c*x^6])/(36*a^2*
x^6) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(72*a^3*x^3) + (b*(5*b^2
 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*
a^(7/2))
```

Rubi [A] time = 0.157619, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 744, 834, 806, 724, 206}

$$-\frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(9*a*x^9) + (5*b*Sqrt[a + b*x^3 + c*x^6])/(36*a^2*
x^6) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(72*a^3*x^3) + (b*(5*b^2
 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*
a^(7/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} - \frac{\text{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{9a}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}\left(15b^2-16ac\right)+\frac{5bcx}{2}}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{18a^2}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3} - \frac{\left(b\left(5b^2-12ac\right)\right)}{}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3} + \frac{\left(b\left(5b^2-12ac\right)\right)}{}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3} + \frac{b\left(5b^2-12ac\right)\text{t}}{}$$

Mathematica [A] time = 0.0793621, size = 112, normalized size = 0.77

$$\frac{\sqrt{a+bx^3+cx^6}\left(-8a^2+2a\left(5bx^3+8cx^6\right)-15b^2x^6\right)}{72a^3x^9} + \frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 15*b^2*x^6 + 2*a*(5*b*x^3 + 8*c*x^6)))/(
72*a^3*x^9) + (b*(5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a +
 b*x^3 + c*x^6]])])/(48*a^(7/2))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x^{10}} \frac{1}{\sqrt{cx^6 + bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85863, size = 613, normalized size = 4.23

$$\left[-\frac{3\left(5\,b^3 - 12\,abc\right)\sqrt{a}x^9 \log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right) + 4\left(\left(15\,ab^2 - 16\,a^2c\right)x^6 - 10\,a^2bx^3 + 8\,a^3\right)\sqrt{cx^6}}{288\,a^4x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x
^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((15
*a*b^2 - 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^
4*x^9), -1/144*(3*(5*b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b
*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^2
- 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^9)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{10}\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**10*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^10), x)
```

3.228 $\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=192

$$-\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}}+\frac{5b\left(21b^2-44ac\right)\sqrt{a+bx^3+cx^6}}{576a^4x^3}-\frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6}$$

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(12*a*x^12) + (7*b*Sqrt[a + b*x^3 + c*x^6])/(72*a^
2*x^9) - ((35*b^2 - 36*a*c)*Sqrt[a + b*x^3 + c*x^6])/(288*a^3*x^6) + (5*b*(
21*b^2 - 44*a*c)*Sqrt[a + b*x^3 + c*x^6])/(576*a^4*x^3) - ((35*b^4 - 120*a*
b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6
])])/(384*a^(9/2))
```

Rubi [A] time = 0.233341, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 744, 834, 806, 724, 206}

$$-\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}}+\frac{5b\left(21b^2-44ac\right)\sqrt{a+bx^3+cx^6}}{576a^4x^3}-\frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```


```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(12*a*x^12) + (7*b*Sqrt[a + b*x^3 + c*x^6])/(72*a^
2*x^9) - ((35*b^2 - 36*a*c)*Sqrt[a + b*x^3 + c*x^6])/(288*a^3*x^6) + (5*b*(
21*b^2 - 44*a*c)*Sqrt[a + b*x^3 + c*x^6])/(576*a^4*x^3) - ((35*b^4 - 120*a*
b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6
])])/(384*a^(9/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
```

```
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} - \frac{\operatorname{Subst}\left(\int \frac{\frac{7b}{2}+3cx}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{12a}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{4}(35b^2-36ac)+7bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{36a^2}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} - \frac{\operatorname{Subst}\left(\int \frac{\frac{5}{8}b(21b$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b(21b^2-44ac$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b(21b^2-44ac$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b(21b^2-44ac$$

Mathematica [A] time = 0.101818, size = 141, normalized size = 0.73

$$\frac{\sqrt{a+bx^3+cx^6}\left(8a^2\left(7bx^3+9cx^6\right)-48a^3-10abx^6\left(7b+22cx^3\right)+105b^3x^9\right)}{576a^4x^{12}} - \frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{}{2\cdot}\right.}{384a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 + 105*b^3*x^9 - 10*a*b*x^6*(7*b + 22*c*x^
3) + 8*a^2*(7*b*x^3 + 9*c*x^6)))/(576*a^4*x^12) - ((35*b^4 - 120*a*b^2*c +
48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(38
4*a^(9/2))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{1}{x^{13}} \frac{1}{\sqrt{cx^6 + bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.1326, size = 764, normalized size = 3.98

$$\left[\frac{3\left(35\,b^4 - 120\,ab^2c + 48\,a^2c^2\right)\sqrt{a}x^{12}\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right) + 4\left(5\left(21\,ab^3 - 44\,a^2bc\right)x^9 + 56\,a^3\right.}{2304\,a^5x^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*
a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*
a^2)/x^6) + 4*(5*(21*a*b^3 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2
 - 36*a^3*c)*x^6 - 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12), 1/1152*(3*(
35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*
x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(5*(21*a*b^3
 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2 - 36*a^3*c)*x^6 - 48*a^4)
*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**13*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^13), x)
```

3.229 $\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=140

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.125674, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}\\ &= \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.0952551, size = 168, normalized size = 1.2

$$\frac{x^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{4\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2
, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int x^3\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\sqrt{cx^6+bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^6 + bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

3.230 $\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=140

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.0909508, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}\\ &= \frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.0846593, size = 168, normalized size = 1.2

$$\frac{x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{2\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2
, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\sqrt{cx^6+bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

3.231 $\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=135

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

Rubi [A] time = 0.0639999, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1348, 429}

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}\\ &= \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.0601751, size = 163, normalized size = 1.21

$$\frac{x\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2,
1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*
a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{cx^6+bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

3.232 $\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-1/3, 1/2, 1/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*
a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^3 + c*x^6]))
```

Rubi [A] time = 0.118718, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-1/3, 1/2, 1/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*
a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^3 + c*x^6]))
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}\\ &= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [B] time = 0.371287, size = 343, normalized size = 2.49

$$\frac{8cx^6\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+5bx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{2}{3}\right.}{20ax\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (-20*(a + b*x^3 + c*x^6) + 5*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 8*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*a*x*Sqrt[a + b*x^
3 + c*x^6])
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}}{cx^8+bx^5+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c*x^8 + b*x^5 + a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

3.233 $\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

```
[Out] -(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[-2/3, 1/2, 1/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.119539, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[-2/3, 1/2, 1/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*Sqrt[a + b*x^3 + c*x^6])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}\\ &= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [B] time = 0.332502, size = 342, normalized size = 2.44

$$\frac{cx^6\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-2bx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1(}{8ax^2\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (-4*(a + b*x^3 + c*x^6) - 2*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(8*a*x^2*Sqrt[a + b*x^3
+ c*x^6])
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}}{cx^9+bx^6+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c*x^9 + b*x^6 + a*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

3.234 $\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2-12ac\right)\right)\sqrt{a+bx^3+cx^6}}{12c^3\left(b^2-4ac\right)}+\frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}+\frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (2*x^9*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*x^6*
Sqrt[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52*a*c) - 2*c*
(5*b^2 - 12*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c^3*(b^2 - 4*a*c)) + ((5
*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(
8*c^(7/2))
```

Rubi [A] time = 0.228535, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 738, 832, 779, 621, 206}

$$-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2-12ac\right)\right)\sqrt{a+bx^3+cx^6}}{12c^3\left(b^2-4ac\right)}+\frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}+\frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^9*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*x^6*
Sqrt[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52*a*c) - 2*c*
(5*b^2 - 12*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c^3*(b^2 - 4*a*c)) + ((5
*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(
8*c^(7/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
```

```
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^4}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\operatorname{Subst}\left(\int \frac{x^2(6a+3bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3\left(b^2-4ac\right)}$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{2\operatorname{Subst}\left(\int \frac{x\left(-6ab-\frac{3}{2}\left(5b^2-12ac\right)x\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{9c\left(b^2-4ac\right)}$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12ac\right)x^3\right)\sqrt{}}{12c^3\left(b^2-4ac\right)}$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12ac\right)x^3\right)\sqrt{}}{12c^3\left(b^2-4ac\right)}$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12ac\right)x^3\right)\sqrt{}}{12c^3\left(b^2-4ac\right)}$$

Mathematica [A] time = 0.191698, size = 181, normalized size = 0.93

$$\frac{\frac{2\sqrt{c}\left(4a^2c\left(6cx^3-13b\right)+a\left(-62b^2cx^3+15b^3-20bc^2x^6+8c^3x^9\right)+b^2x^3\left(15b^2+5bcx^3-2c^2x^6\right)\right)}{\sqrt{a+bx^3+cx^6}}-3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{7/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x^3) + b^2*x^3*(15*b^2 + 5*b*c*x^3 - 2*c^
2*x^6) + a*(15*b^3 - 62*b^2*c*x^3 - 20*b*c^2*x^6 + 8*c^3*x^9)))/Sqrt[a + b*
x^3 + c*x^6] - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(7/2)*(-b^2 + 4*a*c))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int x^{14}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27616, size = 1272, normalized size = 6.52

$$\left[-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^6+5\,ab^4-24\,a^2b^2c+16\,a^3c^2+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^3\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,b\right.}{48\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6 + 5*a*b^4 - 24*a^2*b^2
*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^3)*sqrt(c)*log(-8*c
^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c)
- 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*b*c^3)*x^6 - 15*
a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^3)*sqrt(c
*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^6 + (b^3*
c^4 - 4*a*b*c^5)*x^3), -1/24*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6
+ 5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)
*x^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c
^2*x^6 + b*c*x^3 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*
```

```
b*c^3)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*
c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*
c^6)*x^6 + (b^3*c^4 - 4*a*b*c^5)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**14/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.235 $\int \frac{x^{11}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{\left(-8ac+3b^2-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)}+\frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

```
[Out] (2*x^6*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + ((3*b^2 -
 8*a*c - 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(3*c^2*(b^2 - 4*a*c)) - (b*Arc
Tanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2*c^(5/2))
```

Rubi [A] time = 0.110573, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 738, 779, 621, 206}

$$\frac{\left(-8ac+3b^2-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)}+\frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```


```
[Out] (2*x^6*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + ((3*b^2 -
 8*a*c - 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(3*c^2*(b^2 - 4*a*c)) - (b*Arc
Tanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2*c^(5/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\operatorname{Subst}\left(\int \frac{x(4a+2bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3\left(b^2-4ac\right)}\\
&= \frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{\left(3b^2-8ac-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)} - \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx\right.}{2c^2}\\
&= \frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{\left(3b^2-8ac-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)} - \frac{b\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x,\right.}{c^2}\\
&= \frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{\left(3b^2-8ac-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)} - \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right.}{2c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.11925, size = 137, normalized size = 1.

$$
\frac{\frac{2\sqrt{c}\left(8a^2c+a\left(-3b^2+10bcx^3+4c^2x^6\right)-b^2x^3\left(3b+cx^3\right)\right)}{\sqrt{a+bx^3+cx^6}}+3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{5/2}\left(4ac-b^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(8*a^2*c - b^2*x^3*(3*b + c*x^3) + a*(-3*b^2 + 10*b*c*x^3 + 4*c
^2*x^6)))/Sqrt[a + b*x^3 + c*x^6] + 3*b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)
/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(6*c^(5/2)*(-b^2 + 4*a*c))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$
\int x^{11}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99479, size = 981, normalized size = 7.16

$$\left[\frac{3\left(\left(b^{3}c-4\,abc^{2}\right)x^{6}+ab^{3}-4\,a^{2}bc+\left(b^{4}-4\,ab^{2}c\right)x^{3}\right)\sqrt{c}\log\left(-8\,c^{2}x^{6}-8\,bcx^{3}-b^{2}+4\,\sqrt{cx^{6}+bx^{3}+a}\left(2\,cx^{3}+b\right)\sqrt{c}-\right.}{12\left(\left(b^{2}c^{4}-4\,ac^{5}\right)x^{6}+ab^{2}c^{3}-4\,a^{2}c^{4}+\left(b^{3}c^{3}-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x
^3)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2
*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a
^2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4
*a*c^5)*x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3), 1/6*(3*((
b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^3)*sqrt(-c
)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*
x^3 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a^2*c^2 + (3*b^3*c
 - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4*a*c^5)*x^6 + a*b
^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(a+bx^{3}+cx^{6}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**11/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(cx^{6}+bx^{3}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^11/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.236 $\int \frac{x^8}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

```
[Out] (2*x^3*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*Sqrt
[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]
*Sqrt[a + b*x^3 + c*x^6])]/(3*c^(3/2))
```

Rubi [A] time = 0.0895328, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 738, 640, 621, 206}

$$\frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^3*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*Sqrt
[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]
*Sqrt[a + b*x^3 + c*x^6])]/(3*c^(3/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\operatorname{Subst}\left(\int \frac{2a+bx}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3\left(b^2-4ac\right)}\\ &= \frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3c}\\ &= \frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} + \frac{2\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3c}\\ &= \frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} + \frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.109485, size = 107, normalized size = 0.89

$$\frac{\frac{2\sqrt{c}\left(a\left(b-2cx^3\right)+b^2x^3\right)}{\sqrt{a+bx^3+cx^6}} - \left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(b^2*x^3 + a*(b - 2*c*x^3)))/Sqrt[a + b*x^3 + c*x^6] - (b^2 - 4
*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(3*c^(3/2
)*(-b^2 + 4*a*c))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int x^8\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82198, size = 833, normalized size = 6.94

$$\left[\frac{\left(\left(b^{2}c - 4\,ac^{2}\right)x^{6} + \left(b^{3} - 4\,abc\right)x^{3} + ab^{2} - 4\,a^{2}c\right)\sqrt{c}\log\left(-8\,c^{2}x^{6} - 8\,bcx^{3} - b^{2} - 4\,\sqrt{cx^{6} + bx^{3} + a}\left(2\,cx^{3} + b\right)\sqrt{c} - 4\,ac\right.}{6\left(\left(b^{2}c^{3} - 4\,ac^{4}\right)x^{6} + ab^{2}c^{2} - 4\,a^{2}c^{3} + \left(b^{3}c^{2} - 4\,abc^{3}\right)x^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(
c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3 + a*
b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a*b*c
^3)*x^3), -1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^
2*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^
2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3
+ a*b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a
*b*c^3)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{8}}{\left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**8/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{8}}{\left(cx^{6} + bx^{3} + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.237 $\int \frac{x^5}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.0293345, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1357, 636}

$$\frac{2\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{2\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.0989696, size = 41, normalized size = 1.05

$$-\frac{2\left(2a+bx^3\right)}{3\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*(2*a + b*x^3))/(3*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [A] time = 0.005, size = 38, normalized size = 1.

$$-\frac{2bx^3+4a}{12ac-3b^2}\frac{1}{\sqrt{cx^6+bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] -2/3/(c*x^6+b*x^3+a)^(1/2)*(b*x^3+2*a)/(4*a*c-b^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68782, size = 144, normalized size = 3.69

$$\frac{2\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)}{3\left(\left(b^2c-4ac^2\right)x^6+\left(b^3-4abc\right)x^3+ab^2-4a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)/((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4
*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**5/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [A] time = 1.45755, size = 61, normalized size = 1.56

$$\frac{2\left(\frac{bx^3}{b^2-4\,ac}+\frac{2\,a}{b^2-4\,ac}\right)}{3\,\sqrt{cx^6+bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*x^3/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

3.238 $\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\left(b+2cx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.0247371, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1352, 613}

$$-\frac{2\left(b+2cx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)\\ &= -\frac{2\left(b+2cx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.0236791, size = 38, normalized size = 1.

$$-\frac{2\left(b+2cx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [A] time = 0.007, size = 37, normalized size = 1.

$$\frac{4\,c x^3+2\,b}{12\,ac-3\,b^2}\frac{1}{\sqrt{c x^6+b x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] 2/3/(c*x^6+b*x^3+a)^(1/2)*(2*c*x^3+b)/(4*a*c-b^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68983, size = 146, normalized size = 3.84

$$-\frac{2\,\sqrt{c x^6+b x^3+a}\left(2\,c x^3+b\right)}{3\left(\left(b^2c-4\,ac^2\right)x^6+\left(b^3-4\,abc\right)x^3+ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)/((b^2*c - 4*a*c^2)*x^6 + (b^3 -
4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [A] time = 1.48891, size = 61, normalized size = 1.61

$$-\frac{2\left(\frac{2\,cx^3}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}\right)}{3\,\sqrt{cx^6+bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(2*c*x^3/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

3.239 $\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{2\left(-2ac+b^2+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - A
rcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*a^(3/2))
```

Rubi [A] time = 0.077819, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 740, 12, 724, 206}

$$\frac{2\left(-2ac+b^2+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - A
rcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*a^(3/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\operatorname{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)}\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a}\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3a}\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.124951, size = 92, normalized size = 1.

$$\frac{1}{3}\left(\frac{2\left(-2ac+b^2+bcx^3\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{a^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((2*(b^2 - 2*a*c + b*c*x^3))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - Ar
cTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/a^(3/2))/3
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0092, size = 837, normalized size = 9.1

$$\left[\frac{\left(\left(b^{2}c-4\,ac^{2}\right)x^{6}+\left(b^{3}-4\,abc\right)x^{3}+ab^{2}-4\,a^{2}c\right)\sqrt{a}\log\left(-\frac{\left(b^{2}+4\,ac\right)x^{6}+8\,abx^{3}-4\,\sqrt{cx^{6}+bx^{3}+a}\left(bx^{3}+2\,a\right)\sqrt{a}+8\,a^{2}}{x^{6}}\right)+4\,\sqrt{cx^{6}+bx^{3}}}{6\left(\left(a^{2}b^{2}c-4\,a^{3}c^{2}\right)x^{6}+a^{3}b^{2}-4\,a^{4}c+\left(a^{2}b^{3}-4\,a^{3}bc\right)x^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(
a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b^2
- 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3 - 4
*a^3*b*c)*x^3), 1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 -
 4*a^2*c)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a
)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b^2
 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3 -
4*a^3*b*c)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+bx^{3}+cx^{6}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{6}+bx^{3}+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x), x)
```

3.240 $\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2x^3\left(b^2-4ac\right)}+\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}+\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6])
 - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^3 + c*x^6])/(3*a^2*(b^2 - 4*a*c)*x^3) + (b
*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2*a^(5/2))
```

Rubi [A] time = 0.124031, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 740, 806, 724, 206}

$$-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2x^3\left(b^2-4ac\right)}+\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}+\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6])
 - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^3 + c*x^6])/(3*a^2*(b^2 - 4*a*c)*x^3) + (b
*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2*a^(5/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^3\right)\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{3a\left(b^2-4ac\right)}\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,\ldots\right.}{2a^2}\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\ldots\right.}{a^2}\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0845054, size = 137, normalized size = 0.96

$$\frac{\frac{2\sqrt{a}\left(-4a^2c+a\left(b^2-10bcx^3-8c^2x^6\right)+3b^2x^3\left(b+cx^3\right)\right)}{x^3\sqrt{a+bx^3+cx^6}} - 3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-4*a^2*c + 3*b^2*x^3*(b + c*x^3) + a*(b^2 - 10*b*c*x^3 - 8*c^2
*x^6)))/(x^3*Sqrt[a + b*x^3 + c*x^6]) - 3*b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*
x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(6*a^(5/2)*(-b^2 + 4*a*c))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13855, size = 1026, normalized size = 7.23

$$\left[\frac{3\left(\left(b^3c-4\,abc^2\right)x^9+\left(b^4-4\,ab^2c\right)x^6+\left(ab^3-4\,a^2bc\right)x^3\right)\sqrt{a}\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)-4\,\left(\left(3\right.\right.}{12\,\left(\left(a^3b^2c-4\,a^4c^2\right)x^9+\left(a^3b^3-4\,a^4bc\right)x^6+\left(a^4b^2-4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^3 - 4*a^2*
b*c)*x^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^
3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2*c - 8*a^2*c^2)*x^6
 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))
/((a^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^
5*c)*x^3), -1/6*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^
3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*
a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)*x^6 + a
^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a
^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)
*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^4), x)
```

3.241 $\int \frac{1}{x^7(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=198

$$\frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3x^3(b^2-4ac)}-\frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{6a^2x^6(b^2-4ac)}-\frac{(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}+\frac{2(-2ac}{3ax^6(b^2-4a}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^6*Sqrt[a + b*x^3 + c*x^6])
 - ((5*b^2 - 12*a*c)*Sqrt[a + b*x^3 + c*x^6])/(6*a^2*(b^2 - 4*a*c)*x^6) + (
b*(15*b^2 - 52*a*c)*Sqrt[a + b*x^3 + c*x^6])/(12*a^3*(b^2 - 4*a*c)*x^3) - (
(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])
/(8*a^(7/2))
```

Rubi [A] time = 0.20426, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 740, 834, 806, 724, 206}

$$\frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3x^3(b^2-4ac)}-\frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{6a^2x^6(b^2-4ac)}-\frac{(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}+\frac{2(-2ac}{3ax^6(b^2-4a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^6*Sqrt[a + b*x^3 + c*x^6])
 - ((5*b^2 - 12*a*c)*Sqrt[a + b*x^3 + c*x^6])/(6*a^2*(b^2 - 4*a*c)*x^6) + (
b*(15*b^2 - 52*a*c)*Sqrt[a + b*x^3 + c*x^6])/(12*a^3*(b^2 - 4*a*c)*x^3) - (
(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])
/(8*a^(7/2))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
```

```
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5b^2+12ac\right)-2bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{4}b\left(15b^2-52ac\right)-}{x^2\sqrt{a+bx}}\right.}{3a^2\left(b^2-\right.}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3}}{12a^3\left(b^2-4ac\right)x^3}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3}}{12a^3\left(b^2-4ac\right)x^3}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3}}{12a^3\left(b^2-4ac\right)x^3}$$

Mathematica [A] time = 0.129268, size = 179, normalized size = 0.9

$$\frac{\frac{2\sqrt{a}\left(2a^2\left(b^2+10bcx^3-12c^2x^6\right)-8a^3c+abx^3\left(-5b^2+62bcx^3+52c^2x^6\right)-15b^3x^6\left(b+cx^3\right)\right)}{x^6\sqrt{a+bx^3+cx^6}}+3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{7/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-8*a^3*c - 15*b^3*x^6*(b + c*x^3) + 2*a^2*(b^2 + 10*b*c*x^3 -
12*c^2*x^6) + a*b*x^3*(-5*b^2 + 62*b*c*x^3 + 52*c^2*x^6)))/(x^6*Sqrt[a + b*
x^3 + c*x^6]) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(
2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(7/2)*(-b^2 + 4*a*c))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^7}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.67217, size = 1319, normalized size = 6.66

$$\left[-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^{12}+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^9+\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2\right)x^6\right)\sqrt{a}\log\left(-\frac{\left(b^2+4\,ac\right)\cdots}{\cdots}\right)\cdots}{48\left(\left(a^4b^2c-\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c
 + 16*a^2*b*c^2)*x^9 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(a)*l
og(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a
)*sqrt(a) + 8*a^2)/x^6) - 4*((15*a*b^3*c - 52*a^2*b*c^2)*x^9 + (15*a*b^4 -
```

```
62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*a^3*b^2 + 8*a^4*c + 5*(a^2*b^3 - 4*a^3*b
*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^4*b^2*c - 4*a^5*c^2)*x^12 + (a^4*b^3
- 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a^6*c)*x^6), 1/24*(3*((5*b^4*c - 24*a*b^2*c
^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^9 + (5*a*b^4
- 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 +
a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3*c - 52*
a^2*b*c^2)*x^9 + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*a^3*b^2 + 8
*a^4*c + 5*(a^2*b^3 - 4*a^3*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^4*b^2*c
- 4*a^5*c^2)*x^12 + (a^4*b^3 - 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a^6*c)*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^7), x)
```

3.242 $\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=256

$$-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{a+bx^3+cx^6}}{72a^4x^3\left(b^2-4ac\right)}+\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{36a^3x^6\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2x^9\left(b^2-4ac\right)}+\cdots$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^9*Sqrt[a + b*x^3 + c*x^6])
 - ((7*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(9*a^2*(b^2 - 4*a*c)*x^9) + (
b*(35*b^2 - 116*a*c)*Sqrt[a + b*x^3 + c*x^6])/(36*a^3*(b^2 - 4*a*c)*x^6) -
((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a + b*x^3 + c*x^6])/(72*a^4*(b^
2 - 4*a*c)*x^3) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])])/(48*a^(9/2))
```

Rubi [A] time = 0.285943, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 740, 834, 806, 724, 206}

$$-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{a+bx^3+cx^6}}{72a^4x^3\left(b^2-4ac\right)}+\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{36a^3x^6\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2x^9\left(b^2-4ac\right)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```


```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^9*Sqrt[a + b*x^3 + c*x^6])
 - ((7*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(9*a^2*(b^2 - 4*a*c)*x^9) + (
b*(35*b^2 - 116*a*c)*Sqrt[a + b*x^3 + c*x^6])/(36*a^3*(b^2 - 4*a*c)*x^6) -
((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a + b*x^3 + c*x^6])/(72*a^4*(b^
2 - 4*a*c)*x^3) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])])/(48*a^(9/2))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-7b^2+16ac\right)-3bcx}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{4}b\left(35b^2-\right.}{x^3}\right.}{9a^2\left(\right.}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b\left(35b^2-116ac\right)\sqrt{a}}{36a^3\left(b^2-4a\right.}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b\left(35b^2-116ac\right)\sqrt{a}}{36a^3\left(b^2-4a\right.}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b\left(35b^2-116ac\right)\sqrt{a}}{36a^3\left(b^2-4a\right.}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b\left(35b^2-116ac\right)\sqrt{a}}{36a^3\left(b^2-4a\right.}$$

Mathematica [A] time = 0.182314, size = 223, normalized size = 0.87

$$\frac{\frac{2\sqrt{a}\left(2a^2x^3\left(-86b^2cx^3-7b^3+244bc^2x^6+128c^3x^9\right)+8a^3\left(b^2+7bcx^3+16c^2x^6\right)-32a^4c+5ab^2x^6\left(7b^2-106bcx^3-92c^2x^6\right)+105b^4x^9\left(b+cx^3\right)\right)}{x^9\sqrt{a+bx^3+cx^6}} - 15b\left(48a^2c^2-40\right.}{144a^{9/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-32*a^4*c + 105*b^4*x^9*(b + c*x^3) + 5*a*b^2*x^6*(7*b^2 - 106
*b*c*x^3 - 92*c^2*x^6) + 8*a^3*(b^2 + 7*b*c*x^3 + 16*c^2*x^6) + 2*a^2*x^3*(
-7*b^3 - 86*b^2*c*x^3 + 244*b*c^2*x^6 + 128*c^3*x^9)))/(x^9*Sqrt[a + b*x^3
+ c*x^6]) - 15*b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2
*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(144*a^(9/2)*(-b^2 + 4*a*c))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{x^{10}}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.65787, size = 1550, normalized size = 6.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b
^4*c + 48*a^2*b^2*c^2)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*
sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*
x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((105*a*b^4*c - 460*a^2*b^2*c^2 + 256*
a^3*c^3)*x^12 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*a^2*b
^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^3*b^3
- 4*a^4*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^5*b^2*c - 4*a^6*c^2)*x^15 +
(a^5*b^3 - 4*a^6*b*c)*x^12 + (a^6*b^2 - 4*a^7*c)*x^9), -1/144*(15*((7*b^5*c
 - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2
)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*sqrt(-a)*arctan(1/2*s
qrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) +
2*((105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^12 + (105*a*b^5 - 530*a^
2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x
^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^3*b^3 - 4*a^4*b*c)*x^3)*sqrt(c*x^6 + b*x^
3 + a))/((a^5*b^2*c - 4*a^6*c^2)*x^15 + (a^5*b^3 - 4*a^6*b*c)*x^12 + (a^6*b
^2 - 4*a^7*c)*x^9)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{10}\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**10*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^10), x)
```

3.243 $\int \frac{x^3}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{3};\frac{3}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/3, 3/2, 3/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.132141, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{3};\frac{3}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/3, 3/2, 3/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^3 + c*x^6])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}$$

$$= \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{3};\frac{3}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] time = 0.331652, size = 340, normalized size = 2.38

$$\frac{x\left(cx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+2b\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\right.}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(-2*(b + 2*c*x^3) + 2*b*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^
3 + c*x^6])
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int x^3\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}\,x^3}{c^2x^{12}+2\,bcx^9+\left(b^2+2\,ac\right)x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x^3/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*
x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**3/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.244 $\int \frac{x}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/3, 3/2, 3/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.0967523, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/3, 3/2, 3/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^3 + c*x^6])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}$$
$$= \frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] time = 0.495649, size = 362, normalized size = 2.53

$$\frac{x^2\left(8bcx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+5\left(4ac+b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4a}}{\sqrt{b^2-}}}\right.}{30a\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*(-20*(b^2 - 2*a*c + b*c*x^3) + 5*(b^2 + 4*a*c)*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 8*b*c*x^3*Sqrt[(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3,
(-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(
30*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int x\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}\,x}{c^2x^{12}+2\,bcx^9+\left(b^2+2\,ac\right)x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^
6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.245 $\int \frac{1}{(a+bx^3+cx^6)^{3/2}} dx$

Optimal. Leaf size=138

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[1/3, 3/2, 3/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.0672494, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1348, 429}

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(-3/2),x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[1/3, 3/2, 3/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^3 + c*x^6])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}\\ &= \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [B] time = 0.468416, size = 359, normalized size = 2.6

$$\frac{x\left(bcx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-2\left(b^2-8ac\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4a}}{\sqrt{b^2}}}\right.}{6a\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(-3/2),x]
```

```
[Out] (x*(-4*(b^2 - 2*a*c + b*c*x^3) - 2*(b^2 - 8*a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + b*c*x^3*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(6*a*(
-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}}{c^2x^{12}+2\,bcx^9+\left(b^2+2\,ac\right)x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6
+ 2*a*b*x^3 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

3.246 $\int \frac{1}{x^2(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-1/3, 3/2, 3/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*
a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*x^3 + c*x^6]))
```

Rubi [A] time = 0.127745, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-1/3, 3/2, 3/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*
a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*x^3 + c*x^6]))
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}$$

$$= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] time = 0.761432, size = 407, normalized size = 2.89

$$-\frac{5bx^3\left(12ac-5b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-4\left(60a^2c+2cx^6\left(5b^2-16ac\right)\right.}{60a^2x\left(b^2-4a\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -(5*b*(-5*b^2 + 12*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 4*(60*a^2*c - 25*b^2*x^3*(b + c*x^3) + 5*
a*(-3*b^2 + 18*b*c*x^3 + 16*c^2*x^6) + 2*c*(5*b^2 - 16*a*c)*x^6*Sqrt[(b - S
qrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(60*a
^2*(b^2 - 4*a*c)*x*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}}{c^2x^{14}+2\,bcx^{11}+\left(b^2+2\,ac\right)x^8+2\,abx^5+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^14 + 2*b*c*x^11 + (b^2 + 2*a*c)*x^8
 + 2*a*b*x^5 + a^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

3.247 $\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

```
[Out] -(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[-2/3, 3/2, 3/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*x^2*Sqrt[a + b*x^3 + c*x^6]
)
```

Rubi [A] time = 0.128424, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[-2/3, 3/2, 3/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*x^2*Sqrt[a + b*x^3 + c*x^6]
)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}$$

$$= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] time = 0.647967, size = 405, normalized size = 2.83

$$\frac{-48a^2c+cx^6\left(20ac-7b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+2bx^3\left(7b^2-36ac\right)}{24a^2x^2\left(4ac-b\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (-48*a^2*c + 28*b^2*x^3*(b + c*x^3) + 4*a*(3*b^2 - 24*b*c*x^3 - 20*c^2*x^6)
 + 2*b*(7*b^2 - 36*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*(-7*b^2 + 20*a*c)*x^6*Sqrt[(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(24*a^2*(-b^
2 + 4*a*c)*x^2*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}}{c^2x^{15}+2\,bcx^{12}+\left(b^2+2\,ac\right)x^9+2\,abx^6+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^15 + 2*b*c*x^12 + (b^2 + 2*a*c)*x^9
 + 2*a*b*x^6 + a^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

3.248 $\int (dx)^m \left(a+bx^3+cx^6\right)^2 dx$

Optimal. Leaf size=101

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{2ab(dx)^{m+4}}{d^4(m+4)}+\frac{2bc(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(4 + m))/(d^4*(4 + m)) + ((b
^2 + 2*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*c*(d*x)^(10 + m))/(d^10*(10
 + m)) + (c^2*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rubi [A] time = 0.0609639, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1353}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{2ab(dx)^{m+4}}{d^4(m+4)}+\frac{2bc(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(4 + m))/(d^4*(4 + m)) + ((b
^2 + 2*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*c*(d*x)^(10 + m))/(d^10*(10
 + m)) + (c^2*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rule 1353

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+bx^3+cx^6\right)^2 dx &= \int \left(a^2(dx)^m+\frac{2ab(dx)^{3+m}}{d^3}+\frac{\left(b^2+2ac\right)(dx)^{6+m}}{d^6}+\frac{2bc(dx)^{9+m}}{d^9}+\frac{c^2(dx)^{12+m}}{d^{12}}\right) dx\\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{4+m}}{d^4(4+m)}+\frac{\left(b^2+2ac\right)(dx)^{7+m}}{d^7(7+m)}+\frac{2bc(dx)^{10+m}}{d^{10}(10+m)}+\frac{c^2(dx)^{13+m}}{d^{13}(13+m)}\end{aligned}$$

Mathematica [A] time = 0.0715919, size = 70, normalized size = 0.69

$$x(dx)^m\left(\frac{a^2}{m+1}+\frac{x^6\left(2ac+b^2\right)}{m+7}+\frac{2abx^3}{m+4}+\frac{2bcx^9}{m+10}+\frac{c^2x^{12}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^3)/(4 + m) + ((b^2 + 2*a*c)*x^6)/(7 + m)
+ (2*b*c*x^9)/(10 + m) + (c^2*x^12)/(13 + m))
```

Maple [B] time = 0.007, size = 301, normalized size = 3.

$$\frac{\left(c^2m^4x^{12} + 22\,c^2m^3x^{12} + 159\,c^2m^2x^{12} + 2\,bcm^4x^9 + 418\,c^2mx^{12} + 50\,bcm^3x^9 + 280\,c^2x^{12} + 390\,bcm^2x^9 + 2\,acm^4x^6 + b^2m \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^2,x)
```

```
[Out] x*(c^2*m^4*x^12+22*c^2*m^3*x^12+159*c^2*m^2*x^12+2*b*c*m^4*x^9+418*c^2*m*x^
12+50*b*c*m^3*x^9+280*c^2*x^12+390*b*c*m^2*x^9+2*a*c*m^4*x^6+b^2*m^4*x^6+10
70*b*c*m*x^9+56*a*c*m^3*x^6+28*b^2*m^3*x^6+728*b*c*x^9+498*a*c*m^2*x^6+249*
b^2*m^2*x^6+2*a*b*m^4*x^3+1484*a*c*m*x^6+742*b^2*m*x^6+62*a*b*m^3*x^3+1040*
a*c*x^6+520*b^2*x^6+642*a*b*m^2*x^3+a^2*m^4+2402*a*b*m*x^3+34*a^2*m^3+1820*
a*b*x^3+411*a^2*m^2+2074*a^2*m+3640*a^2)*(d*x)^m/(13+m)/(10+m)/(7+m)/(4+m)/
(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.58221, size = 599, normalized size = 5.93

$$\frac{\left(\left(c^2m^4 + 22\,c^2m^3 + 159\,c^2m^2 + 418\,c^2m + 280\,c^2\right)x^{13} + 2\left(bcm^4 + 25\,bcm^3 + 195\,bcm^2 + 535\,bcm + 364\,bc\right)x^{10} + \left(\left(b^2 + \cdots\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 22*c^2*m^3 + 159*c^2*m^2 + 418*c^2*m + 280*c^2)*x^13 + 2*(b*c*m
^4 + 25*b*c*m^3 + 195*b*c*m^2 + 535*b*c*m + 364*b*c)*x^10 + ((b^2 + 2*a*c)*
m^4 + 28*(b^2 + 2*a*c)*m^3 + 249*(b^2 + 2*a*c)*m^2 + 520*b^2 + 1040*a*c + 7
42*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 31*a*b*m^3 + 321*a*b*m^2 + 1201*a*b*
m + 910*a*b)*x^4 + (a^2*m^4 + 34*a^2*m^3 + 411*a^2*m^2 + 2074*a^2*m + 3640*
a^2)*x)*(d*x)^m/(m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)
```

Sympy [A] time = 6.02886, size = 1510, normalized size = 14.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(12*x**12) - 2*a*b/(9*x**9) - a*c/(3*x**6) - b**2/(6*x**6
) - 2*b*c/(3*x**3) + c**2*log(x))/d**13, Eq(m, -13)), ((-a**2/(9*x**9) - a*
b/(3*x**6) - 2*a*c/(3*x**3) - b**2/(3*x**3) + 2*b*c*log(x) + c**2*x**3/3)/d
**10, Eq(m, -10)), ((-a**2/(6*x**6) - 2*a*b/(3*x**3) + 2*a*c*log(x) + b**2*
log(x) + 2*b*c*x**3/3 + c**2*x**6/6)/d**7, Eq(m, -7)), ((-a**2/(3*x**3) + 2
*a*b*log(x) + 2*a*c*x**3/3 + b**2*x**3/3 + b*c*x**6/3 + c**2*x**9/9)/d**4,
Eq(m, -4)), ((a**2*log(x) + 2*a*b*x**3/3 + a*c*x**6/3 + b**2*x**6/6 + 2*b*c
*x**9/9 + c**2*x**12/12)/d, Eq(m, -1)), (a**2*d**m*m**4*x*x**m/(m**5 + 35*m
**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 34*a**2*d**m*m**3*x*x**m/(m**
5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 411*a**2*d**m*m**2*x*
x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2074*a**2*d*
*m*m*x*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 3640*
a**2*d**m*x*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) +
2*a*b*d**m*m**4*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
 3640) + 62*a*b*d**m*m**3*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2
+ 5714*m + 3640) + 642*a*b*d**m*m**2*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 +
 2485*m**2 + 5714*m + 3640) + 2402*a*b*d**m*m*x**4*x**m/(m**5 + 35*m**4 + 4
45*m**3 + 2485*m**2 + 5714*m + 3640) + 1820*a*b*d**m*x**4*x**m/(m**5 + 35*m
**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*a*c*d**m*m**4*x**7*x**m/(m*
*5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 56*a*c*d**m*m**3*x**
7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 498*a*c*d*
*m*m**2*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) +
 1484*a*c*d**m*m*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m
+ 3640) + 1040*a*c*d**m*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 +
5714*m + 3640) + b**2*d**m*m**4*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485
*m**2 + 5714*m + 3640) + 28*b**2*d**m*m**3*x**7*x**m/(m**5 + 35*m**4 + 445*
m**3 + 2485*m**2 + 5714*m + 3640) + 249*b**2*d**m*m**2*x**7*x**m/(m**5 + 35
*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 742*b**2*d**m*m*x**7*x**m/(
m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 520*b**2*d**m*x**7
*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*b*c*d**m*
m**4*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 5
0*b*c*d**m*m**3*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m
+ 3640) + 390*b*c*d**m*m**2*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m*
*2 + 5714*m + 3640) + 1070*b*c*d**m*m*x**10*x**m/(m**5 + 35*m**4 + 445*m**3
 + 2485*m**2 + 5714*m + 3640) + 728*b*c*d**m*x**10*x**m/(m**5 + 35*m**4 + 4
45*m**3 + 2485*m**2 + 5714*m + 3640) + c**2*d**m*m**4*x**13*x**m/(m**5 + 35
*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 22*c**2*d**m*m**3*x**13*x**
m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 159*c**2*d**m*m
**2*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 41
8*c**2*d**m*m*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
3640) + 280*c**2*d**m*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5
714*m + 3640), True))
```

Giac [B] time = 1.16794, size = 606, normalized size = 6.

$$\frac{(dx)^m c^2m^4x^{13} + 22\,(dx)^m c^2m^3x^{13} + 159\,(dx)^m c^2m^2x^{13} + 2\,(dx)^m bcm^4x^{10} + 418\,(dx)^m c^2mx^{13} + 50\,(dx)^m bcm^3x^{10} -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^2*m^4*x^13 + 22*(d*x)^m*c^2*m^3*x^13 + 159*(d*x)^m*c^2*m^2*x^13
+ 2*(d*x)^m*b*c*m^4*x^10 + 418*(d*x)^m*c^2*m*x^13 + 50*(d*x)^m*b*c*m^3*x^10
 + 280*(d*x)^m*c^2*x^13 + 390*(d*x)^m*b*c*m^2*x^10 + (d*x)^m*b^2*m^4*x^7 +
2*(d*x)^m*a*c*m^4*x^7 + 1070*(d*x)^m*b*c*m*x^10 + 28*(d*x)^m*b^2*m^3*x^7 +
56*(d*x)^m*a*c*m^3*x^7 + 728*(d*x)^m*b*c*x^10 + 249*(d*x)^m*b^2*m^2*x^7 + 4
98*(d*x)^m*a*c*m^2*x^7 + 2*(d*x)^m*a*b*m^4*x^4 + 742*(d*x)^m*b^2*m*x^7 + 14
```

```
84*(d*x)^m*a*c*m*x^7 + 62*(d*x)^m*a*b*m^3*x^4 + 520*(d*x)^m*b^2*x^7 + 1040*
(d*x)^m*a*c*x^7 + 642*(d*x)^m*a*b*m^2*x^4 + (d*x)^m*a^2*m^4*x + 2402*(d*x)^
m*a*b*m*x^4 + 34*(d*x)^m*a^2*m^3*x + 1820*(d*x)^m*a*b*x^4 + 411*(d*x)^m*a^2
*m^2*x + 2074*(d*x)^m*a^2*m*x + 3640*(d*x)^m*a^2*x)/(m^5 + 35*m^4 + 445*m^3
 + 2485*m^2 + 5714*m + 3640)
```

3.249 $\int (dx)^m \left(a+bx^3+cx^6\right) dx$

Optimal. Leaf size=52

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+7}}{d^7(m+7)}$$

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(4 + m))/(d^4*(4 + m)) + (c*(d*x)^
(7 + m))/(d^7*(7 + m))
```

Rubi [A] time = 0.0206515, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+7}}{d^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(4 + m))/(d^4*(4 + m)) + (c*(d*x)^
(7 + m))/(d^7*(7 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+bx^3+cx^6\right)dx&=\int\left(a(dx)^m+\frac{b(dx)^{3+m}}{d^3}+\frac{c(dx)^{6+m}}{d^6}\right)dx\\&=\frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{4+m}}{d^4(4+m)}+\frac{c(dx)^{7+m}}{d^7(7+m)}\end{aligned}$$

Mathematica [A] time = 0.0295413, size = 35, normalized size = 0.67

$$x(dx)^m\left(\frac{a}{m+1}+\frac{bx^3}{m+4}+\frac{cx^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

```
[Out] x*(d*x)^m*(a/(1 + m) + (b*x^3)/(4 + m) + (c*x^6)/(7 + m))
```

Maple [A] time = 0.003, size = 78, normalized size = 1.5

$$\frac{\left(cm^2x^6+5\,cmx^6+4\,cx^6+bm^2x^3+8\,bmx^3+7\,bx^3+am^2+11\,am+28\,a\right)x\,(dx)^m}{(7+m)\,(4+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a),x)
```

```
[Out] x*(c*m^2*x^6+5*c*m*x^6+4*c*x^6+b*m^2*x^3+8*b*m*x^3+7*b*x^3+a*m^2+11*a*m+28*
a)*(d*x)^m/(7+m)/(4+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57497, size = 162, normalized size = 3.12

$$\frac{\left(\left(cm^2 + 5\,cm + 4\,c\right)x^7 + \left(bm^2 + 8\,bm + 7\,b\right)x^4 + \left(am^2 + 11\,am + 28\,a\right)x\right)(dx)^m}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 5*c*m + 4*c)*x^7 + (b*m^2 + 8*b*m + 7*b)*x^4 + (a*m^2 + 11*a*m +
28*a)*x)*(d*x)^m/(m^3 + 12*m^2 + 39*m + 28)
```

Sympy [A] time = 1.50703, size = 314, normalized size = 6.04

$$\begin{cases} \frac{-\frac{a}{6x^6} - \frac{b}{3x^3} + c\log(x)}{d^7} \\ \frac{-\frac{a}{3x^3} + b\log(x) + \frac{cx^3}{3}}{d^4} \\ \frac{a\log(x) + \frac{bx^3}{3} + \frac{cx^6}{6}}{d} \\ \frac{ad^m m^2 x x^m}{m^3+12m^2+39m+28} + \frac{11ad^m m x x^m}{m^3+12m^2+39m+28} + \frac{28ad^m x x^m}{m^3+12m^2+39m+28} + \frac{bd^m m^2 x^4 x^m}{m^3+12m^2+39m+28} + \frac{8bd^m m x^4 x^m}{m^3+12m^2+39m+28} + \frac{7bd^m x^4 x^m}{m^3+12m^2+39m+28} + \frac{cd^m m^2 x^7}{m^3+12m^2+3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a),x)
```

```
[Out] Piecewise(((-a/(6*x**6) - b/(3*x**3) + c*log(x))/d**7, Eq(m, -7)), ((-a/(3*
x**3) + b*log(x) + c*x**3/3)/d**4, Eq(m, -4)), ((a*log(x) + b*x**3/3 + c*x*
*6/6)/d, Eq(m, -1)), (a*d**m*m**2*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 11*
a*d**m*m*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 28*a*d**m*x*x**m/(m**3 + 12*
m**2 + 39*m + 28) + b*d**m*m**2*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 8*
b*d**m*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 7*b*d**m*x**4*x**m/(m**3
+ 12*m**2 + 39*m + 28) + c*d**m*m**2*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28)
 + 5*c*d**m*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*c*d**m*x**7*x**m/(
```

```
m**3 + 12*m**2 + 39*m + 28), True))
```

Giac [B] time = 1.09214, size = 161, normalized size = 3.1

$$\frac{(dx)^m cm^2x^7 + 5 (dx)^m cmx^7 + 4 (dx)^m cx^7 + (dx)^m bm^2x^4 + 8 (dx)^m bmx^4 + 7 (dx)^m bx^4 + (dx)^m am^2x + 11 (dx)^m am}{m^3 + 12 m^2 + 39 m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*c*m^2*x^7 + 5*(d*x)^m*c*m*x^7 + 4*(d*x)^m*c*x^7 + (d*x)^m*b*m^2*x^
4 + 8*(d*x)^m*b*m*x^4 + 7*(d*x)^m*b*x^4 + (d*x)^m*a*m^2*x + 11*(d*x)^m*a*m*
x + 28*(d*x)^m*a*x)/(m^3 + 12*m^2 + 39*m + 28)
```

3.250 $\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=173

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m
)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(
1 + m))
```

Rubi [A] time = 0.237544, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1375, 364}

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m
)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(
1 + m))
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx &= \frac{c\int \frac{(dx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}\\ &= \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)}-\frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}\end{aligned}$$

Mathematica [C] time = 0.0433496, size = 59, normalized size = 0.34

$$\frac{x(dx)^m\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{{}_2F_1\left(1,m+1;m+2;\frac{x}{\#1}\right)}{\#1^3b+2a}\&\right]}{3(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

```
[Out] (x*(d*x)^m*RootSum[a + b*#1^3 + c*#1^6 & , Hypergeometric2F1[1, 1 + m, 2 +
m, x/#1]/(2*a + b*#1^3) & ])/(3*(1 + m))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^6+b*x^3+a),x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{cx^6+bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

3.251 $\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^2} dx$

Optimal. Leaf size=315

$$\frac{c(dx)^{m+1}\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right){}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{3ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(2-m)\sqrt{b^2-4\right.}{3ad(}$$

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*d*(a + b*x^3 + c
*x^6)) + (c*(b^2*(2 - m) + b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c])])/(3*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m))
 - (c*(b^2*(2 - m) - b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*x)^(1
+ m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c])])/(3*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rubi [A] time = 0.713023, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1366, 1510, 364}

$$\frac{c(dx)^{m+1}\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right){}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{3ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(2-m)\sqrt{b^2-4\right.}{3ad(}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*d*(a + b*x^3 + c
*x^6)) + (c*(b^2*(2 - m) + b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c])])/(3*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m))
 - (c*(b^2*(2 - m) - b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*x)^(1
+ m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c])])/(3*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rule 1366

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(p
+ 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0
] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^2}\,dx = \frac{(dx)^{1+m}\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)d\left(a+bx^3+cx^6\right)} - \frac{\int \frac{(dx)^m\left(-b^2(2-m)+2ac(5-m)-bc(2-m)x^3\right)}{a+bx^3+cx^6}\,dx}{3a\left(b^2-4ac\right)}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)d\left(a+bx^3+cx^6\right)} - \frac{\left(c\left(b^2(2-m)-b\sqrt{b^2-4ac}(2-m)-4ac(5-m)\right)\right)\int \frac{(dx)}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-}}}{6a\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)d\left(a+bx^3+cx^6\right)} + \frac{c\left(b^2(2-m)+b\sqrt{b^2-4ac}(2-m)-4ac(5-m)\right)(dx)^{1+m}\,{}_2F_1}{3a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+}$$

Mathematica [C] time = 0.0783945, size = 78, normalized size = 0.25

$$\frac{x(dx)^m F_1\left(\frac{m+1}{3};2,2;\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{a^2(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] (x*(d*x)^m*AppellF1[(1 + m)/3, 2, 2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(a^2*(1 + m))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^6+bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^6+b*x^3+a)^2,x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^6+bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{c^2x^{12} + 2\,bcx^9 + \left(b^2 + 2\,ac\right)x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6 + 2*a*b*x^3 + a^
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^6 + bx^3 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)
```

3.252 $\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=158

$$\frac{a(dx)^{m+1}\sqrt{a+bx^3+cx^6}F_1\left(\frac{m+1}{3};-\frac{3}{2},-\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -3/2, -3/2, (4
 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.152443, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{a(dx)^{m+1}\sqrt{a+bx^3+cx^6}F_1\left(\frac{m+1}{3};-\frac{3}{2},-\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -3/2, -3/2, (4
 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int (dx)^m \left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2} dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{a(dx)^{1+m}\sqrt{a+bx^3+cx^6}F_1\left(\frac{1+m}{3};-\frac{3}{2},-\frac{3}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.284247, size = 357, normalized size = 2.26

$$\frac{x(dx)^m\sqrt{a+bx^3+cx^6}\left(a\left(m^2+11m+28\right)F_1\left(\frac{m+1}{3};-\frac{1}{2},-\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+(m+1)x^3\left(c(m+4)x^3F_1\left(\frac{m}{}\right.\right.\right.}{(m+1)(m+4)(m+7)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2c}{b-\sqrt{b^2-4ac}}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^3 + c*x^6]*(a*(28 + 11*m + m^2)*AppellF1[(1 + m)/3,
 -1/2, -1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
 Sqrt[b^2 - 4*a*c])] + (1 + m)*x^3*(b*(7 + m)*AppellF1[(4 + m)/3, -1/2, -1/
2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
- 4*a*c])] + c*(4 + m)*x^3*AppellF1[(7 + m)/3, -1/2, -1/2, (10 + m)/3, (-2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))/((1
+ m)*(4 + m)*(7 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^6+bx^3+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} (dx)^m , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

3.253 $\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}F_1\left(\frac{m+1}{3};-\frac{1}{2},-\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4 +
 m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.139226, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}F_1\left(\frac{m+1}{3};-\frac{1}{2},-\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4 +
 m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx = \frac{\sqrt{a+bx^3+cx^6}\int (dx)^m\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{(dx)^{1+m}\sqrt{a+bx^3+cx^6}F_1\left(\frac{1+m}{3};-\frac{1}{2},-\frac{1}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A] time = 0.112619, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\sqrt{a+bx^3+cx^6}F_1\left(\frac{m+1}{3};-\frac{1}{2},-\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4 + m)/
3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int (dx)^m\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6+bx^3+a}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^6+bx^3+a}\,(dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \sqrt{a + bx^3 + cx^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6 + bx^3 + a}\,(dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

3.254 $\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{3};\frac{1}{2},\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^3+cx^6}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 +
m)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.132958, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{3};\frac{1}{2},\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 +
m)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}$$
$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{3};\frac{1}{2},\frac{1}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^3+cx^6}}$$

Mathematica [A] time = 0.1209, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{m+1}{3};\frac{1}{2},\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1
 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)
/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx)^m}{\sqrt{cx^6+bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^6 + bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

3.255 $\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^{3/2}} \, dx$

Optimal. Leaf size=160

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{3};\frac{3}{2},\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^3+cx^6}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*(1
+ m)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.14082, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{3};\frac{3}{2},\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*(1
+ m)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{3};\frac{3}{2},\frac{3}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(1+m)\sqrt{a+bx^3+cx^6}}$$

Mathematica [A] time = 0.176848, size = 221, normalized size = 1.38

$$\frac{x(dx)^m\left(\sqrt{b^2-4ac}-b-2cx^3\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{3/2}F_1\left(\frac{m+1}{3};\frac{3}{2},\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\left(\sqrt{b^2-4ac}-b\right)\left(a+bx^3+cx^6\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]))^(3/2)*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqr
t[b^2 - 4*a*c])*(1 + m)*(a + b*x^3 + c*x^6)^(3/2))
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int (dx)^m\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^6 + bx^3 + a}\,(dx)^m}{c^2x^{12} + 2\,bcx^9 + \left(b^2 + 2\,ac\right)x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a
*c)*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.256 $\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=155

$$\frac{(dx)^{m+1}\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{m+1}{3};-p,-p;\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3,
 (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(
d*(1 + m)*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.101441, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{m+1}{3};-p,-p;\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3,
 (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(
d*(1 + m)*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int (dx)^m \left(1+\frac{2cx^3}{b-\sqrt{b^2-}}\right.$$

$$=\frac{(dx)^{1+m}\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1+m}{3};-p,-p;\frac{4+m}{3};-\frac{2cx}{b-\sqrt{b^2}}\right.}{d(1+m)}$$

Mathematica [A] time = 0.249083, size = 179, normalized size = 1.15

$$\frac{x(dx)^m\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{m+1}{3};-p,-p;\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3, (-2
*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((1 +
 m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqr
t[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^6+bx^3+a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^p (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6+bx^3+a\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

3.257 $\int x^8 \left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=224

$$\frac{2^p\left(2ac-b^2(p+2)\right)\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c^2(p+1)(2p+3)\sqrt{b^2-4ac}}-\frac{b(p+2)\left(a+b\right.}{6c^2(p+1)(}$$

```
[Out] -(b*(2 + p)*(a + b*x^3 + c*x^6)^(1 + p))/(6*c^2*(1 + p)*(3 + 2*p)) + (x^3*(
a + b*x^3 + c*x^6)^(1 + p))/(3*c*(3 + 2*p)) + (2^p*(2*a*c - b^2*(2 + p))*(-
((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3
+ c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(3*c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2
*p))
```

Rubi [A] time = 0.243492, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1357, 742, 640, 624}

$$\frac{2^p\left(2ac-b^2(p+2)\right)\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c^2(p+1)(2p+3)\sqrt{b^2-4ac}}-\frac{b(p+2)\left(a+b\right.}{6c^2(p+1)(}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] -(b*(2 + p)*(a + b*x^3 + c*x^6)^(1 + p))/(6*c^2*(1 + p)*(3 + 2*p)) + (x^3*(
a + b*x^3 + c*x^6)^(1 + p))/(3*c*(3 + 2*p)) + (2^p*(2*a*c - b^2*(2 + p))*(-
((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3
+ c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(3*c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2
*p))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int x^8 \left(a+bx^3+cx^6\right)^p\,dx = \frac{1}{3}\text{Subst}\left(\int x^2\left(a+bx+cx^2\right)^p\,dx, x, x^3\right)$$
$$= \frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)} + \frac{\text{Subst}\left(\int(-a-b(2+p)x)\left(a+bx+cx^2\right)^p\,dx, x, x^3\right)}{3c(3+2p)}$$
$$= -\frac{b(2+p)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)} + \frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)} - \frac{\left(2ac-b^2(2+p)\right)\text{Subst}\left(\int\left(a+bx\right.\right.}{6c^2(3+2p)}$$
$$= -\frac{b(2+p)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)} + \frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)} + \frac{2^p\left(2ac-b^2(2+p)\right)\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right.}{3}$$

Mathematica [C] time = 0.203059, size = 162, normalized size = 0.72

$$\frac{1}{9}x^9\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(3;-p,-p;4;-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^9*(a + b*x^3 + c*x^6)^p*AppellF1[3, -p, -p, 4, (-2*c*x^3)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(9*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^8\left(cx^6+bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^6+bx^3+a\right)^p x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^p x^8, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^p x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

3.258 $\int x^5 \left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=161

$$\frac{b2^p\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c(p+1)\sqrt{b^2-4ac}}+\frac{\left(a+bx^3+cx^6\right)^{p+1}}{6c(p+1)}$$

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(6*c*(1 + p)) + (2^p*b*(-((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hyper
geometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^
2 - 4*a*c])])/(3*c*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Rubi [A] time = 0.116984, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1357, 640, 624}

$$\frac{b2^p\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c(p+1)\sqrt{b^2-4ac}}+\frac{\left(a+bx^3+cx^6\right)^{p+1}}{6c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3 + c*x^6)^p,x]
```


```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(6*c*(1 + p)) + (2^p*b*(-((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hyper
geometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^
2 - 4*a*c])])/(3*c*Sqrt[b^2 - 4*a*c]*(1 + p))
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int x^5\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{3}\operatorname{Subst}\left(\int x\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)$$
$$=\frac{\left(a+bx^3+cx^6\right)^{1+p}}{6c(1+p)}-\frac{b\operatorname{Subst}\left(\int\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)}{6c}$$
$$=\frac{\left(a+bx^3+cx^6\right)^{1+p}}{6c(1+p)}+\frac{2^pb\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}{}_2F_1\left(-p,1+p;2+p;\right.}{3c\sqrt{b^2-4ac}(1+p)}$$

Mathematica [C] time = 0.21815, size = 162, normalized size = 1.01

$$\frac{1}{6}x^6\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(2;-p,-p;3;-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx}{\sqrt{b^2-4a}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^p*AppellF1[2, -p, -p, 3, (-2*c*x^3)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(6*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int x^5\left(cx^6+bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^6+bx^3+a\right)^px^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(cx^6+bx^3+a\right)^px^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^p x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

3.259 $\int x^2 \left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=130

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^3+cx^6\right)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3(p+1)\sqrt{b^2-4ac}}$$

```
[Out] -(2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 -
p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqr
t[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(3*Sqrt[b^2 - 4*a*c]*(1 +
 p))
```

Rubi [A] time = 0.0733837, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1352, 624}

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^3+cx^6\right)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] -(2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 -
p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqr
t[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(3*Sqrt[b^2 - 4*a*c]*(1 +
 p))
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^3+cx^6\right)^p\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)\\ &= -\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}(1+p)}\end{aligned}$$

Mathematica [A] time = 0.099367, size = 138, normalized size = 1.06

$$\frac{2^{p-1}\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\,{}_2F_1\left(-p,p+1;p+2;\frac{-2cx^3-b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(a + b*x^3 + c*x^6)^p*Hyperge
ometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)/(2*Sqrt[b^2
 - 4*a*c])])/(3*c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a
*c])^p)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^2\left(cx^6+bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^6+bx^3+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^6+bx^3+a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

3.260 $\int x^4 \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=138

$$\frac{1}{5}x^5\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.0954244, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$\frac{1}{5}x^5\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^3+cx^6\right)^p\,dx &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int x^4\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p \\ &= \frac{1}{5}x^5\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2}{b-\sqrt{}}\right.\end{aligned}$$

Mathematica [A] time = 0.173094, size = 166, normalized size = 1.2

$$\frac{1}{5}x^5\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(5*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x^4 \left(c x^6 + b x^3 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^4*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^6 + b x^3 + a\right)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c x^6 + b x^3 + a\right)^p x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

3.261 $\int x^3 \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=138

$$\frac{1}{4}x^4\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.$$

```
[Out] (x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.0909034, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$\frac{1}{4}x^4\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^3+cx^6\right)^p\,dx &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int x^3\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right.\\ &= \frac{1}{4}x^4\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{\ }{b-}\right.\end{aligned}$$

Mathematica [A] time = 0.182851, size = 166, normalized size = 1.2

$$\frac{1}{4}x^4\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2c}{\sqrt{b^2-4}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int x^{3}\left(c x^{6}+b x^{3}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^3*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^{6}+b x^{3}+a\right)^{p} x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c x^{6}+b x^{3}+a\right)^{p} x^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

3.262 $\int x\left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=138

$$\frac{1}{2}x^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.0699415, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1385, 510}

$$\frac{1}{2}x^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x\left(a+bx^3+cx^6\right)^p\,dx=\left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p(1$$

$$=\frac{1}{2}x^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2c}{b-\sqrt{b}}\right.$$

Mathematica [A] time = 0.193634, size = 166, normalized size = 1.2

$$\frac{1}{2}x^2\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(2*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int x \left(c x^{6}+b x^{3}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^{6}+b x^{3}+a\right)^{p} x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c x^{6}+b x^{3}+a\right)^{p} x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

3.263 $\int \left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=133

$$x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b - Sqrt[b^
2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.0588891, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1348, 429}

$$x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b - Sqrt[b^
2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+bx^3+cx^6\right)^p\,dx=\left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1\right.$$

$$=x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx}{b-\sqrt{b^2}}\right.$$

Mathematica [A] time = 0.178785, size = 161, normalized size = 1.21

$$x\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \left(c x^6 + b x^3 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^6 + b x^3 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c x^6 + b x^3 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p, x)
```

3.264 $\int \frac{(a+bx^3+cx^6)^p}{x} \, dx$

Optimal. Leaf size=157

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3p}$$

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -(b - S
qrt[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*p*((b
- Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(c*x^3))^p)
```

Rubi [A] time = 0.141581, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1357, 758, 133}

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3p}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -(b - S
qrt[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*p*((b
- Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(c*x^3))^p)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^p}{x}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^p}{x}\,dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\left(2^{2p}\left(\frac{1}{x^3}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\text{Sub}$$

$$= \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2\ldots}}{2cx\ldots}\right.}{3p}$$

Mathematica [A] time = 0.204122, size = 157, normalized size = 1.

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{\sqrt{b^2-4ac}-b}{2cx^3}\right)}{3p}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -(b + S
qrt[b^2 - 4*a*c])/(2*c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*p*((b
- Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(c*x^3))^p)
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

3.265 $\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx$

Optimal. Leaf size=136

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

```
[Out] -(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Rubi [A] time = 0.0891608, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^2,x]
```

```
[Out] -(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3+cx^6\right)^p}{x^2}\,dx &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^p}{x^2}\,dx\\ &= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.175245, size = 164, normalized size = 1.21

$$-\frac{\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^2,x]
```

```
[Out] -(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^2,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^6 + bx^3 + a)^p}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

3.266 $\int \frac{\left(a+bx^3+cx^6\right)^p}{x^3}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b - Sqrt[b^
2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*(1 + (2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.0893756, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^3,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b - Sqrt[b^
2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*(1 + (2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^p}{x^3}\,dx = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.}{x^3}$$

$$= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.}{2x^2}$$

Mathematica [A] time = 0.199373, size = 166, normalized size = 1.2

$$-\frac{\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^3,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(2*x^2*((b - Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^3,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

3.267 $\int \frac{(a+bx^3+cx^6)^p}{x^4} dx$

Optimal. Leaf size=164

$$-\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-2p)x^3}$$

```
[Out] -(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p), -(b - Sqrt
[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*(1 - 2*p)
*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(c*x^3))^p)
```

Rubi [A] time = 0.126654, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1357, 758, 133}

$$-\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-2p)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^4,x]
```

```
[Out] -(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p), -(b - Sqrt
[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*(1 - 2*p)
*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(c*x^3))^p)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^p}{x^4}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^p}{x^2}\,dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\left(2^{2p}\left(\frac{1}{x^3}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\operatorname{Subst}\right.$$

$$= -\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4a}}{2cx^3}\right.}{3(1-2p)x^3}$$

Mathematica [A] time = 0.215915, size = 162, normalized size = 0.99

$$\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(1-2p;-p,-p;2-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{\sqrt{b^2-4ac}-b}{2cx^3}\right)}{3(2p-1)x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^4,x]
```

```
[Out] (4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -(b + Sqrt[b^
2 - 4*a*c])/(2*c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*(-1 + 2*p)*x
^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] +
2*c*x^3)/(c*x^3))^p)
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

3.268 $\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*x^4*(1 + (2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.090244, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^5,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*x^4*(1 + (2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3+cx^6\right)^p}{x^5}\,dx &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2c}{b+\sqrt{b^2}}\right.}{x^5}\\ &= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4a}}\right.}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.200615, size = 166, normalized size = 1.2

$$-\frac{\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{4x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^5,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*x^4*((b - Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^5,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^6 + bx^3 + a)^p}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

3.269 $\int \frac{\left(a+bx^3+cx^6\right)^p}{x^6} \, dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*x^5*(1 + (2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.0886634, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^6,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*x^5*(1 + (2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3+cx^6\right)^p}{x^6} \, dx &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^p}{x^6} \, dx\\ &= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.17918, size = 166, normalized size = 1.2

$$-\frac{\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{5x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^6,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(5*x^5*((b - Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^6,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

3.270 $\int \frac{(a+bx^3+cx^6)^p}{x^7} dx$

Optimal. Leaf size=168

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-p)x^6}$$

```
[Out] -(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2*(1 - p), -p, -p, 3 - 2*p, -
(b - Sqrt[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*
(1 - p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rubi [A] time = 0.126097, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1357, 758, 133}

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-p)x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^7,x]
```


```
[Out] -(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2*(1 - p), -p, -p, 3 - 2*p, -
(b - Sqrt[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*
(1 - p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(c*x^3))^p)
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^p}{x^7}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^p}{x^3}\,dx, x, x^3\right)$$
$$= -\left(\frac{1}{3}\left(2^{2p}\left(\frac{1}{x^3}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\text{Sub}\right.$$
$$= -\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(2(1-p);-p,-p;3-2p;-\right.}{3(1-p)x^6}$$

Mathematica [A] time = 0.233864, size = 164, normalized size = 0.98

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(2-2p;-p,-p;3-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{\sqrt{b^2-4ac}-b}{2cx^3}\right)}{3(p-1)x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^7,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -(b
+ Sqrt[b^2 - 4*a*c])/(2*c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*(-1
 + p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(c*x^3))^p)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^7,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

3.271 $\int \frac{x^m}{1+2x^4+x^8} dx$

Optimal. Leaf size=32

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};-x^4\right)}{m+1}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, -x^4])/(1 + m)
```

Rubi [A] time = 0.0077789, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};-x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 + 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, -x^4])/(1 + m)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{1+2x^4+x^8}\,dx &= \int \frac{x^m}{\left(1+x^4\right)^2}\,dx\\ &= \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{4};\frac{5+m}{4};-x^4\right)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.006883, size = 34, normalized size = 1.06

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+1}{4}+1;-x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(1 + 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, -x^4])/(1 + m)
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 + 2x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8+2*x^4+1),x)
```

```
[Out] int(x^m/(x^8+2*x^4+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 + 2x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{x^8 + 2x^4 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 + 2*x^4 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(x^4 + 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8+2*x**4+1),x)
```

```
[Out] Integral(x**m/(x**4 + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 + 2\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

3.272 $\int \frac{x^9}{1+2x^4+x^8} dx$

Optimal. Leaf size=30

$$-\frac{x^6}{4\left(x^4+1\right)}+\frac{3x^2}{4}-\frac{3}{4}\tan^{-1}\left(x^2\right)$$

```
[Out] (3*x^2)/4 - x^6/(4*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rubi [A] time = 0.0123403, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 288, 321, 203}

$$-\frac{x^6}{4\left(x^4+1\right)}+\frac{3x^2}{4}-\frac{3}{4}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + 2*x^4 + x^8),x]

[Out] (3*x^2)/4 - x^6/(4*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{1+2x^4+x^8}\,dx &= \int \frac{x^9}{\left(1+x^4\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, x^2\right) \\
&= -\frac{x^6}{4\left(1+x^4\right)} + \frac{3}{4}\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, x^2\right) \\
&= \frac{3x^2}{4} - \frac{x^6}{4\left(1+x^4\right)} - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\
&= \frac{3x^2}{4} - \frac{x^6}{4\left(1+x^4\right)} - \frac{3}{4}\tan^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0143612, size = 24, normalized size = 0.8

$$\frac{1}{4}\left(x^2\left(\frac{1}{x^4+1}+2\right)-3\tan^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + 2*x^4 + x^8),x]
```

```
[Out] (x^2*(2 + (1 + x^4)^(-1)) - 3*ArcTan[x^2])/4
```

Maple [A] time = 0.006, size = 25, normalized size = 0.8

$$\frac{x^2}{2}+\frac{x^2}{4\,x^4+4}-\frac{3\,\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+2*x^4+1),x)
```

```
[Out] 1/2*x^2+1/4*x^2/(x^4+1)-3/4*arctan(x^2)
```

Maxima [A] time = 1.50002, size = 32, normalized size = 1.07

$$\frac{1}{2}\,x^2+\frac{x^2}{4\left(x^4+1\right)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

Fricas [A] time = 1.46552, size = 77, normalized size = 2.57

$$\frac{2\,x^6+3\,x^2-3\left(x^4+1\right)\arctan\left(x^2\right)}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x^6 + 3*x^2 - 3*(x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] time = 0.127484, size = 22, normalized size = 0.73

$$\frac{x^2}{2}+\frac{x^2}{4x^4+4}-\frac{3\,\mathrm{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+2*x**4+1),x)
```

```
[Out] x**2/2 + x**2/(4*x**4 + 4) - 3*atan(x**2)/4
```

Giac [A] time = 1.11941, size = 32, normalized size = 1.07

$$\frac{1}{2}\,x^2+\frac{x^2}{4\left(x^4+1\right)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

3.273 $\int \frac{x^7}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=22

$$\frac{1}{4\left(x^4+1\right)}+\frac{1}{4}\log\left(x^4+1\right)$$

```
[Out] 1/(4*(1 + x^4)) + Log[1 + x^4]/4
```

Rubi [A] time = 0.01037, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 43}

$$\frac{1}{4\left(x^4+1\right)}+\frac{1}{4}\log\left(x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 + x^4)) + Log[1 + x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1+2x^4+x^8}\,dx &= \int \frac{x^7}{\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(1+x)^2}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)dx,x,x^4\right)\\
&= \frac{1}{4\left(1+x^4\right)}+\frac{1}{4}\log\left(1+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0066118, size = 18, normalized size = 0.82

$$\frac{1}{4}\left(\frac{1}{x^4+1}+\log\left(x^4+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((1 + x^4)^(-1) + Log[1 + x^4])/4
```

Maple [A] time = 0.007, size = 19, normalized size = 0.9

$$\frac{1}{4x^4+4}+\frac{\ln\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+2*x^4+1),x)
```

```
[Out] 1/4/(x^4+1)+1/4*ln(x^4+1)
```

Maxima [A] time = 0.971477, size = 24, normalized size = 1.09

$$\frac{1}{4\left(x^4+1\right)}+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

Fricas [A] time = 1.43578, size = 59, normalized size = 2.68

$$\frac{\left(x^4+1\right)\log\left(x^4+1\right)+1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*((x^4 + 1)*log(x^4 + 1) + 1)/(x^4 + 1)
```

Sympy [A] time = 0.110301, size = 15, normalized size = 0.68

$$\frac{\log\left(x^4+1\right)}{4}+\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+2*x**4+1),x)
```

```
[Out] log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

Giac [A] time = 1.12836, size = 24, normalized size = 1.09

$$\frac{1}{4\left(x^4+1\right)}+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

3.274 $\int \frac{x^5}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=23

$$\frac{1}{4}\tan^{-1}\left(x^2\right)-\frac{x^2}{4\left(x^4+1\right)}$$

```
[Out] -x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rubi [A] time = 0.0093071, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {28, 275, 288, 203}

$$\frac{1}{4}\tan^{-1}\left(x^2\right)-\frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{1+2x^4+x^8}\,dx &= \int \frac{x^5}{\left(1+x^4\right)^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx,x,x^2\right)\\ &= -\frac{x^2}{4\left(1+x^4\right)}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)\\ &= -\frac{x^2}{4\left(1+x^4\right)}+\frac{1}{4}\tan^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0096704, size = 23, normalized size = 1.

$$\frac{1}{4}\tan^{-1}\left(x^2\right)-\frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Maple [A] time = 0.004, size = 20, normalized size = 0.9

$$-\frac{x^2}{4\,x^4+4}+\frac{\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+2*x^4+1),x)
```

```
[Out] -1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Maxima [A] time = 1.53714, size = 26, normalized size = 1.13

$$-\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Fricas [A] time = 1.42827, size = 62, normalized size = 2.7

$$-\frac{x^2-\left(x^4+1\right)\arctan\left(x^2\right)}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*(x^2 - (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] time = 0.121301, size = 15, normalized size = 0.65

$$-\frac{x^2}{4x^4+4}+\frac{\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8+2*x**4+1),x)
```

```
[Out] -x**2/(4*x**4 + 4) + atan(x**2)/4
```

Giac [A] time = 1.09713, size = 26, normalized size = 1.13

$$-\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

3.275 $\int \frac{x^3}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{4(x^4+1)}$$

[Out] -1/(4*(1 + x^4))

Rubi [A] time = 0.0028654, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 261}

$$-\frac{1}{4(x^4+1)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 + 2*x^4 + x^8),x]

[Out] -1/(4*(1 + x^4))

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1+2x^4+x^8}\,dx &= \int \frac{x^3}{\left(1+x^4\right)^2}\,dx\\ &= -\frac{1}{4\left(1+x^4\right)}\end{aligned}$$

Mathematica [A] time = 0.0020691, size = 11, normalized size = 1.

$$-\frac{1}{4(x^4+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(1 + 2*x^4 + x^8),x]

[Out] -1/(4*(1 + x^4))

Maple [A] time = 0.004, size = 10, normalized size = 0.9

$$-\frac{1}{4 x^4 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+2*x^4+1),x)
```

```
[Out] -1/4/(x^4+1)
```

Maxima [A] time = 0.959297, size = 12, normalized size = 1.09

$$-\frac{1}{4 \left(x^4 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 + 1)
```

Fricas [A] time = 1.40466, size = 22, normalized size = 2.

$$-\frac{1}{4 \left(x^4 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4/(x^4 + 1)
```

Sympy [A] time = 0.099485, size = 8, normalized size = 0.73

$$-\frac{1}{4x^4 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+2*x**4+1),x)
```

```
[Out] -1/(4*x**4 + 4)
```

Giac [A] time = 1.10829, size = 12, normalized size = 1.09

$$-\frac{1}{4 \left(x^4 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4/(x^4 + 1)
```

3.276 $\int \frac{x}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\tan^{-1}\left(x^2\right)$$

```
[Out] x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rubi [A] time = 0.0070609, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {28, 275, 199, 203}

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+2x^4+x^8}\,dx &= \int \frac{x}{\left(1+x^4\right)^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{4\left(1+x^4\right)}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{4\left(1+x^4\right)}+\frac{1}{4}\tan^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0058095, size = 20, normalized size = 0.87

$$\frac{1}{4}\left(\frac{x^2}{x^4+1}+\tan^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + 2*x^4 + x^8),x]
```

```
[Out] (x^2/(1 + x^4) + ArcTan[x^2])/4
```

Maple [A] time = 0.004, size = 20, normalized size = 0.9

$$\frac{x^2}{4\,x^4+4}+\frac{\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+2*x^4+1),x)
```

```
[Out] 1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Maxima [A] time = 1.45571, size = 26, normalized size = 1.13

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Fricas [A] time = 1.4642, size = 61, normalized size = 2.65

$$\frac{x^2+\left(x^4+1\right)\arctan\left(x^2\right)}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*(x^2 + (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] time = 0.121276, size = 15, normalized size = 0.65

$$\frac{x^2}{4x^4+4}+\frac{\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+2*x**4+1),x)
```

```
[Out] x**2/(4*x**4 + 4) + atan(x**2)/4
```

Giac [A] time = 1.09231, size = 26, normalized size = 1.13

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

3.277 $\int \frac{1}{x(1+2x^4+x^8)} \, dx$

Optimal. Leaf size=24

$$\frac{1}{4(x^4+1)} - \frac{1}{4}\log\left(x^4+1\right) + \log(x)$$

```
[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Rubi [A] time = 0.0118203, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 44}

$$\frac{1}{4(x^4+1)} - \frac{1}{4}\log\left(x^4+1\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + 2*x^4 + x^8)),x]
```

```
[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1+2x^4+x^8\right)} \, dx &= \int \frac{1}{x\left(1+x^4\right)^2} \, dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1+x)^2} \, dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{-1-x} + \frac{1}{x} - \frac{1}{(1+x)^2}\right) dx, x, x^4\right) \\
&= \frac{1}{4\left(1+x^4\right)} + \log(x) - \frac{1}{4}\log\left(1+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0094163, size = 24, normalized size = 1.

$$\frac{1}{4\left(x^4+1\right)}-\frac{1}{4}\log\left(x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + 2*x^4 + x^8)),x]
```

```
[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Maple [A] time = 0.013, size = 21, normalized size = 0.9

$$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+2*x^4+1),x)
```

```
[Out] 1/4/(x^4+1)+ln(x)-1/4*ln(x^4+1)
```

Maxima [A] time = 1.03468, size = 32, normalized size = 1.33

$$\frac{1}{4\left(x^4+1\right)}-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

Fricas [A] time = 1.41221, size = 89, normalized size = 3.71

$$-\frac{\left(x^4+1\right)\log\left(x^4+1\right)-4\left(x^4+1\right)\log(x)-1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*((x^4 + 1)*log(x^4 + 1) - 4*(x^4 + 1)*log(x) - 1)/(x^4 + 1)
```

Sympy [A] time = 0.131085, size = 19, normalized size = 0.79

$$\log(x)-\frac{\log\left(x^4+1\right)}{4}+\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+2*x**4+1),x)
```

```
[Out] log(x) - log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

Giac [A] time = 1.10797, size = 39, normalized size = 1.62

$$\frac{x^4+2}{4\left(x^4+1\right)}-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(x^4 + 2)/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

3.278 $\int \frac{1}{x^3(1+2x^4+x^8)} \, dx$

Optimal. Leaf size=30

$$\frac{1}{4x^2(x^4+1)} - \frac{3}{4x^2} - \frac{3}{4}\tan^{-1}(x^2)$$

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rubi [A] time = 0.0123191, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 290, 325, 203}

$$\frac{1}{4x^2(x^4+1)} - \frac{3}{4x^2} - \frac{3}{4}\tan^{-1}(x^2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + 2*x^4 + x^8)),x]

[Out] -3/(4*x^2) + 1/(4*x^2*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^3\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{1}{4x^2\left(1+x^4\right)}+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{3}{4x^2}+\frac{1}{4x^2\left(1+x^4\right)}-\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)\\
&= -\frac{3}{4x^2}+\frac{1}{4x^2\left(1+x^4\right)}-\frac{3}{4}\tan^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0115794, size = 30, normalized size = 1.

$$-\frac{x^2}{4\left(x^4+1\right)}-\frac{1}{2x^2}+\frac{3}{4}\tan^{-1}\left(\frac{1}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - x^2/(4*(1 + x^4)) + (3*ArcTan[x^(-2)])/4
```

Maple [A] time = 0.008, size = 25, normalized size = 0.8

$$-\frac{1}{2\,x^2}-\frac{x^2}{4\,x^4+4}-\frac{3\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+2*x^4+1),x)
```

```
[Out] -1/2/x^2-1/4*x^2/(x^4+1)-3/4*arctan(x^2)
```

Maxima [A] time = 1.49736, size = 34, normalized size = 1.13

$$-\frac{3\,x^4+2}{4\left(x^6+x^2\right)}-\frac{3}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

Fricas [A] time = 1.4231, size = 78, normalized size = 2.6

$$-\frac{3\,x^4+3\,\left(x^6+x^2\right)\arctan\left(x^2\right)+2}{4\,\left(x^6+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*(3*x^4 + 3*(x^6 + x^2)*arctan(x^2) + 2)/(x^6 + x^2)
```

Sympy [A] time = 0.1484, size = 26, normalized size = 0.87

$$-\frac{3x^4+2}{4x^6+4x^2}-\frac{3\,\mathrm{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+2*x**4+1),x)
```

```
[Out] -(3*x**4 + 2)/(4*x**6 + 4*x**2) - 3*atan(x**2)/4
```

Giac [A] time = 1.11739, size = 34, normalized size = 1.13

$$-\frac{3\,x^4+2}{4\,\left(x^6+x^2\right)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

3.279 $\int \frac{1}{x^5(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{4\left(x^4+1\right)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

```
[Out] -1/(4*x^4) - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Rubi [A] time = 0.0162449, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 44}

$$-\frac{1}{4\left(x^4+1\right)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + 2*x^4 + x^8)),x]

[Out] -1/(4*x^4) - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^5\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(1+x)^2}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{2}{x}+\frac{1}{(1+x)^2}+\frac{2}{1+x}\right)dx, x, x^4\right)\\
&= -\frac{1}{4x^4}-\frac{1}{4\left(1+x^4\right)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0118251, size = 33, normalized size = 1.

$$-\frac{1}{4\left(x^4+1\right)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Maple [A] time = 0.013, size = 28, normalized size = 0.9

$$-\frac{1}{4\,x^4}-\frac{1}{4\,x^4+4}-2\,\ln(x)+\frac{\ln\left(x^4+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+2*x^4+1),x)
```

```
[Out] -1/4/x^4-1/4/(x^4+1)-2*ln(x)+1/2*ln(x^4+1)
```

Maxima [A] time = 1.00189, size = 45, normalized size = 1.36

$$-\frac{2\,x^4+1}{4\left(x^8+x^4\right)}+\frac{1}{2}\,\log\left(x^4+1\right)-\frac{1}{2}\,\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

Fricas [A] time = 1.43895, size = 111, normalized size = 3.36

$$-\frac{2\,x^4-2\left(x^8+x^4\right)\log\left(x^4+1\right)+8\left(x^8+x^4\right)\log(x)+1}{4\left(x^8+x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^4 - 2*(x^8 + x^4)*log(x^4 + 1) + 8*(x^8 + x^4)*log(x) + 1)/(x^8 +
 x^4)
```

Sympy [A] time = 0.158586, size = 29, normalized size = 0.88

$$-\frac{2x^4+1}{4x^8+4x^4}-2\log(x)+\frac{\log\left(x^4+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+2*x**4+1),x)
```

```
[Out] -(2*x**4 + 1)/(4*x**8 + 4*x**4) - 2*log(x) + log(x**4 + 1)/2
```

Giac [A] time = 1.07711, size = 45, normalized size = 1.36

$$-\frac{2x^4+1}{4\left(x^8+x^4\right)}+\frac{1}{2}\log\left(x^4+1\right)-\frac{1}{2}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

3.280 $\int \frac{1}{x^7(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{4x^6(x^4+1)}+\frac{5}{4x^2}-\frac{5}{12x^6}+\frac{5}{4}\tan^{-1}(x^2)$$

```
[Out] -5/(12*x^6) + 5/(4*x^2) + 1/(4*x^6*(1 + x^4)) + (5*ArcTan[x^2])/4
```

Rubi [A] time = 0.0157296, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 290, 325, 203}

$$\frac{1}{4x^6(x^4+1)}+\frac{5}{4x^2}-\frac{5}{12x^6}+\frac{5}{4}\tan^{-1}(x^2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + 2*x^4 + x^8)),x]

[Out] -5/(12*x^6) + 5/(4*x^2) + 1/(4*x^6*(1 + x^4)) + (5*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^7\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{1}{4x^6\left(1+x^4\right)}+\frac{5}{4}\operatorname{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{5}{12x^6}+\frac{1}{4x^6\left(1+x^4\right)}-\frac{5}{4}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{1}{4x^6\left(1+x^4\right)}+\frac{5}{4}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)\\
&= -\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{1}{4x^6\left(1+x^4\right)}+\frac{5}{4}\tan^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0107823, size = 33, normalized size = 0.89

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{x^2}-\frac{1}{6x^6}-\frac{5}{4}\tan^{-1}\left(\frac{1}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) + x^(-2) + x^2/(4*(1 + x^4)) - (5*ArcTan[x^(-2)])/4
```

Maple [A] time = 0.011, size = 28, normalized size = 0.8

$$-\frac{1}{6x^6}+x^{-2}+\frac{x^2}{4x^4+4}+\frac{5\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+2*x^4+1),x)
```

```
[Out] -1/6/x^6+1/x^2+1/4*x^2/(x^4+1)+5/4*arctan(x^2)
```

Maxima [A] time = 1.4855, size = 41, normalized size = 1.11

$$\frac{15x^8+10x^4-2}{12\left(x^{10}+x^6\right)}+\frac{5}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*(15*x^8 + 10*x^4 - 2)/(x^10 + x^6) + 5/4*arctan(x^2)
```

Fricas [A] time = 1.42714, size = 96, normalized size = 2.59

$$\frac{15\,x^8+10\,x^4+15\left(x^{10}+x^6\right)\arctan\left(x^2\right)-2}{12\left(x^{10}+x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*(15*x^8 + 10*x^4 + 15*(x^10 + x^6)*arctan(x^2) - 2)/(x^10 + x^6)
```

Sympy [A] time = 0.18952, size = 29, normalized size = 0.78

$$\frac{5\,\mathrm{atan}\left(x^2\right)}{4}+\frac{15x^8+10x^4-2}{12x^{10}+12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+2*x**4+1),x)
```

```
[Out] 5*atan(x**2)/4 + (15*x**8 + 10*x**4 - 2)/(12*x**10 + 12*x**6)
```

Giac [A] time = 1.14105, size = 42, normalized size = 1.14

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{6\,x^4-1}{6\,x^6}+\frac{5}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^2/(x^4 + 1) + 1/6*(6*x^4 - 1)/x^6 + 5/4*arctan(x^2)
```

3.281 $\int \frac{x^8}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=104

$$-\frac{x^5}{4\left(x^4+1\right)}+\frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{5x}{4}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] (5*x)/4 - x^5/(4*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*Ar
cTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]
) - (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi [A] time = 0.054602, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 288, 321, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^5}{4\left(x^4+1\right)}+\frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{5x}{4}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] (5*x)/4 - x^5/(4*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*Ar
cTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]
) - (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{1+2x^4+x^8}\,dx &= \int \frac{x^8}{\left(1+x^4\right)^2}\,dx\\
&= -\frac{x^5}{4\left(1+x^4\right)}+\frac{5}{4}\int \frac{x^4}{1+x^4}\,dx\\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}-\frac{5}{4}\int \frac{1}{1+x^4}\,dx\\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}-\frac{5}{8}\int \frac{1-x^2}{1+x^4}\,dx-\frac{5}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}-\frac{5}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx-\frac{5}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx+\frac{5\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}}+\frac{5\int \frac{\sqrt{2}-}{-1+\sqrt{2}}}{16\sqrt{}}\\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}+\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{8\sqrt{2}}\\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}\right.}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0704203, size = 94, normalized size = 0.9

$$\frac{1}{32}\left(\frac{8x}{x^4+1}+5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+32x+10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)-10\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] (32*x + (8*x)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*Arc
Tan[1 + Sqrt[2]*x] + 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 5*Sqrt[2]*Log[1 +
 Sqrt[2]*x + x^2])/32
```

Maple [A] time = 0.007, size = 69, normalized size = 0.7

$$x+\frac{x}{4x^4+4}-\frac{5\sqrt{2}}{32}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)-\frac{5\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{16}-\frac{5\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+2*x^4+1),x)
```

```
[Out] x+1/4*x/(x^4+1)-5/32*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))-5/16*a
rctan(1+x*2^(1/2))*2^(1/2)-5/16*arctan(-1+x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.49095, size = 112, normalized size = 1.08

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{5}{32}\sqrt{2}\log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

Fricas [A] time = 1.51484, size = 392, normalized size = 3.77

$$\frac{32x^5+20\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+20\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}\right.}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/32*(32*x^5 + 20*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 +
sqrt(2)*x + 1) - 1) + 20*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt
```

```
(x^2 - sqrt(2)*x + 1) + 1) - 5*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) +
 5*sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1) + 40*x)/(x^4 + 1)
```

Sympy [A] time = 0.185903, size = 90, normalized size = 0.87

$$x + \frac{x}{4x^4 + 4} + \frac{5\sqrt{2}\log\left(x^2 - \sqrt{2}x + 1\right)}{32} - \frac{5\sqrt{2}\log\left(x^2 + \sqrt{2}x + 1\right)}{32} - \frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x - 1\right)}{16} - \frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x + 1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+2*x**4+1),x)
```

```
[Out] x + x/(4*x**4 + 4) + 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 5*sqrt(2)*log
(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*sqrt(2)*at
an(sqrt(2)*x + 1)/16
```

Giac [A] time = 1.10433, size = 112, normalized size = 1.08

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x + \sqrt{2}\right)\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x - \sqrt{2}\right)\right) - \frac{5}{32}\sqrt{2}\log\left(x^2 + \sqrt{2}x + 1\right) + \frac{5}{32}\sqrt{2}\log\left(x^2 - \sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

3.282 $\int \frac{x^6}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=99

$$-\frac{x^3}{4\left(x^4+1\right)}+\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -x^3/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 +
Sqrt[2]*x])/(8*Sqrt[2]) + (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) - (3*Lo
g[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi [A] time = 0.0509516, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {28, 288, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^3}{4\left(x^4+1\right)}+\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x^3/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 +
Sqrt[2]*x])/(8*Sqrt[2]) + (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) - (3*Lo
g[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{1+2x^4+x^8}\,dx &= \int \frac{x^6}{\left(1+x^4\right)^2}\,dx \\
&= -\frac{x^3}{4\left(1+x^4\right)} + \frac{3}{4}\int \frac{x^2}{1+x^4}\,dx \\
&= -\frac{x^3}{4\left(1+x^4\right)} - \frac{3}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{3}{8}\int \frac{1+x^2}{1+x^4}\,dx \\
&= -\frac{x^3}{4\left(1+x^4\right)} + \frac{3}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{3}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \frac{3\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} + \frac{3\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}}{16\sqrt{2}} \\
&= -\frac{x^3}{4\left(1+x^4\right)} + \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} - \\
&= -\frac{x^3}{4\left(1+x^4\right)} - \frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}x+\right.}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0595656, size = 93, normalized size = 0.94

$$\frac{1}{32}\left(-\frac{8x^3}{x^4+1}+3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+6\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((-8*x^3)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1
+ Sqrt[2]*x] + 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 3*Sqrt[2]*Log[1 + Sqrt[
```

```
2]*x + x^2])/32
```

Maple [A] time = 0.006, size = 70, normalized size = 0.7

$$-\frac{x^3}{4x^4+4}+\frac{3\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{16}+\frac{3\sqrt{2}}{32}\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+\frac{3\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+2*x^4+1),x)
```

```
[Out] -1/4*x^3/(x^4+1)+3/16*arctan(-1+x*2^(1/2))*2^(1/2)+3/32*2^(1/2)*ln((1+x^2-x
*2^(1/2))/(1+x^2+x*2^(1/2)))+3/16*arctan(1+x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.45502, size = 113, normalized size = 1.14

$$-\frac{x^3}{4\left(x^4+1\right)}+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{3}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A] time = 1.64108, size = 382, normalized size = 3.86

$$-\frac{8x^3+12\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+12\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}\right.}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(8*x^3 + 12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 +
sqrt(2)*x + 1) - 1) + 12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt
(x^2 - sqrt(2)*x + 1) + 1) + 3*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) -
 3*sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1))/(x^4 + 1)
```

Sympy [A] time = 0.177453, size = 90, normalized size = 0.91

$$-\frac{x^3}{4x^4+4}+\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 + 4) + 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 3*sqrt(2)*log
(x**2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*at
an(sqrt(2)*x + 1)/16
```

Giac [A] time = 1.11589, size = 113, normalized size = 1.14

$$-\frac{x^3}{4\left(x^4+1\right)} + \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{3}{32}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

3.283 $\int \frac{x^4}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=97

$$-\frac{x}{4\left(x^4+1\right)}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -x/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x
]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) + Log[1 + Sqrt[2]*x +
 x^2]/(16*Sqrt[2])
```

Rubi [A] time = 0.0511466, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {28, 288, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x}{4\left(x^4+1\right)}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x
]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) + Log[1 + Sqrt[2]*x +
 x^2]/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{1+2x^4+x^8}\,dx &= \int \frac{x^4}{\left(1+x^4\right)^2}\,dx\\
&= -\frac{x}{4\left(1+x^4\right)}+\frac{1}{4}\int \frac{1}{1+x^4}\,dx\\
&= -\frac{x}{4\left(1+x^4\right)}+\frac{1}{8}\int \frac{1-x^2}{1+x^4}\,dx+\frac{1}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{x}{4\left(1+x^4\right)}+\frac{1}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{1}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx-\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}}-\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{16\sqrt{2}}\\
&= -\frac{x}{4\left(1+x^4\right)}-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{8\sqrt{2}}-\text{Subst}\\
&= -\frac{x}{4\left(1+x^4\right)}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}}-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0719407, size = 90, normalized size = 0.93

$$\frac{1}{32}\left(-\frac{8x}{x^4+1}-\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+2\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((-8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x +
```

```
x^2])/32
```

Maple [A] time = 0.005, size = 68, normalized size = 0.7

$$-\frac{x}{4x^4+4}+\frac{\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{16}+\frac{\sqrt{2}}{32}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+2*x^4+1),x)
```

```
[Out] -1/4*x/(x^4+1)+1/16*arctan(-1+x*2^(1/2))*2^(1/2)+1/32*2^(1/2)*ln((1+x^2+x*2
^(1/2))/(1+x^2-x*2^(1/2)))+1/16*arctan(1+x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.46151, size = 111, normalized size = 1.14

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{32}\sqrt{2}\log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

Fricas [A] time = 1.80504, size = 371, normalized size = 3.82

$$-\frac{4\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+4\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\sqrt{}}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x
 + 1) - 1) + 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqr
t(2)*x + 1) + 1) - sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + sqrt(2)*(x^
4 + 1)*log(x^2 - sqrt(2)*x + 1) + 8*x)/(x^4 + 1)
```

Sympy [A] time = 0.173247, size = 82, normalized size = 0.85

$$-\frac{x}{4x^4+4}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+2*x**4+1),x)
```

```
[Out] -x/(4*x**4 + 4) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + sqrt(2)*log(x**2 +
 sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt(2)*
x + 1)/16
```

Giac [A] time = 1.0967, size = 111, normalized size = 1.14

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

3.284 $\int \frac{x^2}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=99

$$\frac{x^3}{4\left(x^4+1\right)}+\frac{\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] x^3/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*
x]/(8*Sqrt[2]) + Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*x
+ x^2]/(16*Sqrt[2])
```

Rubi [A] time = 0.050228, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {28, 290, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^3}{4\left(x^4+1\right)}+\frac{\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*
x]/(8*Sqrt[2]) + Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*x
+ x^2]/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \int \frac{x^2}{\left(1+x^4\right)^2}\,dx$$

$$= \frac{x^3}{4\left(1+x^4\right)} + \frac{1}{4}\int \frac{x^2}{1+x^4}\,dx$$

$$= \frac{x^3}{4\left(1+x^4\right)} - \frac{1}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{8}\int \frac{1+x^2}{1+x^4}\,dx$$

$$= \frac{x^3}{4\left(1+x^4\right)} + \frac{1}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} + \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{16\sqrt{2}}$$

$$= \frac{x^3}{4\left(1+x^4\right)} + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{x^3}{4\left(1+x^4\right)} - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

Mathematica [A] time = 0.0495853, size = 92, normalized size = 0.93

$$\frac{1}{32}\left(\frac{8x^3}{x^4+1} + \sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - 2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right) + 2\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((8*x^3)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*x
```

```
+ x^2])/32
```

Maple [A] time = 0.005, size = 70, normalized size = 0.7

$$\frac{x^3}{4x^4+4}+\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{16}+\frac{\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{16}+\frac{\sqrt{2}}{32}\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+2*x^4+1),x)
```

```
[Out] 1/4*x^3/(x^4+1)+1/16*arctan(1+x*2^(1/2))*2^(1/2)+1/16*arctan(-1+x*2^(1/2))*
2^(1/2)+1/32*2^(1/2)*ln((1+x^2-x*2^(1/2))/(1+x^2+x*2^(1/2)))
```

Maxima [A] time = 1.4797, size = 113, normalized size = 1.14

$$\frac{x^3}{4\left(x^4+1\right)}+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A] time = 1.65132, size = 373, normalized size = 3.77

$$\frac{8x^3-4\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-4\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 - 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sq
rt(2)*x + 1) - 1) - 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^
2 - sqrt(2)*x + 1) + 1) - sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + sqrt
(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1))/(x^4 + 1)
```

Sympy [A] time = 0.178476, size = 83, normalized size = 0.84

$$\frac{x^3}{4x^4+4}+\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+2*x**4+1),x)
```

```
[Out] x**3/(4*x**4 + 4) + sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - sqrt(2)*log(x**2
 + sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt(2
)*x + 1)/16
```

<u>Giac</u> [A] time = 1.10743, size = 113, normalized size = 1.14

$$\frac{x^3}{4\left(x^4+1\right)} + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{32}\sqrt{2}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

3.285 $\int \frac{1}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=97

$$\frac{x}{4\left(x^4+1\right)}-\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] x/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 + Sqr
t[2]*x])/(8*Sqrt[2]) - (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) + (3*Log[1
 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi [A] time = 0.0469694, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {28, 199, 211, 1165, 628, 1162, 617, 204}

$$\frac{x}{4\left(x^4+1\right)}-\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^4 + x^8)^(-1),x]
```

```
[Out] x/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 + Sqr
t[2]*x])/(8*Sqrt[2]) - (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) + (3*Log[1
 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1+2x^4+x^8}\,dx &= \int \frac{1}{\left(1+x^4\right)^2}\,dx\\
&= \frac{x}{4\left(1+x^4\right)}+\frac{3}{4}\int \frac{1}{1+x^4}\,dx\\
&= \frac{x}{4\left(1+x^4\right)}+\frac{3}{8}\int \frac{1-x^2}{1+x^4}\,dx+\frac{3}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= \frac{x}{4\left(1+x^4\right)}+\frac{3}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{3}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx-\frac{3\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}}-\frac{3\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,d
\\
&= \frac{x}{4\left(1+x^4\right)}-\frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{3\,S
\\
&= \frac{x}{4\left(1+x^4\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}}-\frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}x+x^2\right.}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0495273, size = 91, normalized size = 0.94

$$\frac{1}{32}\left(\frac{8x}{x^4+1}-3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+6\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^4 + x^8)^(-1),x]
```

```
[Out] ((8*x)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1 + S
qrt[2]*x] - 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 3*Sqrt[2]*Log[1 + Sqrt[2]*
```

```
x + x^2])/32
```

Maple [A] time = 0.005, size = 68, normalized size = 0.7

$$\frac{x}{4x^4+4}+\frac{3\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{16}+\frac{3\sqrt{2}}{32}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+\frac{3\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+2*x^4+1),x)
```

```
[Out] 1/4*x/(x^4+1)+3/16*arctan(-1+x*2^(1/2))*2^(1/2)+3/32*2^(1/2)*ln((1+x^2+x*2^
(1/2))/(1+x^2-x*2^(1/2)))+3/16*arctan(1+x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.50572, size = 111, normalized size = 1.14

$$\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{3}{32}\sqrt{2}\log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

Fricas [A] time = 1.5888, size = 379, normalized size = 3.91

$$-\frac{12\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+12\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*
x + 1) - 1) + 12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - s
qrt(2)*x + 1) + 1) - 3*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + 3*sqrt(
2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1) - 8*x)/(x^4 + 1)
```

Sympy [A] time = 0.180556, size = 88, normalized size = 0.91

$$\frac{x}{4x^4+4}-\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+2*x**4+1),x)
```

```
[Out] x/(4*x**4 + 4) - 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 3*sqrt(2)*log(x**
2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*atan(s
qrt(2)*x + 1)/16
```

Giac [A] time = 1.10491, size = 111, normalized size = 1.14

$$\frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)+\frac{3}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{3}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

3.286 $\int \frac{1}{x^2\left(1+2x^4+x^8\right)} \, dx$

Optimal. Leaf size=106

$$\frac{1}{4x\left(x^4+1\right)}-\frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{5}{4x}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -5/(4*x) + 1/(4*x*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*A
rcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2
]) + (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi [A] time = 0.0515349, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 290, 325, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{4x\left(x^4+1\right)}-\frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{5}{4x}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -5/(4*x) + 1/(4*x*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*A
rcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2
]) + (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^2\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4x\left(1+x^4\right)}+\frac{5}{4}\int \frac{1}{x^2\left(1+x^4\right)}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}-\frac{5}{4}\int \frac{x^2}{1+x^4}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}+\frac{5}{8}\int \frac{1-x^2}{1+x^4}\,dx-\frac{5}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}-\frac{5}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx-\frac{5}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx-\frac{5\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}}-\frac{5 \ldots}{}\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}-\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1 \ldots\right.}{8\sqrt{2}}\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{5\log\left(\ldots\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.0699025, size = 98, normalized size = 0.92

$$\frac{1}{32}\left(-\frac{8x^3}{x^4+1}-5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{32}{x}+10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)-10\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x - (8*x^3)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*
ArcTan[1 + Sqrt[2]*x] - 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 5*Sqrt[2]*Log[
1 + Sqrt[2]*x + x^2])/32
```

Maple [A] time = 0.008, size = 75, normalized size = 0.7

$$-x^{-1}-\frac{x^3}{4x^4+4}-\frac{5\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{16}-\frac{5\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{16}-\frac{5\sqrt{2}}{32}\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+2*x^4+1),x)
```

```
[Out] -1/x-1/4*x^3/(x^4+1)-5/16*arctan(1+x*2^(1/2))*2^(1/2)-5/16*arctan(-1+x*2^(1
/2))*2^(1/2)-5/32*2^(1/2)*ln((1+x^2-x*2^(1/2))/(1+x^2+x*2^(1/2)))
```

Maxima [A] time = 1.47945, size = 119, normalized size = 1.12

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{5}{32}\sqrt{2}\log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

Fricas [A] time = 1.56656, size = 390, normalized size = 3.68

$$-\frac{40x^4-20\sqrt{2}\left(x^5+x\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-20\sqrt{2}\left(x^5+x\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+\right.}{32\left(x^5+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(40*x^4 - 20*sqrt(2)*(x^5 + x)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 +
 sqrt(2)*x + 1) - 1) - 20*sqrt(2)*(x^5 + x)*arctan(-sqrt(2)*x + sqrt(2)*sqr
```

```
t(x^2 - sqrt(2)*x + 1) + 1) - 5*sqrt(2)*(x^5 + x)*log(x^2 + sqrt(2)*x + 1)
+ 5*sqrt(2)*(x^5 + x)*log(x^2 - sqrt(2)*x + 1) + 32)/(x^5 + x)
```

Sympy [A] time = 0.213433, size = 95, normalized size = 0.9

$$-\frac{5x^4+4}{4x^5+4x}-\frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}-\frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+2*x**4+1),x)
```

```
[Out] -(5*x**4 + 4)/(4*x**5 + 4*x) - 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 5*s
qrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*
sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Giac [A] time = 1.09023, size = 119, normalized size = 1.12

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

3.287 $\int \frac{1}{x^4(1+2x^4+x^8)} dx$

Optimal. Leaf size=106

$$\frac{1}{4x^3(x^4+1)} - \frac{7}{12x^3} + \frac{7\log(x^2-\sqrt{2}x+1)}{16\sqrt{2}} - \frac{7\log(x^2+\sqrt{2}x+1)}{16\sqrt{2}} + \frac{7\tan^{-1}(1-\sqrt{2}x)}{8\sqrt{2}} - \frac{7\tan^{-1}(\sqrt{2}x+1)}{8\sqrt{2}}$$

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 + x^4)) + (7*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) -
 (7*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (7*Log[1 - Sqrt[2]*x + x^2])/(16*S
qrt[2]) - (7*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi [A] time = 0.0521337, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 290, 325, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{4x^3(x^4+1)} - \frac{7}{12x^3} + \frac{7\log(x^2-\sqrt{2}x+1)}{16\sqrt{2}} - \frac{7\log(x^2+\sqrt{2}x+1)}{16\sqrt{2}} + \frac{7\tan^{-1}(1-\sqrt{2}x)}{8\sqrt{2}} - \frac{7\tan^{-1}(\sqrt{2}x+1)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 + x^4)) + (7*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) -
 (7*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (7*Log[1 - Sqrt[2]*x + x^2])/(16*S
qrt[2]) - (7*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^4\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^3\left(1+x^4\right)}+\frac{7}{4}\int \frac{1}{x^4\left(1+x^4\right)}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}-\frac{7}{4}\int \frac{1}{1+x^4}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}-\frac{7}{8}\int \frac{1-x^2}{1+x^4}\,dx-\frac{7}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}-\frac{7}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx-\frac{7}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx+\frac{7\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}}+\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}+\frac{7\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{7\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{7\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x\right.}{8\sqrt{2}}\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}+\frac{7\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{7\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}}+\frac{7\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}-7\mathrm{l}
\end{aligned}
$$

Mathematica [A] time = 0.0738577, size = 96, normalized size = 0.91

$$\frac{1}{96}\left(-\frac{24x}{x^4+1}-\frac{32}{x^3}+21\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-21\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+42\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)-42\sqrt{2}\tan^{-1}\left(\sqrt{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x^3 - (24*x)/(1 + x^4) + 42*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 42*Sqrt[2]
*ArcTan[1 + Sqrt[2]*x] + 21*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 21*Sqrt[2]*L
og[1 + Sqrt[2]*x + x^2])/96
```

Maple [A] time = 0.008, size = 73, normalized size = 0.7

$$-\frac{1}{3x^3}-\frac{x}{4x^4+4}-\frac{7\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{16}-\frac{7\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{16}-\frac{7\sqrt{2}}{32}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+2*x^4+1),x)
```

```
[Out] -1/3/x^3-1/4*x/(x^4+1)-7/16*arctan(1+x*2^(1/2))*2^(1/2)-7/16*arctan(-1+x*2^
(1/2))*2^(1/2)-7/32*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))
```

Maxima [A] time = 1.49873, size = 122, normalized size = 1.15

$$-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{7}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{7}{32}\sqrt{2}\log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/12*(7*x^4 + 4)/(x^7 + x^3)
```

Fricas [A] time = 1.52469, size = 406, normalized size = 3.83

$$-\frac{56x^4-84\sqrt{2}\left(x^7+x^3\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-84\sqrt{2}\left(x^7+x^3\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x\cdots}\right.}{96\left(x^7+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/96*(56*x^4 - 84*sqrt(2)*(x^7 + x^3)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2
 + sqrt(2)*x + 1) - 1) - 84*sqrt(2)*(x^7 + x^3)*arctan(-sqrt(2)*x + sqrt(2)
```

```
*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 21*sqrt(2)*(x^7 + x^3)*log(x^2 + sqrt(2)*
x + 1) - 21*sqrt(2)*(x^7 + x^3)*log(x^2 - sqrt(2)*x + 1) + 32)/(x^7 + x^3)
```

Sympy [A] time = 0.219176, size = 97, normalized size = 0.92

$$-\frac{7x^4+4}{12x^7+12x^3}+\frac{7\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{7\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}-\frac{7\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{7\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+2*x**4+1),x)
```

```
[Out] -(7*x**4 + 4)/(12*x**7 + 12*x**3) + 7*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32
- 7*sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 7*sqrt(2)*atan(sqrt(2)*x - 1)/16
 - 7*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Giac [A] time = 1.12046, size = 117, normalized size = 1.1

$$-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{7}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{7}{32}\sqrt{2}\log\left(x^2-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1) - 1/3/x^3
```

3.288 $\int \frac{1}{x^6(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=113

$$\frac{1}{4x^5(x^4+1)}-\frac{9}{20x^5}+\frac{9\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{9\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{9}{4x}-\frac{9\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{9\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -9/(20*x^5) + 9/(4*x) + 1/(4*x^5*(1 + x^4)) - (9*ArcTan[1 - Sqrt[2]*x])/(8*
Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (9*Log[1 - Sqrt[2]*x + x
^2])/(16*Sqrt[2]) - (9*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi [A] time = 0.0543694, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 290, 325, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{4x^5(x^4+1)}-\frac{9}{20x^5}+\frac{9\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{9\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{9}{4x}-\frac{9\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{9\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -9/(20*x^5) + 9/(4*x) + 1/(4*x^5*(1 + x^4)) - (9*ArcTan[1 - Sqrt[2]*x])/(8*
Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (9*Log[1 - Sqrt[2]*x + x
^2])/(16*Sqrt[2]) - (9*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^6\left(1+x^4\right)^2}\,dx \\
&= \frac{1}{4x^5\left(1+x^4\right)} + \frac{9}{4}\int \frac{1}{x^6\left(1+x^4\right)}\,dx \\
&= -\frac{9}{20x^5} + \frac{1}{4x^5\left(1+x^4\right)} - \frac{9}{4}\int \frac{1}{x^2\left(1+x^4\right)}\,dx \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5\left(1+x^4\right)} + \frac{9}{4}\int \frac{x^2}{1+x^4}\,dx \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5\left(1+x^4\right)} - \frac{9}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{9}{8}\int \frac{1+x^2}{1+x^4}\,dx \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5\left(1+x^4\right)} + \frac{9}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{9}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \frac{9\int \frac{\sqrt{2}-}{-1-\sqrt{}}}{16\sqrt{}} \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5\left(1+x^4\right)} + \frac{9\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{9\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{9\,\mathrm{Subst}\left(\int \right.}{} \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5\left(1+x^4\right)} - \frac{9\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{9\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{9\log\left(1-\sqrt{2}x+\right.}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0776377, size = 103, normalized size = 0.91

$$
\frac{1}{160}\left(\frac{40x^3}{x^4+1} - \frac{32}{x^5} + 45\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - 45\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{320}{x} - 90\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right) + 90\sqrt{2}\tan\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x^5 + 320/x + (40*x^3)/(1 + x^4) - 90*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] +
90*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + 45*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 45
*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/160
```

Maple [A] time = 0.01, size = 80, normalized size = 0.7

$$
-\frac{1}{5x^5} + 2x^{-1} + \frac{x^3}{4x^4+4} + \frac{9\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{16} + \frac{9\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{16} + \frac{9\sqrt{2}}{32}\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8+2*x^4+1),x)
```

```
[Out] -1/5/x^5+2/x+1/4*x^3/(x^4+1)+9/16*arctan(1+x*2^(1/2))*2^(1/2)+9/16*arctan(-
1+x*2^(1/2))*2^(1/2)+9/32*2^(1/2)*ln((1+x^2-x*2^(1/2))/(1+x^2+x*2^(1/2)))
```

Maxima [A] time = 1.47472, size = 128, normalized size = 1.13

$$
\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{9}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{9}{32}\sqrt{2}\log\left(x^2-\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 9/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/20*(45*x^8 + 36*x^4 - 4)/(x^9 + x^5)
```

Fricas [A] time = 1.57116, size = 424, normalized size = 3.75

$$\frac{360\,x^8 + 288\,x^4 - 180\,\sqrt{2}\left(x^9 + x^5\right)\arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 + \sqrt{2}x + 1} - 1\right) - 180\,\sqrt{2}\left(x^9 + x^5\right)\arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2}\right.}{160\left(x^9 + x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/160*(360*x^8 + 288*x^4 - 180*sqrt(2)*(x^9 + x^5)*arctan(-sqrt(2)*x + sqrt
(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 180*sqrt(2)*(x^9 + x^5)*arctan(-sqrt(2
)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 45*sqrt(2)*(x^9 + x^5)*log(x
^2 + sqrt(2)*x + 1) + 45*sqrt(2)*(x^9 + x^5)*log(x^2 - sqrt(2)*x + 1) - 32)
/(x^9 + x^5)
```

Sympy [A] time = 0.245177, size = 102, normalized size = 0.9

$$\frac{9\sqrt{2}\log\left(x^2 - \sqrt{2}x + 1\right)}{32} - \frac{9\sqrt{2}\log\left(x^2 + \sqrt{2}x + 1\right)}{32} + \frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x - 1\right)}{16} + \frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x + 1\right)}{16} + \frac{45x^8 + 36x^4 - 4}{20x^9 + 20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8+2*x**4+1),x)
```

```
[Out] 9*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 9*sqrt(2)*log(x**2 + sqrt(2)*x + 1
)/32 + 9*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 9*sqrt(2)*atan(sqrt(2)*x + 1)/16
+ (45*x**8 + 36*x**4 - 4)/(20*x**9 + 20*x**5)
```

Giac [A] time = 1.12316, size = 130, normalized size = 1.15

$$\frac{x^3}{4\left(x^4 + 1\right)} + \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x + \sqrt{2}\right)\right) + \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x - \sqrt{2}\right)\right) - \frac{9}{32}\sqrt{2}\log\left(x^2 + \sqrt{2}x + 1\right) + \frac{9}{32}\sqrt{2}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 9/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/5*(10*x^4 - 1)/x^5
```

3.289 $\int \frac{1}{x^8(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=113

$$\frac{1}{4x^7\left(x^4+1\right)}+\frac{11}{12x^3}-\frac{11}{28x^7}-\frac{11\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{11\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{11\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{11\tan^{-1}\left(\sqrt{2}x\right.}{8\sqrt{2}}$$

```
[Out] -11/(28*x^7) + 11/(12*x^3) + 1/(4*x^7*(1 + x^4)) - (11*ArcTan[1 - Sqrt[2]*x
])/(8*Sqrt[2]) + (11*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (11*Log[1 - Sqrt[
2]*x + x^2])/(16*Sqrt[2]) + (11*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi [A] time = 0.0548459, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 290, 325, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{4x^7\left(x^4+1\right)}+\frac{11}{12x^3}-\frac{11}{28x^7}-\frac{11\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{11\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{11\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{11\tan^{-1}\left(\sqrt{2}x\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -11/(28*x^7) + 11/(12*x^3) + 1/(4*x^7*(1 + x^4)) - (11*ArcTan[1 - Sqrt[2]*x
])/(8*Sqrt[2]) + (11*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (11*Log[1 - Sqrt[
2]*x + x^2])/(16*Sqrt[2]) + (11*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^8\left(1+2x^4+x^8\right)}\,dx = \int \frac{1}{x^8\left(1+x^4\right)^2}\,dx$$
$$= \frac{1}{4x^7\left(1+x^4\right)} + \frac{11}{4}\int \frac{1}{x^8\left(1+x^4\right)}\,dx$$
$$= -\frac{11}{28x^7} + \frac{1}{4x^7\left(1+x^4\right)} - \frac{11}{4}\int \frac{1}{x^4\left(1+x^4\right)}\,dx$$
$$= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7\left(1+x^4\right)} + \frac{11}{4}\int \frac{1}{1+x^4}\,dx$$
$$= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7\left(1+x^4\right)} + \frac{11}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{11}{8}\int \frac{1+x^2}{1+x^4}\,dx$$
$$= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7\left(1+x^4\right)} + \frac{11}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{11}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx - \frac{11\int \frac{}{-1}}{1}$$
$$= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7\left(1+x^4\right)} - \frac{11\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{11\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{11\,\text{Subst}}{}$$
$$= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7\left(1+x^4\right)} - \frac{11\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{11\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} - \frac{11\log\left(1-\sqrt{}\right.}{16\sqrt{2}}$$

Mathematica [A] time = 0.0813733, size = 101, normalized size = 0.89

$$\frac{1}{672}\left(\frac{168x}{x^4+1}+\frac{448}{x^3}-\frac{96}{x^7}-231\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+231\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-462\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+462\sqrt{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-96/x^7 + 448/x^3 + (168*x)/(1 + x^4) - 462*Sqrt[2]*ArcTan[1 - Sqrt[2]*x]
+ 462*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] - 231*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2]
+ 231*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/672
```

Maple [A] time = 0.007, size = 78, normalized size = 0.7

$$-\frac{1}{7x^7}+\frac{2}{3x^3}+\frac{x}{4x^4+4}+\frac{11\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{16}+\frac{11\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{16}+\frac{11\sqrt{2}}{32}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8+2*x^4+1),x)
```

```
[Out] -1/7/x^7+2/3/x^3+1/4*x/(x^4+1)+11/16*arctan(1+x*2^(1/2))*2^(1/2)+11/16*arct
an(-1+x*2^(1/2))*2^(1/2)+11/32*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2
)))
```

Maxima [A] time = 1.49561, size = 128, normalized size = 1.13

$$\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{11}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{11}{32}\sqrt{2}\log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/32
*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/84*(77*x^8 + 44*x^4 - 12)/(x^11 + x^7
)
```

Fricas [A] time = 1.64168, size = 433, normalized size = 3.83

$$\frac{616\,x^8+352\,x^4-924\,\sqrt{2}\left(x^{11}+x^7\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-924\,\sqrt{2}\left(x^{11}+x^7\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{\right.}{672\left(x^{11}+x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/672*(616*x^8 + 352*x^4 - 924*sqrt(2)*(x^11 + x^7)*arctan(-sqrt(2)*x + sqr
t(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 924*sqrt(2)*(x^11 + x^7)*arctan(-sqrt
(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 231*sqrt(2)*(x^11 + x^7)*l
og(x^2 + sqrt(2)*x + 1) - 231*sqrt(2)*(x^11 + x^7)*log(x^2 - sqrt(2)*x + 1)
 - 96)/(x^11 + x^7)
```

Sympy [A] time = 0.24912, size = 102, normalized size = 0.9

$$-\frac{11\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{11\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{11\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{11\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}+\frac{77x^8+44x^4}{84x^{11}+84}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8+2*x**4+1),x)
```

```
[Out] -11*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 11*sqrt(2)*log(x**2 + sqrt(2)*x
+ 1)/32 + 11*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 11*sqrt(2)*atan(sqrt(2)*x + 1
)/16 + (77*x**8 + 44*x**4 - 12)/(84*x**11 + 84*x**7)
```

Giac [A] time = 1.14518, size = 127, normalized size = 1.12

$$\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{11}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{11}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/32
*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1) + 1/21*(14*x^4 - 3)/x^7
```

3.290 $\int \frac{x^m}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=30

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};x^4\right)}{m+1}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4])/(1 + m)
```

Rubi [A] time = 0.0056888, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 - 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4])/(1 + m)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{1-2x^4+x^8}\,dx &= \int \frac{x^m}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{4};\frac{5+m}{4};x^4\right)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.006415, size = 32, normalized size = 1.07

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+1}{4}+1;x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(1 - 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, x^4])/(1 + m)
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 - 2x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8-2*x^4+1),x)
```

```
[Out] int(x^m/(x^8-2*x^4+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 - 2x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{x^8 - 2x^4 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 - 2*x^4 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(x-1)^2 (x+1)^2 \left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8-2*x**4+1),x)
```

```
[Out] Integral(x**m/((x - 1)**2*(x + 1)**2*(x**2 + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 - 2x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

3.291 $\int \frac{x^9}{1-2x^4+x^8} dx$

Optimal. Leaf size=32

$$\frac{x^6}{4\left(1-x^4\right)}+\frac{3x^2}{4}-\frac{3}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] (3*x^2)/4 + x^6/(4*(1 - x^4)) - (3*ArcTanh[x^2])/4
```

Rubi [A] time = 0.0130546, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 288, 321, 207}

$$\frac{x^6}{4\left(1-x^4\right)}+\frac{3x^2}{4}-\frac{3}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - 2*x^4 + x^8),x]
```

```
[Out] (3*x^2)/4 + x^6/(4*(1 - x^4)) - (3*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^9}{1-2x^4+x^8}\,dx &= \int \frac{x^9}{\left(-1+x^4\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{x^6}{4\left(1-x^4\right)} + \frac{3}{4}\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, x^2\right) \\
&= \frac{3x^2}{4} + \frac{x^6}{4\left(1-x^4\right)} + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right) \\
&= \frac{3x^2}{4} + \frac{x^6}{4\left(1-x^4\right)} - \frac{3}{4}\tanh^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0236068, size = 39, normalized size = 1.22

$$\frac{1}{8}\left(2\left(\frac{1}{1-x^4}+2\right)x^2+3\log\left(1-x^2\right)-3\log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - 2*x^4 + x^8),x]
```

```
[Out] (2*x^2*(2 + (1 - x^4)^(-1)) + 3*Log[1 - x^2] - 3*Log[1 + x^2])/8
```

Maple [A] time = 0.009, size = 41, normalized size = 1.3

$$\frac{x^2}{2}-\frac{1}{8\,x^2+8}-\frac{3\,\ln\left(x^2+1\right)}{8}-\frac{1}{8\,x^2-8}+\frac{3\,\ln\left(x^2-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8-2*x^4+1),x)
```

```
[Out] 1/2*x^2-1/8/(x^2+1)-3/8*ln(x^2+1)-1/8/(x^2-1)+3/8*ln(x^2-1)
```

Maxima [A] time = 1.01735, size = 46, normalized size = 1.44

$$\frac{1}{2}\,x^2-\frac{x^2}{4\left(x^4-1\right)}-\frac{3}{8}\,\log\left(x^2+1\right)+\frac{3}{8}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(x^2 - 1)
```

Fricas [A] time = 1.45559, size = 115, normalized size = 3.59

$$\frac{4\,x^{6}-6\,x^{2}-3\left(x^{4}-1\right)\log\left(x^{2}+1\right)+3\left(x^{4}-1\right)\log\left(x^{2}-1\right)}{8\left(x^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/8*(4*x^6 - 6*x^2 - 3*(x^4 - 1)*log(x^2 + 1) + 3*(x^4 - 1)*log(x^2 - 1))/(
x^4 - 1)
```

Sympy [A] time = 0.126685, size = 34, normalized size = 1.06

$$\frac{x^{2}}{2}-\frac{x^{2}}{4x^{4}-4}+\frac{3\log\left(x^{2}-1\right)}{8}-\frac{3\log\left(x^{2}+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8-2*x**4+1),x)
```

```
[Out] x**2/2 - x**2/(4*x**4 - 4) + 3*log(x**2 - 1)/8 - 3*log(x**2 + 1)/8
```

Giac [A] time = 1.1067, size = 47, normalized size = 1.47

$$\frac{1}{2}\,x^{2}-\frac{x^{2}}{4\left(x^{4}-1\right)}-\frac{3}{8}\,\log\left(x^{2}+1\right)+\frac{3}{8}\,\log\left(\left|x^{2}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(abs(x^2 - 1))
```

3.292 $\int \frac{x^7}{1-2x^4+x^8} dx$

Optimal. Leaf size=26

$$\frac{1}{4\left(1-x^4\right)}+\frac{1}{4}\log\left(1-x^4\right)$$

```
[Out] 1/(4*(1 - x^4)) + Log[1 - x^4]/4
```

Rubi [A] time = 0.0128445, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 43}

$$\frac{1}{4\left(1-x^4\right)}+\frac{1}{4}\log\left(1-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 - x^4)) + Log[1 - x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1-2x^4+x^8}\,dx &= \int \frac{x^7}{\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{(-1+x)^2}\,dx,x,x^4\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^2}+\frac{1}{-1+x}\right)dx,x,x^4\right)\\
&= \frac{1}{4\left(1-x^4\right)}+\frac{1}{4}\log\left(1-x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0068021, size = 22, normalized size = 0.85

$$\frac{1}{4}\log\left(x^4-1\right)-\frac{1}{4\left(x^4-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - 2*x^4 + x^8),x]
```

```
[Out] -1/(4*(-1 + x^4)) + Log[-1 + x^4]/4
```

Maple [A] time = 0.006, size = 19, normalized size = 0.7

$$-\frac{1}{4x^4-4}+\frac{\ln\left(x^4-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8-2*x^4+1),x)
```

```
[Out] -1/4/(x^4-1)+1/4*ln(x^4-1)
```

Maxima [A] time = 0.975217, size = 24, normalized size = 0.92

$$-\frac{1}{4\left(x^4-1\right)}+\frac{1}{4}\log\left(x^4-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 - 1) + 1/4*log(x^4 - 1)
```

Fricas [A] time = 1.4481, size = 59, normalized size = 2.27

$$\frac{\left(x^4-1\right)\log\left(x^4-1\right)-1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*((x^4 - 1)*log(x^4 - 1) - 1)/(x^4 - 1)
```

Sympy [A] time = 0.110643, size = 15, normalized size = 0.58

$$\frac{\log\left(x^4-1\right)}{4}-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8-2*x**4+1),x)
```

```
[Out] log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

Giac [A] time = 1.10895, size = 26, normalized size = 1.

$$-\frac{1}{4\left(x^{4}-1\right)}+\frac{1}{4}\log\left(\left|x^{4}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4/(x^4 - 1) + 1/4*log(abs(x^4 - 1))
```

3.293 $\int \frac{x^5}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{4\left(1-x^4\right)}-\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] x^2/(4*(1 - x^4)) - ArcTanh[x^2]/4
```

Rubi [A] time = 0.0101594, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {28, 275, 288, 207}

$$\frac{x^2}{4\left(1-x^4\right)}-\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - 2*x^4 + x^8),x]

[Out] x^2/(4*(1 - x^4)) - ArcTanh[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = \int \frac{x^5}{\left(-1+x^4\right)^2}\,dx$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2}\,dx, x, x^2\right)$$
$$= \frac{x^2}{4\left(1-x^4\right)} + \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right)$$
$$= \frac{x^2}{4\left(1-x^4\right)} - \frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Mathematica [A] time = 0.0114759, size = 33, normalized size = 1.32

$$\frac{1}{8}\left(-\frac{2x^2}{x^4-1} + \log\left(1-x^2\right) - \log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-2*x^2)/(-1 + x^4) + Log[1 - x^2] - Log[1 + x^2])/8
```

Maple [A] time = 0.007, size = 36, normalized size = 1.4

$$-\frac{1}{8\,x^2+8} - \frac{\ln\left(x^2+1\right)}{8} - \frac{1}{8\,x^2-8} + \frac{\ln\left(x^2-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8-2*x^4+1),x)
```

```
[Out] -1/8/(x^2+1)-1/8*ln(x^2+1)-1/8/(x^2-1)+1/8*ln(x^2-1)
```

Maxima [A] time = 0.991136, size = 39, normalized size = 1.56

$$-\frac{x^2}{4\left(x^4-1\right)} - \frac{1}{8}\,\log\left(x^2+1\right) + \frac{1}{8}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(x^2 - 1)
```

Fricas [B] time = 1.6353, size = 100, normalized size = 4.

$$-\frac{2\,x^2 + \left(x^4-1\right)\log\left(x^2+1\right) - \left(x^4-1\right)\log\left(x^2-1\right)}{8\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^2 + (x^4 - 1)*log(x^2 + 1) - (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

Sympy [A] time = 0.123338, size = 26, normalized size = 1.04

$$-\frac{x^2}{4x^4-4}+\frac{\log\left(x^2-1\right)}{8}-\frac{\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8-2*x**4+1),x)
```

```
[Out] -x**2/(4*x**4 - 4) + log(x**2 - 1)/8 - log(x**2 + 1)/8
```

Giac [A] time = 1.08954, size = 41, normalized size = 1.64

$$-\frac{x^2}{4\left(x^4-1\right)}-\frac{1}{8}\log\left(x^2+1\right)+\frac{1}{8}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(abs(x^2 - 1))
```

3.294 $\int \frac{x^3}{1-2x^4+x^8} dx$

Optimal. Leaf size=13

$$\frac{1}{4(1-x^4)}$$

```
[Out] 1/(4*(1 - x^4))
```

Rubi [A] time = 0.0025553, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 261}

$$\frac{1}{4(1-x^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 - x^4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1-2x^4+x^8} dx &= \int \frac{x^3}{\left(-1+x^4\right)^2} dx\\ &= \frac{1}{4(1-x^4)}\end{aligned}$$

Mathematica [A] time = 0.0026446, size = 11, normalized size = 0.85

$$-\frac{1}{4(x^4-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - 2*x^4 + x^8),x]
```

```
[Out] -1/(4*(-1 + x^4))
```

Maple [A] time = 0.001, size = 10, normalized size = 0.8

$$-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-2*x^4+1),x)
```

```
[Out] -1/4/(x^4-1)
```

Maxima [A] time = 1.00088, size = 12, normalized size = 0.92

$$-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 - 1)
```

Fricas [A] time = 1.67114, size = 22, normalized size = 1.69

$$-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4/(x^4 - 1)
```

Sympy [A] time = 0.101148, size = 8, normalized size = 0.62

$$-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-2*x**4+1),x)
```

```
[Out] -1/(4*x**4 - 4)
```

Giac [A] time = 1.09683, size = 12, normalized size = 0.92

$$-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4/(x^4 - 1)
```

3.295 $\int \frac{x}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=25

$$\frac{x^2}{4\left(1-x^4\right)}+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] x^2/(4*(1 - x^4)) + ArcTanh[x^2]/4
```

Rubi [A] time = 0.0081537, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {28, 275, 199, 207}

$$\frac{x^2}{4\left(1-x^4\right)}+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 - x^4)) + ArcTanh[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{1-2x^4+x^8}\,dx &= \int \frac{x}{\left(-1+x^4\right)^2}\,dx \\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\left(-1+x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{x^2}{4\left(1-x^4\right)} - \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right) \\
&= \frac{x^2}{4\left(1-x^4\right)} + \frac{1}{4}\tanh^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0083112, size = 33, normalized size = 1.32

$$\frac{1}{8}\left(-\frac{2x^2}{x^4-1}-\log\left(1-x^2\right)+\log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-2*x^2)/(-1 + x^4) - Log[1 - x^2] + Log[1 + x^2])/8
```

Maple [A] time = 0.009, size = 36, normalized size = 1.4

$$-\frac{1}{8x^2+8}+\frac{\ln\left(x^2+1\right)}{8}-\frac{1}{8x^2-8}-\frac{\ln\left(x^2-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8-2*x^4+1),x)
```

```
[Out] -1/8/(x^2+1)+1/8*ln(x^2+1)-1/8/(x^2-1)-1/8*ln(x^2-1)
```

Maxima [A] time = 1.00547, size = 39, normalized size = 1.56

$$-\frac{x^2}{4\left(x^4-1\right)}+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

Fricas [B] time = 1.52176, size = 100, normalized size = 4.

$$-\frac{2x^2-\left(x^4-1\right)\log\left(x^2+1\right)+\left(x^4-1\right)\log\left(x^2-1\right)}{8\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^2 - (x^4 - 1)*log(x^2 + 1) + (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

Sympy [A] time = 0.126262, size = 26, normalized size = 1.04

$$-\frac{x^2}{4x^4-4}-\frac{\log\left(x^2-1\right)}{8}+\frac{\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8-2*x**4+1),x)
```

```
[Out] -x**2/(4*x**4 - 4) - log(x**2 - 1)/8 + log(x**2 + 1)/8
```

Giac [A] time = 1.12905, size = 41, normalized size = 1.64

$$-\frac{x^2}{4\left(x^4-1\right)}+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(abs(x^2 - 1))
```

3.296 $\int \frac{1}{x(1-2x^4+x^8)} dx$

Optimal. Leaf size=28

$$\frac{1}{4(1-x^4)} - \frac{1}{4}\log(1-x^4) + \log(x)$$

```
[Out] 1/(4*(1 - x^4)) + Log[x] - Log[1 - x^4]/4
```

Rubi [A] time = 0.0147175, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 44}

$$\frac{1}{4(1-x^4)} - \frac{1}{4}\log(1-x^4) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - 2*x^4 + x^8)),x]
```

```
[Out] 1/(4*(1 - x^4)) + Log[x] - Log[1 - x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1-2x^4+x^8)} dx &= \int \frac{1}{x(-1+x^4)^2} dx \\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^2 x} dx, x, x^4\right) \\
&= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{1}{1-x} + \frac{1}{(-1+x)^2} + \frac{1}{x}\right) dx, x, x^4\right) \\
&= \frac{1}{4(1-x^4)} + \log(x) - \frac{1}{4}\log(1-x^4)
\end{aligned}$$

Mathematica [A] time = 0.0083742, size = 26, normalized size = 0.93

$$-\frac{1}{4\left(x^4-1\right)}-\frac{1}{4}\log\left(1-x^4\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/(4*(-1 + x^4)) + Log[x] - Log[1 - x^4]/4
```

Maple [A] time = 0.012, size = 47, normalized size = 1.7

$$\frac{1}{8x^2+8}-\frac{\ln\left(x^2+1\right)}{4}+\ln(x)+\frac{1}{16+16x}-\frac{\ln(1+x)}{4}-\frac{1}{16x-16}-\frac{\ln(x-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8-2*x^4+1),x)
```

```
[Out] 1/8/(x^2+1)-1/4*ln(x^2+1)+ln(x)+1/16/(1+x)-1/4*ln(1+x)-1/16/(x-1)-1/4*ln(x-
1)
```

Maxima [A] time = 0.988116, size = 32, normalized size = 1.14

$$-\frac{1}{4\left(x^4-1\right)}-\frac{1}{4}\log\left(x^4-1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 - 1) - 1/4*log(x^4 - 1) + 1/4*log(x^4)
```

Fricas [A] time = 1.47772, size = 89, normalized size = 3.18

$$-\frac{\left(x^4-1\right)\log\left(x^4-1\right)-4\left(x^4-1\right)\log(x)+1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*((x^4 - 1)*log(x^4 - 1) - 4*(x^4 - 1)*log(x) + 1)/(x^4 - 1)
```

Sympy [A] time = 0.131994, size = 19, normalized size = 0.68

$$\log(x)-\frac{\log\left(x^4-1\right)}{4}-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8-2*x**4+1),x)
```

```
[Out] log(x) - log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

Giac [A] time = 1.08469, size = 41, normalized size = 1.46

$$\frac{x^4-2}{4\left(x^4-1\right)}+\frac{1}{4}\log\left(x^4\right)-\frac{1}{4}\log\left(\left|x^4-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(x^4 - 2)/(x^4 - 1) + 1/4*log(x^4) - 1/4*log(abs(x^4 - 1))
```

3.297 $\int \frac{1}{x^3(1-2x^4+x^8)} dx$

Optimal. Leaf size=32

$$\frac{1}{4x^2(1-x^4)} - \frac{3}{4x^2} + \frac{3}{4}\tanh^{-1}(x^2)$$

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 - x^4)) + (3*ArcTanh[x^2])/4
```

Rubi [A] time = 0.0126115, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 290, 325, 207}

$$\frac{1}{4x^2(1-x^4)} - \frac{3}{4x^2} + \frac{3}{4}\tanh^{-1}(x^2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 - x^4)) + (3*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^3\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(-1+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{1}{4x^2\left(1-x^4\right)}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{x^2\left(-1+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{3}{4x^2}+\frac{1}{4x^2\left(1-x^4\right)}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,x^2\right)\\
&= -\frac{3}{4x^2}+\frac{1}{4x^2\left(1-x^4\right)}+\frac{3}{4}\tanh^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.017206, size = 41, normalized size = 1.28

$$\frac{1}{8}\left(\frac{4-6x^4}{x^2\left(x^4-1\right)}-3\log\left(1-x^2\right)+3\log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - 2*x^4 + x^8)),x]
```

```
[Out] ((4 - 6*x^4)/(x^2*(-1 + x^4)) - 3*Log[1 - x^2] + 3*Log[1 + x^2])/8
```

Maple [A] time = 0.017, size = 50, normalized size = 1.6

$$-\frac{1}{8x^2+8}+\frac{3\ln\left(x^2+1\right)}{8}-\frac{1}{2x^2}+\frac{1}{16+16x}-\frac{3\ln\left(1+x\right)}{8}-\frac{1}{16x-16}-\frac{3\ln\left(x-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8-2*x^4+1),x)
```

```
[Out] -1/8/(x^2+1)+3/8*ln(x^2+1)-1/2/x^2+1/16/(1+x)-3/8*ln(1+x)-1/16/(x-1)-3/8*ln
(x-1)
```

Maxima [A] time = 1.01926, size = 50, normalized size = 1.56

$$-\frac{3x^4-2}{4\left(x^6-x^2\right)}+\frac{3}{8}\log\left(x^2+1\right)-\frac{3}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(x^2 - 1)
```

Fricas [B] time = 1.42239, size = 119, normalized size = 3.72

$$-\frac{6\,x^4-3\left(x^6-x^2\right)\log\left(x^2+1\right)+3\left(x^6-x^2\right)\log\left(x^2-1\right)-4}{8\left(x^6-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/8*(6*x^4 - 3*(x^6 - x^2)*log(x^2 + 1) + 3*(x^6 - x^2)*log(x^2 - 1) - 4)/
(x^6 - x^2)
```

Sympy [A] time = 0.155546, size = 36, normalized size = 1.12

$$-\frac{3x^4-2}{4x^6-4x^2}-\frac{3\log\left(x^2-1\right)}{8}+\frac{3\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8-2*x**4+1),x)
```

```
[Out] -(3*x**4 - 2)/(4*x**6 - 4*x**2) - 3*log(x**2 - 1)/8 + 3*log(x**2 + 1)/8
```

Giac [A] time = 1.12248, size = 51, normalized size = 1.59

$$-\frac{3\,x^4-2}{4\left(x^6-x^2\right)}+\frac{3}{8}\,\log\left(x^2+1\right)-\frac{3}{8}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(abs(x^2 - 1))
```

3.298 $\int \frac{1}{x^5(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{4\left(1-x^4\right)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

```
[Out] -1/(4*x^4) + 1/(4*(1 - x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

Rubi [A] time = 0.0185109, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 44}

$$\frac{1}{4\left(1-x^4\right)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - 2*x^4 + x^8)),x]
```


```
[Out] -1/(4*x^4) + 1/(4*(1 - x^4)) + 2*Log[x] - Log[1 - x^4]/2
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^5\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^2x^2}\,dx, x, x^4\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{x^2}+\frac{2}{x}\right)dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4\left(1-x^4\right)}+2\log(x)-\frac{1}{2}\log\left(1-x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0126081, size = 35, normalized size = 0.95

$$-\frac{1}{4\left(x^4-1\right)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - 1/(4*(-1 + x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

Maple [A] time = 0.016, size = 54, normalized size = 1.5

$$\frac{1}{8x^2+8}-\frac{\ln\left(x^2+1\right)}{2}-\frac{1}{4x^4}+2\ln(x)+\frac{1}{16+16x}-\frac{\ln(1+x)}{2}-\frac{1}{16x-16}-\frac{\ln(x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8-2*x^4+1),x)
```

```
[Out] 1/8/(x^2+1)-1/2*ln(x^2+1)-1/4/x^4+2*ln(x)+1/16/(1+x)-1/2*ln(1+x)-1/16/(x-1)
-1/2*ln(x-1)
```

Maxima [A] time = 0.974259, size = 47, normalized size = 1.27

$$-\frac{2x^4-1}{4\left(x^8-x^4\right)}-\frac{1}{2}\log\left(x^4-1\right)+\frac{1}{2}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^4 - 1)/(x^8 - x^4) - 1/2*log(x^4 - 1) + 1/2*log(x^4)
```

Fricas [A] time = 1.45873, size = 111, normalized size = 3.

$$-\frac{2x^4+2\left(x^8-x^4\right)\log\left(x^4-1\right)-8\left(x^8-x^4\right)\log(x)-1}{4\left(x^8-x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^4 + 2*(x^8 - x^4)*log(x^4 - 1) - 8*(x^8 - x^4)*log(x) - 1)/(x^8 -
 x^4)
```

Sympy [A] time = 0.160786, size = 29, normalized size = 0.78

$$-\frac{2x^4-1}{4x^8-4x^4}+2\log(x)-\frac{\log\left(x^4-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8-2*x**4+1),x)
```

```
[Out] -(2*x**4 - 1)/(4*x**8 - 4*x**4) + 2*log(x) - log(x**4 - 1)/2
```

Giac [A] time = 1.11053, size = 49, normalized size = 1.32

$$-\frac{2\,x^4-1}{4\left(x^8-x^4\right)}+\frac{1}{2}\,\log\left(x^4\right)-\frac{1}{2}\,\log\left(\left|x^4-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^4 - 1)/(x^8 - x^4) + 1/2*log(x^4) - 1/2*log(abs(x^4 - 1))
```

3.299 $\int \frac{1}{x^7(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=39

$$\frac{1}{4x^6(1-x^4)}-\frac{5}{4x^2}-\frac{5}{12x^6}+\frac{5}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] -5/(12*x^6) - 5/(4*x^2) + 1/(4*x^6*(1 - x^4)) + (5*ArcTanh[x^2])/4
```

Rubi [A] time = 0.0171345, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 290, 325, 207}

$$\frac{1}{4x^6(1-x^4)}-\frac{5}{4x^2}-\frac{5}{12x^6}+\frac{5}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -5/(12*x^6) - 5/(4*x^2) + 1/(4*x^6*(1 - x^4)) + (5*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^7\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(-1+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{1}{4x^6\left(1-x^4\right)}-\frac{5}{4}\,\text{Subst}\left(\int \frac{1}{x^4\left(-1+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{5}{12x^6}+\frac{1}{4x^6\left(1-x^4\right)}-\frac{5}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(-1+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{5}{12x^6}-\frac{5}{4x^2}+\frac{1}{4x^6\left(1-x^4\right)}-\frac{5}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,x^2\right)\\
&= -\frac{5}{12x^6}-\frac{5}{4x^2}+\frac{1}{4x^6\left(1-x^4\right)}+\frac{5}{4}\tanh^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0146504, size = 49, normalized size = 1.26

$$-\frac{x^2}{4\left(x^4-1\right)}-\frac{1}{x^2}-\frac{1}{6x^6}-\frac{5}{8}\log\left(1-x^2\right)+\frac{5}{8}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) - x^(-2) - x^2/(4*(-1 + x^4)) - (5*Log[1 - x^2])/8 + (5*Log[1 + 
x^2])/8
```

Maple [A] time = 0.017, size = 55, normalized size = 1.4

$$-\frac{1}{8\,x^2+8}+\frac{5\,\ln\left(x^2+1\right)}{8}-\frac{1}{6\,x^6}-x^{-2}+\frac{1}{16+16\,x}-\frac{5\,\ln(1+x)}{8}-\frac{1}{16\,x-16}-\frac{5\,\ln(x-1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8-2*x^4+1),x)
```

```
[Out] -1/8/(x^2+1)+5/8*ln(x^2+1)-1/6/x^6-1/x^2+1/16/(1+x)-5/8*ln(1+x)-1/16/(x-1)-
5/8*ln(x-1)
```

Maxima [A] time = 1.01256, size = 57, normalized size = 1.46

$$-\frac{15\,x^8-10\,x^4-2}{12\left(x^{10}-x^6\right)}+\frac{5}{8}\,\log\left(x^2+1\right)-\frac{5}{8}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*(15*x^8 - 10*x^4 - 2)/(x^10 - x^6) + 5/8*log(x^2 + 1) - 5/8*log(x^2 -
 1)
```

Fricas [B] time = 1.46567, size = 140, normalized size = 3.59

$$-\frac{30\,x^8 - 20\,x^4 - 15\,\left(x^{10} - x^6\right)\log\left(x^2 + 1\right) + 15\,\left(x^{10} - x^6\right)\log\left(x^2 - 1\right) - 4}{24\,\left(x^{10} - x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(30*x^8 - 20*x^4 - 15*(x^10 - x^6)*log(x^2 + 1) + 15*(x^10 - x^6)*log
(x^2 - 1) - 4)/(x^10 - x^6)
```

Sympy [A] time = 0.187277, size = 41, normalized size = 1.05

$$-\frac{5\log\left(x^2 - 1\right)}{8} + \frac{5\log\left(x^2 + 1\right)}{8} - \frac{15x^8 - 10x^4 - 2}{12x^{10} - 12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8-2*x**4+1),x)
```

```
[Out] -5*log(x**2 - 1)/8 + 5*log(x**2 + 1)/8 - (15*x**8 - 10*x**4 - 2)/(12*x**10
- 12*x**6)
```

Giac [A] time = 1.12038, size = 57, normalized size = 1.46

$$-\frac{x^2}{4\left(x^4 - 1\right)} - \frac{6\,x^4 + 1}{6\,x^6} + \frac{5}{8}\,\log\left(x^2 + 1\right) - \frac{5}{8}\,\log\left(\left|x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^2/(x^4 - 1) - 1/6*(6*x^4 + 1)/x^6 + 5/8*log(x^2 + 1) - 5/8*log(abs(x
^2 - 1))
```

3.300 $\int \frac{x^8}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=34

$$\frac{x^5}{4\left(1-x^4\right)}+\frac{5x}{4}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)$$

```
[Out] (5*x)/4 + x^5/(4*(1 - x^4)) - (5*ArcTan[x])/8 - (5*ArcTanh[x])/8
```

Rubi [A] time = 0.0090223, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 288, 321, 212, 206, 203}

$$\frac{x^5}{4\left(1-x^4\right)}+\frac{5x}{4}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - 2*x^4 + x^8),x]
```

```
[Out] (5*x)/4 + x^5/(4*(1 - x^4)) - (5*ArcTan[x])/8 - (5*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{1-2x^4+x^8}\,dx &= \int \frac{x^8}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{x^5}{4\left(1-x^4\right)}+\frac{5}{4}\int \frac{x^4}{-1+x^4}\,dx\\ &= \frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}+\frac{5}{4}\int \frac{1}{-1+x^4}\,dx\\ &= \frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{8}\int \frac{1}{1-x^2}\,dx-\frac{5}{8}\int \frac{1}{1+x^2}\,dx\\ &= \frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0157707, size = 38, normalized size = 1.12

$$-\frac{x}{4\left(x^4-1\right)}+x+\frac{5}{16}\log(1-x)-\frac{5}{16}\log(x+1)-\frac{5}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - 2*x^4 + x^8),x]
```

```
[Out] x - x/(4*(-1 + x^4)) - (5*ArcTan[x])/8 + (5*Log[1 - x])/16 - (5*Log[1 + x])
/16
```

Maple [A] time = 0.013, size = 43, normalized size = 1.3

$$x+\frac{x}{8\,x^2+8}-\frac{5\arctan(x)}{8}-\frac{1}{16+16\,x}-\frac{5\ln(1+x)}{16}-\frac{1}{16\,x-16}+\frac{5\ln(x-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8-2*x^4+1),x)
```

```
[Out] x+1/8*x/(x^2+1)-5/8*arctan(x)-1/16/(1+x)-5/16*ln(1+x)-1/16/(x-1)+5/16*ln(x-
1)
```

Maxima [A] time = 1.47366, size = 38, normalized size = 1.12

$$x-\frac{x}{4\left(x^4-1\right)}-\frac{5}{8}\arctan(x)-\frac{5}{16}\log(x+1)+\frac{5}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(x + 1) + 5/16*log(x - 1)
```

Fricas [B] time = 1.56054, size = 144, normalized size = 4.24

$$\frac{16\,x^5 - 10\left(x^4 - 1\right)\arctan\left(x\right) - 5\left(x^4 - 1\right)\log\left(x + 1\right) + 5\left(x^4 - 1\right)\log\left(x - 1\right) - 20\,x}{16\left(x^4 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/16*(16*x^5 - 10*(x^4 - 1)*arctan(x) - 5*(x^4 - 1)*log(x + 1) + 5*(x^4 - 1
)*log(x - 1) - 20*x)/(x^4 - 1)
```

Sympy [A] time = 0.159471, size = 32, normalized size = 0.94

$$x - \frac{x}{4x^4 - 4} + \frac{5\log\left(x - 1\right)}{16} - \frac{5\log\left(x + 1\right)}{16} - \frac{5\operatorname{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-2*x**4+1),x)
```

```
[Out] x - x/(4*x**4 - 4) + 5*log(x - 1)/16 - 5*log(x + 1)/16 - 5*atan(x)/8
```

Giac [A] time = 1.13069, size = 41, normalized size = 1.21

$$x - \frac{x}{4\left(x^4 - 1\right)} - \frac{5}{8}\arctan\left(x\right) - \frac{5}{16}\log\left(|x + 1|\right) + \frac{5}{16}\log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(abs(x + 1)) + 5/16*log(abs(x
 - 1))
```

3.301 $\int \frac{x^6}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=29

$$\frac{x^3}{4\left(1-x^4\right)}+\frac{3}{8}\tan^{-1}(x)-\frac{3}{8}\tanh^{-1}(x)$$

```
[Out] x^3/(4*(1 - x^4)) + (3*ArcTan[x])/8 - (3*ArcTanh[x])/8
```

Rubi [A] time = 0.0077958, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 288, 298, 203, 206}

$$\frac{x^3}{4\left(1-x^4\right)}+\frac{3}{8}\tan^{-1}(x)-\frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 - x^4)) + (3*ArcTan[x])/8 - (3*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{1-2x^4+x^8}\,dx &= \int \frac{x^6}{\left(-1+x^4\right)^2}\,dx\\
&= \frac{x^3}{4\left(1-x^4\right)}+\frac{3}{4}\int \frac{x^2}{-1+x^4}\,dx\\
&= \frac{x^3}{4\left(1-x^4\right)}-\frac{3}{8}\int \frac{1}{1-x^2}\,dx+\frac{3}{8}\int \frac{1}{1+x^2}\,dx\\
&= \frac{x^3}{4\left(1-x^4\right)}+\frac{3}{8}\tan^{-1}(x)-\frac{3}{8}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0162542, size = 35, normalized size = 1.21

$$\frac{1}{16}\left(-\frac{4x^3}{x^4-1}+3\log(1-x)-3\log(x+1)+6\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-4*x^3)/(-1 + x^4) + 6*ArcTan[x] + 3*Log[1 - x] - 3*Log[1 + x])/16
```

Maple [A] time = 0.011, size = 42, normalized size = 1.5

$$-\frac{x}{8\,x^2+8}+\frac{3\,\arctan(x)}{8}-\frac{1}{16+16\,x}-\frac{3\,\ln(1+x)}{16}-\frac{1}{16\,x-16}+\frac{3\,\ln(x-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8-2*x^4+1),x)
```

```
[Out] -1/8*x/(x^2+1)+3/8*arctan(x)-1/16/(1+x)-3/16*ln(1+x)-1/16/(x-1)+3/16*ln(x-1
)
```

Maxima [A] time = 1.49676, size = 39, normalized size = 1.34

$$-\frac{x^3}{4\left(x^4-1\right)}+\frac{3}{8}\arctan(x)-\frac{3}{16}\log(x+1)+\frac{3}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(x + 1) + 3/16*log(x - 1)
```

Fricas [B] time = 1.45841, size = 134, normalized size = 4.62

$$-\frac{4\,x^3-6\left(x^4-1\right)\arctan(x)+3\left(x^4-1\right)\log(x+1)-3\left(x^4-1\right)\log(x-1)}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 - 6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)
*log(x - 1))/(x^4 - 1)
```

Sympy [A] time = 0.157046, size = 32, normalized size = 1.1

$$-\frac{x^3}{4x^4-4}+\frac{3\log(x-1)}{16}-\frac{3\log(x+1)}{16}+\frac{3\,\mathrm{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8-2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 - 4) + 3*log(x - 1)/16 - 3*log(x + 1)/16 + 3*atan(x)/8
```

Giac [A] time = 1.10297, size = 42, normalized size = 1.45

$$-\frac{x^3}{4\left(x^4-1\right)}+\frac{3}{8}\arctan(x)-\frac{3}{16}\log(|x+1|)+\frac{3}{16}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(abs(x + 1)) + 3/16*log(abs(x
- 1))
```

3.302 $\int \frac{x^4}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=27

$$\frac{x}{4(1-x^4)} - \frac{1}{8}\tan^{-1}(x) - \frac{1}{8}\tanh^{-1}(x)$$

```
[Out] x/(4*(1 - x^4)) - ArcTan[x]/8 - ArcTanh[x]/8
```

Rubi [A] time = 0.0073036, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 288, 212, 206, 203}

$$\frac{x}{4(1-x^4)} - \frac{1}{8}\tan^{-1}(x) - \frac{1}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - 2*x^4 + x^8),x]
```

```
[Out] x/(4*(1 - x^4)) - ArcTan[x]/8 - ArcTanh[x]/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = \int \frac{x^4}{\left(-1+x^4\right)^2}\,dx$$
$$= \frac{x}{4\left(1-x^4\right)} + \frac{1}{4}\int \frac{1}{-1+x^4}\,dx$$
$$= \frac{x}{4\left(1-x^4\right)} - \frac{1}{8}\int \frac{1}{1-x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x^2}\,dx$$
$$= \frac{x}{4\left(1-x^4\right)} - \frac{1}{8}\tan^{-1}(x) - \frac{1}{8}\tanh^{-1}(x)$$

Mathematica [A] time = 0.0135865, size = 31, normalized size = 1.15

$$\frac{1}{16}\left(-\frac{4x}{x^4-1}+\log(1-x)-\log(x+1)-2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-4*x)/(-1 + x^4) - 2*ArcTan[x] + Log[1 - x] - Log[1 + x])/16
```

Maple [A] time = 0.012, size = 42, normalized size = 1.6

$$\frac{x}{8\,x^2+8}-\frac{\arctan(x)}{8}-\frac{1}{16+16\,x}-\frac{\ln(1+x)}{16}-\frac{1}{16\,x-16}+\frac{\ln(x-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8-2*x^4+1),x)
```

```
[Out] 1/8*x/(x^2+1)-1/8*arctan(x)-1/16/(1+x)-1/16*ln(1+x)-1/16/(x-1)+1/16*ln(x-1)
```

Maxima [A] time = 1.54614, size = 36, normalized size = 1.33

$$-\frac{x}{4\left(x^4-1\right)}-\frac{1}{8}\arctan(x)-\frac{1}{16}\log(x+1)+\frac{1}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(x + 1) + 1/16*log(x - 1)
```

Fricas [B] time = 1.50616, size = 126, normalized size = 4.67

$$-\frac{2\left(x^4-1\right)\arctan(x)+\left(x^4-1\right)\log(x+1)-\left(x^4-1\right)\log(x-1)+4\,x}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log(x - 1)
+ 4*x)/(x^4 - 1)
```

Sympy [A] time = 0.149363, size = 26, normalized size = 0.96

$$-\frac{x}{4x^4-4}+\frac{\log(x-1)}{16}-\frac{\log(x+1)}{16}-\frac{\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-2*x**4+1),x)
```

```
[Out] -x/(4*x**4 - 4) + log(x - 1)/16 - log(x + 1)/16 - atan(x)/8
```

Giac [A] time = 1.09093, size = 39, normalized size = 1.44

$$-\frac{x}{4\left(x^4-1\right)}-\frac{1}{8}\arctan(x)-\frac{1}{16}\log(|x+1|)+\frac{1}{16}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(abs(x + 1)) + 1/16*log(abs(x -
1))
```

3.303 $\int \frac{x^2}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=29

$$\frac{x^3}{4\left(1-x^4\right)}-\frac{1}{8}\tan^{-1}(x)+\frac{1}{8}\tanh^{-1}(x)$$

```
[Out] x^3/(4*(1 - x^4)) - ArcTan[x]/8 + ArcTanh[x]/8
```

Rubi [A] time = 0.0083581, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 290, 298, 203, 206}

$$\frac{x^3}{4\left(1-x^4\right)}-\frac{1}{8}\tan^{-1}(x)+\frac{1}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 - x^4)) - ArcTan[x]/8 + ArcTanh[x]/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{1-2x^4+x^8}\,dx &= \int \frac{x^2}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{x^3}{4\left(1-x^4\right)}-\frac{1}{4}\int \frac{x^2}{-1+x^4}\,dx\\ &= \frac{x^3}{4\left(1-x^4\right)}+\frac{1}{8}\int \frac{1}{1-x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x^3}{4\left(1-x^4\right)}-\frac{1}{8}\tan^{-1}(x)+\frac{1}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0120117, size = 33, normalized size = 1.14

$$\frac{1}{16}\left(-\frac{4x^3}{x^4-1}-\log(1-x)+\log(x+1)-2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-4*x^3)/(-1 + x^4) - 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/16
```

Maple [A] time = 0.01, size = 42, normalized size = 1.5

$$-\frac{x}{8x^2+8}-\frac{\arctan(x)}{8}-\frac{1}{16+16x}+\frac{\ln(1+x)}{16}-\frac{1}{16x-16}-\frac{\ln(x-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8-2*x^4+1),x)
```

```
[Out] -1/8*x/(x^2+1)-1/8*arctan(x)-1/16/(1+x)+1/16*ln(1+x)-1/16/(x-1)-1/16*ln(x-1
)
```

Maxima [A] time = 1.50077, size = 39, normalized size = 1.34

$$-\frac{x^3}{4\left(x^4-1\right)}-\frac{1}{8}\arctan(x)+\frac{1}{16}\log(x+1)-\frac{1}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(x + 1) - 1/16*log(x - 1)
```

Fricas [B] time = 1.51697, size = 128, normalized size = 4.41

$$-\frac{4x^3+2\left(x^4-1\right)\arctan(x)-\left(x^4-1\right)\log(x+1)+\left(x^4-1\right)\log(x-1)}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 + 2*(x^4 - 1)*arctan(x) - (x^4 - 1)*log(x + 1) + (x^4 - 1)*log
(x - 1))/(x^4 - 1)
```

Sympy [A] time = 0.148697, size = 27, normalized size = 0.93

$$-\frac{x^3}{4x^4-4}-\frac{\log(x-1)}{16}+\frac{\log(x+1)}{16}-\frac{\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 - 4) - log(x - 1)/16 + log(x + 1)/16 - atan(x)/8
```

Giac [A] time = 1.11053, size = 42, normalized size = 1.45

$$-\frac{x^3}{4\left(x^4-1\right)}-\frac{1}{8}\arctan(x)+\frac{1}{16}\log(|x+1|)-\frac{1}{16}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(abs(x + 1)) - 1/16*log(abs(x
- 1))
```

3.304 $\int \frac{1}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=27

$$\frac{x}{4\left(1-x^4\right)}+\frac{3}{8}\tan^{-1}(x)+\frac{3}{8}\tanh^{-1}(x)$$

```
[Out] x/(4*(1 - x^4)) + (3*ArcTan[x])/8 + (3*ArcTanh[x])/8
```

Rubi [A] time = 0.0053873, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {28, 199, 212, 206, 203}

$$\frac{x}{4\left(1-x^4\right)}+\frac{3}{8}\tan^{-1}(x)+\frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^4 + x^8)^(-1),x]
```

```
[Out] x/(4*(1 - x^4)) + (3*ArcTan[x])/8 + (3*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1-2x^4+x^8}\,dx &= \int \frac{1}{\left(-1+x^4\right)^2}\,dx\\
&= \frac{x}{4\left(1-x^4\right)} - \frac{3}{4}\int \frac{1}{-1+x^4}\,dx\\
&= \frac{x}{4\left(1-x^4\right)} + \frac{3}{8}\int \frac{1}{1-x^2}\,dx + \frac{3}{8}\int \frac{1}{1+x^2}\,dx\\
&= \frac{x}{4\left(1-x^4\right)} + \frac{3}{8}\tan^{-1}(x) + \frac{3}{8}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.00972, size = 33, normalized size = 1.22

$$
\frac{1}{16}\left(-\frac{4x}{x^4-1} - 3\log(1-x) + 3\log(x+1) + 6\tan^{-1}(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^4 + x^8)^(-1),x]
```

```
[Out] ((-4*x)/(-1 + x^4) + 6*ArcTan[x] - 3*Log[1 - x] + 3*Log[1 + x])/16
```

Maple [A] time = 0.012, size = 42, normalized size = 1.6

$$
\frac{x}{8\,x^2+8} + \frac{3\,\arctan(x)}{8} - \frac{1}{16+16\,x} + \frac{3\,\ln(1+x)}{16} - \frac{1}{16\,x-16} - \frac{3\,\ln(x-1)}{16}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-2*x^4+1),x)
```

```
[Out] 1/8*x/(x^2+1)+3/8*arctan(x)-1/16/(1+x)+3/16*ln(1+x)-1/16/(x-1)-3/16*ln(x-1)
```

Maxima [A] time = 1.51261, size = 36, normalized size = 1.33

$$
-\frac{x}{4\left(x^4-1\right)} + \frac{3}{8}\arctan(x) + \frac{3}{16}\log(x+1) - \frac{3}{16}\log(x-1)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(x + 1) - 3/16*log(x - 1)
```

Fricas [B] time = 1.48817, size = 130, normalized size = 4.81

$$
\frac{6\left(x^4-1\right)\arctan(x) + 3\left(x^4-1\right)\log(x+1) - 3\left(x^4-1\right)\log(x-1) - 4\,x}{16\left(x^4-1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)*log(x -
1) - 4*x)/(x^4 - 1)
```

Sympy [A] time = 0.15957, size = 31, normalized size = 1.15

$$-\frac{x}{4x^4-4}-\frac{3\log(x-1)}{16}+\frac{3\log(x+1)}{16}+\frac{3\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-2*x**4+1),x)
```

```
[Out] -x/(4*x**4 - 4) - 3*log(x - 1)/16 + 3*log(x + 1)/16 + 3*atan(x)/8
```

Giac [A] time = 1.11741, size = 39, normalized size = 1.44

$$-\frac{x}{4\left(x^4-1\right)}+\frac{3}{8}\arctan(x)+\frac{3}{16}\log(|x+1|)-\frac{3}{16}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(abs(x + 1)) - 3/16*log(abs(x -
1))
```

3.305 $\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx$

Optimal. Leaf size=36

$$\frac{1}{4x\left(1-x^4\right)}-\frac{5}{4x}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{8}\tanh^{-1}(x)$$

```
[Out] -5/(4*x) + 1/(4*x*(1 - x^4)) - (5*ArcTan[x])/8 + (5*ArcTanh[x])/8
```

Rubi [A] time = 0.0101916, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 290, 325, 298, 203, 206}

$$\frac{1}{4x\left(1-x^4\right)}-\frac{5}{4x}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -5/(4*x) + 1/(4*x*(1 - x^4)) - (5*ArcTan[x])/8 + (5*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^2\left(-1+x^4\right)^2}\,dx\\ &= \frac{1}{4x\left(1-x^4\right)}-\frac{5}{4}\int \frac{1}{x^2\left(-1+x^4\right)}\,dx\\ &= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}-\frac{5}{4}\int \frac{x^2}{-1+x^4}\,dx\\ &= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}+\frac{5}{8}\int \frac{1}{1-x^2}\,dx-\frac{5}{8}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0166815, size = 40, normalized size = 1.11

$$\frac{1}{16}\left(-\frac{4x^3}{x^4-1}-\frac{16}{x}-5\log(1-x)+5\log(x+1)-10\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-16/x - (4*x^3)/(-1 + x^4) - 10*ArcTan[x] - 5*Log[1 - x] + 5*Log[1 + x])/1
6
```

Maple [A] time = 0.014, size = 47, normalized size = 1.3

$$-\frac{x}{8x^2+8}-\frac{5\arctan(x)}{8}-x^{-1}-\frac{1}{16+16x}+\frac{5\ln(1+x)}{16}-\frac{1}{16x-16}-\frac{5\ln(x-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8-2*x^4+1),x)
```

```
[Out] -1/8*x/(x^2+1)-5/8*arctan(x)-1/x-1/16/(1+x)+5/16*ln(1+x)-1/16/(x-1)-5/16*ln
(x-1)
```

Maxima [A] time = 1.49071, size = 47, normalized size = 1.31

$$-\frac{5x^4-4}{4\left(x^5-x\right)}-\frac{5}{8}\arctan(x)+\frac{5}{16}\log(x+1)-\frac{5}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(x + 1) - 5/16*log(x -
 1)
```

Fricas [B] time = 1.50341, size = 143, normalized size = 3.97

$$-\frac{20 x^4 + 10 \left(x^5 - x\right) \arctan\left(x\right) - 5 \left(x^5 - x\right) \log\left(x + 1\right) + 5 \left(x^5 - x\right) \log\left(x - 1\right) - 16}{16 \left(x^5 - x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(20*x^4 + 10*(x^5 - x)*arctan(x) - 5*(x^5 - x)*log(x + 1) + 5*(x^5 -
x)*log(x - 1) - 16)/(x^5 - x)
```

Sympy [A] time = 0.184574, size = 37, normalized size = 1.03

$$-\frac{5x^4 - 4}{4x^5 - 4x} - \frac{5 \log\left(x - 1\right)}{16} + \frac{5 \log\left(x + 1\right)}{16} - \frac{5 \operatorname{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8-2*x**4+1),x)
```

```
[Out] -(5*x**4 - 4)/(4*x**5 - 4*x) - 5*log(x - 1)/16 + 5*log(x + 1)/16 - 5*atan(x
)/8
```

Giac [A] time = 1.1382, size = 50, normalized size = 1.39

$$-\frac{5 x^4 - 4}{4 \left(x^5 - x\right)} - \frac{5}{8} \arctan\left(x\right) + \frac{5}{16} \log\left(|x + 1|\right) - \frac{5}{16} \log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(abs(x + 1)) - 5/16*lo
g(abs(x - 1))
```

3.306 $\int \frac{1}{x^4(1-2x^4+x^8)} dx$

Optimal. Leaf size=36

$$\frac{1}{4x^3(1-x^4)} - \frac{7}{12x^3} + \frac{7}{8}\tan^{-1}(x) + \frac{7}{8}\tanh^{-1}(x)$$

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 - x^4)) + (7*ArcTan[x])/8 + (7*ArcTanh[x])/8
```

Rubi [A] time = 0.0088647, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 290, 325, 212, 206, 203}

$$\frac{1}{4x^3(1-x^4)} - \frac{7}{12x^3} + \frac{7}{8}\tan^{-1}(x) + \frac{7}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 - x^4)) + (7*ArcTan[x])/8 + (7*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^4\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^3\left(1-x^4\right)}-\frac{7}{4}\int \frac{1}{x^4\left(-1+x^4\right)}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1-x^4\right)}-\frac{7}{4}\int \frac{1}{-1+x^4}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1-x^4\right)}+\frac{7}{8}\int \frac{1}{1-x^2}\,dx+\frac{7}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1-x^4\right)}+\frac{7}{8}\tan^{-1}(x)+\frac{7}{8}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0186844, size = 38, normalized size = 1.06

$$\frac{1}{48}\left(-\frac{12x}{x^4-1}-\frac{16}{x^3}-21\log(1-x)+21\log(x+1)+42\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-16/x^3 - (12*x)/(-1 + x^4) + 42*ArcTan[x] - 21*Log[1 - x] + 21*Log[1 + x]
)/48
```

Maple [A] time = 0.015, size = 47, normalized size = 1.3

$$\frac{x}{8\,x^2+8}+\frac{7\arctan(x)}{8}-\frac{1}{3\,x^3}-\frac{1}{16+16\,x}+\frac{7\ln(1+x)}{16}-\frac{1}{16\,x-16}-\frac{7\ln(x-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8-2*x^4+1),x)
```

```
[Out] 1/8*x/(x^2+1)+7/8*arctan(x)-1/3/x^3-1/16/(1+x)+7/16*ln(1+x)-1/16/(x-1)-7/16
*ln(x-1)
```

Maxima [A] time = 1.60184, size = 50, normalized size = 1.39

$$-\frac{7\,x^4-4}{12\left(x^7-x^3\right)}+\frac{7}{8}\arctan(x)+\frac{7}{16}\log(x+1)-\frac{7}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*(7*x^4 - 4)/(x^7 - x^3) + 7/8*arctan(x) + 7/16*log(x + 1) - 7/16*log(
x - 1)
```

Fricas [B] time = 1.46941, size = 157, normalized size = 4.36

$$-\frac{28\,x^4 - 42\left(x^7 - x^3\right)\arctan\left(x\right) - 21\left(x^7 - x^3\right)\log\left(x+1\right) + 21\left(x^7 - x^3\right)\log\left(x-1\right) - 16}{48\left(x^7 - x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/48*(28*x^4 - 42*(x^7 - x^3)*arctan(x) - 21*(x^7 - x^3)*log(x + 1) + 21*(
x^7 - x^3)*log(x - 1) - 16)/(x^7 - x^3)
```

Sympy [A] time = 0.187099, size = 39, normalized size = 1.08

$$-\frac{7x^4 - 4}{12x^7 - 12x^3} - \frac{7\log\left(x-1\right)}{16} + \frac{7\log\left(x+1\right)}{16} + \frac{7\,\mathrm{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8-2*x**4+1),x)
```

```
[Out] -(7*x**4 - 4)/(12*x**7 - 12*x**3) - 7*log(x - 1)/16 + 7*log(x + 1)/16 + 7*a
tan(x)/8
```

Giac [A] time = 1.11398, size = 46, normalized size = 1.28

$$-\frac{x}{4\left(x^4-1\right)} - \frac{1}{3\,x^3} + \frac{7}{8}\arctan\left(x\right) + \frac{7}{16}\log\left(|x+1|\right) - \frac{7}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/3/x^3 + 7/8*arctan(x) + 7/16*log(abs(x + 1)) - 7/16*lo
g(abs(x - 1))
```

3.307 $\int \frac{1}{x^6(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=43

$$\frac{1}{4x^5(1-x^4)}-\frac{9}{20x^5}-\frac{9}{4x}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)$$

```
[Out] -9/(20*x^5) - 9/(4*x) + 1/(4*x^5*(1 - x^4)) - (9*ArcTan[x])/8 + (9*ArcTanh[
x])/8
```

Rubi [A] time = 0.0122302, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 290, 325, 298, 203, 206}

$$\frac{1}{4x^5(1-x^4)}-\frac{9}{20x^5}-\frac{9}{4x}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -9/(20*x^5) - 9/(4*x) + 1/(4*x^5*(1 - x^4)) - (9*ArcTan[x])/8 + (9*ArcTanh[
x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^6\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4}\int \frac{1}{x^6\left(-1+x^4\right)}\,dx\\
&= -\frac{9}{20x^5}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4}\int \frac{1}{x^2\left(-1+x^4\right)}\,dx\\
&= -\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4}\int \frac{x^2}{-1+x^4}\,dx\\
&= -\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}+\frac{9}{8}\int \frac{1}{1-x^2}\,dx-\frac{9}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0209894, size = 51, normalized size = 1.19

$$-\frac{x^3}{4\left(x^4-1\right)}-\frac{1}{5x^5}-\frac{2}{x}-\frac{9}{16}\log(1-x)+\frac{9}{16}\log(x+1)-\frac{9}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - 2/x - x^3/(4*(-1 + x^4)) - (9*ArcTan[x])/8 - (9*Log[1 - x])/16
 + (9*Log[1 + x])/16
```

Maple [A] time = 0.016, size = 52, normalized size = 1.2

$$-\frac{x}{8\,x^2+8}-\frac{9\arctan(x)}{8}-\frac{1}{5\,x^5}-2\,x^{-1}-\frac{1}{16+16\,x}+\frac{9\ln(1+x)}{16}-\frac{1}{16\,x-16}-\frac{9\ln(x-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8-2*x^4+1),x)
```

```
[Out] -1/8*x/(x^2+1)-9/8*arctan(x)-1/5/x^5-2/x-1/16/(1+x)+9/16*ln(1+x)-1/16/(x-1)
-9/16*ln(x-1)
```

Maxima [A] time = 1.51062, size = 57, normalized size = 1.33

$$-\frac{45\,x^8-36\,x^4-4}{20\left(x^9-x^5\right)}-\frac{9}{8}\,\arctan\left(x\right)+\frac{9}{16}\,\log\left(x+1\right)-\frac{9}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/20*(45*x^8 - 36*x^4 - 4)/(x^9 - x^5) - 9/8*arctan(x) + 9/16*log(x + 1) -
 9/16*log(x - 1)
```

Fricas [B] time = 1.47535, size = 171, normalized size = 3.98

$$-\frac{180\,x^8-144\,x^4+90\left(x^9-x^5\right)\arctan\left(x\right)-45\left(x^9-x^5\right)\log\left(x+1\right)+45\left(x^9-x^5\right)\log\left(x-1\right)-16}{80\left(x^9-x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/80*(180*x^8 - 144*x^4 + 90*(x^9 - x^5)*arctan(x) - 45*(x^9 - x^5)*log(x
+ 1) + 45*(x^9 - x^5)*log(x - 1) - 16)/(x^9 - x^5)
```

Sympy [A] time = 0.223132, size = 44, normalized size = 1.02

$$-\frac{9\log\left(x-1\right)}{16}+\frac{9\log\left(x+1\right)}{16}-\frac{9\,\mathrm{atan}\left(x\right)}{8}-\frac{45x^8-36x^4-4}{20x^9-20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8-2*x**4+1),x)
```

```
[Out] -9*log(x - 1)/16 + 9*log(x + 1)/16 - 9*atan(x)/8 - (45*x**8 - 36*x**4 - 4)/
(20*x**9 - 20*x**5)
```

Giac [A] time = 1.1035, size = 58, normalized size = 1.35

$$-\frac{x^3}{4\left(x^4-1\right)}-\frac{10\,x^4+1}{5\,x^5}-\frac{9}{8}\,\arctan\left(x\right)+\frac{9}{16}\,\log\left(|x+1|\right)-\frac{9}{16}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/5*(10*x^4 + 1)/x^5 - 9/8*arctan(x) + 9/16*log(abs(x
+ 1)) - 9/16*log(abs(x - 1))
```

3.308 $\int \frac{1}{x^8(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=43

$$\frac{1}{4x^7(1-x^4)} - \frac{11}{12x^3} - \frac{11}{28x^7} + \frac{11}{8}\tan^{-1}(x) + \frac{11}{8}\tanh^{-1}(x)$$

```
[Out] -11/(28*x^7) - 11/(12*x^3) + 1/(4*x^7*(1 - x^4)) + (11*ArcTan[x])/8 + (11*A
rcTanh[x])/8
```

Rubi [A] time = 0.0120157, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 290, 325, 212, 206, 203}

$$\frac{1}{4x^7(1-x^4)} - \frac{11}{12x^3} - \frac{11}{28x^7} + \frac{11}{8}\tan^{-1}(x) + \frac{11}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -11/(28*x^7) - 11/(12*x^3) + 1/(4*x^7*(1 - x^4)) + (11*ArcTan[x])/8 + (11*A
rcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^8\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^7\left(1-x^4\right)}-\frac{11}{4}\int \frac{1}{x^8\left(-1+x^4\right)}\,dx\\
&= -\frac{11}{28x^7}+\frac{1}{4x^7\left(1-x^4\right)}-\frac{11}{4}\int \frac{1}{x^4\left(-1+x^4\right)}\,dx\\
&= -\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}-\frac{11}{4}\int \frac{1}{-1+x^4}\,dx\\
&= -\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}+\frac{11}{8}\int \frac{1}{1-x^2}\,dx+\frac{11}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}+\frac{11}{8}\tan^{-1}(x)+\frac{11}{8}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0194211, size = 43, normalized size = 1.

$$\frac{1}{336}\left(-\frac{84x}{x^4-1}-\frac{224}{x^3}-\frac{48}{x^7}-231\log(1-x)+231\log(x+1)+462\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-48/x^7 - 224/x^3 - (84*x)/(-1 + x^4) + 462*ArcTan[x] - 231*Log[1 - x] + 2
31*Log[1 + x])/336
```

Maple [A] time = 0.015, size = 52, normalized size = 1.2

$$\frac{x}{8x^2+8}+\frac{11\arctan(x)}{8}-\frac{1}{7x^7}-\frac{2}{3x^3}-\frac{1}{16+16x}+\frac{11\ln(1+x)}{16}-\frac{1}{16x-16}-\frac{11\ln(x-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8-2*x^4+1),x)
```

```
[Out] 1/8*x/(x^2+1)+11/8*arctan(x)-1/7/x^7-2/3/x^3-1/16/(1+x)+11/16*ln(1+x)-1/16/
(x-1)-11/16*ln(x-1)
```

Maxima [A] time = 1.57545, size = 57, normalized size = 1.33

$$-\frac{77\,x^{8}-44\,x^{4}-12}{84\,\left(x^{11}-x^{7}\right)}+\frac{11}{8}\,\arctan\left(x\right)+\frac{11}{16}\,\log\left(x+1\right)-\frac{11}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/84*(77*x^8 - 44*x^4 - 12)/(x^11 - x^7) + 11/8*arctan(x) + 11/16*log(x +
1) - 11/16*log(x - 1)
```

Fricas [B] time = 1.51699, size = 182, normalized size = 4.23

$$-\frac{308\,x^{8}-176\,x^{4}-462\,\left(x^{11}-x^{7}\right)\arctan\left(x\right)-231\,\left(x^{11}-x^{7}\right)\log\left(x+1\right)+231\,\left(x^{11}-x^{7}\right)\log\left(x-1\right)-48}{336\,\left(x^{11}-x^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/336*(308*x^8 - 176*x^4 - 462*(x^11 - x^7)*arctan(x) - 231*(x^11 - x^7)*l
og(x + 1) + 231*(x^11 - x^7)*log(x - 1) - 48)/(x^11 - x^7)
```

Sympy [A] time = 0.229206, size = 44, normalized size = 1.02

$$-\frac{11\log\left(x-1\right)}{16}+\frac{11\log\left(x+1\right)}{16}+\frac{11\,\mathrm{atan}\left(x\right)}{8}-\frac{77x^{8}-44x^{4}-12}{84x^{11}-84x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-2*x**4+1),x)
```

```
[Out] -11*log(x - 1)/16 + 11*log(x + 1)/16 + 11*atan(x)/8 - (77*x**8 - 44*x**4 -
12)/(84*x**11 - 84*x**7)
```

Giac [A] time = 1.07282, size = 55, normalized size = 1.28

$$-\frac{x}{4\left(x^{4}-1\right)}-\frac{14\,x^{4}+3}{21\,x^{7}}+\frac{11}{8}\,\arctan\left(x\right)+\frac{11}{16}\,\log\left(|x+1|\right)-\frac{11}{16}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/21*(14*x^4 + 3)/x^7 + 11/8*arctan(x) + 11/16*log(abs(x
 + 1)) - 11/16*log(abs(x - 1))
```

3.309 $\int \frac{x^m}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=163

$$\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b - S
qrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1 + m)) - (
2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b + Sq
rt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m))
```

Rubi [A] time = 0.140995, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1375, 364}

$$\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b - S
qrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1 + m)) - (
2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b + Sq
rt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m))
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m}{a+bx^4+cx^8}\,dx &= \frac{c\int \frac{x^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{x^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{2cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)}-\frac{2cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+m)}
\end{aligned}$$

Mathematica [C] time = 0.0435743, size = 58, normalized size = 0.36

$$\frac{x^{m+1}\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{{}_2F_1\left(1,m+1;m+2;\frac{x}{\#1}\right)}{\#1^4b+2a}\&\right]}{4(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(a + b*x^4 + c*x^8),x]
```

```
[Out] (x^(1 + m)*RootSum[a + b*#1^4 + c*#1^8 & , Hypergeometric2F1[1, 1 + m, 2 +
m, x/#1]/(2*a + b*#1^4) & ])/(4*(1 + m))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \frac{x^m}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c*x^8+b*x^4+a),x)
```

```
[Out] int(x^m/(c*x^8+b*x^4+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{cx^8+bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral(x^m/(c*x^8 + b*x^4 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

3.310 $\int \frac{x^{11}}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=81

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}+\frac{x^4}{4c}$$

```
[Out] x^4/(4*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

Rubi [A] time = 0.0837916, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1357, 703, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^4/(4*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{a+bx^4+cx^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^4\right)\\ &= \frac{x^4}{4c}+\frac{\operatorname{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^4\right)}{4c}\\ &= \frac{x^4}{4c}-\frac{b\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8c^2}+\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8c^2}\\ &= \frac{x^4}{4c}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}-\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4c^2}\\ &= \frac{x^4}{4c}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}\end{aligned}$$

Mathematica [A] time = 0.05244, size = 78, normalized size = 0.96

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-b\log\left(a+bx^4+cx^8\right)+2cx^4}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*c*x^4 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

Maple [A] time = 0.004, size = 111, normalized size = 1.4

$$\frac{x^4}{4c}-\frac{b\ln\left(cx^8+bx^4+a\right)}{8c^2}-\frac{a}{2c}\arctan\left((2cx^4+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{b^2}{4c^2}\arctan\left((2cx^4+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*x^4/c-1/8*b*ln(c*x^8+b*x^4+a)/c^2-1/2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^4+b)/(4*a*c-b^2)^(1/2))*a+1/4/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*
a*c-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90724, size = 556, normalized size = 6.86

$$\left[\frac{2\left(b^2c-4\,ac^2\right)x^4-\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-\left(b^3-4\,abc\right)\log\left(cx^8+bx^4+a\right)}{8\left(b^2c^2-4\,ac^3\right)}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(b^2*c - 4*a*c^2)*x^4 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b
*x^4 + a)) - (b^3 - 4*a*b*c)*log(c*x^8 + b*x^4 + a))/(b^2*c^2 - 4*a*c^3), 1
/8*(2*(b^2*c - 4*a*c^2)*x^4 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^8 +
 b*x^4 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 2.90928, size = 316, normalized size = 3.9

$$\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{8c^2\left(4ac-b^2\right)}\right)\log\left(x^4+\frac{-ab-16ac^2\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{8c^2(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{8c^2(4ac-b^2)}\right)}{2ac-b^2}\right)+\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))*
log(x**4 + (-a*b - 16*a*c**2*(-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(8*c**2*(4*a*c - b**2))) + 4*b**2*c*(-b/(8*c**2) - sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(8*c**2) + s
qrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))*log(x**4 + (-a*b
 - 16*a*c**2*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a
*c - b**2))) + 4*b**2*c*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**4/(4*c)
```

Giac [A] time = 7.44507, size = 101, normalized size = 1.25

$$\frac{x^4}{4\,c} - \frac{b\log\left(cx^8 + bx^4 + a\right)}{8\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4/c - 1/8*b*log(c*x^8 + b*x^4 + a)/c^2 + 1/4*(b^2 - 2*a*c)*arctan((2*
c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.311 $\int \frac{x^9}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=192

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

```
[Out] x^2/(2*c) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^
2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]])
```

Rubi [A] time = 0.338914, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1359, 1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^2/(2*c) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^
2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx, x, x^2\right)$$

$$= \frac{x^2}{2c} - \frac{\operatorname{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx, x, x^2\right)}{2c}$$

$$= \frac{x^2}{2c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{4c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right.}{4c}$$

$$= \frac{x^2}{2c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.132932, size = 210, normalized size = 1.09

$$\frac{-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + 2\sqrt{c}x^2}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*Sqrt[c]*x^2 - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

Maple [B] time = 0.035, size = 360, normalized size = 1.9

$$\frac{x^2}{2\,c} - \frac{\sqrt{2}b}{4\,c}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}} + \frac{\sqrt{2}a}{2}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/2*x^2/c-1/4/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
```

```
^(1/2))*a-1/4/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+1/4/c*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*b+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/4/c/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^4 + a)*x/(c*x^8 + b*x^4 + a), x)/c
```

Fricas [B] time = 1.7006, size = 2192, normalized size = 11.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 -
 a^2*c)*x^2 + 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b
*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3
*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*
a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^
3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^
3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4
*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
- 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqr
t(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 + 1/2*sqr
t(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4
*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a
*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 -
 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b
^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^
6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - 2*x^2)/c
```

Sympy [A] time = 3.20674, size = 134, normalized size = 0.7

$$\mathrm{RootSum}\left(t^4\left(4096a^2c^5 - 2048ab^2c^4 + 256b^4c^3\right) + t^2\left(192a^2bc^2 - 112ab^3c + 16b^5\right) + a^3, \left(t \mapsto t\log\left(x^2 + \frac{256t^3abc^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**2*c**5 - 2048*a*b**2*c**4 + 256*b**4*c**3) + _t**2*(
192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + a**3, Lambda(_t, _t*log(x**2 +
(256*_t**3*a*b*c**4 - 64*_t**3*b**3*c**3 - 8*_t*a**2*c**2 + 16*_t*a*b**2*c
- 4*_t*b**4)/(a**2*c - a*b**2)))) + x**2/(2*c)
```

Giac [C] time = 8.02828, size = 1721, normalized size = 8.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - ((a*c^3)^(1/4)*b*c*x^4*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
)))) - (a*c^3)^(1/4)*b*c*x^4*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) +
 (a*c^3)^(1/4)*a*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*
sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(
1/4)*a*c*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*si
nh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan((x^2 - (a/c)^
(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))/((a/c)^(1/4)*si
n(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*b*c
*abs(c) - (b^2 - 4*a*c)*c^2) - ((a*c^3)^(1/4)*b*c*x^4*cosh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*b*c*x^4*sin(1/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c))))) - (a*c^3)^(1/4)*a*c*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*a
rctan((x^2 - (a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sq
rt(b^2 - 4*a*c)*b*c*abs(c) - (b^2 - 4*a*c)*c^2) + 1/2*((a*c^3)^(1/4)*b*c*x^
4*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*b*c*x^4*cos(
5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a*c*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*c*cos(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c))))))*log(x^4 - 2*x^2*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))) + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*b*c*abs(c) - (b
^2 - 4*a*c)*c^2) + 1/2*((a*c^3)^(1/4)*b*c*x^4*cos(1/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) - (a*c^3)^(1/4)*b*c*x^4*cos(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))) + (a*c^3)^(1/4)*a*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
- (a*c^3)^(1/4)*a*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(x^4 -
 2*x^2*(a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + s
```

```
qrt(a/c))/(sqrt(b^2 - 4*a*c)*b*c*abs(c) - (b^2 - 4*a*c)*c^2)
```

3.312 $\int \frac{x^7}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^4 + c*x^8]/(8*c)
```

Rubi [A] time = 0.0578285, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1357, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^4 + c*x^8]/(8*c)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,x^4\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^4\right)}{8c}$$
$$= \frac{\log\left(a+bx^4+cx^8\right)}{8c} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^4\right)}{4c}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

Mathematica [A] time = 0.025042, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^4+cx^8\right)-\frac{2b\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^4 + c*x^8])/(8*c)
```

Maple [A] time = 0.003, size = 60, normalized size = 1.

$$\frac{\ln\left(cx^8+bx^4+a\right)}{8c} - \frac{b}{4c}\arctan\left((2cx^4+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/8*ln(c*x^8+b*x^4+a)/c-1/4*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c
-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61185, size = 443, normalized size = 7.03

$$\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^8+bx^4+a\right)}{8\left(b^2c-4\,ac^2\right)},\ \frac{2\sqrt{-b^2+4\,ac}b\arctan\left(-\frac{\left(2\,cx^4+b\right)\sqrt{\ }}{b^2-}\right.}{8\left(b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x
^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) + (b^2 - 4*a*c)*log(c*x^8 +
 b*x^4 + a))/(b^2*c - 4*a*c^2), 1/8*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^8 + b*x^4
+ a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] time = 1.72497, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{8c\left(4ac-b^2\right)}+\frac{1}{8c}\right)\log\left(x^4+\frac{-16ac\left(-\frac{b\sqrt{-4ac+b^2}}{8c\left(4ac-b^2\right)}+\frac{1}{8c}\right)+2a+4b^2\left(-\frac{b\sqrt{-4ac+b^2}}{8c\left(4ac-b^2\right)}+\frac{1}{8c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{8c\left(4ac-b^2\right)}+\frac{1}{8c}\right)\log\left(x^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*c
*(-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(-b
*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b) + (b*sqrt(-4*a*c +
 b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*c*(b*sqrt(-4*a*c +
 b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(b*sqrt(-4*a*c + b**2
)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b)
```

Giac [A] time = 7.94798, size = 80, normalized size = 1.27

$$-\frac{b\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\sqrt{-b^2+4\,ac}c}+\frac{\log\left(cx^8+bx^4+a\right)}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
8*log(c*x^8 + b*x^4 + a)/c
```

3.313 $\int \frac{x^5}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -(Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) + (Sqrt[b + Sqrt[b^2 -
4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[
2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.126211, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1359, 1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] -(Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) + (Sqrt[b + Sqrt[b^2 -
4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[
2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx,x,x^2\right)$$
$$= \frac{1}{4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)+\frac{1}{4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{}{\frac{b}{2}+}\right.$$
$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.0926503, size = 171, normalized size = 1.08

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 -
4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(
Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[
b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

Maple [A] time = 0.016, size = 216, normalized size = 1.4

$$\frac{\sqrt{2}}{4}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}+\frac{\sqrt{2}b}{4}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-1/4*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] time = 1.52664, size = 1214, normalized size = 7.64

$$\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2+\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\right.}{\sqrt{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) -
1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) -
1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) +
1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3))
```

Sympy [A] time = 2.21134, size = 76, normalized size = 0.48

$$\operatorname{RootSum}\left(t^4\left(4096a^2c^3-2048ab^2c^2+256b^4c\right)+t^2\left(-64abc+16b^3\right)+a,\left(t\mapsto t\log\left(512t^3ac^2-128t^3b^2c-4tb+x^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**2*c**3 - 2048*a*b**2*c**2 + 256*b**4*c) + _t**2*(-64
*a*b*c + 16*b**3) + a, Lambda(_t, _t*log(512*_t**3*a*c**2 - 128*_t**3*b**2*
c - 4*_t*b + x**2)))
```

Giac [C] time = 7.82648, size = 1403, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*
a*c)*b)*x^4*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4
```

```
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b
^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*x^4*sin(5/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan((x^2 - (a/c)^(1/4)*cos(5/4*pi +
 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))/((a/c)^(1/4)*sin(5/4*pi + 1/2*arc
sin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c - 4*a^2*c^2) + 1/4*(((a*c^3)^(1
/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*x^4*cosh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(
1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*x^4*sin(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c))))))*arctan((x^2 - (a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c
)*b/(a*abs(c))))))/(a*b^2*c - 4*a^2*c^2) - 1/8*(((a*c^3)^(1/4)*b^2 - 4*(a*c
^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*x^4*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^
3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*x^4*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
))))*log(x^4 - 2*x^2*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))) + sqrt(a/c))/(a*b^2*c - 4*a^2*c^2) - 1/8*(((a*c^3)^(1/4)*b^2 - 4
*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*x^4*cos(1/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c +
(a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*x^4*cos(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c))))))*log(x^4 - 2*x^2*(a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))) + sqrt(a/c))/(a*b^2*c - 4*a^2*c^2)
```

3.314 $\int \frac{x^3}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

[Out] -ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]/(2*Sqrt[b^2 - 4*a*c])

Rubi [A] time = 0.034339, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*x^4 + c*x^8),x]

[Out] -ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]/(2*Sqrt[b^2 - 4*a*c])

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+bx^4+cx^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^4\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^4\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.0119278, size = 42, normalized size = 1.11

$$\frac{\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^4 + c*x^8),x]
```

```
[Out] ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]]/(2*Sqrt[-b^2 + 4*a*c])
```

Maple [A] time = 0.003, size = 37, normalized size = 1.

$$\frac{1}{2}\arctan\left((2\,cx^4+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51326, size = 296, normalized size = 7.79

$$\left[\frac{\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac-(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{4\,\sqrt{b^2-4\,ac}}, -\frac{\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{2\,(b^2-4\,ac)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c - (2*c*x^4 + b)*sqrt(b^2 - 4*
a*c))/(c*x^8 + b*x^4 + a))/sqrt(b^2 - 4*a*c), -1/2*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] time = 0.698491, size = 131, normalized size = 3.45

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**4 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4 + sqrt(-1/(4*a*c - b**2))*log(x**4 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4
```

Giac [A] time = 7.50366, size = 49, normalized size = 1.29

$$\frac{\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.315 $\int \frac{x}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=154

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*ArcTan[(Sqrt[2]
*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.0940859, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1359, 1093, 205}

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] (Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*ArcTan[(Sqrt[2]
*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x}{a+bx^4+cx^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a+bx^2+cx^4}\,dx, x, x^2\right)$$
$$= \frac{c\operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{2\sqrt{b^2-4ac}} - \frac{c\operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{2\sqrt{b^2-4ac}}$$
$$= \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.084659, size = 133, normalized size = 0.86

$$\frac{\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] (Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/Sqrt[b
- Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[2]*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.013, size = 120, normalized size = 0.8

$$-\frac{c\sqrt{2}}{2}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}} - \frac{c\sqrt{2}}{2}\operatorname{Artanh}\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/2*c/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] time = 1.58348, size = 1345, normalized size = 8.73

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(cx^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(c*x^2 + 1/2*sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/s
qrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c
))/(a*b^2 - 4*a^2*c))) + 1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^
2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c
 - (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c
)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/4*sqrt(1/2)*sqrt(-(b - (
a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2 + 1/
2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqr
t(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/
4*sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 -
4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt
(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/
(a*b^2 - 4*a^2*c)))
```

Sympy [A] time = 2.45914, size = 88, normalized size = 0.57

$$\operatorname{RootSum}\left(t^4\left(4096a^3c^2-2048a^2b^2c+256ab^4\right)+t^2\left(-64abc+16b^3\right)+c,\left(t\mapsto t\log\left(x^2+\frac{256t^3a^2bc-64t^3ab^3+8tac-}{c}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**3*c**2 - 2048*a**2*b**2*c + 256*a*b**4) + _t**2*(-64
*a*b*c + 16*b**3) + c, Lambda(_t, _t*log(x**2 + (256*_t**3*a**2*b*c - 64*_t
**3*a*b**3 + 8*_t*a*c - 4*_t*b**2)/c)))
```

Giac [C] time = 7.99019, size = 1370, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*
a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 -
```

```
 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c))))))*arctan((x^2 - (a/c)^(1/4)*cos(5/4*pi + 1/2*arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))/((a/c)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))))/(a*b^2*c - 4*a^2*c^2) + 1/4*(((a*c^3)^(1/4)*b^2
- 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (
a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
))))*arctan((x^2 - (a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
))/(a*b^2*c - 4*a^2*c^2) - 1/8*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c +
(a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
)))) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 -
4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(x^4 - 2*x^2*(a/
c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + sqrt(a/c))/
(a*b^2*c - 4*a^2*c^2) - 1/8*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*
c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a
*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(x^4 - 2*x^2*(a/c)^
(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + sqrt(a/c))/(a*
b^2*c - 4*a^2*c^2)
```

3.316 $\int \frac{1}{x(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^4 + c*x^8]/(8*a)
```

Rubi [A] time = 0.0683157, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1357, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^4 + c*x^8]/(8*a)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^4\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)}{4a} + \frac{\operatorname{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^4\right)}{4a}\\
&= \frac{\log(x)}{a} - \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8a} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8a}\\
&= \frac{\log(x)}{a} - \frac{\log\left(a+bx^4+cx^8\right)}{8a} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log\left(a+bx^4+cx^8\right)}{8a}
\end{aligned}$$

Mathematica [C] time = 0.0232458, size = 66, normalized size = 0.96

$$\frac{\log(x)}{a} - \frac{\operatorname{RootSum}\left[\#1^4 b + \#1^8 c + a\&, \frac{\#1^4 c\log(x-\#1)+b\log(x-\#1)}{2\#1^4 c+b}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] Log[x]/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]*#
1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

Maple [A] time = 0.007, size = 66, normalized size = 1.

$$\frac{\ln(x)}{a} - \frac{\ln\left(cx^8+bx^4+a\right)}{8a} - \frac{b}{4a}\arctan\left((2cx^4+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^8+b*x^4+a),x)
```

```
[Out] ln(x)/a-1/8*ln(c*x^8+b*x^4+a)/a-1/4/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b
)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80138, size = 510, normalized size = 7.39

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^8+bx^4+a\right)+8\left(b^2-4\,ac\right)\log\left(x\right)}{8\left(ab^2-4\,a^2c\right)},\ \frac{2\sqrt{-b^2+4\,\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x
^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^2 - 4*a*c)*log(c*x^8 +
 b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/8*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^8 + b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] time = 5.46836, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)\log\left(x^4+\frac{-16a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)+4ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-16*a**
2*c*(-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(-b*
sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c))
+ (b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-16*a*
*2*c*(b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(b*s
qrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c)) +
 log(x)/a
```

Giac [A] time = 7.70529, size = 92, normalized size = 1.33

$$-\frac{b \arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a}-\frac{\log\left(cx^8+bx^4+a\right)}{8\,a}+\frac{\log\left(x^4\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
8*log(c*x^8 + b*x^4 + a)/a + 1/4*log(x^4)/a
```

3.317 $\int \frac{1}{x^3(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=184

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.224378, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1359, 1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(2*a*x^2) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx,x,x^2\right)\\ &= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx,x,x^2\right)}{2a}\\ &= -\frac{1}{2ax^2}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)}{4a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\right.}{4a}\\ &= -\frac{1}{2ax^2}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [C] time = 0.0319162, size = 75, normalized size = 0.41

$$-\frac{\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&,\frac{\#1^4 c\log(x-\#1)+b\log(x-\#1)}{\#1^2 b+2\#1^6 c}\&\right]}{4a}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(2*a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #
1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

Maple [A] time = 0.017, size = 240, normalized size = 1.3

$$-\frac{c\sqrt{2}}{4\,a}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}+\frac{c\sqrt{2}b}{4\,a}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/4*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b+1/4*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/4*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b-1/2/a/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2/(a*x^2)
```

Fricas [B] time = 1.82014, size = 2314, normalized size = 12.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2
*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4
 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a
^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt
(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 - 1/2*sq
rt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b
*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*
c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c +
 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
 - sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^
2 - a*c^3)*x^2 - 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 -
6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*
c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x^2)
```

Sympy [A] time = 3.64692, size = 153, normalized size = 0.83

$$\operatorname{RootSum}\left(t^{4}\left(4096a^{5}c^{2} - 2048a^{4}b^{2}c + 256a^{3}b^{4}\right) + t^{2}\left(192a^{2}bc^{2} - 112ab^{3}c + 16b^{5}\right) + c^{3}, \left(t \mapsto t\log\left(x^{2} + \frac{-512t^{3}a^{5}c^{2} + 3}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**5*c**2 - 2048*a**4*b**2*c + 256*a**3*b**4) + _t**2*(
192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + c**3, Lambda(_t, _t*log(x**2 +
(-512*_t**3*a**5*c**2 + 384*_t**3*a**4*b**2*c - 64*_t**3*a**3*b**4 - 20*_t*
```

```
a**2*b*c**2 + 20*_t*a*b**3*c - 4*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(2*a*
x**2)
```

Giac [C] time = 7.44808, size = 1721, normalized size = 9.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -((a*c^3)^(1/4)*a*c*x^4*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c
^3)^(1/4)*a*c*x^4*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1
/4)*a*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*b*si
n(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan((x^2 - (a/c)^(1/4)*cos(5
/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))/((a/c)^(1/4)*sin(5/4*pi +
1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*a*b*abs(a) - (
b^2 - 4*a*c)*a^2) - ((a*c^3)^(1/4)*a*c*x^4*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c))))) - (a*c^3)^(1/4)*a*c*x^4*sin(1/4*pi + 1/2*real_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c))))) + (a*c^3)^(1/4)*a*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (
a*c^3)^(1/4)*a*b*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan((x^2
- (a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))/((a/c)^
(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*
a*c)*a*b*abs(a) - (b^2 - 4*a*c)*a^2) + 1/2*((a*c^3)^(1/4)*a*c*x^4*cos(5/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*c*x^4*cos(5/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a*b*cos(5/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*b*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))))*log(x^4 - 2*x^2*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))) + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*a*b*abs(a) - (b^2 - 4*a*c)
*a^2) + 1/2*((a*c^3)^(1/4)*a*c*x^4*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
)))) - (a*c^3)^(1/4)*a*c*x^4*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) +
 (a*c^3)^(1/4)*a*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(
1/4)*a*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*si
nh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(x^4 - 2*x^2*(a/c
)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + sqrt(a/c))/(
sqrt(b^2 - 4*a*c)*a*b*abs(a) - (b^2 - 4*a*c)*a^2) - 1/2/(a*x^2)
```

3.318 $\int \frac{1}{x^5(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}$$

```
[Out] -1/(4*a*x^4) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^4 + c*x^8])/(8*a^2
)
```

Rubi [A] time = 0.123629, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(4*a*x^4) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^4 + c*x^8])/(8*a^2
)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^4\right)\\
&= -\frac{1}{4ax^4}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx, x, x^4\right)}{4a}\\
&= -\frac{1}{4ax^4}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx, x, x^4\right)}{4a}\\
&= -\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, x^4\right)}{4a^2}\\
&= -\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8a^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8a^2}\\
&= -\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4a^2}\\
&= -\frac{1}{4ax^4}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}
\end{aligned}
$$

Mathematica [C] time = 0.0310873, size = 92, normalized size = 1.03

$$
\frac{\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&,\frac{\#1^4 bc\log(x-\#1)-ac\log(x-\#1)+b^2\log(x-\#1)}{2\#1^4 c+b}\&\right]}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(4*a*x^4) - (b*Log[x])/a^2 + RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log[x
- #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a^2)
```

Maple [A] time = 0.009, size = 119, normalized size = 1.3

$$-\frac{1}{4\,a x^4}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(c x^8+b x^4+a\right)}{8\,a^2}-\frac{c}{2\,a}\arctan\left((2\,c x^4+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2}{4\,a^2}\arctan\left((2\,c x^4+b)\frac{1}{\sqrt{4\,a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4/a/x^4-b*ln(x)/a^2+1/8*b*ln(c*x^8+b*x^4+a)/a^2-1/2/a/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))*c+1/4/a^2/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^4+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.5792, size = 664, normalized size = 7.46

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,x^4\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-\left(b^3-4\,abc\right)x^4\log\left(cx^8+bx^4+a\right)+8\left(b^3-4\,abc\right)x^4}{8\left(a^2b^2-4\,a^3c\right)x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2
 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^3 - 4
*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4), -1/8*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^4*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 7.50556, size = 127, normalized size = 1.43

$$\frac{b\log\left(cx^8+bx^4+a\right)}{8\,a^2}-\frac{b\log\left(x^4\right)}{4\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a^2}+\frac{bx^4-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*b*log(c*x^8 + b*x^4 + a)/a^2 - 1/4*b*log(x^4)/a^2 + 1/4*(b^2 - 2*a*c)*a
rctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/4*(b*x
^4 - a)/(a^2*x^4)
```

3.319 $\int \frac{x^{10}}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=381

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{b}{\sqrt{}}\right.}{2}$$

```
[Out] x^3/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2 - 4*a
*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(
1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*
c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b
^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 0.64723, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1367, 1510, 298, 205, 208}

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{b}{\sqrt{}}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^3/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2 - 4*a
*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(
1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*
c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b
^2 - 4*a*c])^(1/4))
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
```

```
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \frac{x^3}{3c} - \frac{\int \frac{x^2\left(3a+3bx^4\right)}{a+bx^4+cx^8}\,dx}{3c}$$
$$= \frac{x^3}{3c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}$$
$$= \frac{x^3}{3c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}c^{3/2}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}c^{3/2}} + \frac{\left(b+\frac{b}{\sqrt{}}\right.}{}$$
$$= \frac{x^3}{3c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\mathrm{t}}{2\,2^{3/4}c^{7/4}}$$

Mathematica [C] time = 0.0411818, size = 70, normalized size = 0.18

$$\frac{4x^3-3\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&,\frac{\#1^4 b\log(x-\#1)+a\log(x-\#1)}{2\#1^5 c+\#1 b}\&\right]}{12c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^4 + c*x^8),x]
```

```
[Out] (4*x^3 - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#
1^4)/(b*#1 + 2*c*#1^5) & ])/(12*c)
```

Maple [C] time = 0.037, size = 63, normalized size = 0.2

$$\frac{x^3}{3c}-\frac{1}{4c}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)}\frac{\left(\_R^6 b+\_R^2 a\right)\ln(x-\_R)}{2\_R^7 c+\_R^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/3*x^3/c-1/4/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z
^8*c+_Z^4*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 11.0777, size = 13450, normalized size = 35.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^3 + 12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*
c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6
*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7
 - 8*a*b^2*c^8 + 16*a^2*c^9)))*arctan(-1/2*((b^6*c^7 - 10*a*b^4*c^8 + 32*a^
2*b^2*c^9 - 32*a^3*c^10)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^
4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)) - (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^
2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4)*x + sqrt(1/2)*(b^9 - 9*a*b^7*c + 26*a^2*b
^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^
2*c^9 - 32*a^3*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6
*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15
 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt((2*(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*
b^2*c^2 - a^6*c^3)*x^2 - sqrt(1/2)*(b^11 - 12*a*b^9*c + 53*a^2*b^7*c^2 - 10
3*a^3*b^5*c^3 + 79*a^4*b^3*c^4 - 12*a^5*b*c^5 - (b^8*c^7 - 13*a*b^6*c^8 + 6
0*a^2*b^4*c^9 - 112*a^3*b^2*c^10 + 64*a^4*c^11)*sqrt((b^12 - 10*a*b^10*c +
37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6
)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 -
 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2
*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b
^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c
^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^3*b^6 - 5*a^
4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c
+ 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt(
(b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^
3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^2*b^6 - 5*a^3*b^4*c + 6
*a^4*b^2*c^2 - a^5*c^3)) - 12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*
a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12
```

- 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*arctan(1/2*(sqrt(1/2)*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt((2*(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*x^2 - sqrt(1/2)*(b^11 - 12*a*b^9*c + 53*a^2*b^7*c^2 - 103*a^3*b^5*c^3 + 79*a^4*b^3*c^4 - 12*a^5*b*c^5 + (b^8*c^7 - 13*a*b^6*c^8 + 60*a^2*b^4*c^9 - 112*a^3*b^2*c^10 + 64*a^4*c^11)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)) - ((b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)) + (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4)*x)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))))/(a^2*b^6 - 5*a^3*b^4*c + 6*a^4*b^2*c^2 - a^5*c^3)) - 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*x) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8

```
*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3
*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*
c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9
)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*
b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^
6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^1
5 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))
- (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*x) - 3*c*sqrt(sqrt(1/2)
*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2
*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c
^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 +
 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*lo
g(1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 +
553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^
7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 -
320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48
*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a
^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12
- 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b
^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17
)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*
b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 1
0*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*
c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))
/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2
*c^2 - a^8*c^3)*x) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a
*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b
^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c
 + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 +
 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9
- 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt
(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7
 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*
b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2
*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 -
8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^
3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4
*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^
9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*x))/c
```

Sympy [A] time = 46.0003, size = 360, normalized size = 0.94

$$\operatorname{RootSum}\left(t^{8}\left(16777216a^{4}c^{11} - 16777216a^{3}b^{2}c^{10} + 6291456a^{2}b^{4}c^{9} - 1048576ab^{6}c^{8} + 65536b^{8}c^{7}\right) + t^{4}\left(-28672a^{5}bc^{5} + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**11 - 16777216*a**3*b**2*c**10 + 6291456*a**
2*b**4*c**9 - 1048576*a*b**6*c**8 + 65536*b**8*c**7) + _t**4*(-28672*a**5*b
```

```
*c**5 + 71680*a**4*b**3*c**4 - 59136*a**3*b**5*c**3 + 22016*a**2*b**7*c**2
- 3840*a*b**9*c + 256*b**11) + a**7, Lambda(_t, _t*log(x + (5242880*_t**7*a
**5*b*c**12 - 9175040*_t**7*a**4*b**3*c**11 + 5963776*_t**7*a**3*b**5*c**10
 - 1851392*_t**7*a**2*b**7*c**9 + 278528*_t**7*a*b**9*c**8 - 16384*_t**7*b*
*11*c**7 + 512*_t**3*a**7*c**7 - 9344*_t**3*a**6*b**2*c**6 + 29184*_t**3*a*
*5*b**4*c**5 - 35392*_t**3*a**4*b**6*c**4 + 20992*_t**3*a**3*b**8*c**3 - 65
28*_t**3*a**2*b**10*c**2 + 1024*_t**3*a*b**12*c - 64*_t**3*b**14)/(a**8*c**
3 - 6*a**7*b**2*c**2 + 5*a**6*b**4*c - a**5*b**6)))) + x**3/(3*c)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{10}}{cx^8 + bx^4 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^10/(c*x^8 + b*x^4 + a), x)
```

3.320 $\int \frac{x^8}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=376

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\cdots$$

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4
*a*c])^(3/4))
```

Rubi [A] time = 0.573854, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1367, 1422, 212, 208, 205}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4
*a*c])^(3/4))
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
```

```
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{a+bx^4+cx^8}\,dx &= \frac{x}{c}-\frac{\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx}{c}\\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}\\
&= \frac{x}{c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}}+\frac{\left(b+\frac{b^2}{\sqrt{b^2}}\right.\cdots}{\cdots}\\
&= \frac{x}{c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan\cdots}{2\sqrt[4]{2}c^{5/4}\left(-b\cdots\right.}
\end{aligned}$$

Mathematica [C] time = 0.0377287, size = 70, normalized size = 0.19

$$\frac{x}{c}-\frac{\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&,\frac{\#1^4 b\log(x-\#1)+a\log(x-\#1)}{\#1^3 b+2\#1^7 c}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] time = 0.004, size = 59, normalized size = 0.2

$$\frac{x}{c}+\frac{1}{4c}\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}b-a\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^8+b*x^4+a),x)
```

```
[Out] x/c+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*
b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 4.35127, size = 10701, normalized size = 28.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/4*(2*sqrt(1/2)*((b^10*c^
5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128
*a^5*c^10)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^11 - 1
3*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c
^5)*x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*
c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^
6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
+ 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (
b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*sqrt((2*(a^2*b^4 - 3*a^3
*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^4*c^2 - 30*a^
3*b^2*c^3 + 8*a^4*c^4 + (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b
*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^
6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*
```

b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
+ 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/
(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(a^4*b^
4 - 3*a^5*b^2*c + a^6*c^2)) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/
4*(2*sqrt(1/2)*((b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8
+ 352*a^4*b^2*c^9 - 128*a^5*c^10)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 6
4*a^3*c^13)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*
a^4*b^3*c^4 - 32*a^5*b*c^5)*x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 -
5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)) + (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c
^4 - 32*a^5*b*c^5 - (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4
*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*
a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*
b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*sqrt((2*(a^
2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^
4*c^2 - 30*a^3*b^2*c^3 + 8*a^4*c^4 - (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c
^7 - 64*a^3*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt
(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)))/(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))/(a^4*b^4 - 3*a^5*b^2*c + a
^6*c^2)) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5
- 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3
*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*
a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3
*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6
*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log
((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2
- 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2

```
*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqr
t(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 -
7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*
c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6
*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqr
t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 +
16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c
 + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^
5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8
*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - 4*x)/c
```

Sympy [A] time = 15.8387, size = 218, normalized size = 0.58

$$\operatorname{RootSum}\left(t^8\left(16777216a^4c^9 - 16777216a^3b^2c^8 + 6291456a^2b^4c^7 - 1048576ab^6c^6 + 65536b^8c^5\right) + t^4\left(20480a^4bc^4 - 307\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**9 - 16777216*a**3*b**2*c**8 + 6291456*a**2*
b**4*c**7 - 1048576*a*b**6*c**6 + 65536*b**8*c**5) + _t**4*(20480*a**4*b*c*
*4 - 30720*a**3*b**3*c**3 + 15616*a**2*b**5*c**2 - 3328*a*b**7*c + 256*b**9
) + a**5, Lambda(_t, _t*log(x + (16384*_t**5*a**2*b*c**7 - 8192*_t**5*a*b**
3*c**6 + 1024*_t**5*b**5*c**5 - 8*_t*a**3*c**3 + 36*_t*a**2*b**2*c**2 - 24*
_t*a*b**4*c + 4*_t*b**6)/(a**3*c**2 - 3*a**2*b**2*c + a*b**4)))) + x/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^8/(c*x^8 + b*x^4 + a), x)
```

3.321 $\int \frac{x^6}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=325

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\cdots\right.}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4a\cdots}}$$

```
[Out] -((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b + Sqrt[b^2 -
 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(3/4)*Arc
Tanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4
)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^(1/4)*c^(
1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*
c])
```

Rubi [A] time = 0.306896, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1374, 298, 205, 208}

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\cdots\right.}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4a\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b + Sqrt[b^2 -
 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(3/4)*Arc
Tanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4
)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^(1/4)*c^(
1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*
c])
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+c\ldots}$$

$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}+\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}-\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\ldots}{\ldots}$$

$$= -\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-b-\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.0244725, size = 44, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{\#1^3\log(x-\#1)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1^3)/(b + 2*c*#1^4) & ]/4
```

Maple [C] time = 0.003, size = 43, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\_R^6\ln(x-\_R)}{2\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] time = 2.92876, size = 8154, normalized size = 25.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*arctan(1/2*((b^5*c^3
 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)) + (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^5*c^3 - 8*a*b^3*c^4 + 1
6*a^2*b*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*
a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt((2*(a*b^2 - a^2*c)*x^2 - sqrt(1/2)*(b^5 -
5*a*b^3*c + 4*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3
*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2
*c^8 - 64*a^3*c^9)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2
*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2
*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(a*b^2 - a^2*c)
))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^
8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(a*b^2 - a^2*c)) +
 sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*arctan(1/2*(sqrt(1/2
)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 +
16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a
^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt((2*(
a*b^2 - a^2*c)*x^2 - sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (b^6*c^3 -
12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^3 - 3*a
*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b
^2*c^4 + 16*a^2*c^5)))/(a*b^2 - a^2*c)) + ((b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*
b*c^5)*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*
b^2*c^8 - 64*a^3*c^9)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x)*sqrt(sqrt(1/2)*sq
rt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))))/(a*b^2 - a^2*c)) + 1/4*sqrt(sqrt(1/2)
*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))
/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c
+ 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5
- 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 -
 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8
*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 +
16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a
^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2
 - a^3*c)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2
```

```
*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7
 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*lo
g(-1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^
3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^
2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3
*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a
*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(
b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^
3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2
*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^
3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b
^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
 - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c
^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c
)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 1
6*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*s
qrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a
*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*s
qrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c -
 (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4
 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x)
```

Sympy [A] time = 7.0665, size = 230, normalized size = 0.71

$$\text{RootSum}\left(t^8\left(16777216a^4c^7 - 16777216a^3b^2c^6 + 6291456a^2b^4c^5 - 1048576ab^6c^4 + 65536b^8c^3\right) + t^4\left(-12288a^3bc^3 + 10\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**7 - 16777216*a**3*b**2*c**6 + 6291456*a**2*
b**4*c**5 - 1048576*a*b**6*c**4 + 65536*b**8*c**3) + _t**4*(-12288*a**3*b*c
**3 + 10240*a**2*b**3*c**2 - 2816*a*b**5*c + 256*b**7) + a**3, Lambda(_t, _
t*log(x + (2097152*_t**7*a**4*c**7 - 2621440*_t**7*a**3*b**2*c**6 + 1179648
*_t**7*a**2*b**4*c**5 - 229376*_t**7*a*b**6*c**4 + 16384*_t**7*b**8*c**3 -
1280*_t**3*a**3*b*c**3 + 1600*_t**3*a**2*b**3*c**2 - 576*_t**3*a*b**5*c + 6
4*_t**3*b**7)/(a**3*c - a**2*b**2))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

3.322 $\int \frac{x^4}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=325

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 -
 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 -
4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcT
anh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)
*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c
])
```

Rubi [A] time = 0.29584, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1374, 212, 208, 205}

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 -
 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 -
4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcT
anh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)
*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c
])
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}$$

$$= \frac{\sqrt{-b-\sqrt{b^2-4ac}}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{b^2-4ac}} + \frac{\sqrt{-b-\sqrt{b^2-4ac}}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{b^2-4ac}} - \frac{\sqrt{-}}{}$$

$$= \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[4]{-b-\sqrt{b^2}}}{}$$

Mathematica [C] time = 0.0216347, size = 42, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&,\frac{\#1\log(x-\#1)}{2\#1^4 c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1)/(b + 2*c*#1^4) & ]/4
```

Maple [C] time = 0.002, size = 43, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\text{RootOf}(\_Z^8c+\_Z^4b+a)}\frac{\_R^4\ln(x-\_R)}{2\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] time = 2.04456, size = 5315, normalized size = 16.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)))*arctan(1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x^2 + sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(-(b + (b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*((b^4 - 8*a*b^2
*c + 16*a^2*c^2)*x - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*x/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sqrt
(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + sqrt(s
qrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))
)*arctan(-1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*sqrt(x^2 + sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b - (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(-(b - (b^4*c - 8*a*b
^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*((b^4 - 8*a*b^2*c + 1
6*a^2*c^2)*x + (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x/sqr
t(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b - (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b -
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + 1/4*sqrt(sqr
t(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*
log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x - (b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)))*log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sq
rt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b
^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))
/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1/4*sqrt(sqr
```

```
t(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*
log(x - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
```

Sympy [A] time = 3.81215, size = 126, normalized size = 0.39

$$\operatorname{RootSum}\left(t^8\left(16777216a^4c^5 - 16777216a^3b^2c^4 + 6291456a^2b^4c^3 - 1048576ab^6c^2 + 65536b^8c\right) + t^4\left(4096a^2bc^2 - 204\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**5 - 16777216*a**3*b**2*c**4 + 6291456*a**2*
b**4*c**3 - 1048576*a*b**6*c**2 + 65536*b**8*c) + _t**4*(4096*a**2*b*c**2 -
 2048*a*b**3*c + 256*b**5) + a, Lambda(_t, _t*log(-32768*_t**5*a**2*c**3 +
16384*_t**5*a*b**2*c**2 - 2048*_t**5*b**4*c - 4*_t*b + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

3.323 $\int \frac{x^2}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=315

$$-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\tanh^{-1}\left(\cdots\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\cdots}}$$

```
[Out] -((c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(
3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (c^(1/4)*ArcTan[(
2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a
*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) - (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 0.289891, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1375, 298, 205, 208}

$$-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\tanh^{-1}\left(\cdots\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(
3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (c^(1/4)*ArcTan[(
2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a
*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) - (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = \frac{c\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{c}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}} - \frac{\sqrt{c}\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}} - \frac{\sqrt{c}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}} +$$

$$= -\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.0230013, size = 43, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&,\frac{\log (x-\#1)}{2 \#1^5 c+\#1 b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1 + 2*c*#1^5) & ]/4
```

Maple [C] time = 0.003, size = 43, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\mathrm{RootOf}(\_Z^8c+\_Z^4b+a)}\frac{\_R^2\ln(x-\_R)}{2\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] time = 2.29391, size = 5895, normalized size = 18.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3
*c^2)))*arctan(-(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*x*sqrt(sqrt(1/2)*sqrt(-(
b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4
*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/sqrt(a^2*b^6 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) + sqrt(1/2)*(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a
^2*b^2*c + 16*a^3*c^2)))*sqrt((2*c*x^2 - sqrt(1/2)*(b^3 - 4*a*b*c + (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^
2*b^2*c + 16*a^3*c^2)))/c)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3)) - sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/
sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2)))*arctan(-((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*x/sqrt(a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - sqrt(1/2)*(a*b^4 - 8*
a^2*b^2*c + 16*a^3*c^2)*sqrt((2*c*x^2 - sqrt(1/2)*(b^3 - 4*a*b*c - (a*b^6 -
 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2
*b^2*c + 16*a^3*c^2)))/c)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c +
16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*
b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a
^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b +
(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4
- 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sqrt(sqrt(1/2
)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-
1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^
3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)) + c*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2
*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a
*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sqrt(sqrt(1/2)*
```

```
sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/
2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*
b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(
a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c
 + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a
^3*c^2)) + c*x)
```

Sympy [A] time = 3.49312, size = 172, normalized size = 0.55

$$\mathrm{RootSum}\left(t^8\left(16777216a^5c^4 - 16777216a^4b^2c^3 + 6291456a^3b^4c^2 - 1048576a^2b^6c + 65536ab^8\right) + t^4\left(4096a^2bc^2 - 20\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**5*c**4 - 16777216*a**4*b**2*c**3 + 6291456*a**3*
b**4*c**2 - 1048576*a**2*b**6*c + 65536*a*b**8) + _t**4*(4096*a**2*b*c**2 -
 2048*a*b**3*c + 256*b**5) + c, Lambda(_t, _t*log(x + (1048576*_t**7*a**4*b
*c**3 - 786432*_t**7*a**3*b**3*c**2 + 196608*_t**7*a**2*b**5*c - 16384*_t**
7*a*b**7 - 512*_t**3*a**2*c**2 + 384*_t**3*a*b**2*c - 64*_t**3*b**4)/c)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^8 + bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

3.324 $\int \frac{1}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=315

$$\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tan}{\sqrt[4]{2}\sqrt{b^2-4}}$$

```
[Out] (c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/
4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4
*a*c])^(3/4)) - (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 0.304007, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1347, 212, 208, 205}

$$\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tan}{\sqrt[4]{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4 + c*x^8)^(-1),x]
```

```
[Out] (c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/
4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4
*a*c])^(3/4)) - (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{c\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{c\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} - \frac{c\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}} - \cdots$$

$$= \frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\tanh^{-1}\left(\cdots\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\cdots\right.}$$

Mathematica [C] time = 0.0254798, size = 45, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&,\frac{\log(x-\#1)}{\#1^3 b+2\#1^7 c}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4 + c*x^8)^(-1),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1^3 + 2*c*#1^7) & ]/4
```

Maple [C] time = 0.003, size = 40, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8 c+\_Z^4 b+a\right)}\frac{\ln(x-\_R)}{2\_R^7 c+\_R^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(1/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] time = 3.01051, size = 8162, normalized size = 25.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(1/4*(2*sqrt(1
/2)*((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c
^4)*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)) - (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3)*x)*
sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) + (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 -
 16*a^3*b*c^3 - (a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3
+ 128*a^7*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))*sqrt((2
*(b^2*c^2 - a*c^3)*x^2 + sqrt(1/2)*(b^6 - 7*a*b^4*c + 14*a^2*b^2*c^2 - 8*a^
3*c^3 - (a^3*b^7 - 12*a^4*b^5*c + 48*a^5*b^3*c^2 - 64*a^6*b*c^3)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^2*c^2 - a*c^3)))*sqrt(sqrt(1
/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^2*c^2 - a*c^3)) + sqrt(sqrt(1
/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(1/4*(2*sqrt(1/2)*((a^3*b^
8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*x*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)) + (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3)*x)*sqrt(sqrt(1/
2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)
))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 -
 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)) - (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (a^3*b^8 - 14*a^4
*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqr
t(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64
*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (
a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c +
 16*a^5*c^2))*sqrt((2*(b^2*c^2 - a*c^3)*x^2 + sqrt(1/2)*(b^6 - 7*a*b^4*c +
14*a^2*b^2*c^2 - 8*a^3*c^3 + (a^3*b^7 - 12*a^4*b^5*c + 48*a^5*b^3*c^2 - 64*
a^6*b*c^3)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^
```

```
8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^2*c^2 -
 a*c^3)))/(b^2*c^2 - a*c^3)) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a
^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(
b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
+ (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqr
t(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*
b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*
b^2*c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2
*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)
*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 -
3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4
*a^2*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(
1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))))
```

Sympy [A] time = 6.57737, size = 177, normalized size = 0.56

$$\mathrm{RootSum}\left(t^8\left(16777216a^7c^4 - 16777216a^6b^2c^3 + 6291456a^5b^4c^2 - 1048576a^4b^6c + 65536a^3b^8\right) + t^4\left(-12288a^3bc^3 + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**7*c**4 - 16777216*a**6*b**2*c**3 + 6291456*a**5*
b**4*c**2 - 1048576*a**4*b**6*c + 65536*a**3*b**8) + _t**4*(-12288*a**3*b*c
**3 + 10240*a**2*b**3*c**2 - 2816*a*b**5*c + 256*b**7) + c**3, Lambda(_t, _
t*log(x + (16384*_t**5*a**5*b*c**2 - 8192*_t**5*a**4*b**3*c + 1024*_t**5*a*
*3*b**5 + 8*_t*a**2*c**2 - 16*_t*a*b**2*c + 4*_t*b**4)/(a*c**2 - b**2*c))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^8 + bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^8 + b*x^4 + a), x)
```

3.325 $\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=363

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

```
[Out] -(1/(a*x)) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4
)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sq
rt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^
(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(
1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 0.411679, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1368, 1510, 298, 205, 208}

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(1/(a*x)) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4
)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sq
rt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^
(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(
1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
```

```
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx = -\frac{1}{ax}+\frac{\int \frac{x^2\left(-b-cx^4\right)}{a+bx^4+cx^8}\,dx}{a}$$

$$= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}$$

$$= -\frac{1}{ax}+\frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}a}-\frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}}{2\sqrt{2}a}$$

$$= -\frac{1}{ax}-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}+\frac{\sqrt[4]{c}\,(}{}$$

Mathematica [C] time = 0.0382723, size = 71, normalized size = 0.2

$$-\frac{\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^5c+\#1b}\&\right]}{4a}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(1/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]
*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

Maple [C] time = 0.006, size = 63, normalized size = 0.2

$$-\frac{1}{4a}\sum_{\_R=\text{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(\_R^6c+\_R^2b\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4
*b+a))-1/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 6.4674, size = 10831, normalized size = 29.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-1/2*((a^5*b^5 - 8*a^6*b
^3*c + 16*a^7*b*c^2)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) +
 (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*x - sqrt(1/2)*(b^6 - 7*a*b^
4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*s
qrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6
- 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((2*(b^4*c^3 - 3*a*b
^2*c^4 + a^2*c^5)*x^2 - sqrt(1/2)*(b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 43*a
^3*b^3*c^3 + 12*a^4*b*c^4 + (a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*
a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)
))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10
*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^
2*c + 16*a^7*c^2)))/(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)))*sqrt(sqrt(1/2)*sqrt
(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2)))/(b^4*c - 3*a*b^2*c^2 + a^2*c^3)) - 4*a*x*sqrt(sqrt(1/2)*sqrt(-
(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt(
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12
*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2)))*arctan(-1/2*(sqrt(1/2)*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3
*c^3 - (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^
2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2
```

```
+ (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 -
 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt((2*(b^4*c^3 - 3
*a*b^2*c^4 + a^2*c^5)*x^2 - sqrt(1/2)*(b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 -
43*a^3*b^3*c^3 + 12*a^4*b*c^4 - (a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 -
112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*
c^3)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a
^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^
6*b^2*c + 16*a^7*c^2)))/(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)) + ((a^5*b^5 - 8*
a^6*b^3*c + 16*a^7*b*c^2)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*x)*sqrt(sqrt(1/2)*sqrt
(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2))))/(b^4*c - 3*a*b^2*c^2 + a^2*c^3)) - a*x*sqrt(sqrt(1/2)*sqrt(-(
b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*
a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a
^7*c^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b
^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c + 98*a^7
*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48
*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(
-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt
((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 1
2*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16
*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) + a*x*sqrt(sqrt(1/2)*sqrt
(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2)))*log(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a
^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c + 98
*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11
*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b
^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*s
qrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c
+ 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) - a*x*sqrt(sqrt(1/2)*
sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c
 + 16*a^7*c^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 13
8*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c +
 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*
a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))
```

```
)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^
2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*
b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2
*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) + a*x*sqrt(sqrt(1/
2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10
*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^
2*c + 16*a^7*c^2)))*log(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2
- 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8
*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5
- 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11
*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^
7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6
*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) + 4)/(a*x)
```

Sympy [A] time = 16.9495, size = 304, normalized size = 0.84

$$\mathrm{RootSum}\Big(t^8\left(16777216a^9c^4-16777216a^8b^2c^3+6291456a^7b^4c^2-1048576a^6b^6c+65536a^5b^8\right)+t^4\left(20480a^4bc^4-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**9*c**4 - 16777216*a**8*b**2*c**3 + 6291456*a**7*
b**4*c**2 - 1048576*a**6*b**6*c + 65536*a**5*b**8) + _t**4*(20480*a**4*b*c*
*4 - 30720*a**3*b**3*c**3 + 15616*a**2*b**5*c**2 - 3328*a*b**7*c + 256*b**9
) + c**5, Lambda(_t, _t*log(x + (-2097152*_t**7*a**10*c**5 + 5767168*_t**7*
a**9*b**2*c**4 - 4587520*_t**7*a**8*b**4*c**3 + 1605632*_t**7*a**7*b**6*c**
2 - 262144*_t**7*a**6*b**8*c + 16384*_t**7*a**5*b**10 - 2304*_t**3*a**5*b*c
**5 + 8256*_t**3*a**4*b**3*c**4 - 8832*_t**3*a**3*b**5*c**3 + 4032*_t**3*a*
*2*b**7*c**2 - 832*_t**3*a*b**9*c + 64*_t**3*b**11)/(a**2*c**6 - 3*a*b**2*c
**5 + b**4*c**4)))) - 1/(a*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^8+bx^4+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^8 + b*x^4 + a)*x^2), x)
```

3.326 $\int \frac{1}{x^4(a+bx^4+cx^8)} dx$

Optimal. Leaf size=365

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] -1/(3*a*x^3) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3
/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (
c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a
*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 0.398978, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1368, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(3*a*x^3) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3
/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (
c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a
*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
```

```
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^4+cx^8\right)}\,dx = -\frac{1}{3ax^3}+\frac{\int \frac{-3b-3cx^4}{a+bx^4+cx^8}\,dx}{3a}$$

$$= -\frac{1}{3ax^3}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}$$

$$= -\frac{1}{3ax^3}+\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}}+\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{\ldots}}}{2a\sqrt{-b-\sqrt{b^2-4ac}}}$$

$$= -\frac{1}{3ax^3}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] time = 0.0433128, size = 75, normalized size = 0.21

$$-\frac{\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{\#1^3b+2\#1^7c}\&\right]}{4a}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(3*a*x^3) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #
1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*a)
```

Maple [C] time = 0.006, size = 62, normalized size = 0.2

$$-\frac{1}{3\,ax^3}+\frac{1}{4\,a}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(-\_R^4c-b\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/3/a/x^3+1/4/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*
c+_Z^4*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 7.61082, size = 13495, normalized size = 36.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/12*(12*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a
^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c +
37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6
)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8
*a^8*b^2*c + 16*a^9*c^2)))*arctan(-1/4*(2*sqrt(1/2)*((a^7*b^11 - 17*a^8*b^9
*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5
)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4
*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^
2 - 64*a^17*c^3)) + (b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^
3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7)*x)*sq
rt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48
*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) - (b^1
4 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 45
7*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (a^7*b^11 - 17*a^8*b^9*c + 1
13*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*sqrt
((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 1
2*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a
^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4
 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^
15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9
*c^2))*sqrt((2*(b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*x^2 + sqrt
(1/2)*(b^12 - 13*a*b^10*c + 64*a^2*b^8*c^2 - 147*a^3*b^6*c^3 + 156*a^4*b^4*
c^4 - 66*a^5*b^2*c^5 + 8*a^6*c^6 + (a^7*b^9 - 14*a^8*b^7*c + 72*a^9*b^5*c^2
 - 160*a^10*b^3*c^3 + 128*a^11*b*c^4)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^
6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c
```

+ 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt
((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 1
2*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a
^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(b^6*c^4 - 5*a*b^4*c^5 +
6*a^2*b^2*c^6 - a^3*c^7)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b
^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10
*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c
^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/
(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(b^6*c^5 - 5*a*b^4*c^6 + 6*a^2*b^2*c
^7 - a^3*c^8)) - 12*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^
3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*
a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(
a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*arctan(-1/4*(2*sqrt(1/2)*((a^7*b^11 -
17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320
*a^12*b*c^5)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*
a^16*b^2*c^2 - 64*a^17*c^3)) - (b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328
*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7
*c^7)*x)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2
*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^1
4*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b
^2*c + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
+ (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b
^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)) + (b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3
+ 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (a^7*
b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4
- 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c
^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c +
48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 1
4*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^
12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^
5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*
c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a
^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12
- 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b
^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3
)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2))*sqrt((2*(b^6*c^4 - 5*a*b^4*c^5 +
6*a^2*b^2*c^6 - a^3*c^7)*x^2 + sqrt(1/2)*(b^12 - 13*a*b^10*c + 64*a^2*b^8*c
^2 - 147*a^3*b^6*c^3 + 156*a^4*b^4*c^4 - 66*a^5*b^2*c^5 + 8*a^6*c^6 - (a^7*
b^9 - 14*a^8*b^7*c + 72*a^9*b^5*c^2 - 160*a^10*b^3*c^3 + 128*a^11*b*c^4)*sq
rt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64
*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b
^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*
a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a
^9*c^2)))/(b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)))/(b^6*c^5 - 5*
a*b^4*c^6 + 6*a^2*b^2*c^7 - a^3*c^8)) - 3*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 -
7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9
*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b
^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*
c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-(b^6*c^2 -
5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x + 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b
^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^
2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6

```
*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c
 + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c +
 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((
b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*
a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^1
7*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*a*x^3*sqrt(sqrt(1/2)*s
qrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2
*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 4
8*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(
-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x - 1/2*(b^9 - 9*a*b^7*c
 + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^4*c
+ 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12
*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 -
7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^
4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c
^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) - 3*a*x^3*sqrt(
sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4
- 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*
a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^1
5*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x + 1/2*(b^9
- 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (a^7*b^6 - 10
*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^
14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sq
rt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48
*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*
a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
- (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b
^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x
- 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (a
^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(s
qrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 -
 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15
*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c
^2)))) - 4)/(a*x^3)
```

Sympy [A] time = 39.6987, size = 277, normalized size = 0.76

$$\mathrm{RootSum}\Big(t^8\left(16777216a^{11}c^4 - 16777216a^{10}b^2c^3 + 6291456a^9b^4c^2 - 1048576a^8b^6c + 65536a^7b^8\right) + t^4\left(-28672a^5bc^5 + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**11*c**4 - 16777216*a**10*b**2*c**3 + 6291456*a**
9*b**4*c**2 - 1048576*a**8*b**6*c + 65536*a**7*b**8) + _t**4*(-28672*a**5*b
```

```
*c**5 + 71680*a**4*b**3*c**4 - 59136*a**3*b**5*c**3 + 22016*a**2*b**7*c**2
- 3840*a*b**9*c + 256*b**11) + c**7, Lambda(_t, _t*log(x + (32768*_t**5*a**
10*c**3 - 32768*_t**5*a**9*b**2*c**2 + 10240*_t**5*a**8*b**4*c - 1024*_t**5
*a**7*b**6 - 36*_t*a**4*b*c**4 + 120*_t*a**3*b**3*c**3 - 108*_t*a**2*b**5*c
**2 + 36*_t*a*b**7*c - 4*_t*b**9)/(a**3*c**5 - 6*a**2*b**2*c**4 + 5*a*b**4*
c**3 - b**6*c**2)))) - 1/(3*a*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^8 + bx^4 + a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^8 + b*x^4 + a)*x^4), x)
```

3.327 $\int \frac{x^m}{1+x^4+x^8} \, dx$

Optimal. Leaf size=127

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 - I*Sqr
t[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(I - Sqrt[3])*(1
 + m))
```

Rubi [A] time = 0.0779896, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1375, 364}

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 + x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 - I*Sqr
t[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(I - Sqrt[3])*(1
 + m))
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{1+x^4+x^8} \, dx &= -\frac{i\int \frac{x^m}{\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^4} \, dx}{\sqrt{3}}+\frac{i\int \frac{x^m}{\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^4} \, dx}{\sqrt{3}}\\ &= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}\end{aligned}$$

Mathematica [C] time = 0.40294, size = 212, normalized size = 1.67

$$\frac{1}{4}x^{m+1}\left(\frac{x^2\text{RootSum}\left[\#1^4-\#1^2+1\&,\frac{{}_2F_1\left(1,m+3;m+4;\frac{x}{\#1}\right)}{\#1^2-2}\&\right]}{m+3}-\frac{\text{RootSum}\left[\#1^4-\#1^2+1\&,\frac{{}_2F_1\left(1,m+1;m+2;\frac{x}{\#1}\right)}{\#1^2-2}\&\right]}{m+1}+\frac{3i(\sqrt{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(1 + x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*(((3*I)*(I + Sqrt[3])*Hypergeometric2F1[1, 1 + m, 2 + m, -((-1)^
(1/3)*x)] + (3*I)*(I + Sqrt[3])*Hypergeometric2F1[1, 1 + m, 2 + m, (-1)^(1/
3)*x] - 6*(Hypergeometric2F1[1, 1 + m, 2 + m, -((-1)^(2/3)*x)] + Hypergeome
tric2F1[1, 1 + m, 2 + m, (-1)^(2/3)*x]))/(6*(-2 + (-1)^(1/3))*(1 + m)) - Ro
otSum[1 - #1^2 + #1^4 & , Hypergeometric2F1[1, 1 + m, 2 + m, x/#1]/(-2 + #1
^2) & ]/(1 + m) + (x^2*RootSum[1 - #1^2 + #1^4 & , Hypergeometric2F1[1, 3 +
 m, 4 + m, x/#1]/(-2 + #1^2) & ])/(3 + m)))/4
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8+x^4+1),x)
```

```
[Out] int(x^m/(x^8+x^4+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 + x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{x^8+x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 + x^4 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(x^2-x+1)(x^2+x+1)(x^4-x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8+x**4+1),x)
```

```
[Out] Integral(x**m/((x**2 - x + 1)*(x**2 + x + 1)*(x**4 - x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 + x^4 + 1), x)
```

3.328 $\int \frac{x^{11}}{1+x^4+x^8} \, dx$

Optimal. Leaf size=44

$$\frac{x^4}{4}-\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8
```

Rubi [A] time = 0.0367014, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1357, 703, 634, 618, 204, 628}

$$\frac{x^4}{4}-\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 + x^4 + x^8),x]
```

```
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{1+x^4+x^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{1+x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4}-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^4\right)-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4}-\frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^4\right)\\
&= \frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1+x^4+x^8\right)
\end{aligned}$$

Mathematica [A] time = 0.0111154, size = 44, normalized size = 1.

$$\frac{x^4}{4}-\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 + x^4 + x^8),x]
```

```
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8
```

Maple [A] time = 0.002, size = 36, normalized size = 0.8

$$\frac{x^4}{4}-\frac{\ln\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+x^4+1),x)
```

```
[Out] 1/4*x^4-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.47132, size = 47, normalized size = 1.07

$$\frac{1}{4}x^4-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

Fricas [A] time = 1.45751, size = 109, normalized size = 2.48

$$\frac{1}{4}x^4 - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right) - \frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

Sympy [A] time = 0.134792, size = 42, normalized size = 0.95

$$\frac{x^4}{4} - \frac{\log\left(x^8+x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+x**4+1),x)
```

```
[Out] x**4/4 - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3
)/12
```

Giac [A] time = 1.09179, size = 47, normalized size = 1.07

$$\frac{1}{4}x^4 - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right) - \frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

3.329 $\int \frac{x^9}{1+x^4+x^8} dx$

Optimal. Leaf size=54

$$\frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] x^2/2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/Sqrt[3
]]/(2*Sqrt[3])
```

Rubi [A] time = 0.0577644, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1359, 1122, 1161, 618, 204}

$$\frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + x^4 + x^8),x]
```

```
[Out] x^2/2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/Sqrt[3
]]/(2*Sqrt[3])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{1+x^4+x^8}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{1+x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{x^2}{2}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{x^2}{2}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{2}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right)\\
&= \frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.173811, size = 98, normalized size = 1.81

$$\frac{x^2}{2}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x^2\right)}{2\sqrt{6+6i\sqrt{3}}}-\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x^2\right)}{2\sqrt{6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/(1 + x^4 + x^8),x]
```

```
[Out] x^2/2 - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 + (6*I)*Sq
rt[3]]) - ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 - (6*I)*
Sqrt[3]])
```

Maple [A] time = 0.005, size = 43, normalized size = 0.8

$$\frac{x^2}{2}-\frac{\sqrt{3}}{6}\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)-\frac{\sqrt{3}}{6}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+x^4+1),x)
```

```
[Out] 1/2*x^2-1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)-1/6*3^(1/2)*arctan(1/3*(2
*x^2-1)*3^(1/2))
```

Maxima [A] time = 1.49526, size = 57, normalized size = 1.06

$$\frac{1}{2}x^2-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x^2 - 1))
```

Fricas [A] time = 1.43287, size = 128, normalized size = 2.37

$$\frac{1}{2}x^2 - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x^2\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(x^6 + 2x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x^2) - 1/6*sqrt(3)*arctan(1/3*sqrt
(3)*(x^6 + 2*x^2))
```

Sympy [A] time = 0.137822, size = 51, normalized size = 0.94

$$\frac{x^2}{2} + \frac{\sqrt{3}\left(-2\operatorname{atan}\left(\frac{\sqrt{3}x^2}{3}\right) - 2\operatorname{atan}\left(\frac{\sqrt{3}x^6}{3} + \frac{2\sqrt{3}x^2}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(
3)*x**2/3))/12
```

Giac [A] time = 1.10704, size = 57, normalized size = 1.06

$$\frac{1}{2}x^2 - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2 + 1\right)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2 - 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x^2 - 1))
```

3.330 $\int \frac{x^7}{1+x^4+x^8} \, dx$

Optimal. Leaf size=37

$$\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 + x^4 + x^8]/8
```

Rubi [A] time = 0.0310579, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 + x^4 + x^8]/8
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{1+x^4+x^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{1+x+x^2}\,dx,x,x^4\right)\\
&= -\left(\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^4\right)\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^4\right)\\
&= \frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1+x^4+x^8\right)
\end{aligned}
$$

Mathematica [A] time = 0.008011, size = 37, normalized size = 1.

$$\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 + x^4 + x^8]/8
```

Maple [A] time = 0.003, size = 31, normalized size = 0.8

$$\frac{\ln\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.49953, size = 41, normalized size = 1.11

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)+\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

Fricas [A] time = 1.48928, size = 97, normalized size = 2.62

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)+\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

Sympy [A] time = 0.130603, size = 37, normalized size = 1.

$$\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+x**4+1),x)
```

```
[Out] log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/12
```

Giac [A] time = 1.09363, size = 41, normalized size = 1.11

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)+\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

3.331 $\int \frac{x^5}{1+x^4+x^8}\,dx$

Optimal. Leaf size=75

$$\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*S
qrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]/8
```

Rubi [A] time = 0.0765716, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1359, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*S
qrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]/8
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{1+x^4+x^8}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{1+x^2+x^4}\,dx, x, x^2\right)\\
&= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^2+x^4}\,dx, x, x^2\right)\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1+2x}{-1-x-x^2}\,dx, x, x^2\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1-2x}{-1+x-x^2}\,dx, x, x^2\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-x+x\cdots}\right.\\
&= \frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right)-\frac{1}{4}\operatorname{Subst}\left(\int \cdots\right.\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)\end{aligned}$$

Mathematica [C] time = 0.121438, size = 94, normalized size = 1.25

$$\frac{\sqrt{1-i\sqrt{3}}\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x^2\right)+\sqrt{1+i\sqrt{3}}\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x^2\right)}{4\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/(1 + x^4 + x^8),x]
```

```
[Out] (Sqrt[1 - I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2] + Sqrt[1
 + I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(4*Sqrt[6])
```

Maple [A] time = 0.003, size = 62, normalized size = 0.8

$$-\frac{\ln\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)+\frac{\ln\left(x^4-x^2+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+x^4+1),x)
```

[Out] -1/8*ln(x^4+x^2+1)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)+1/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))

Maxima [A] time = 1.50898, size = 82, normalized size = 1.09

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+x^4+1),x, algorithm="maxima")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)

Fricas [A] time = 1.52828, size = 193, normalized size = 2.57

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+x^4+1),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)

Sympy [A] time = 0.191331, size = 76, normalized size = 1.01

$$\frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(x**8+x**4+1),x)

[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12

Giac [A] time = 1.10643, size = 82, normalized size = 1.09

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+x^4+1),x, algorithm="giac")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)

3.332 $\int \frac{x^3}{1+x^4+x^8} \, dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.0212884, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1352, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1+x^4+x^8} \, dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2} \, dx, x, x^4\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2} \, dx, x, 1+2x^4\right)\right)\\ &= \frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0056358, size = 23, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A] time = 0.002, size = 19, normalized size = 0.8

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+x^4+1),x)
```

```
[Out] 1/6*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.51776, size = 24, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Fricas [A] time = 1.45901, size = 61, normalized size = 2.65

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Sympy [A] time = 0.116641, size = 26, normalized size = 1.13

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/6
```

Giac [A] time = 1.16556, size = 24, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

3.333 $\int \frac{x}{1+x^4+x^8} dx$

Optimal. Leaf size=75

$$-\frac{1}{8}\log\left(x^4-x^2+1\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*S
qrt[3]) - Log[1 - x^2 + x^4]/8 + Log[1 + x^2 + x^4]/8
```

Rubi [A] time = 0.0636766, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1359, 1094, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^4-x^2+1\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*S
qrt[3]) - Log[1 - x^2 + x^4]/8 + Log[1 + x^2 + x^4]/8
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+x^4+x^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2+x^4}\,dx,x,x^2\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx,x,x^2\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1+x}{1+x+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)-\frac{1}{8}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^2\right)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,d\right.\\ &= -\frac{1}{8}\log\left(1-x^2+x^4\right)+\frac{1}{8}\log\left(1+x^2+x^4\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)-\frac{1}{4}\,\text{Subst}\left(\int\right.\\ &= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1-x^2+x^4\right)+\frac{1}{8}\log\left(1+x^2+x^4\right)\end{aligned}$$

Mathematica [C] time = 0.0501971, size = 79, normalized size = 1.05

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x^2\right)-\sqrt{1+i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x^2\right)\right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(1 + x^4 + x^8),x]
```

```
[Out] ((I/2)*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x^2)/2] - Sqrt[1 + I*Sqr
t[3]]*ArcTan[((I + Sqrt[3])*x^2)/2]))/Sqrt[6]
```

Maple [A] time = 0.004, size = 62, normalized size = 0.8

$$\frac{\ln\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)-\frac{\ln\left(x^4-x^2+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^4+x^2+1)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)-1/8*ln(x^4-x^2
+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Maxima [A] time = 1.50983, size = 82, normalized size = 1.09

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

Fricas [A] time = 1.46677, size = 193, normalized size = 2.57

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

Sympy [A] time = 0.190986, size = 76, normalized size = 1.01

$$-\frac{\log\left(x^4-x^2+1\right)}{8}+\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+x**4+1),x)
```

```
[Out] -log(x**4 - x**2 + 1)/8 + log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x
**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

Giac [A] time = 1.10085, size = 82, normalized size = 1.09

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

3.334 $\int \frac{1}{x(1+x^4+x^8)}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 + x^4 + x^8]/8
```

Rubi [A] time = 0.0347782, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^4 + x^8)),x]
```

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 + x^4 + x^8]/8
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1+x^4+x^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\left(1+x+x^2\right)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^4\right)-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1+x^4+x^8\right)
\end{aligned}$$

Mathematica [C] time = 0.0858239, size = 138, normalized size = 3.54

$$\frac{1}{24}\left(-\sqrt{3}\left(\sqrt{3}-i\right)\log\left(x^2-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)-\sqrt{3}\left(\sqrt{3}+i\right)\log\left(x^2+\frac{1}{2}i\left(\sqrt{3}+i\right)\right)-3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)+24\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^4 + x^8)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
 + 24*Log[x] - Sqrt[3]*(-I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^2] - Sqr
t[3]*(I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] - 3*Log[1 - x + x^2] - 3*
Log[1 + x + x^2])/24
```

Maple [B] time = 0.009, size = 87, normalized size = 2.2

$$-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\ln(x)-\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\ln\left(x^4-x^2+1\right)}{8}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+ln(x)-1/8*ln(x^2-
x+1)+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/8*ln(x^4-x^2+1)-1/12*3^(1/2
)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Maxima [A] time = 1.5315, size = 49, normalized size = 1.26

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{8}\log\left(x^8+x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1/
4*log(x^4)
```

Fricas [A] time = 1.47372, size = 109, normalized size = 2.79

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{8}\log\left(x^8+x^4+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + lo
g(x)
```

Sympy [A] time = 0.150425, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+x**4+1),x)
```

```
[Out] log(x) - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3
)/12
```

Giac [A] time = 1.08753, size = 49, normalized size = 1.26

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{8}\log\left(x^8+x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1/
4*log(x^4)
```

3.335 $\int \frac{1}{x^3(1+x^4+x^8)}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/(2*x^2) + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/S
qrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.0515594, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1359, 1123, 1161, 618, 204}

$$-\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/S
qrt[3]]/(2*Sqrt[3])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(1+x^4+x^8\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1+x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{-1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^2\right)\\
&= -\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.0507518, size = 100, normalized size = 1.85

$$\frac{1}{12}\left(-\frac{6}{x^2}+i\sqrt{3}\log\left(2x^2-i\sqrt{3}-1\right)-i\sqrt{3}\log\left(2x^2+i\sqrt{3}-1\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] (-6/x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] + I*Sqrt[3]*Log[-1 - I*Sqrt[3] + 2*x^2] - I*Sqrt[3]*Log[-1 + I*Sq
rt[3] + 2*x^2])/12
```

Maple [A] time = 0.006, size = 57, normalized size = 1.1

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\sqrt{3}}{6}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+x^4+1),x)
```

```
[Out] 1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))-1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/2/x^2
```

Maxima [A] time = 1.51486, size = 57, normalized size = 1.06

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^2 - 1)) - 1/2/x^2
```

Fricas [A] time = 1.4664, size = 135, normalized size = 2.5

$$-\frac{\sqrt{3}x^2 \arctan\left(\frac{1}{3}\sqrt{3}x^2\right) + \sqrt{3}x^2 \arctan\left(\frac{1}{3}\sqrt{3}\left(x^6 + 2x^2\right)\right) + 3}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(3)*x^2*arctan(1/3*sqrt(3)*x^2) + sqrt(3)*x^2*arctan(1/3*sqrt(3)*
(x^6 + 2*x^2)) + 3)/x^2
```

Sympy [A] time = 0.161669, size = 53, normalized size = 0.98

$$\frac{\sqrt{3}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^2}{3}\right) - 2\,\mathrm{atan}\left(\frac{\sqrt{3}x^6}{3} + \frac{2\sqrt{3}x^2}{3}\right)\right)}{12} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(3)*x**2/3
))/12 - 1/(2*x**2)
```

Giac [A] time = 1.11086, size = 57, normalized size = 1.06

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^2 - 1)) - 1/2/x^2
```

3.336 $\int \frac{1}{x^5(1+x^4+x^8)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\log(x)$$

```
[Out] -1/(4*x^4) - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[x] + Log[1 + x^4
 + x^8]/8
```

Rubi [A] time = 0.0512834, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[x] + Log[1 + x^4
 + x^8]/8
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(1+x^4+x^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1+x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{-1-x}{x\left(1+x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\operatorname{Subst}\left(\int \left(-\frac{1}{x}+\frac{x}{1+x+x^2}\right)dx,x,x^4\right)\\
&= -\frac{1}{4x^4}-\log(x)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{1+x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}-\log(x)-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^4\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}-\log(x)+\frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^4\right)\\
&= -\frac{1}{4x^4}-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\log(x)+\frac{1}{8}\log\left(1+x^4+x^8\right)
\end{aligned}$$

Mathematica [C] time = 0.104257, size = 141, normalized size = 2.94

$$\frac{1}{24}\left(-\frac{6}{x^4}+\sqrt{3}\left(\sqrt{3}+i\right)\log\left(x^2-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)+\sqrt{3}\left(\sqrt{3}-i\right)\log\left(x^2+\frac{1}{2}i\left(\sqrt{3}+i\right)\right)+3\log\left(x^2-x+1\right)+3\log\left(x^2+x+1\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] (-6/x^4 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] - 24*Log[x] + Sqrt[3]*(I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^
2] + Sqrt[3]*(-I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] + 3*Log[1 - x +
x^2] + 3*Log[1 + x + x^2])/24
```

Maple [B] time = 0.009, size = 94, normalized size = 2.

$$\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{1}{4x^4}-\ln(x)+\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\ln\left(x^4-x^2\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4/x^4-ln(x)+1/8*
ln(x^2-x+1)+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/8*ln(x^4-x^2+1)-1/12
*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Maxima [A] time = 1.47343, size = 55, normalized size = 1.15

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/4/x^4 + 1/8*log(x^8 + x^4
 + 1) - 1/4*log(x^4)
```

Fricas [A] time = 1.45547, size = 143, normalized size = 2.98

$$-\frac{2\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-3x^4\log\left(x^8+x^4+1\right)+24x^4\log\left(x\right)+6}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 3*x^4*log(x^8 + x^4
+ 1) + 24*x^4*log(x) + 6)/x^4
```

Sympy [A] time = 0.180634, size = 48, normalized size = 1.

$$-\log\left(x\right)+\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+x**4+1),x)
```

```
[Out] -log(x) + log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/
3)/12 - 1/(4*x**4)
```

Giac [A] time = 1.1091, size = 62, normalized size = 1.29

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)+\frac{x^4-1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/4*(x^4 - 1)/x^4 + 1/8*log
(x^8 + x^4 + 1) - 1/4*log(x^4)
```

3.337 $\int \frac{1}{x^7(1+x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$\frac{1}{2x^2}-\frac{1}{6x^6}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -1/(6*x^6) + 1/(2*x^2) - ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(
1 + 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]
/8
```

Rubi [A] time = 0.0962304, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1359, 1123, 1281, 12, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{2x^2}-\frac{1}{6x^6}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) + 1/(2*x^2) - ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(
1 + 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]
/8
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^7\left(1+x^4+x^8\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2+x^4\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{6x^6}+\frac{1}{6}\,\text{Subst}\left(\int \frac{-3-3x^2}{x^2\left(1+x^2+x^4\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{1}{6}\,\text{Subst}\left(\int -\frac{3x^2}{1+x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1+x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{8}\,\text{Subst}\left(\int \frac{1+2x}{-1-x-x^2}\,dx,x,x^2\right)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1-2x}{-1+x-x^2}\,dx,x,x^2\right)+\frac{1}{8}$$
$$= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\right.$$
$$= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)$$

Mathematica [C] time = 0.111959, size = 142, normalized size = 1.6

$$\frac{1}{24}\left(\frac{12}{x^2}-\frac{4}{x^6}+\sqrt{3}\left(\sqrt{3}-i\right)\log\left(x^2-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)+\sqrt{3}\left(\sqrt{3}+i\right)\log\left(x^2+\frac{1}{2}i\left(\sqrt{3}+i\right)\right)-3\log\left(x^2-x+1\right)-3\log\left(x^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] (-4/x^6 + 12/x^2 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[
(1 + 2*x)/Sqrt[3]] + Sqrt[3]*(-I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^2]
 + Sqrt[3]*(I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] - 3*Log[1 - x + x^2
] - 3*Log[1 + x + x^2])/24
```

Maple [A] time = 0.008, size = 95, normalized size = 1.1

$$-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\ln\left(x^4-\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/6/x^6+1/2/x^2-1
/8*ln(x^2-x+1)+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/8*ln(x^4-x^2+1)+1
/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Maxima [A] time = 1.51521, size = 99, normalized size = 1.11

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{3x^4-1}{6x^6}-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x
^4 - x^2 + 1)
```

Fricas [A] time = 1.54108, size = 234, normalized size = 2.63

$$\frac{2\sqrt{3}x^6\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+2\sqrt{3}x^6\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-3x^6\log\left(x^4+x^2+1\right)+3x^6\log\left(x^4-x^2+1\right)+12}{24x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(3)*x^6*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 2*sqrt(3)*x^6*arctan(
1/3*sqrt(3)*(2*x^2 - 1)) - 3*x^6*log(x^4 + x^2 + 1) + 3*x^6*log(x^4 - x^2 +
 1) + 12*x^4 - 4)/x^6
```

Sympy [A] time = 0.242809, size = 88, normalized size = 0.99

$$\frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}+\frac{3x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x*
*2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12 + (3*x
**4 - 1)/(6*x**6)
```

Giac [A] time = 1.09196, size = 99, normalized size = 1.11

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{3x^4-1}{6x^6}-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x
^4 - x^2 + 1)
```

3.338 $\int \frac{x^8}{1+x^4+x^8}\,dx$

Optimal. Leaf size=141

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{\right.$$

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 - ArcTa
n[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^
2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 +
Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi [A] time = 0.0973638, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1367, 1419, 1094, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + x^4 + x^8),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 - ArcTa
n[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^
2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 +
Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{1+x^4+x^8}\,dx &= x-\int \frac{1+x^4}{1+x^4+x^8}\,dx\\
&= x-\frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx\\
&= x-\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx-\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}\\
&= x-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{}{1-\sqrt{}}\\
&= x+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\frac{1}{4}\text{Subst}\Big(\\
&= x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-x+x^2\right)-
\end{aligned}$$

Mathematica [C] time = 0.281502, size = 139, normalized size = 0.99

$$\frac{1}{24}\left(3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)+24x-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{i\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{-6+6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8/(1 + x^4 + x^8),x]
```

```
[Out] ((-I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[-6 + (6*I)*Sqrt[3]] + (I*ArcTan[(
(1 + I*Sqrt[3])*x)/2])/Sqrt[-6 - (6*I)*Sqrt[3]] + (24*x - 2*Sqrt[3]*ArcTan[
(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*Log[1 - x + x
```

```
^2] - 3*Log[1 + x + x^2])/24
```

Maple [A] time = 0.036, size = 110, normalized size = 0.8

$$x - \frac{\ln\left(x^2+x+1\right)}{8} - \frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right) + \frac{\ln\left(x^2-x+1\right)}{8} - \frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) - \frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+x^4+1),x)
```

```
[Out] x-1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/8*ln(x^2-x+1)-
1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/24*ln(1+x^2+x*3^(1/2))*3^(1/2)-1
/4*arctan(2*x+3^(1/2))+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/4*arctan(2*x-3^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx - \frac{1}{8}\log\left(x^2+x+1\right) + \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + x - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x +
 1) + 1/8*log(x^2 - x + 1)
```

Fricas [A] time = 1.60605, size = 720, normalized size = 5.11

$$\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x + \frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2} - \sqrt{3}\right) + \frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x + \frac{1}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sq
rt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 1/12*sqrt(6)
*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3
)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - 1/48*sqrt(6)*sqrt(2)*lo
g(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(2
)*x + 2*x^2 + 2) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 -
x + 1)
```

Sympy [C] time = 0.692784, size = 192, normalized size = 1.36

$$x + \left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)\log\left(x - 1 - \frac{\sqrt{3}i}{3} - 9216\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)\log\left(x - 1 - 9216\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)^5 + \frac{\sqrt{3}i}{3}\right) + \left(-\frac{1}{8} + \frac{\sqrt{3}}{24}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+x**4+1),x)
```

```
[Out] x + (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/24
)**5) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + sqr
t(3)*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 + sq
rt(3)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/
24)**5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*lo
g(-9216*_t**5 - 8*_t + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{x^8 + x^4 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^8/(x^8 + x^4 + 1), x)
```

3.339 $\int \frac{x^6}{1+x^4+x^8} \, dx$

Optimal. Leaf size=88

$$\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sq
rt[3])
```

Rubi [A] time = 0.0578484, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1372, 1164, 628, 1161, 618, 204}

$$\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sq
rt[3])
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
```

```
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{1+x^4+x^8}\,dx &= -\left(\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\
&= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}\\
&= \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{-3\cdots}\right.\\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0220992, size = 68, normalized size = 0.77

$$\frac{\log\left(-x^2+\sqrt{3}x-1\right)-\log\left(x^2+\sqrt{3}x+1\right)+2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + x^4 + x^8),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[-1 + Sqrt
[3]*x - x^2] - Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

Maple [A] time = 0.01, size = 67, normalized size = 0.8

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+x^4+1),x)
```

```
[Out] 1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\int\frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Fricas [A] time = 1.50494, size = 212, normalized size = 2.41

$$\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(x^3+2\,x\right)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right)+\frac{1}{12}\,\sqrt{3}\log\left(\frac{x^4+5\,x^2-2\,\sqrt{3}\left(x^3+x\right)+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2
+ 1))
```

Sympy [A] time = 0.169756, size = 82, normalized size = 0.93

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 +
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2
```

Giac [A] time = 1.09372, size = 97, normalized size = 1.1

$$\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{12}\,\sqrt{3}\log\left(\frac{\left|2\,x-2\,\sqrt{3}+\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{3}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/12*sqrt(3)*log(abs(2*x - 2*sqrt(3) + 2/x)/abs(2*x + 2*sqrt(3
) + 2/x))
```

3.340 $\int \frac{x^4}{1+x^4+x^8} \, dx$

Optimal. Leaf size=140

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right.$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1
 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/8
 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt
[3]*x + x^2]/(8*Sqrt[3])
```

Rubi [A] time = 0.106539, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1
 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/8
 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt
[3]*x + x^2]/(8*Sqrt[3])
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{1+x^4+x^8}\,dx &= \frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx - \frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx \\
&= -\left(\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right) + \frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx + \frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx - \frac{1}{4}\int \frac{1+x^2}{1+x^2+x^4}\,dx \\
&= -\left(\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx\right) - \frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx - \frac{1}{8}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x+x^2}\,dx + \frac{1}{8}\int \\
&= -\frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right) + \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{1}{4}\,\mathrm{Subst}\Big(\\
&= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right) - \frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}
\end{aligned}
$$

Mathematica [C] time = 0.166229, size = 135, normalized size = 0.96

$$
\frac{1}{24}\left(-3\log\left(x^2-x+1\right)+3\log\left(x^2+x+1\right)-2i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)+2i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{}\right.\right.\right.
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/(1 + x^4 + x^8),x]
```

```
[Out] ((-2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] + (2*I)*Sqrt
[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTan[(-1 +
 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x + x^2] +
 3*Log[1 + x + x^2])/24
```

Maple [A] time = 0.013, size = 109, normalized size = 0.8

$$\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/8*ln(x^2-x+1)-1/
12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/24*ln(1+x^2+x*3^(1/2))*3^(1/2)+1/4
*arctan(2*x+3^(1/2))+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/4*arctan(2*x-3^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\int\frac{x^2}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x + 1
) - 1/8*log(x^2 - x + 1)
```

Fricas [A] time = 1.63692, size = 716, normalized size = 5.11

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*s
qrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/12*sqrt(6
)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(
3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - 1/48*sqrt(6)*sqrt(2)*l
og(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(
2)*x + 2*x^2 + 2) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x +
 1)
```

Sympy [C] time = 0.700254, size = 197, normalized size = 1.41

$$\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+x**4+1),x)
```

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24)
**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 - s
qrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-1/8
 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sq
rt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_
t, _t*log(-18432*_t**5 - 4*_t + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{x^8 + x^4 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^4/(x^8 + x^4 + 1), x)
```

3.341 $\int \frac{x^2}{1+x^4+x^8}\,dx$

Optimal. Leaf size=140

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1
 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^2]/8
 - Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqrt
[3]*x + x^2]/(8*Sqrt[3])
```

Rubi [A] time = 0.0831656, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1373, 1094, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^4 + x^8),x]
```


```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1
 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^2]/8
 - Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqrt
[3]*x + x^2]/(8*Sqrt[3])
```


Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx - \frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx$$

$$= -\left(\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx\right) - \frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx + \frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}$$

$$= -\left(\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx\right) + \frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x+x^2}\,dx - \frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{8}\int \frac{\ }{1-}$$

$$= \frac{1}{8}\log\left(1-x+x^2\right) - \frac{1}{8}\log\left(1+x+x^2\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{1}{4}\text{Subst}\Big($$

$$= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right) + \frac{1}{8}\log\left(1-x+x^2\right) - \frac{1}{8}$$

Mathematica [C] time = 0.161568, size = 135, normalized size = 0.96

$$\frac{1}{48}\left(6\log\left(x^2-x+1\right) - 6\log\left(x^2+x+1\right) + 4i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right) - 4i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(1 + x^4 + x^8),x]
```

```
[Out] ((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (4*I)*Sqrt[
-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 4*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 6*Log[1 - x + x^2] -
6*Log[1 + x + x^2])/48
```

Maple [A] time = 0.009, size = 109, normalized size = 0.8

$$-\frac{\ln\left(x^2+x+1\right)}{8} - \frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right) + \frac{\ln\left(x^2-x+1\right)}{8} - \frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) + \frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/8*ln(x^2-x+1)-1
/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/24*ln(1+x^2+x*3^(1/2))*3^(1/2)+1/
4*arctan(2*x+3^(1/2))-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/4*arctan(2*x-3^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x + 1)
+ 1/8*log(x^2 - x + 1)
```

Fricas [A] time = 1.63165, size = 716, normalized size = 5.11

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*s
qrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/12*sqrt(6
)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(
3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + 1/48*sqrt(6)*sqrt(2)*l
og(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(
2)*x + 2*x^2 + 2) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x +
 1)
```

Sympy [C] time = 0.694468, size = 214, normalized size = 1.53

$$\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-192\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-192\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3+442368\left(-\frac{1}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x + 442368*(-1/8 - sqrt(3)*I/24)**7 - 192*(-1/8 -
 sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 192*(-1/8 + sqrt(3)*I/24
)**3 + 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x + 4423
68*(1/8 - sqrt(3)*I/24)**7 - 192*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*
I/24)*log(x - 192*(1/8 + sqrt(3)*I/24)**3 + 442368*(1/8 + sqrt(3)*I/24)**7)
 + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(442368*_t**7 - 192*
```

```
_t**3 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^8 + x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^8 + x^4 + 1), x)
```

3.342 $\int \frac{1}{1+x^4+x^8} \, dx$

Optimal. Leaf size=88

$$-\frac{\log \left(x^2-\sqrt{3} x+1\right)}{4 \sqrt{3}}+\frac{\log \left(x^2+\sqrt{3} x+1\right)}{4 \sqrt{3}}-\frac{\tan ^{-1}\left(\frac{1-2 x}{\sqrt{3}}\right)}{2 \sqrt{3}}+\frac{\tan ^{-1}\left(\frac{2 x+1}{\sqrt{3}}\right)}{2 \sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sq
rt[3])
```

Rubi [A] time = 0.0516773, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {1346, 1164, 628, 1161, 618, 204}

$$-\frac{\log \left(x^2-\sqrt{3} x+1\right)}{4 \sqrt{3}}+\frac{\log \left(x^2+\sqrt{3} x+1\right)}{4 \sqrt{3}}-\frac{\tan ^{-1}\left(\frac{1-2 x}{\sqrt{3}}\right)}{2 \sqrt{3}}+\frac{\tan ^{-1}\left(\frac{2 x+1}{\sqrt{3}}\right)}{2 \sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4 + x^8)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sq
rt[3])
```

Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
```

```
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^4+x^8}\,dx &= \frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\ &= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx-\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}\\ &= -\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\frac{1}{2}\text{Subst}\left(\int\right.\\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0167882, size = 68, normalized size = 0.77

$$\frac{-\log\left(-x^2+\sqrt{3}x-1\right)+\log\left(x^2+\sqrt{3}x+1\right)+2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4 + x^8)^(-1),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[-1 + Sqrt
[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

Maple [A] time = 0.009, size = 67, normalized size = 0.8

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+x^4+1),x)
```

```
[Out] 1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{2}\int\frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Fricas [A] time = 1.48196, size = 212, normalized size = 2.41

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2x\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{x^4+5x^2+2\sqrt{3}\left(x^3+x\right)+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2
+ 1))
```

Sympy [A] time = 0.171143, size = 82, normalized size = 0.93

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12}-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 -
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2
```

Giac [A] time = 1.11414, size = 97, normalized size = 1.1

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\sqrt{3}\log\left(\frac{\left|2x-2\sqrt{3}+\frac{2}{x}\right|}{\left|2x+2\sqrt{3}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/12*sqrt(3)*log(abs(2*x - 2*sqrt(3) + 2/x)/abs(2*x + 2*sqrt(3
) + 2/x))
```

3.343 $\int \frac{1}{x^2(1+x^4+x^8)} dx$

Optimal. Leaf size=145

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\right.$$

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 -
 ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 - Log[1 -
x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Lo
g[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi [A] time = 0.111633, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1368, 1506, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x^4 + x^8)),x]
```


```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 -
 ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 - Log[1 -
x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Lo
g[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```


Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1506

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r - (c*d - e*q)*x^(n/2), x])
/(q - r*x^(n/2) + c*x^n), x], x] + Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r +
(c*d - e*q)*x^(n/2), x])/(q + r*x^(n/2) + c*x^n), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(1+x^4+x^8\right)}\,dx &= -\frac{1}{x}+\int \frac{x^2\left(-1-x^4\right)}{1+x^4+x^8}\,dx\\
&= -\frac{1}{x}-\frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx\\
&= -\frac{1}{x}+\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx-\frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx-\frac{1}{4}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\
&= -\frac{1}{x}-\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx-\frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\\
&= -\frac{1}{x}-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)-\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\frac{1}{4}\mathrm{S}\\
&= -\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{1}{8}\log\left(1-x+\right.
\end{aligned}$$

Mathematica [C] time = 0.217175, size = 140, normalized size = 0.97

$$\frac{1}{24}\left(-3\log\left(x^2-x+1\right)+3\log\left(x^2+x+1\right)-\frac{24}{x}+2i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-2i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 + x^4 + x^8)),x]
```

```
[Out] (-24/x + (2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (2*
I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTa
n[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x +
 x^2] + 3*Log[1 + x + x^2])/24
```

Maple [A] time = 0.01, size = 114, normalized size = 0.8

$$\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{24}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/8*ln(x^2-x+1)-1/
12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/24*ln(1+x^2+x*3^(1/2))*3^(1/2)-1/4
*arctan(2*x+3^(1/2))-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/4*arctan(2*x-3^(1/2
))-1/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{x}-\frac{1}{2}\int\frac{x^2}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/x - 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 +
 x + 1) - 1/8*log(x^2 - x + 1)
```

Fricas [A] time = 1.68442, size = 720, normalized size = 4.97

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1
/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 4*sqrt(
6)*sqrt(3)*sqrt(2)*x*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sq
rt(3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + sqrt(6)*sqrt(2)*x*l
og(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(6)*sqrt(2)*x*log(-sqrt(6)*sqrt(2)*
```

```
x + 2*x^2 + 2) - 4*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) - 4*sqrt(3)*x*ar
ctan(1/3*sqrt(3)*(2*x - 1)) + 6*x*log(x^2 + x + 1) - 6*x*log(x^2 - x + 1) -
 48)/x
```

Sympy [C] time = 0.722479, size = 218, normalized size = 1.5

$$\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{7}-384\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{3}-442368\left(-\frac{1}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x - 442368*(-1/8 - sqrt(3)*I/24)**7 - 384*(-1/8 -
 sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 384*(-1/8 + sqrt(3)*I/24
)**3 - 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x - 4423
68*(1/8 - sqrt(3)*I/24)**7 - 384*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*
I/24)*log(x - 384*(1/8 + sqrt(3)*I/24)**3 - 442368*(1/8 + sqrt(3)*I/24)**7)
 + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-442368*_t**7 - 384
*_t**3 + x))) - 1/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{8}+x^{4}+1\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^8 + x^4 + 1)*x^2), x)
```

3.344 $\int \frac{1}{x^4(1+x^4+x^8)}\,dx$

Optimal. Leaf size=147

$$-\frac{1}{3x^3}+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-}$$

```
[Out] -1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/
4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1
 - x + x^2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) -
 Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi [A] time = 0.0991428, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1368, 1419, 1094, 634, 618, 204, 628}

$$-\frac{1}{3x^3}+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/
4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1
 - x + x^2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) -
 Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(1+x^4+x^8\right)}\,dx &= -\frac{1}{3x^3}+\frac{1}{3}\int \frac{-3-3x^4}{1+x^4+x^8}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx-\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}\\
&= -\frac{1}{3x^3}-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{8}\\
&= -\frac{1}{3x^3}+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\frac{1}{4}\\
&= -\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-x\right.
\end{aligned}$$

Mathematica [C] time = 0.307162, size = 148, normalized size = 1.01

$$\frac{1}{24}\left(-\frac{8}{x^3}+3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)-\frac{4i\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}i\left(\sqrt{3}+i\right)}}+\frac{4i\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-\frac{1}{6}i\left(\sqrt{3}-i\right)}}-2\sqrt{3}\tan^{-1}\left(\frac{2x}{\ldots}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] (-8/x^3 - ((4*I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[(I/6)*(I + Sqrt[3])] +
 ((4*I)*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[(-I/6)*(-I + Sqrt[3])] - 2*Sqrt
[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*Lo
```

```
g[1 - x + x^2] - 3*Log[1 + x + x^2])/24
```

Maple [A] time = 0.011, size = 114, normalized size = 0.8

$$-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/8*ln(x^2-x+1)-1
/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/24*ln(1+x^2+x*3^(1/2))*3^(1/2)-1/
4*arctan(2*x+3^(1/2))+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/4*arctan(2*x-3^(1/
2))-1/3/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{3x^3}-\frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx-\frac{1}{8}\log\left(x^2+x+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/3/x^3 - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2
 + x + 1) + 1/8*log(x^2 - x + 1)
```

Fricas [B] time = 1.89447, size = 744, normalized size = 5.06

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x^3\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x^3\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^3*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x +
 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 4*sqr
t(6)*sqrt(3)*sqrt(2)*x^3*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6
)*sqrt(3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - sqrt(6)*sqrt(2)
*x^3*log(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(6)*sqrt(2)*x^3*log(-sqrt(6)*
sqrt(2)*x + 2*x^2 + 2) - 4*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x + 1)) - 4*sq
rt(3)*x^3*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*x^3*log(x^2 + x + 1) + 6*x^3*lo
g(x^2 - x + 1) - 16)/x^3
```

Sympy [C] time = 0.733959, size = 197, normalized size = 1.34

$$\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{5}\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{5}+\frac{\sqrt{3}i}{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+x**4+1),x)
```

```
[Out] (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/24)**5
) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + sqrt(3)
*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 + sqrt(3
)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/24)*
*5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-9
216*_t**5 - 8*_t + x))) - 1/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{8}+x^{4}+1\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^8 + x^4 + 1)*x^4), x)
```

3.345 $\int \frac{1}{x^6(1+x^4+x^8)} dx$

Optimal. Leaf size=98

$$-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/(5*x^5) + x^(-1) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2
*x)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + S
qrt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi [A] time = 0.0838172, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {1368, 1504, 12, 1372, 1164, 628, 1161, 618, 204}

$$-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) + x^(-1) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2
*x)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + S
qrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
```

```
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^6\left(1+x^4+x^8\right)}\,dx = -\frac{1}{5x^5}+\frac{1}{5}\int \frac{-5-5x^4}{x^2\left(1+x^4+x^8\right)}\,dx$$
$$= -\frac{1}{5x^5}+\frac{1}{x}-\frac{1}{5}\int -\frac{5x^6}{1+x^4+x^8}\,dx$$
$$= -\frac{1}{5x^5}+\frac{1}{x}+\int \frac{x^6}{1+x^4+x^8}\,dx$$
$$= -\frac{1}{5x^5}+\frac{1}{x}-\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx$$
$$= -\frac{1}{5x^5}+\frac{1}{x}+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}$$
$$= -\frac{1}{5x^5}+\frac{1}{x}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right.$$
$$= -\frac{1}{5x^5}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

Mathematica [A] time = 0.0381827, size = 95, normalized size = 0.97

$$\frac{1}{60}\left(-\frac{12}{x^5}+5\sqrt{3}\log\left(-x^2+\sqrt{3}x-1\right)-5\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{60}{x}+10\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+10\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] (-12/x^5 + 60/x + 10*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Sqrt[3]*ArcTan
[(1 + 2*x)/Sqrt[3]] + 5*Sqrt[3]*Log[-1 + Sqrt[3]*x - x^2] - 5*Sqrt[3]*Log[1
 + Sqrt[3]*x + x^2])/60
```

Maple [A] time = 0.013, size = 75, normalized size = 0.8

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{1}{5x^5}+x^{-1}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8+x^4+1),x)
```

```
[Out] 1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/5/x^5+1/x+1/6*3^(1/2)*arctan(1/3*
(2*x-1)*3^(1/2))-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)+1/12*ln(1+x^2-x*3^(1/2))*
3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{5x^4-1}{5x^5}+\frac{1}{2}\int \frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/5*(5*x^4 - 1)/x^5 + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1),
 x)
```

Fricas [A] time = 1.46082, size = 255, normalized size = 2.6

$$\frac{10\,\sqrt{3}x^5\arctan\left(\frac{1}{3}\,\sqrt{3}\left(x^3+2\,x\right)\right)+10\,\sqrt{3}x^5\arctan\left(\frac{1}{3}\,\sqrt{3}x\right)+5\,\sqrt{3}x^5\log\left(\frac{x^4+5\,x^2-2\,\sqrt{3}\left(x^3+x\right)+1}{x^4-x^2+1}\right)+60\,x^4-12}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/60*(10*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 10*sqrt(3)*x^5*arcta
n(1/3*sqrt(3)*x) + 5*sqrt(3)*x^5*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1
)/(x^4 - x^2 + 1)) + 60*x^4 - 12)/x^5
```

Sympy [A] time = 0.214127, size = 94, normalized size = 0.96

$$\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{5x^4-1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 +
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 + (5*x**4 - 1)/(5*x**5)
```

Giac [A] time = 1.10526, size = 113, normalized size = 1.15

$$\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{12}\,\sqrt{3}\log\left(\frac{\left|2\,x-2\,\sqrt{3}+\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{3}+\frac{2}{x}\right|}\right)+\frac{5\,x^4-1}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/12*sqrt(3)*log(abs(2*x - 2*sqrt(3) + 2/x)/abs(2*x + 2*sqrt(3
) + 2/x)) + 1/5*(5*x^4 - 1)/x^5
```

3.346 $\int \frac{1}{x^8(1+x^4+x^8)}\,dx$

Optimal. Leaf size=154

$$\frac{1}{3x^3}-\frac{1}{7x^7}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}$$

```
[Out] -1/(7*x^7) + 1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqr
t[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*
x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(
8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi [A] time = 0.141826, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1368, 1504, 12, 1373, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{3x^3}-\frac{1}{7x^7}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 + x^4 + x^8)),x]
```


```
[Out] -1/(7*x^7) + 1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqr
t[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*
x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(
8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```


Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^8\left(1+x^4+x^8\right)}\,dx = -\frac{1}{7x^7}+\frac{1}{7}\int \frac{-7-7x^4}{x^4\left(1+x^4+x^8\right)}\,dx$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{21}\int -\frac{21x^4}{1+x^4+x^8}\,dx$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}+\int \frac{x^4}{1+x^4+x^8}\,dx$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx-\frac{1}{4}\int \frac{1+}{1+x}$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx-\frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x}$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x\right.}{8\sqrt{3}}$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{1}{8}$$

Mathematica [C] time = 0.356026, size = 171, normalized size = 1.11

$$\frac{1}{3x^3}-\frac{1}{7x^7}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{2\sqrt{-6+6i\sqrt{3}}}+\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right.}{2\sqrt{-6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^8*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) + 1/(3*x^3) + ((I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2])/(2*S
qrt[-6 + (6*I)*Sqrt[3]]) + ((-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])/(
2*Sqrt[-6 - (6*I)*Sqrt[3]]) - ArcTan[(-1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcT
an[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8
```

Maple [A] time = 0.011, size = 119, normalized size = 0.8

$$\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\ln\left(1+x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/7/x^7+1/3/x^3-1/
8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/24*ln(1+x^2+x*3^(1
/2))*3^(1/2)+1/4*arctan(2*x+3^(1/2))+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/4*a
rctan(2*x-3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{7x^4-3}{21x^7}+\frac{1}{2}\int \frac{x^2}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/21*(7*x^4 - 3)/x^7 + 1/2*integrate(x^2/(x^4 - x^2 + 1), x
) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Fricas [B] time = 1.62196, size = 774, normalized size = 5.03

$$-\frac{28\sqrt{6}\sqrt{3}\sqrt{2}x^7\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+28\sqrt{6}\sqrt{3}\sqrt{2}x^7\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/336*(28*sqrt(6)*sqrt(3)*sqrt(2)*x^7*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*
x + 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 28
*sqrt(6)*sqrt(3)*sqrt(2)*x^7*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sq
rt(6)*sqrt(3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + 7*sqrt(6)*s
qrt(2)*x^7*log(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 7*sqrt(6)*sqrt(2)*x^7*log(-
sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 28*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(2*x + 1
)) + 28*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(2*x - 1)) - 42*x^7*log(x^2 + x + 1)
 + 42*x^7*log(x^2 - x + 1) - 112*x^4 + 48)/x^7
```

Sympy [C] time = 0.750027, size = 209, normalized size = 1.36

$$\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8+x**4+1),x)
```

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24)
**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 - s
qrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-1/8
 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sq
rt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_
t, _t*log(-18432*_t**5 - 4*_t + x))) + (7*x**4 - 3)/(21*x**7)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^8+x^4+1)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^8 + x^4 + 1)*x^8), x)
```

3.347 $\int \frac{x^m}{1-x^4+x^8}\,dx$

Optimal. Leaf size=127

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(1 - I*Sqrt
[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
(1 + m)/4, (5 + m)/4, (2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(I - Sqrt[3])*(1 +
 m))
```

Rubi [A] time = 0.0486989, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1375, 364}

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 - x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(1 - I*Sqrt
[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
(1 + m)/4, (5 + m)/4, (2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(I - Sqrt[3])*(1 +
 m))
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{1-x^4+x^8}\,dx &= -\frac{i\int \frac{x^m}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}}+\frac{i\int \frac{x^m}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}}\\ &= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}\end{aligned}$$

Mathematica [C] time = 0.0374923, size = 52, normalized size = 0.41

$$-\frac{x^{m+1}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{{}_2F_1\left(1,m+1;m+2;\frac{x}{\#1}\right)}{\#1^4-2}\&\right]}{4(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(1 - x^4 + x^8),x]
```

```
[Out] -(x^(1 + m)*RootSum[1 - #1^4 + #1^8 & , Hypergeometric2F1[1, 1 + m, 2 + m,
x/#1]/(-2 + #1^4) & ])/(4*(1 + m))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8-x^4+1),x)
```

```
[Out] int(x^m/(x^8-x^4+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 - x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{x^8-x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 - x^4 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8-x**4+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 - x^4 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 - x^4 + 1), x)
```

3.348 $\int \frac{x^{11}}{1-x^4+x^8} \, dx$

Optimal. Leaf size=46

$$\frac{x^4}{4}+\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] x^4/4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rubi [A] time = 0.0401795, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1357, 703, 634, 618, 204, 628}

$$\frac{x^4}{4}+\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 - x^4 + x^8),x]
```

```
[Out] x^4/4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{1-x^4+x^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4}-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= \frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [A] time = 0.0130415, size = 46, normalized size = 1.

$$\frac{x^4}{4}+\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 - x^4 + x^8),x]
```

```
[Out] x^4/4 - ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Maple [A] time = 0.003, size = 38, normalized size = 0.8

$$\frac{x^4}{4}+\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8-x^4+1),x)
```

```
[Out] 1/4*x^4+1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A] time = 1.47832, size = 50, normalized size = 1.09

$$\frac{1}{4}x^4-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

Fricas [A] time = 1.44576, size = 109, normalized size = 2.37

$$\frac{1}{4}x^4 - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right) + \frac{1}{8}\log\left(x^8 - x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

Sympy [A] time = 0.139121, size = 42, normalized size = 0.91

$$\frac{x^4}{4} + \frac{\log\left(x^8 - x^4 + 1\right)}{8} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8-x**4+1),x)
```

```
[Out] x**4/4 + log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

Giac [A] time = 1.13786, size = 50, normalized size = 1.09

$$\frac{1}{4}x^4 - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right) + \frac{1}{8}\log\left(x^8 - x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

3.349 $\int \frac{x^9}{1-x^4+x^8} \, dx$

Optimal. Leaf size=57

$$\frac{x^2}{2}+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

```
[Out] x^2/2 + Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]
/(4*Sqrt[3])
```

Rubi [A] time = 0.0431245, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1359, 1122, 1164, 628}

$$\frac{x^2}{2}+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - x^4 + x^8),x]
```

```
[Out] x^2/2 + Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]
/(4*Sqrt[3])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^9}{1-x^4+x^8} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{x^4}{1-x^2+x^4} \, dx, x, x^2\right)$$
$$= \frac{x^2}{2} - \frac{1}{2} \text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4} \, dx, x, x^2\right)$$
$$= \frac{x^2}{2} + \frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2} \, dx, x, x^2\right)}{4\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2} \, dx, x, x^2\right)}{4\sqrt{3}}$$
$$= \frac{x^2}{2} + \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}$$

Mathematica [A] time = 0.0158855, size = 55, normalized size = 0.96

$$\frac{1}{12} \left(6x^2 + \sqrt{3} \log\left(-x^4+\sqrt{3}x^2-1\right) - \sqrt{3} \log\left(x^4+\sqrt{3}x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - x^4 + x^8),x]
```

```
[Out] (6*x^2 + Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] - Sqrt[3]*Log[1 + Sqrt[3]*x^2
+ x^4])/12
```

Maple [A] time = 0.006, size = 44, normalized size = 0.8

$$\frac{x^2}{2} + \frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12} - \frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8-x^4+1),x)
```

```
[Out] 1/2*x^2+1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)-1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} x^2 + \int \frac{\left(x^4-1\right)x}{x^8-x^4+1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 1.45492, size = 117, normalized size = 2.05

$$\frac{1}{2} x^2 + \frac{1}{12} \sqrt{3} \log\left(\frac{x^8+5x^4-2\sqrt{3}\left(x^6+x^2\right)+1}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/12*sqrt(3)*log((x^8 + 5*x^4 - 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 -
 x^4 + 1))
```

Sympy [A] time = 0.125956, size = 48, normalized size = 0.84

$$\frac{x^2}{2}+\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}-\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8-x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 - sqrt(3)*log(x**4 + sqrt(
3)*x**2 + 1)/12
```

Giac [B] time = 1.27482, size = 348, normalized size = 6.11

$$\frac{1}{2}x^2-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt
(6) + sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2
))/(sqrt(6) + sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6)
+ sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x -
sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/96*(sqrt(6) + 3*sqrt(2))*log(x^
2 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/96*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/96*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) - sqrt(2)) + 1) + 1/96*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) - sqrt(2)) + 1)
```

3.350 $\int \frac{x^7}{1-x^4+x^8} \, dx$

Optimal. Leaf size=39

$$\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rubi [A] time = 0.0329002, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{1-x^4+x^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{1}{8}\log\left(1-x^4+x^8\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}
$$

Mathematica [A] time = 0.0078897, size = 39, normalized size = 1.

$$\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Maple [A] time = 0.002, size = 33, normalized size = 0.9

$$\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8-x^4+1),x)
```

```
[Out] 1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A] time = 1.48736, size = 43, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

Fricas [A] time = 1.49932, size = 96, normalized size = 2.46

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

Sympy [A] time = 0.132473, size = 37, normalized size = 0.95

$$\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8-x**4+1),x)
```

```
[Out] log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

Giac [A] time = 1.14537, size = 43, normalized size = 1.1

$$\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

3.351 $\int \frac{x^5}{1-x^4+x^8} dx$

Optimal. Leaf size=82

$$\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

```
[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 + Log[1 - Sqrt[3]*x^
2 + x^4]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rubi [A] time = 0.0710082, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1359, 1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 + Log[1 - Sqrt[3]*x^
2 + x^4]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right)\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, x^2\right) + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, x^2\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x}\right.}{8\sqrt{3}}$$
$$= \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x^2\right) - \frac{1}{4}\,\text{Subst}$$
$$= -\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right) + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right) + \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}$$

Mathematica [C] time = 0.136915, size = 98, normalized size = 1.2

$$\frac{\sqrt{-1-i\sqrt{3}}\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)+\sqrt{-1+i\sqrt{3}}\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)}{4\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x^2)/2] + Sqrt[
-1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x^2)/2])/(4*Sqrt[6])
```

Maple [A] time = 0.006, size = 65, normalized size = 0.8

$$\frac{\arctan\left(2\,x^2-\sqrt{3}\right)}{4} + \frac{\arctan\left(2\,x^2+\sqrt{3}\right)}{4} + \frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{24} - \frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8-x^4+1),x)
```

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))+1/24*ln(1+x^4-x^2*3^(1/
2))*3^(1/2)-1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{x^8 - x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 1.54999, size = 545, normalized size = 6.65

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2 + \frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4 + \sqrt{6}\sqrt{2}x^2 + 2} - \sqrt{3}\right) - \frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3
*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) - 1/12*sq
rt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)
*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) - 1/48*sqrt(6)*sq
rt(2)*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) + 1/48*sqrt(6)*sqrt(2)*log(2*x^4
 - sqrt(6)*sqrt(2)*x^2 + 2)
```

Sympy [A] time = 0.200349, size = 70, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^4 - \sqrt{3}x^2 + 1\right)}{24} - \frac{\sqrt{3}\log\left(x^4 + \sqrt{3}x^2 + 1\right)}{24} + \frac{\operatorname{atan}\left(2x^2 - \sqrt{3}\right)}{4} + \frac{\operatorname{atan}\left(2x^2 + \sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 - sqrt(3)*log(x**4 + sqrt(3)*x**2 +
 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

Giac [B] time = 1.26237, size = 342, normalized size = 4.17

$$\frac{1}{48}\left(\sqrt{6} - 3\sqrt{2}\right)\arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{48}\left(\sqrt{6} - 3\sqrt{2}\right)\arctan\left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{48}\left(\sqrt{6} + 3\sqrt{2}\right)\arctan\left(\frac{4x + \sqrt{}}{\sqrt{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/96*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/96*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/96*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/96*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

3.352 $\int \frac{x^3}{1-x^4+x^8}\,dx$

Optimal. Leaf size=23

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.0223594, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 204}

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1-x^4+x^8}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0061327, size = 23, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A] time = 0.001, size = 19, normalized size = 0.8

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-x^4+1),x)
```

```
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A] time = 1.50008, size = 24, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Fricas [A] time = 1.47503, size = 61, normalized size = 2.65

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Sympy [A] time = 0.123665, size = 26, normalized size = 1.13

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/6
```

Giac [A] time = 1.11272, size = 24, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

3.353 $\int \frac{x}{1-x^4+x^8} dx$

Optimal. Leaf size=82

$$-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

```
[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1 - Sqrt[3]*x^
2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rubi [A] time = 0.0571404, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1359, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1 - Sqrt[3]*x^
2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1-x^4+x^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx,x,x^2\right)}{4\sqrt{3}}+\frac{\text{Subst}\left(\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx,x,x^2\right)}{4\sqrt{3}}\\
&= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx,x,x^2\right)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx,x,x^2\right)-\frac{\text{Subst}\left(\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx,\right.}{8\sqrt{3}}\\
&= -\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x^2\right)-\frac{1}{4}\,\text{Subst}\Big(\\
&= -\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)-\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.0555856, size = 83, normalized size = 1.01

$$\frac{i\left(\sqrt{-1-i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)-\sqrt{-1+i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)\right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(1 - x^4 + x^8),x]
```

```
[Out] ((I/2)*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x^2)/2] - Sqrt[-1 + I*
Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x^2)/2]))/Sqrt[6]
```

Maple [A] time = 0.008, size = 65, normalized size = 0.8

$$\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{24}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8-x^4+1),x)
```

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))-1/24*ln(1+x^4-x^2*3^(1/
2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 1.59651, size = 545, normalized size = 6.65

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3
*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) - 1/12*sq
rt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)
*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + 1/48*sqrt(6)*sq
rt(2)*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - 1/48*sqrt(6)*sqrt(2)*log(2*x^4
 - sqrt(6)*sqrt(2)*x^2 + 2)
```

Sympy [A] time = 0.195369, size = 70, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}+\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x/(x^8 - x^4 + 1), x)
```

3.354 $\int \frac{1}{x(1-x^4+x^8)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 + x^8]/8
```

Rubi [A] time = 0.0378776, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^4 + x^8)),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 + x^8]/8
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\left(1-x+x^2\right)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \log(x)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [C] time = 0.0130928, size = 55, normalized size = 1.34

$$\log(x)-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^4 + x^8)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-1
+ 2*#1^4) & ]/4
```

Maple [A] time = 0.007, size = 35, normalized size = 0.9

$$\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8-x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A] time = 1.48284, size = 51, normalized size = 1.24

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/4
*log(x^4)
```

Fricas [A] time = 1.4634, size = 108, normalized size = 2.63

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + log
(x)
```

Sympy [A] time = 0.150035, size = 41, normalized size = 1.

$$\log\left(x\right)-\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

Giac [A] time = 1.13535, size = 51, normalized size = 1.24

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/4
*log(x^4)
```

3.355 $\int \frac{1}{x^3(1-x^4+x^8)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

```
[Out] -1/(2*x^2) - Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 +
 x^4]/(4*Sqrt[3])
```

Rubi [A] time = 0.0419199, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1359, 1123, 1164, 628}

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 +
 x^4]/(4*Sqrt[3])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(1-x^2+x^4\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}$$
$$= -\frac{1}{2x^2}-\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}$$

Mathematica [A] time = 0.0157662, size = 55, normalized size = 0.96

$$\frac{1}{12}\left(-\frac{6}{x^2}-\sqrt{3}\log\left(-x^4+\sqrt{3}x^2-1\right)+\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] (-6/x^2 - Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] + Sqrt[3]*Log[1 + Sqrt[3]*x^2
 + x^4])/12
```

Maple [A] time = 0.01, size = 44, normalized size = 0.8

$$-\frac{1}{2\,x^2}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8-x^4+1),x)
```

```
[Out] -1/2/x^2-1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2\,x^2}-\int \frac{\left(x^4-1\right)x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 1.42929, size = 123, normalized size = 2.16

$$\frac{\sqrt{3}x^2\log\left(\frac{x^8+5\,x^4+2\,\sqrt{3}\left(x^6+x^2\right)+1}{x^8-x^4+1}\right)-6}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(3)*x^2*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4
+ 1)) - 6)/x^2
```

Sympy [A] time = 0.15477, size = 49, normalized size = 0.86

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/12 - 1/(2*x**2)
```

Giac [B] time = 1.27694, size = 348, normalized size = 6.11

$$\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x}{\ }\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/96*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/96*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/96*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/96*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1) - 1/2/x^2
```

3.356 $\int \frac{1}{x^5(1-x^4+x^8)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

```
[Out] -1/(4*x^4) + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4
 + x^8]/8
```

Rubi [A] time = 0.0536337, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4
 + x^8]/8
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(1-x^4+x^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\log(x)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\log(x)-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\
&= -\frac{1}{4x^4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}
$$

Mathematica [C] time = 0.0134839, size = 51, normalized size = 1.06

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{4x^4}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) + Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 +
2*#1^4) & ]/4
```

Maple [A] time = 0.005, size = 40, normalized size = 0.8

$$-\frac{1}{4\,x^4}+\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2\,x^4-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8-x^4+1),x)
```

```
[Out] -1/4/x^4+ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A] time = 1.5457, size = 58, normalized size = 1.21

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4/x^4 - 1/8*log(x^8 - x^4
 + 1) + 1/4*log(x^4)
```

Fricas [A] time = 1.50353, size = 143, normalized size = 2.98

$$-\frac{2\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+3x^4\log\left(x^8-x^4+1\right)-24x^4\log\left(x\right)+6}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 3*x^4*log(x^8 - x^4
+ 1) - 24*x^4*log(x) + 6)/x^4
```

Sympy [A] time = 0.190079, size = 48, normalized size = 1.

$$\log\left(x\right)-\frac{\log\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12 - 1/(4*x**4)
```

Giac [A] time = 1.13813, size = 65, normalized size = 1.35

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{x^4+1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4*(x^4 + 1)/x^4 - 1/8*log
(x^8 - x^4 + 1) + 1/4*log(x^4)
```

3.357 $\int \frac{1}{x^7(1-x^4+x^8)}\,dx$

Optimal. Leaf size=96

$$-\frac{1}{2x^2}-\frac{1}{6x^6}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

```
[Out] -1/(6*x^6) - 1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2
]/4 - Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(
8*Sqrt[3])
```

Rubi [A] time = 0.0943797, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1359, 1123, 1281, 12, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{2x^2}-\frac{1}{6x^6}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) - 1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2
]/4 - Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(
8*Sqrt[3])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^7\left(1-x^4+x^8\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(1-x^2+x^4\right)}\,dx, x, x^2\right)$$
$$= -\frac{1}{6x^6} + \frac{1}{6}\,\text{Subst}\left(\int \frac{3-3x^2}{x^2\left(1-x^2+x^4\right)}\,dx, x, x^2\right)$$
$$= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{1}{6}\,\text{Subst}\left(\int \frac{3x^2}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= -\frac{1}{6x^6} - \frac{1}{2x^2} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, x^2\right) - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, x^2\right) - \frac{\text{Su}}{}$$
$$= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+\right.$$
$$= -\frac{1}{6x^6} - \frac{1}{2x^2} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right) - \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right) - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x\right.}{8\sqrt{3}}$$

Mathematica [C] time = 0.0175828, size = 56, normalized size = 0.58

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{2x^2}-\frac{1}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) - 1/(2*x^2) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1
 + 2*#1^4) & ]/4
```

Maple [A] time = 0.01, size = 75, normalized size = 0.8

$$-\frac{1}{6\,x^6}-\frac{1}{2\,x^2}-\frac{\arctan\left(2\,x^2-\sqrt{3}\right)}{4}-\frac{\arctan\left(2\,x^2+\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{24}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8-x^4+1),x)
```

```
[Out] -1/6/x^6-1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*l
n(1+x^4-x^2*3^(1/2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3\,x^4+1}{6\,x^6}-\int \frac{x^5}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/6*(3*x^4 + 1)/x^6 - integrate(x^5/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 1.57348, size = 576, normalized size = 6.

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x^6\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x^6\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^6*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2
 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) + 4
*sqrt(6)*sqrt(3)*sqrt(2)*x^6*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*
sqrt(6)*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + sqrt(6)*
sqrt(2)*x^6*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(6)*sqrt(2)*x^6*log(
2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) - 24*x^4 - 8)/x^6
```

Sympy [A] time = 0.256651, size = 82, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}-\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}-\frac{3x^4+1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 - (3*x**4 + 1
)/(6*x**6)
```

Giac [B] time = 1.27311, size = 358, normalized size = 3.73

$$-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/96*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/96*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/96*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/96*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/6*(3*x^4 + 1)/x^6
```

3.358 $\int \frac{x^8}{1-x^4+x^8}\,dx$

Optimal. Leaf size=356

$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\right.$$

```
[Out] x + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt
[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 +
 Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3
*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*S
qrt[3*(2 + Sqrt[3])]) - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x
+ x^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 +
(Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + S
qrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.332663, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1367, 1421, 1169, 634, 618, 204, 628}

$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - x^4 + x^8),x]
```

```
[Out] x + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt
[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 +
 Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3
*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*S
qrt[3*(2 + Sqrt[3])]) - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x
+ x^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 +
(Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + S
qrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^8}{1-x^4+x^8}\,dx = x - \int \frac{1-x^4}{1-x^4+x^8}\,dx$$
$$= x + \frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}$$
$$= x - \frac{\int \frac{\sqrt{3(2-\sqrt{3})}-(-2+\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{3(2-\sqrt{3})}+(-2+\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{3(2+\sqrt{3})}-(2+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{3(2+\sqrt{3})}+(2+\sqrt{3})x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$
$$= x + \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx + \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx - \frac{1}{8}\sqrt{\frac{1}{3}}$$
$$= x - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) + \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) + \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\right.}$$
$$= x + \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\right.}$$

Mathematica [C] time = 0.0143236, size = 59, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]+x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - x^4 + x^8),x]
```

```
[Out] x + RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
2*#1^7) & ]/4
```

Maple [C] time = 0.007, size = 44, normalized size = 0.1

$$x+\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(\_R^4-1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8-x^4+1),x)
```

```
[Out] x+1/4*sum((_R^4-1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x+\int \frac{x^4-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] x + integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 1.83303, size = 2319, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 +
2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/4
8*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sq
rt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/96*sq
rt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(
6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + 1/96*sq
rt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(
6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/12*sq
rt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6)*
(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2
) - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqr
t(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3
) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*
sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(
3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
 + sqrt(3) - 2) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqr
t(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt
(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqr
t(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2
) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x
^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)
*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3
)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2) + x
```

Sympy [A] time = 1.25787, size = 26, normalized size = 0.07

$$x + \operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(9216t^5 - 8t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-x**4+1),x)
```

```
[Out] x + RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 -
8*_t + x)))
```

Giac [A] time = 1.14806, size = 343, normalized size = 0.96

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) + x
```

3.359 $\int \frac{x^6}{1-x^4+x^8}\,dx$

Optimal. Leaf size=275

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\text{tan}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rubi [A] time = 0.242665, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1372, 1169, 634, 618, 204, 628}

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\text{tan}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - x^4 + x^8),x]
```


```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```


Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6}{1-x^4+x^8}\,dx = -\frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{2+\sqrt{3}}+\left(1+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2-\sqrt{3}\right)}}$$

$$= \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1+\sqrt{2+\right.}{4\sqrt{6}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\log\left(1-\sqrt{2-\right.}{4\sqrt{6}}$$

Mathematica [C] time = 0.0111492, size = 41, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/4
```

Maple [C] time = 0.008, size = 32, normalized size = 0.1

$$\frac{\sum_{\_R=\mathrm{RootOf}(9\_Z^4+1)} \_R \ln \left(9\_R^3 x - 3\_R^2 + x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{x^8 - x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 1.59413, size = 608, normalized size = 2.21

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) - 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1
) + 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)
```

Sympy [A] time = 0.20892, size = 165, normalized size = 0.6

$$\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
```

```
4*x**2 + 2*sqrt(6)*x + 3))/24 + sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
sqrt(6)*x + 1)/24 - sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

Giac [A] time = 1.15781, size = 277, normalized size = 1.01

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) - 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) + 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.360 $\int \frac{x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=347

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\cdots$$

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])
]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 - Sqr
t[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
+ Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 - Sqrt[3])]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[
3])]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) + Log[
1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) - Log[1 + Sqrt[2 +
 Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rubi [A] time = 0.20877, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])
]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 - Sqr
t[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
+ Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 - Sqrt[3])]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[
3])]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) + Log[
1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) - Log[1 + Sqrt[2 +
 Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{1-x^4+x^8}\,dx &= \frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}\\
&= -\frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} + \frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} + \frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} - \frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}\\
&= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}x-x^2}\cdots}{8\sqrt{3\left(2-\sqrt{3}\right)}\cdots}\\
&= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2\cdots}\right.}{8\sqrt{3\left(2\cdots\right.}}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1-\sqrt{2-\cdots}\right.}{8\sqrt{3\left(2-\cdots\right.}}
\end{aligned}$$

Mathematica [C] time = 0.0109694, size = 39, normalized size = 0.11

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]/4
```

Maple [C] time = 0.005, size = 40, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln(x-\_R)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 1.75063, size = 1913, normalized size = 5.51

$$\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+12x^2+12\right)-\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(-2\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(2*sqrt(6)*
sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) - 1/48*sqrt(6)*(sqrt(3)*
sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqr
t(sqrt(3) + 2) + 12*x^2 + 12) + 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*
sqrt(-4*sqrt(3) + 8)*log(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1
2*x^2 + 12) - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) +
8)*log(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12) - 1/1
2*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*s
```

```
qrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqr
t(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2) - 1/12*sqrt(
6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqr
t(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt
(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) + 1/24*sqrt(6)*sqr
t(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqr
t(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqr
t(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2) + 1/24*sqrt(6)
*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4
*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-
4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt(3) + 2)
```

Sympy [A] time = 1.28636, size = 24, normalized size = 0.07

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-18432t^5 + 4t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4*
_t + x)))
```

Giac [A] time = 1.18939, size = 342, normalized size = 0.99

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{}}{\sqrt{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

3.361 $\int \frac{x^2}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{4}\sqrt{}$$

```
[Out] (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]])
/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[
3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt
[2 - Sqrt[3]]])/4 + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[
3])]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) - Log[
1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) + Log[1 + Sqrt[2 +
 Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rubi [A] time = 0.199669, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1373, 1094, 634, 618, 204, 628}

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{4}\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]])
/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[
3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt
[2 - Sqrt[3]]])/4 + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[
3])]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) - Log[
1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) + Log[1 + Sqrt[2 +
 Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \frac{\int \frac{1}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,d}{8\sqrt{3\left(2-\sqrt{3}\right)}}$$

$$= \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2+\right.}{8\sqrt{3\left(2+\right.}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1-\sqrt{2-\right.}{8\sqrt{3\left(2-\right.}}$$

Mathematica [C] time = 0.011243, size = 40, normalized size = 0.11

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^5-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1 + 2*#1^5) & ]/4
```

Maple [C] time = 0.007, size = 40, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 1.80795, size = 1914, normalized size = 5.39

$$-\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+12x^2+12\right)+\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(2*sqrt(6)
*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) + 1/48*sqrt(6)*(sqrt(3)
*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sq
rt(sqrt(3) + 2) + 12*x^2 + 12) - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))
*sqrt(-4*sqrt(3) + 8)*log(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) +
12*x^2 + 12) + 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) +
 8)*log(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12) - 1/
12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*
sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sq
rt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2) - 1/12*sqrt
(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sq
rt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqr
t(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) + 1/24*sqrt(6)*sq
rt(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sq
rt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sq
rt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2) + 1/24*sqrt(6
)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-
4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(
```

```
-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt(3) + 2)
```

Sympy [A] time = 1.2978, size = 26, normalized size = 0.07

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-442368t^7 - 192t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 - 1
92*_t**3 + x)))
```

Giac [A] time = 1.1451, size = 342, normalized size = 0.96

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{}}{\sqrt{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

3.362 $\int \frac{1}{1-x^4+x^8} dx$

Optimal. Leaf size=275

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\text{ta}}{}-$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rubi [A] time = 0.210548, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1346, 1169, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\text{ta}}{}-$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4 + x^8)^(-1),x]
```


```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```


Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3(2-\sqrt{3})}-(-1+\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2-\sqrt{3})}+(-1+\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})}-(1+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})}+(1+\sqrt{3})x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} +$$

$$= -\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,d}{4\sqrt{6\left(2-\sqrt{3}\right)}}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2}\right.}{4\sqrt{}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\log\left(1-\sqrt{2-}\right.}{4\sqrt{6}}$$

Mathematica [C] time = 0.0108245, size = 42, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.005, size = 30, normalized size = 0.1

$$\frac{\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_R x+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 1.60933, size = 608, normalized size = 2.21

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) + 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1
) - 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)
```

Sympy [A] time = 0.240344, size = 165, normalized size = 0.6

$$\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
```

```
sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

Giac [A] time = 1.14836, size = 277, normalized size = 1.01

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.363 $\int \frac{1}{x^2(1-x^4+x^8)}\,dx$

Optimal. Leaf size=360

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2\cdots}\right.$$

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*
Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]
]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[
3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]
)/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[
(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.238585, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1368, 1506, 1279, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^4 + x^8)),x]
```


```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*
Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]
]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[
3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]
)/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[
(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```


Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1506

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r - (c*d - e*q)*x^(n/2), x])
/(q - r*x^(n/2) + c*x^n), x], x] + Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r +
```

```
(c*d - e*q)*x^(n/2), x])/(q + r*x^(n/2) + c*x^n), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2(1-x^4+x^8)}\,dx = -\frac{1}{x} + \int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{x} + \frac{\int \frac{x^2(\sqrt{3}-2x^2)}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{x^2(\sqrt{3}+2x^2)}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{x} - \frac{\int \frac{-2+\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{2+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{x} - \frac{\int \frac{2\sqrt{2-\sqrt{3}}-(2-\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3(2-\sqrt{3})}} - \frac{\int \frac{2\sqrt{2-\sqrt{3}}+(2-\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3(2-\sqrt{3})}} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}-(-2-\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3(2+\sqrt{3})}} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}+(-2}{1+\sqrt{2+\sqrt{3}}x}}{4\sqrt{3(2+}}$$

$$= -\frac{1}{x} + \frac{1}{8}\sqrt{\frac{1}{3}(7-4\sqrt{3})}\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx + \frac{1}{8}\sqrt{\frac{1}{3}(7-4\sqrt{3})}\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}$$

$$= -\frac{1}{x} + \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}$$

$$= -\frac{1}{x} + \frac{1}{4}\sqrt{\frac{1}{3}(2+\sqrt{3})}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}(2-\sqrt{3})}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{4}$$

Mathematica [C] time = 0.015709, size = 61, normalized size = 0.17

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#
1 + 2*#1^5) & ]/4
```

Maple [C] time = 0.009, size = 52, normalized size = 0.1

$$-\frac{1}{4}\sum_{\_R=\text{RootOf}(\_Z^8-\_Z^4+1)}\frac{(\_R^6-\_R^2)\ln(x-\_R)}{2\_R^7-\_R^3}-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8-x^4+1),x)
```

```
[Out] -1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))-1/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x}-\int \frac{x^6-x^2}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^6 - x^2)/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 1.82402, size = 2327, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/96*(8*sqrt(6)*sqrt(2)*x*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2
 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(s
qrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt
(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) + 8*sqrt(6)*sqrt(2)*x
*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sq
rt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2)
)*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(
sqrt(3) + 2) + sqrt(3) - 2) + 4*sqrt(6)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8)*arct
an(1/12*sqrt(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*s
qrt(-4*sqrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8
) - 1/6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) -
sqrt(3) - 2) + 4*sqrt(6)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)
*sqrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3)
+ 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)
*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2) -
2*sqrt(6)*(sqrt(3)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt(sqrt(3) + 2)*log(12*x^2 +
2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 2*s
qrt(6)*(sqrt(3)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*s
qrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - sqrt(6
)*(sqrt(3)*sqrt(2)*x + 2*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(
6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + sqrt(6)
*(sqrt(3)*sqrt(2)*x + 2*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + 96)/x
```

Sympy [A] time = 1.29061, size = 29, normalized size = 0.08

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-442368t^7 + 384t^3 + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 + 3
84*_t**3 + x))) - 1/x
```

Giac [A] time = 1.20009, size = 348, normalized size = 0.97

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+}{\sqrt{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/x
```

3.364 $\int \frac{1}{x^4(1-x^4+x^8)}\,dx$

Optimal. Leaf size=370

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\right.$$

```
[Out] -1/(3*x^3) - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2
 + Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]
] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.235534, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1368, 1421, 1169, 634, 618, 204, 628}

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^4 + x^8)),x]
```


```
[Out] -1/(3*x^3) - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2
 + Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]
] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```


Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(1-x^4+x^8\right)}\,dx = -\frac{1}{3x^3}+\frac{1}{3}\int \frac{3-3x^4}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}-\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{3x^3}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\cdots$$

$$= -\frac{1}{3x^3}-\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx-\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\cdots$$

$$= -\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)-\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)-\cdots$$

$$= -\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\cdots$$

Mathematica [C] time = 0.0139884, size = 65, normalized size = 0.18

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/
(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.01, size = 50, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))-1/3/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3x^3}-\int\frac{x^4-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 1.80146, size = 2361, normalized size = 6.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/96*(8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^
2 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(
sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqr
t(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) + 8*sqrt(6)*sqrt(2)*
x^3*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)
*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt
(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sq
rt(sqrt(3) + 2) + sqrt(3) - 2) + 4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)
*arctan(1/12*sqrt(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)
```

```
*x)*sqrt(-4*sqrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3
) + 8) - 1/6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) +
8) - sqrt(3) - 2) + 4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arctan(1/12*
sqrt(6)*sqrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*s
qrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*
sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3)
+ 2) + 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*lo
g(12*x^2 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
+ 12) - 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*l
og(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
 + 12) + sqrt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x^3)*sqrt(-4*sqrt(3) + 8)
*log(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) +
 8) + 12) - sqrt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x^3)*sqrt(-4*sqrt(3) +
 8)*log(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3
) + 8) + 12) - 32)/x^3
```

Sympy [A] time = 1.30491, size = 31, normalized size = 0.08

$$\text{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-9216t^5 + 8t + x\right)\right)\right) - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-9216*_t**5 + 8*_
t + x))) - 1/(3*x**3)
```

Giac [A] time = 1.16026, size = 348, normalized size = 0.94

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1) - 1/3/x^3
```

3.365 $\int \frac{1}{x^6(1-x^4+x^8)}\,dx$

Optimal. Leaf size=287

$$-\frac{1}{5x^5}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-$$

```
[Out] -1/(5*x^5) - x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(
2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]
) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTa
n[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2
 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sq
rt[6]) - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 +
Sqrt[3]]*x + x^2]/(4*Sqrt[6])
```

Rubi [A] time = 0.241094, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1368, 1504, 12, 1372, 1169, 634, 618, 204, 628}

$$-\frac{1}{5x^5}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(
2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]
) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTa
n[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2
 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sq
rt[6]) - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 +
Sqrt[3]]*x + x^2]/(4*Sqrt[6])
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(1-x^4+x^8\right)}\,dx &= -\frac{1}{5x^5}+\frac{1}{5}\int \frac{5-5x^4}{x^2\left(1-x^4+x^8\right)}\,dx \\
&= -\frac{1}{5x^5}-\frac{1}{x}-\frac{1}{5}\int \frac{5x^6}{1-x^4+x^8}\,dx \\
&= -\frac{1}{5x^5}-\frac{1}{x}-\int \frac{x^6}{1-x^4+x^8}\,dx \\
&= -\frac{1}{5x^5}-\frac{1}{x}+\frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} \\
&= -\frac{1}{5x^5}-\frac{1}{x}-\frac{\int \frac{\sqrt{2-\sqrt{3}}-(1-\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}+(1-\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}-(1+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+(1+\sqrt{3}}{1+\sqrt{2+\sqrt{3}}x+x^2}}{4\sqrt{3\left(2+\sqrt{}\right.}} \\
&= -\frac{1}{5x^5}-\frac{1}{x}-\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}- \\
&= -\frac{1}{5x^5}-\frac{1}{x}-\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} \\
&= -\frac{1}{5x^5}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-
\end{aligned}$$

Mathematica [C] time = 0.0164146, size = 54, normalized size = 0.19

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{5x^5}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 +
2*#1^4) & ]/4
```

Maple [C] time = 0.008, size = 43, normalized size = 0.2

$$-\frac{1}{5\,x^5}-x^{-1}-\frac{\sum_{\_R=\text{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(9\_R^3x-3\_R^2+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8-x^4+1),x)
```

```
[Out] -1/5/x^5-1/x-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{5x^4+1}{5x^5} - \int \frac{x^6}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*(5*x^4 + 1)/x^5 - integrate(x^6/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 1.55549, size = 653, normalized size = 2.28

$$\frac{20\sqrt{3}\sqrt{2}x^5\arctan\left(-\frac{\sqrt{3}\sqrt{2}(x^3-x)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)+20\sqrt{3}\sqrt{2}x^5\arctan\left(-\frac{\sqrt{3}\sqrt{2}(x^3-x)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}(x^3}{3x^2-2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(20*sqrt(3)*sqrt(2)*x^5*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sq
rt(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3
*x^2 - 2)) + 20*sqrt(3)*sqrt(2)*x^5*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^
2 - sqrt(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x +
2))/(3*x^2 - 2)) + 5*sqrt(3)*sqrt(2)*x^5*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x
) + 3*x^2 + 1) - 5*sqrt(3)*sqrt(2)*x^5*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x)
+ 3*x^2 + 1) - 120*x^4 - 24)/x^5
```

Sympy [A] time = 0.272219, size = 180, normalized size = 0.63

$$\frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 + sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*x**3
+ 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2
- sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x
+ 1)/24 - (5*x**4 + 1)/(5*x**5)
```

Giac [A] time = 1.14284, size = 293, normalized size = 1.02

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt(6
)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*arct
an((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sq
rt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*
sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/5*(5*x^4 + 1)/x^5
```

3.366 $\int \frac{1}{x^8(1-x^4+x^8)} dx$

Optimal. Leaf size=377

$$-\frac{1}{3x^3}-\frac{1}{7x^7}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}}\left(2-\sqrt{3}\right.$$

```
[Out] -1/(7*x^7) - 1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] -
 2*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[
3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt
[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 -
 Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x
^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.286404, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1368, 1504, 12, 1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{3x^3}-\frac{1}{7x^7}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}}\left(2-\sqrt{3}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - x^4 + x^8)),x]
```


```
[Out] -1/(7*x^7) - 1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] -
 2*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[
3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt
[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 -
 Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x
^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```


Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
```

gerQ[p]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8\left(1-x^4+x^8\right)}\,dx &= -\frac{1}{7x^7}+\frac{1}{7}\int \frac{7-7x^4}{x^4\left(1-x^4+x^8\right)}\,dx \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{1}{21}\int \frac{21x^4}{1-x^4+x^8}\,dx \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}-\int \frac{x^4}{1-x^4+x^8}\,dx \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}+\frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}}+\frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\cdots}{8\sqrt{3}} \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+\cdots\right.}{8\sqrt{3\left(2+\sqrt{3}\right)}} \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}\cdots}{\sqrt{2-\cdots}}\right.}{4\sqrt{3\left(2-\cdots\right.}}
\end{aligned}$$

Mathematica [C] time = 0.0147045, size = 54, normalized size = 0.14

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{3x^3}-\frac{1}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) - 1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 +
 2*#1^4) & ]/4
```

Maple [C] time = 0.01, size = 51, normalized size = 0.1

$$-\frac{1}{7\,x^7}-\frac{1}{3\,x^3}-\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln(x-\_R)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8-x^4+1),x)
```

```
[Out] -1/7/x^7-1/3/x^3-1/4*sum(_R^4/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{7\,x^{4}+3}{21\,x^{7}}-\int \frac{x^{4}}{x^{8}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/21*(7*x^4 + 3)/x^7 - integrate(x^4/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 1.75301, size = 1990, normalized size = 5.28

$$56\,\sqrt{6}\sqrt{2}x^{7}\sqrt{\sqrt{3}+2}\arctan\left(-\frac{1}{3}\,\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+\frac{1}{6}\,\sqrt{6}\sqrt{2}\sqrt{2\,\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+12\,x^{2}+12}\sqrt{\sqrt{3}+2}-\sqrt{3}-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/672*(56*sqrt(6)*sqrt(2)*x^7*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)
*sqrt(2)*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*s
qrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2)
+ 56*sqrt(6)*sqrt(2)*x^7*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt
(2)*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(
2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) - 28
*sqrt(6)*sqrt(2)*x^7*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(
2)*x*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(
2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2
) - 28*sqrt(6)*sqrt(2)*x^7*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)
*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3
)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt
(3) + 2) - 224*x^4 - 14*sqrt(6)*(sqrt(3)*sqrt(2)*x^7 - 2*sqrt(2)*x^7)*sqrt(
sqrt(3) + 2)*log(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 1
2) + 14*sqrt(6)*(sqrt(3)*sqrt(2)*x^7 - 2*sqrt(2)*x^7)*sqrt(sqrt(3) + 2)*log
(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) - 7*sqrt(6)*
(sqrt(3)*sqrt(2)*x^7 + 2*sqrt(2)*x^7)*sqrt(-4*sqrt(3) + 8)*log(sqrt(6)*sqrt
(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12) + 7*sqrt(6)*(sqrt(3)*sqrt
(2)*x^7 + 2*sqrt(2)*x^7)*sqrt(-4*sqrt(3) + 8)*log(-sqrt(6)*sqrt(3)*sqrt(2)*
x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12) - 96)/x^7
```

Sympy [A] time = 1.37798, size = 36, normalized size = 0.1

$$\operatorname{RootSum}\left(5308416t^{8}-2304t^{4}+1,\left(t\mapsto t\log\left(18432t^{5}-4t+x\right)\right)\right)-\frac{7x^{4}+3}{21x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(18432*_t**5 - 4*_
t + x))) - (7*x**4 + 3)/(21*x**7)
```

Giac [A] time = 1.16812, size = 358, normalized size = 0.95

$$-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/21*(7*x^4 + 3)/x^7
```

3.367 $\int \frac{x^m}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=117

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(3 - Sqrt[
5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1, (
1 + m)/4, (5 + m)/4, (-2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1 + m
))
```

Rubi [A] time = 0.0837868, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1375, 364}

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 + 3*x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(3 - Sqrt[
5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1, (
1 + m)/4, (5 + m)/4, (-2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1 + m
))
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{1+3x^4+x^8}\,dx&=\frac{\int \frac{x^m}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}-\frac{\int \frac{x^m}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}\\&=\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}\end{aligned}$$

Mathematica [C] time = 0.0413588, size = 54, normalized size = 0.46

$$\frac{x^{m+1}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{{}_2F_1\left(1,m+1;m+2;\frac{x}{\#1}\right)}{3\#1^4+2}\&\right]}{4(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(1 + 3*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*RootSum[1 + 3*#1^4 + #1^8 & , Hypergeometric2F1[1, 1 + m, 2 + m,
 x/#1]/(2 + 3*#1^4) & ])/(4*(1 + m))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8+3*x^4+1),x)
```

```
[Out] int(x^m/(x^8+3*x^4+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{x^8+3x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 + 3*x^4 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8+3*x**4+1),x)
```

```
[Out] Integral(x**m/(x**8 + 3*x**4 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

3.368 $\int \frac{x^{11}}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=62

$$\frac{x^4}{4} - \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right) - \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

```
[Out] x^4/4 - ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] time = 0.0562573, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 703, 632, 31}

$$\frac{x^4}{4} - \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right) - \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 + 3*x^4 + x^8),x]
```

```
[Out] x^4/4 - ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] + 2*x^4])/40
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{1+3x+x^2}\,dx, x, x^4\right)$$
$$= \frac{x^4}{4} + \frac{1}{4}\operatorname{Subst}\left(\int \frac{-1-3x}{1+3x+x^2}\,dx, x, x^4\right)$$
$$= \frac{x^4}{4} + \frac{1}{40}\left(-15+7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right) - \frac{1}{40}\left(15+7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x\right.$$
$$= \frac{x^4}{4} - \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right) - \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

Mathematica [A] time = 0.0321734, size = 57, normalized size = 0.92

$$\frac{1}{40}\left(10x^4+\left(7\sqrt{5}-15\right)\log\left(-2x^4+\sqrt{5}-3\right)-\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 + 3*x^4 + x^8),x]
```

```
[Out] (10*x^4 + (-15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] - (15 + 7*Sqrt[5])*Lo
g[3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.003, size = 38, normalized size = 0.6

$$\frac{x^4}{4} - \frac{3\ln\left(x^8+3x^4+1\right)}{8} - \frac{7\sqrt{5}}{20}\operatorname{Artanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*x^4-3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Maxima [A] time = 1.49733, size = 68, normalized size = 1.1

$$\frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right) - \frac{3}{8}\log\left(x^8+3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 3
/8*log(x^8 + 3*x^4 + 1)
```

Fricas [A] time = 1.48317, size = 157, normalized size = 2.53

$$\frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{2x^8+6x^4-\sqrt{5}\left(2x^4+3\right)+7}{x^8+3x^4+1}\right) - \frac{3}{8}\log\left(x^8+3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 +
 3*x^4 + 1)) - 3/8*log(x^8 + 3*x^4 + 1)
```

Sympy [A] time = 0.142039, size = 60, normalized size = 0.97

$$\frac{x^4}{4} + \left(-\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(-\frac{7\sqrt{5}}{40} - \frac{3}{8}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+3*x**4+1),x)
```

```
[Out] x**4/4 + (-3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (-7*sqrt(5)/40
 - 3/8)*log(x**4 + sqrt(5)/2 + 3/2)
```

Giac [A] time = 1.22097, size = 68, normalized size = 1.1

$$\frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) - \frac{3}{8}\log\left(x^8 + 3x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 3
/8*log(x^8 + 3*x^4 + 1)
```

3.369 $\int \frac{x^9}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=90

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sq
rt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi [A] time = 0.142151, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1359, 1122, 1166, 203}

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + 3*x^4 + x^8),x]
```

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sq
rt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{1+3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{x^2}{2} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1+3x^2}{1+3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{x^2}{2} - \frac{1}{20}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right) - \frac{1}{20}\left(15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,d\right.$$
$$= \frac{x^2}{2} - \frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{20}\sqrt{180-80\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Mathematica [A] time = 0.146126, size = 97, normalized size = 1.08

$$\frac{1}{40}\left(20x^2 - \sqrt{6-2\sqrt{5}}\left(15+7\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \sqrt{2\left(3+\sqrt{5}\right)}\left(7\sqrt{5}-15\right)\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + 3*x^4 + x^8),x]
```

```
[Out] (20*x^2 - Sqrt[6 - 2*Sqrt[5]]*(15 + 7*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]
*x^2] + Sqrt[2*(3 + Sqrt[5])]*(-15 + 7*Sqrt[5])*ArcTan[Sqrt[(3 + Sqrt[5])/2
]*x^2])/40
```

Maple [B] time = 0.041, size = 117, normalized size = 1.3

$$\frac{x^2}{2} + \frac{7\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right) - 3\frac{1}{-2+2\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right) - \frac{7\sqrt{5}}{10+10\sqrt{5}}\arctan\left(4\frac{x^2}{2+2\sqrt{5}}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+3*x^4+1),x)
```

```
[Out] 1/2*x^2+7/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4*x^2/(-2+2*5^(1/2)))-3/(-2+2*5^(
1/2))*arctan(4*x^2/(-2+2*5^(1/2)))-7/5*5^(1/2)/(2+2*5^(1/2))*arctan(4*x^2/(
2+2*5^(1/2)))-3/(2+2*5^(1/2))*arctan(4*x^2/(2+2*5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2 - \int \frac{\left(3x^4+1\right)x}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - integrate((3*x^4 + 1)*x/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] time = 1.58515, size = 456, normalized size = 5.07

$$\frac{1}{2}x^2 - \frac{1}{5}\sqrt{5}\sqrt{-4\sqrt{5}+9}\arctan\left(\frac{1}{4}\sqrt{2x^4-\sqrt{5}+3}\left(3\sqrt{5}\sqrt{2}+7\sqrt{2}\right)\sqrt{-4\sqrt{5}+9} - \frac{1}{2}\left(3\sqrt{5}x^2+7x^2\right)\sqrt{-4\sqrt{5}+9}\right) - \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/5*sqrt(5)*sqrt(-4*sqrt(5) + 9)*arctan(1/4*sqrt(2*x^4 - sqrt(5)
+ 3)*(3*sqrt(5)*sqrt(2) + 7*sqrt(2))*sqrt(-4*sqrt(5) + 9) - 1/2*(3*sqrt(5)*
x^2 + 7*x^2)*sqrt(-4*sqrt(5) + 9)) - 1/5*sqrt(5)*sqrt(4*sqrt(5) + 9)*arctan
(-1/4*(6*sqrt(5)*x^2 - 14*x^2 - sqrt(2*x^4 + sqrt(5) + 3)*(3*sqrt(5)*sqrt(2
) - 7*sqrt(2)))*sqrt(4*sqrt(5) + 9))
```

Sympy [A] time = 0.197984, size = 54, normalized size = 0.6

$$\frac{x^2}{2} + 2\left(\frac{1}{4} - \frac{\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right) - 2\left(\frac{\sqrt{5}}{10} + \frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+3*x**4+1),x)
```

```
[Out] x**2/2 + 2*(1/4 - sqrt(5)/10)*atan(2*x**2/(-1 + sqrt(5))) - 2*(sqrt(5)/10 +
 1/4)*atan(2*x**2/(1 + sqrt(5)))
```

Giac [A] time = 1.23001, size = 89, normalized size = 0.99

$$\frac{1}{2}x^2 - \frac{1}{20}\left(3x^4\left(\sqrt{5}-5\right) + \sqrt{5}-5\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right) - \frac{1}{20}\left(3x^4\left(\sqrt{5}+5\right) + \sqrt{5}+5\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/20*(3*x^4*(sqrt(5) - 5) + sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) +
1)) - 1/20*(3*x^4*(sqrt(5) + 5) + sqrt(5) + 5)*arctan(2*x^2/(sqrt(5) - 1))
```

3.370 $\int \frac{x^7}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=55

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] + 2*x^4])/40
```

Rubi [A] time = 0.0329384, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 632, 31}

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] + 2*x^4])/40
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1+3x^4+x^8} \, dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+3x+x^2} \, dx,x,x^4\right)\\
&= \frac{1}{40}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x} \, dx,x,x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x} \, dx,x,x^4\right)\\
&= \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0227515, size = 53, normalized size = 0.96

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}-3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + S
qrt[5] + 2*x^4])/40
```

Maple [A] time = 0.003, size = 33, normalized size = 0.6

$$\frac{\ln\left(x^8+3x^4+1\right)}{8}+\frac{3\sqrt{5}}{20}\mathrm{Artanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+3*x^4+1),x)
```

```
[Out] 1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Maxima [A] time = 1.48365, size = 61, normalized size = 1.11

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)+\frac{1}{8}\log\left(x^8+3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x^
8 + 3*x^4 + 1)
```

Fricas [A] time = 1.42939, size = 143, normalized size = 2.6

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8+6x^4+\sqrt{5}\left(2x^4+3\right)+7}{x^8+3x^4+1}\right)+\frac{1}{8}\log\left(x^8+3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
)) + 1/8*log(x^8 + 3*x^4 + 1)
```

Sympy [A] time = 0.134588, size = 53, normalized size = 0.96

$$\left(\frac{1}{8} - \frac{3\sqrt{5}}{40}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(\frac{1}{8} + \frac{3\sqrt{5}}{40}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+3*x**4+1),x)
```

```
[Out] (1/8 - 3*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (1/8 + 3*sqrt(5)/40)*log
(x**4 + sqrt(5)/2 + 3/2)
```

Giac [A] time = 1.23648, size = 61, normalized size = 1.11

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^4 - \sqrt{5} + 3}{2\,x^4 + \sqrt{5} + 3}\right) + \frac{1}{8}\log\left(x^8 + 3\,x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x^
8 + 3*x^4 + 1)
```

3.371 $\int \frac{x^5}{1+3x^4+x^8} dx$

Optimal. Leaf size=81

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] (Sqrt[(3 + Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 - (Sqrt[(3 - S
qrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi [A] time = 0.084069, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1359, 1130, 203}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + 3*x^4 + x^8),x]
```

```
[Out] (Sqrt[(3 + Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 - (Sqrt[(3 - S
qrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1+3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{1}{20}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right) + \frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right.$$
$$= \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) - \frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Mathematica [A] time = 0.0468945, size = 75, normalized size = 0.93

$$\frac{2\sqrt{5}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \left(5-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{10\sqrt{6-2\sqrt{5}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + 3*x^4 + x^8),x]
```

```
[Out] (2*Sqrt[5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2] + (5 - 3*Sqrt[5])*ArcTan[Sqrt[
(3 + Sqrt[5])/2]*x^2])/(10*Sqrt[6 - 2*Sqrt[5]])
```

Maple [B] time = 0.017, size = 110, normalized size = 1.4

$$\frac{1}{-2+2\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right) - \frac{3\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right) + \frac{1}{2+2\sqrt{5}}\arctan\left(4\frac{x^2}{2+2\sqrt{5}}\right) + \frac{3\sqrt{5}}{10+10\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+3*x^4+1),x)
```

```
[Out] 1/(-2+2*5^(1/2))*arctan(4*x^2/(-2+2*5^(1/2)))-3/5*5^(1/2)/(-2+2*5^(1/2))*ar
ctan(4*x^2/(-2+2*5^(1/2)))+1/(2+2*5^(1/2))*arctan(4*x^2/(2+2*5^(1/2)))+3/5*
5^(1/2)/(2+2*5^(1/2))*arctan(4*x^2/(2+2*5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] time = 1.59255, size = 517, normalized size = 6.38

$$-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+3}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2x^4+\sqrt{5}+3}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{5}+3} - \frac{1}{20}\sqrt{10}\left(3\sqrt{5}x^2-5x^2\right)\sqrt{\sqrt{5}+3}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(sqrt(5) + 3)*arctan(1/40*sqrt(10)*sqrt(2*x^4 + sqrt(5)
+ 3)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 3) - 1/20*sqrt(10)*(3*s
qrt(5)*x^2 - 5*x^2)*sqrt(sqrt(5) + 3)) + 1/10*sqrt(10)*sqrt(-sqrt(5) + 3)*a
rctan(1/40*sqrt(10)*sqrt(2*x^4 - sqrt(5) + 3)*(3*sqrt(5)*sqrt(2) + 5*sqrt(2
))*sqrt(-sqrt(5) + 3) - 1/20*sqrt(10)*(3*sqrt(5)*x^2 + 5*x^2)*sqrt(-sqrt(5)
 + 3))
```

Sympy [A] time = 0.193788, size = 49, normalized size = 0.6

$$-2\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8+3*x**4+1),x)
```

```
[Out] -2*(1/8 - sqrt(5)/40)*atan(2*x**2/(-1 + sqrt(5))) + 2*(sqrt(5)/40 + 1/8)*at
an(2*x**2/(1 + sqrt(5)))
```

Giac [A] time = 1.2764, size = 63, normalized size = 0.78

$$\frac{1}{20}\,x^4\left(\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\,x^4\left(\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*x^4*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*x^4*(sqrt(5) + 5)
*arctan(2*x^2/(sqrt(5) - 1))
```

3.372 $\int \frac{x^3}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{2x^4+3}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] -ArcTanh[(3 + 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rubi [A] time = 0.0256436, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{2x^4+3}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + 3*x^4 + x^8),x]
```

```
[Out] -ArcTanh[(3 + 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{1+3x^4+x^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+3x+x^2}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, 3+2x^4\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{3+2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.0100449, size = 38, normalized size = 1.65

$$\frac{\log\left(-2x^4+\sqrt{5}-3\right)-\log\left(2x^4+\sqrt{5}+3\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 3*x^4 + x^8),x]
```

```
[Out] (Log[-3 + Sqrt[5] - 2*x^4] - Log[3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

Maple [A] time = 0., size = 19, normalized size = 0.8

$$-\frac{\sqrt{5}}{10}\mathrm{Artanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+3*x^4+1),x)
```

```
[Out] -1/10*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Maxima [A] time = 1.49359, size = 42, normalized size = 1.83

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

Fricas [B] time = 1.45554, size = 107, normalized size = 4.65

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^8+6x^4-\sqrt{5}\left(2x^4+3\right)+7}{x^8+3x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
))
```

Sympy [A] time = 0.121867, size = 42, normalized size = 1.83

$$\frac{\sqrt{5}\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20}-\frac{\sqrt{5}\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+3*x**4+1),x)
```

```
[Out] sqrt(5)*log(x**4 - sqrt(5)/2 + 3/2)/20 - sqrt(5)*log(x**4 + sqrt(5)/2 + 3/2
)/20
```

Giac [A] time = 1.29294, size = 42, normalized size = 1.83

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

3.373 $\int \frac{x}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=75

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

```
[Out] -(ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + Sq
rt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi [A] time = 0.058168, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1359, 1093, 203}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + 3*x^4 + x^8),x]
```

```
[Out] -(ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + Sq
rt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{1+3x^4+x^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}} - \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}}$$
$$= -\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}} + \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Mathematica [A] time = 0.0383284, size = 74, normalized size = 0.99

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}x^2\right)}{\sqrt{10\left(3-\sqrt{5}\right)}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(3 - Sqrt[5])]*x^2]/Sqrt[10*(3 - Sqrt[5])] - ArcTan[Sqrt[2/(3
 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]
```

Maple [A] time = 0.014, size = 60, normalized size = 0.8

$$\frac{2\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right) - \frac{2\sqrt{5}}{10+10\sqrt{5}}\arctan\left(4\frac{x^2}{2+2\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+3*x^4+1),x)
```

```
[Out] 2/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4*x^2/(-2+2*5^(1/2)))-2/5*5^(1/2)/(2+2*5^
(1/2))*arctan(4*x^2/(2+2*5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] time = 1.54003, size = 417, normalized size = 5.56

$$\frac{1}{10}\sqrt{10}\sqrt{-\sqrt{5}+3}\arctan\left(-\frac{1}{10}\sqrt{10}\sqrt{5}x^2\sqrt{-\sqrt{5}+3}+\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^4+\sqrt{5}+3}\sqrt{-\sqrt{5}+3}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+3}\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*sqrt(-sqrt(5) + 3)*arctan(-1/10*sqrt(10)*sqrt(5)*x^2*sqrt(-sq
rt(5) + 3) + 1/20*sqrt(10)*sqrt(5)*sqrt(2)*sqrt(2*x^4 + sqrt(5) + 3)*sqrt(-
sqrt(5) + 3)) - 1/10*sqrt(10)*sqrt(sqrt(5) + 3)*arctan(-1/20*(2*sqrt(10)*sq
rt(5)*x^2 - sqrt(10)*sqrt(5)*sqrt(2)*sqrt(2*x^4 - sqrt(5) + 3))*sqrt(sqrt(5
) + 3))
```

Sympy [A] time = 0.194831, size = 49, normalized size = 0.65

$$2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+3*x**4+1),x)
```

```
[Out] 2*(sqrt(5)/40 + 1/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(1/8 - sqrt(5)/40)*ata
n(2*x**2/(1 + sqrt(5)))
```

Giac [A] time = 1.30564, size = 55, normalized size = 0.73

$$\frac{1}{20}\left(\sqrt{5}-5\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\left(\sqrt{5}+5\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*(sqrt(5) + 5)*arctan(
2*x^2/(sqrt(5) - 1))
```

3.374 $\int \frac{1}{x(1+3x^4+x^8)} dx$

Optimal. Leaf size=57

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)+\log(x)$$

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((5 - 3*Sqrt[5])*L
og[3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] time = 0.0342272, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 705, 29, 632, 31}

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + 3*x^4 + x^8)),x]
```

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((5 - 3*Sqrt[5])*L
og[3 + Sqrt[5] + 2*x^4])/40
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x(1+3x^4+x^8)}dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1+3x+x^2)}dx, x, x^4\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}dx, x, x^4\right) + \frac{1}{4}\text{Subst}\left(\int \frac{-3-x}{1+3x+x^2}dx, x, x^4\right)$$
$$= \log(x) + \frac{1}{40}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}dx, x, x^4\right) - \frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\right.$$
$$= \log(x) - \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right) - \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

Mathematica [A] time = 0.0292216, size = 55, normalized size = 0.96

$$\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}-3\right) + \frac{1}{40}\left(3\sqrt{5}-5\right)\log\left(2x^4+\sqrt{5}+3\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + 3*x^4 + x^8)),x]
```

```
[Out] Log[x] + ((-5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((-5 + 3*Sqrt[5]
)*Log[3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.007, size = 35, normalized size = 0.6

$$-\frac{\ln\left(x^8+3x^4+1\right)}{8} + \frac{3\sqrt{5}}{20}\text{Artanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right) + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+3*x^4+1),x)
```

```
[Out] -1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)+ln(x)
```

Maxima [A] time = 1.48879, size = 69, normalized size = 1.21

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right) - \frac{1}{8}\log\left(x^8+3x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x^
8 + 3*x^4 + 1) + 1/4*log(x^4)
```

Fricas [A] time = 1.48148, size = 155, normalized size = 2.72

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8+6x^4+\sqrt{5}(2x^4+3)+7}{x^8+3x^4+1}\right)-\frac{1}{8}\log\left(x^8+3x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
)) - 1/8*log(x^8 + 3*x^4 + 1) + log(x)
```

Sympy [A] time = 0.152717, size = 58, normalized size = 1.02

$$\log(x)+\left(-\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(-\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+3*x**4+1),x)
```

```
[Out] log(x) + (-3*sqrt(5)/40 - 1/8)*log(x**4 - sqrt(5)/2 + 3/2) + (-1/8 + 3*sqrt
(5)/40)*log(x**4 + sqrt(5)/2 + 3/2)
```

Giac [A] time = 1.24926, size = 69, normalized size = 1.21

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)-\frac{1}{8}\log\left(x^8+3x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x^
8 + 3*x^4 + 1) + 1/4*log(x^4)
```

3.375 $\int \frac{1}{x^3(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

```
[Out] -1/(2*x^2) + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2
- ((3 + Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])
```

Rubi [A] time = 0.0724102, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1359, 1123, 1166, 203}

$$-\frac{1}{2x^2}+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2
- ((3 + Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(1+3x^4+x^8\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(1+3x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{-3-x^2}{1+3x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{20}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)-\frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}
\end{aligned}
$$

$$
= -\frac{1}{2x^2}+\frac{1}{10}\sqrt{45-20\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}
$$

Mathematica [C] time = 0.0176194, size = 65, normalized size = 0.73

$$
-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+3\log(x-\#1)}{2\#1^6+3\#1^2}\&\right]-\frac{1}{2x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^
4)/(3*#1^2 + 2*#1^6) & ]/4
```

Maple [B] time = 0.021, size = 117, normalized size = 1.3

$$
-\frac{1}{-2+2\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right)-\frac{3\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right)-\frac{1}{2+2\sqrt{5}}\arctan\left(4\frac{x^2}{2+2\sqrt{5}}\right)+\frac{3\sqrt{5}}{10+10
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+3*x^4+1),x)
```

```
[Out] -1/(-2+2*5^(1/2))*arctan(4*x^2/(-2+2*5^(1/2)))-3/5*5^(1/2)/(-2+2*5^(1/2))*a
rctan(4*x^2/(-2+2*5^(1/2)))-1/(2+2*5^(1/2))*arctan(4*x^2/(2+2*5^(1/2)))+3/5
*5^(1/2)/(2+2*5^(1/2))*arctan(4*x^2/(2+2*5^(1/2)))-1/2/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
-\frac{1}{2\,x^2}-\int \frac{\left(x^4+3\right)x}{x^8+3\,x^4+1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^4 + 3)*x/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] time = 1.59149, size = 460, normalized size = 5.17

$$-\frac{2\sqrt{5}x^2\sqrt{-4\sqrt{5}+9}\arctan\left(\frac{1}{4}\sqrt{2x^4+\sqrt{5}+3}\left(\sqrt{5}\sqrt{2}+3\sqrt{2}\right)\sqrt{-4\sqrt{5}+9}-\frac{1}{2}\left(\sqrt{5}x^2+3x^2\right)\sqrt{-4\sqrt{5}+9}\right)+2\sqrt{5}x^2\sqrt{}}{10x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*(2*sqrt(5)*x^2*sqrt(-4*sqrt(5) + 9)*arctan(1/4*sqrt(2*x^4 + sqrt(5) +
 3)*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(5) + 9) - 1/2*(sqrt(5)*x^2 +
 3*x^2)*sqrt(-4*sqrt(5) + 9)) + 2*sqrt(5)*x^2*sqrt(4*sqrt(5) + 9)*arctan(-1
/4*(2*sqrt(5)*x^2 - 6*x^2 - sqrt(2*x^4 - sqrt(5) + 3)*(sqrt(5)*sqrt(2) - 3*
sqrt(2)))*sqrt(4*sqrt(5) + 9)) + 5)/x^2
```

Sympy [A] time = 0.230296, size = 56, normalized size = 0.63

$$-2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+3*x**4+1),x)
```

```
[Out] -2*(sqrt(5)/10 + 1/4)*atan(2*x**2/(-1 + sqrt(5))) + 2*(1/4 - sqrt(5)/10)*at
an(2*x**2/(1 + sqrt(5))) - 1/(2*x**2)
```

Giac [A] time = 1.25711, size = 92, normalized size = 1.03

$$-\frac{1}{20}\left(x^4\left(\sqrt{5}-5\right)+3\sqrt{5}-15\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)-\frac{1}{20}\left(x^4\left(\sqrt{5}+5\right)+3\sqrt{5}+15\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*(x^4*(sqrt(5) - 5) + 3*sqrt(5) - 15)*arctan(2*x^2/(sqrt(5) + 1)) - 1/
20*(x^4*(sqrt(5) + 5) + 3*sqrt(5) + 15)*arctan(2*x^2/(sqrt(5) - 1)) - 1/2/x
^2
```

3.376 $\int \frac{1}{x^5(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{4x^4}+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-3\log(x)$$

```
[Out] -1/(4*x^4) - 3*Log[x] + ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((
15 - 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] time = 0.06666, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 709, 800, 632, 31}

$$-\frac{1}{4x^4}+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - 3*Log[x] + ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((
15 - 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(1+3x^4+x^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1+3x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{-3-x}{x\left(1+3x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\operatorname{Subst}\left(\int \left(-\frac{3}{x}+\frac{8+3x}{1+3x+x^2}\right)dx,x,x^4\right)\\
&= -\frac{1}{4x^4}-3\log(x)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{8+3x}{1+3x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}-3\log(x)+\frac{1}{40}\left(15-7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\operatorname{Subst}\left(\int\right.\\
&= -\frac{1}{4x^4}-3\log(x)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0337213, size = 60, normalized size = 0.91

$$\frac{1}{40}\left(-\frac{10}{x^4}+\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}-3\right)+\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-120\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^4 - 120*Log[x] + (15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] + (15 -
7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.007, size = 42, normalized size = 0.6

$$\frac{3\ln\left(x^8+3x^4+1\right)}{8}-\frac{7\sqrt{5}}{20}\operatorname{Artanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)-\frac{1}{4x^4}-3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+3*x^4+1),x)
```

```
[Out] 3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)-1/4/x^4-3*l
n(x)
```

Maxima [A] time = 1.47315, size = 76, normalized size = 1.15

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)-\frac{1}{4x^4}+\frac{3}{8}\log\left(x^8+3x^4+1\right)-\frac{3}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/4/x^4 + 3
/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

Fricas [A] time = 1.66243, size = 193, normalized size = 2.92

$$\frac{7\sqrt{5}x^4\log\left(\frac{2x^8+6x^4-\sqrt{5}(2x^4+3)+7}{x^8+3x^4+1}\right)+15x^4\log\left(x^8+3x^4+1\right)-120x^4\log(x)-10}{40x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(7*sqrt(5)*x^4*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*
x^4 + 1)) + 15*x^4*log(x^8 + 3*x^4 + 1) - 120*x^4*log(x) - 10)/x^4
```

Sympy [A] time = 0.193161, size = 65, normalized size = 0.98

$$-3\log(x)+\left(\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(\frac{3}{8}-\frac{7\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+3*x**4+1),x)
```

```
[Out] -3*log(x) + (3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (3/8 - 7*sqr
t(5)/40)*log(x**4 + sqrt(5)/2 + 3/2) - 1/(4*x**4)
```

Giac [A] time = 1.23136, size = 85, normalized size = 1.29

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)+\frac{3x^4-1}{4x^4}+\frac{3}{8}\log\left(x^8+3x^4+1\right)-\frac{3}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/4*(3*x^4
- 1)/x^4 + 3/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

3.377 $\int \frac{1}{x^7(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=97

$$\frac{3}{2x^2}-\frac{1}{6x^6}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] -1/(6*x^6) + 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sq
rt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]
*x^2])/2
```

Rubi [A] time = 0.134403, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1359, 1123, 1281, 1166, 203}

$$\frac{3}{2x^2}-\frac{1}{6x^6}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) + 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sq
rt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]
*x^2])/2
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(1+3x^4+x^8\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(1+3x^2+x^4\right)}\,dx, x, x^2\right)\\ &= -\frac{1}{6x^6}+\frac{1}{6}\,\text{Subst}\left(\int \frac{-9-3x^2}{x^2\left(1+3x^2+x^4\right)}\,dx, x, x^2\right)\\ &= -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{6}\,\text{Subst}\left(\int \frac{-24-9x^2}{1+3x^2+x^4}\,dx, x, x^2\right)\\ &= -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{20}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)+\frac{1}{20}\left(15+7\sqrt{5}\right)\text{Subst}\Bigg(\\ &= -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{20}\sqrt{1230+550\sqrt{5}}\tan^{-1}\Bigg(\sqrt{\end{aligned}$$

Mathematica [C] time = 0.0194256, size = 73, normalized size = 0.75

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{3\#1^4\log(x-\#1)+8\log(x-\#1)}{2\#1^6+3\#1^2}\&\right]+\frac{3}{2x^2}-\frac{1}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) + 3/(2*x^2) + RootSum[1 + 3*#1^4 + #1^8 & , (8*Log[x - #1] + 3*L
og[x - #1]*#1^4)/(3*#1^2 + 2*#1^6) & ]/4
```

Maple [B] time = 0.016, size = 122, normalized size = 1.3

$$\frac{7\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right)+3\frac{1}{-2+2\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right)-\frac{7\sqrt{5}}{10+10\sqrt{5}}\arctan\left(4\frac{x^2}{2+2\sqrt{5}}\right)+3\frac{}{2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+3*x^4+1),x)
```

```
[Out] 7/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4*x^2/(-2+2*5^(1/2)))+3/(-2+2*5^(1/2))*ar
ctan(4*x^2/(-2+2*5^(1/2)))-7/5*5^(1/2)/(2+2*5^(1/2))*arctan(4*x^2/(2+2*5^(1
/2)))+3/(2+2*5^(1/2))*arctan(4*x^2/(2+2*5^(1/2)))-1/6/x^6+3/2/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{9x^4-1}{6x^6}+\int\frac{(3x^4+8)x}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*(9*x^4 - 1)/x^6 + integrate((3*x^4 + 8)*x/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] time = 1.77081, size = 564, normalized size = 5.81

$$\frac{3\sqrt{10}x^6\sqrt{-55\sqrt{5}+123}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2x^4+\sqrt{5}+3}\left(7\sqrt{5}\sqrt{2}+15\sqrt{2}\right)\sqrt{-55\sqrt{5}+123}-\frac{1}{20}\sqrt{10}\left(7\sqrt{5}x^2+15x^2\right)\sqrt{-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/30*(3*sqrt(10)*x^6*sqrt(-55*sqrt(5) + 123)*arctan(1/40*sqrt(10)*sqrt(2*x^
4 + sqrt(5) + 3)*(7*sqrt(5)*sqrt(2) + 15*sqrt(2))*sqrt(-55*sqrt(5) + 123) -
 1/20*sqrt(10)*(7*sqrt(5)*x^2 + 15*x^2)*sqrt(-55*sqrt(5) + 123)) - 3*sqrt(1
0)*x^6*sqrt(55*sqrt(5) + 123)*arctan(1/40*(sqrt(10)*sqrt(2*x^4 - sqrt(5) +
3)*(7*sqrt(5)*sqrt(2) - 15*sqrt(2)) - 2*sqrt(10)*(7*sqrt(5)*x^2 - 15*x^2))*
sqrt(55*sqrt(5) + 123)) + 45*x^4 - 5)/x^6
```

Sympy [A] time = 0.274987, size = 65, normalized size = 0.67

$$2\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)+\frac{9x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+3*x**4+1),x)
```

```
[Out] 2*(11*sqrt(5)/40 + 5/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(5/8 - 11*sqrt(5)/4
0)*atan(2*x**2/(1 + sqrt(5))) + (9*x**4 - 1)/(6*x**6)
```

Giac [A] time = 1.24186, size = 104, normalized size = 1.07

$$\frac{1}{20}\left(3x^4\left(\sqrt{5}-5\right)+8\sqrt{5}-40\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\left(3x^4\left(\sqrt{5}+5\right)+8\sqrt{5}+40\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)+\frac{9x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*(3*x^4*(sqrt(5) - 5) + 8*sqrt(5) - 40)*arctan(2*x^2/(sqrt(5) + 1)) + 1
/20*(3*x^4*(sqrt(5) + 5) + 8*sqrt(5) + 40)*arctan(2*x^2/(sqrt(5) - 1)) + 1/
6*(9*x^4 - 1)/x^6
```

3.378 $\int \frac{x^8}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=460

$$-\frac{\sqrt[4]{123-55\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right.}\right.}{4\,2^{3/4}\sqrt{5}}$$

```
[Out] x - ((123 - 55*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/
(2*2^(3/4)*Sqrt[5]) + ((123 - 55*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 -
 Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((123 + 55*Sqrt[5])^(1/4)*ArcTan[1
- (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5
])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) -
 ((123 - 55*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^
(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) + ((123 - 55*Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]
) + ((123 + 55*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]
))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*Log[Sq
rt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt
[5])
```

Rubi [A] time = 0.416256, antiderivative size = 440, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1367, 1422, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{984-440\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\sqrt{10}}+\frac{\sqrt[4]{984-440\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\right.}\right.}{8\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + 3*x^4 + x^8),x]
```

```
[Out] x - ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(4*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt
[5])^(1/4)])/(4*Sqrt[10]) + ((123 + 55*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x
)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*Arc
Tan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((984 - 440
*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2
*x^2])/(8*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]
+ 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(8*Sqrt[10]) + ((123 + 55*Sqrt[5])^
(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*
(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = x-\int \frac{1+3x^4}{1+3x^4+x^8}\,dx$$
$$= x-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$
$$= x+\frac{1}{2}\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{2}\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{(1}{}$$
$$= x+\frac{1}{4}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx+\frac{1}{4}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\int \frac{}{\sqrt{\frac{1}{2}\left(3-\right.}}$$
$$= x-\frac{1}{8}\sqrt[4]{\frac{246}{25}-\frac{22}{\sqrt{5}}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)+\frac{1}{8}\sqrt[4]{\frac{246}{25}-\frac{22}{\sqrt{5}}}\log\left(\sqrt{2\left(3-\right.}\right.$$
$$= x-\frac{\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123+55\sqrt{5}}\tan}{2\,2^{3/4}}$$

Mathematica [C] time = 0.0140273, size = 58, normalized size = 0.13

$$x-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{3\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + 3*x^4 + x^8),x]
```

```
[Out] x - RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + 3*Log[x - #1]*#1^4)/(3*#1^
3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.006, size = 46, normalized size = 0.1

$$x+\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(-3\_R^4-1\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+3*x^4+1),x)
```

```
[Out] x+1/4*sum((-3*_R^4-1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x-\int \frac{3\,x^4+1}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] x - integrate((3*x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] time = 1.91333, size = 3565, normalized size = 7.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/80*sqrt(10)*(110*sqrt(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123)*(55*sqrt(5)
- 123)*arctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5
*sqrt(2)*x)*(110*sqrt(5) + 246)^(1/4) - 5*sqrt(110*sqrt(5) + 246)*(3*sqrt(5
) - 7))*(1292*sqrt(5) - 2889)*(110*sqrt(5) + 246)^(5/4)*sqrt(55*sqrt(5) + 1
23) + 1/40*sqrt(10)*(2889*sqrt(5)*x - 6460*x)*(110*sqrt(5) + 246)^(5/4)*sqr
t(55*sqrt(5) + 123) - 1/8*(55*sqrt(5)*sqrt(2) - 123*sqrt(2))*sqrt(110*sqrt(
5) + 246)*sqrt(55*sqrt(5) + 123)) + 1/80*sqrt(10)*(110*sqrt(5) + 246)^(3/4)
*sqrt(55*sqrt(5) + 123)*(55*sqrt(5) - 123)*arctan(1/80*sqrt(10)*sqrt(20*x^2
 - sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(110*sqrt(5) + 246)^(1/4) -
 5*sqrt(110*sqrt(5) + 246)*(3*sqrt(5) - 7))*(1292*sqrt(5) - 2889)*(110*sqrt
(5) + 246)^(5/4)*sqrt(55*sqrt(5) + 123) + 1/40*sqrt(10)*(2889*sqrt(5)*x - 6
460*x)*(110*sqrt(5) + 246)^(5/4)*sqrt(55*sqrt(5) + 123) + 1/8*(55*sqrt(5)*s
qrt(2) - 123*sqrt(2))*sqrt(110*sqrt(5) + 246)*sqrt(55*sqrt(5) + 123)) - 1/8
0*sqrt(10)*(55*sqrt(5) + 123)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^
(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*
sqrt(2)*x)*(-110*sqrt(5) + 246)^(1/4) + 5*(3*sqrt(5) + 7)*sqrt(-110*sqrt(5)
 + 246))*(1292*sqrt(5) + 2889)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)
^(5/4) - 1/40*(sqrt(10)*(2889*sqrt(5)*x + 6460*x)*(-110*sqrt(5) + 246)^(5/4
) + 5*(55*sqrt(5)*sqrt(2) + 123*sqrt(2))*sqrt(-110*sqrt(5) + 246))*sqrt(-55
*sqrt(5) + 123)) - 1/80*sqrt(10)*(55*sqrt(5) + 123)*sqrt(-55*sqrt(5) + 123)
*(-110*sqrt(5) + 246)^(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2 - sqrt(10)*(3*
sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-110*sqrt(5) + 246)^(1/4) + 5*(3*sqrt(5)
+ 7)*sqrt(-110*sqrt(5) + 246))*(1292*sqrt(5) + 2889)*sqrt(-55*sqrt(5) + 123
)*(-110*sqrt(5) + 246)^(5/4) - 1/40*(sqrt(10)*(2889*sqrt(5)*x + 6460*x)*(-1
10*sqrt(5) + 246)^(5/4) - 5*(55*sqrt(5)*sqrt(2) + 123*sqrt(2))*sqrt(-110*sq
rt(5) + 246))*sqrt(-55*sqrt(5) + 123)) - 1/80*sqrt(10)*sqrt(2)*(110*sqrt(5)
 + 246)^(1/4)*log(20*x^2 + sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(11
0*sqrt(5) + 246)^(1/4) - 5*sqrt(110*sqrt(5) + 246)*(3*sqrt(5) - 7)) + 1/80*
sqrt(10)*sqrt(2)*(110*sqrt(5) + 246)^(1/4)*log(20*x^2 - sqrt(10)*(3*sqrt(5)
*sqrt(2)*x - 5*sqrt(2)*x)*(110*sqrt(5) + 246)^(1/4) - 5*sqrt(110*sqrt(5) +
246)*(3*sqrt(5) - 7)) + 1/80*sqrt(10)*sqrt(2)*(-110*sqrt(5) + 246)^(1/4)*lo
g(20*x^2 + sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-110*sqrt(5) + 246
)^(1/4) + 5*(3*sqrt(5) + 7)*sqrt(-110*sqrt(5) + 246)) - 1/80*sqrt(10)*sqrt(
2)*(-110*sqrt(5) + 246)^(1/4)*log(20*x^2 - sqrt(10)*(3*sqrt(5)*sqrt(2)*x +
5*sqrt(2)*x)*(-110*sqrt(5) + 246)^(1/4) + 5*(3*sqrt(5) + 7)*sqrt(-110*sqrt(
5) + 246)) + x
```

Sympy [A] time = 1.17545, size = 29, normalized size = 0.06

$$x + \operatorname{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t\log\left(\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+3*x**4+1),x)
```

```
[Out] x + RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(15360*_t**
5/11 + 1288*_t/55 + x)))
```

Giac [A] time = 1.35593, size = 343, normalized size = 0.75

$$\frac{1}{40}(i+1)\sqrt{25\sqrt{5}-55}\log\left(1730(i+1)x+1730i\sqrt{\sqrt{5}-1}\right)-\frac{1}{40}(i+1)\sqrt{25\sqrt{5}-55}\log\left(1730(i+1)x-1730i\sqrt{\sqrt{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/40*(i + 1)*sqrt(25*sqrt(5) - 55)*log(1730*(i + 1)*x + 1730*i*sqrt(sqrt(5)
 - 1)) - 1/40*(i + 1)*sqrt(25*sqrt(5) - 55)*log(1730*(i + 1)*x - 1730*i*sqr
t(sqrt(5) - 1)) - 1/40*(i - 1)*sqrt(25*sqrt(5) - 55)*log(1730*(i + 1)*x + 1
730*sqrt(sqrt(5) - 1)) + 1/40*(i - 1)*sqrt(25*sqrt(5) - 55)*log(1730*(i + 1
)*x - 1730*sqrt(sqrt(5) - 1)) - 1/40*(i + 1)*sqrt(25*sqrt(5) + 55)*log(850*
(i + 1)*x + 850*i*sqrt(sqrt(5) + 1)) + 1/40*(i + 1)*sqrt(25*sqrt(5) + 55)*l
og(850*(i + 1)*x - 850*i*sqrt(sqrt(5) + 1)) + 1/40*(i - 1)*sqrt(25*sqrt(5)
+ 55)*log(850*(i + 1)*x + 850*sqrt(sqrt(5) + 1)) - 1/40*(i - 1)*sqrt(25*sqr
t(5) + 55)*log(850*(i + 1)*x - 850*sqrt(sqrt(5) + 1)) + x
```

3.379 $\int \frac{x^6}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=431

$$-\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{(3+}{}$$

```
[Out] ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*Sqrt
[10]) - ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(2*Sqrt[10]) - ((3 + Sqrt[5])^(3/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(
1/4)])/(4*2^(1/4)*Sqrt[5]) + ((3 + Sqrt[5])^(3/4)*ArcTan[1 + (2^(3/4)*x)/(3
 + Sqrt[5])^(1/4)])/(4*2^(1/4)*Sqrt[5]) - ((9 - 4*Sqrt[5])^(1/4)*Log[Sqrt[2
*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9
- 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x
+ 2*x^2])/(4*Sqrt[10]) + ((3 + Sqrt[5])^(3/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2
*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(1/4)*Sqrt[5]) - ((3 + Sqrt[5])^(
3/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2
^(1/4)*Sqrt[5])
```

Rubi [A] time = 0.294608, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1374, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{(3+}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*Sqrt
[10]) - ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(2*Sqrt[10]) - ((3 + Sqrt[5])^(3/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(
1/4)])/(4*2^(1/4)*Sqrt[5]) + ((3 + Sqrt[5])^(3/4)*ArcTan[1 + (2^(3/4)*x)/(3
 + Sqrt[5])^(1/4)])/(4*2^(1/4)*Sqrt[5]) - ((9 - 4*Sqrt[5])^(1/4)*Log[Sqrt[2
*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9
- 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x
+ 2*x^2])/(4*Sqrt[10]) + ((3 + Sqrt[5])^(3/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2
*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(1/4)*Sqrt[5]) - ((3 + Sqrt[5])^(
3/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2
^(1/4)*Sqrt[5])
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = -\left(\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} - \frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} + \frac{\left(3+\sqrt{5}\right)\int \cdots}{\cdots}$$

$$= -\frac{\sqrt[4]{9-4\sqrt{5}}\int \frac{\sqrt[4]{2}\left(3-\sqrt{5}\right)+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt{10}} - \frac{\sqrt[4]{9-4\sqrt{5}}\int \frac{\sqrt[4]{2}\left(3-\sqrt{5}\right)-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt{10}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\cdots}{\cdots}$$

$$= -\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\cdots\right.}\right.}{4\sqrt{10}}$$

$$= \frac{\left(3-\sqrt{5}\right)^{3/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{36-16\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\cdots\right)}{4\sqrt[4]{2}\sqrt{5}}$$

Mathematica [C] time = 0.0121073, size = 41, normalized size = 0.1

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4+3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1^3)/(3 + 2*#1^4) & ]/4
```

Maple [C] time = 0.007, size = 40, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^6\ln(x-\_R)}{2\_R^7+3\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(_R^6/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] time = 1.80635, size = 2326, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*sqrt(2)*(4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 + (3*sqrt(
5)*sqrt(2)*x - 7*sqrt(2)*x)*(4*sqrt(5) + 9)^(3/4) - sqrt(4*sqrt(5) + 9)*(sq
rt(5) - 3))*(21*sqrt(5) - 47)*(4*sqrt(5) + 9)^(5/4) - 1/2*(21*sqrt(5)*sqrt(
2)*x - 47*sqrt(2)*x)*(4*sqrt(5) + 9)^(5/4) - 1) + 1/10*sqrt(5)*sqrt(2)*(4*s
qrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 - (3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*
x)*(4*sqrt(5) + 9)^(3/4) - sqrt(4*sqrt(5) + 9)*(sqrt(5) - 3))*(21*sqrt(5) -
 47)*(4*sqrt(5) + 9)^(5/4) - 1/2*(21*sqrt(5)*sqrt(2)*x - 47*sqrt(2)*x)*(4*s
qrt(5) + 9)^(5/4) + 1) + 1/10*sqrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan
(1/2*sqrt(2*x^2 + (3*sqrt(5)*sqrt(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4
) + (sqrt(5) + 3)*sqrt(-4*sqrt(5) + 9))*(21*sqrt(5) + 47)*(-4*sqrt(5) + 9)^
(5/4) - 1/2*(21*sqrt(5)*sqrt(2)*x + 47*sqrt(2)*x)*(-4*sqrt(5) + 9)^(5/4) -
1) + 1/10*sqrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 - (3
*sqrt(5)*sqrt(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + (sqrt(5) + 3)*sq
rt(-4*sqrt(5) + 9))*(21*sqrt(5) + 47)*(-4*sqrt(5) + 9)^(5/4) - 1/2*(21*sqrt
(5)*sqrt(2)*x + 47*sqrt(2)*x)*(-4*sqrt(5) + 9)^(5/4) + 1) + 1/40*sqrt(5)*sq
rt(2)*(4*sqrt(5) + 9)^(1/4)*log(2*x^2 + (3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*x)
*(4*sqrt(5) + 9)^(3/4) - sqrt(4*sqrt(5) + 9)*(sqrt(5) - 3)) - 1/40*sqrt(5)*
sqrt(2)*(4*sqrt(5) + 9)^(1/4)*log(2*x^2 - (3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*
x)*(4*sqrt(5) + 9)^(3/4) - sqrt(4*sqrt(5) + 9)*(sqrt(5) - 3)) + 1/40*sqrt(5
)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*log(2*x^2 + (3*sqrt(5)*sqrt(2)*x + 7*sqrt(
2)*x)*(-4*sqrt(5) + 9)^(3/4) + (sqrt(5) + 3)*sqrt(-4*sqrt(5) + 9)) - 1/40*s
qrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*log(2*x^2 - (3*sqrt(5)*sqrt(2)*x + 7*
sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + (sqrt(5) + 3)*sqrt(-4*sqrt(5) + 9))
```

Sympy [A] time = 1.14604, size = 26, normalized size = 0.06

$$\operatorname{RootSum}\left(40960000t^{8}+115200t^{4}+1,\left(t\mapsto t\log\left(-1792000t^{7}-4920t^{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-1792000*_t**7
 - 4920*_t**3 + x)))
```

Giac [A] time = 1.36248, size = 342, normalized size = 0.79

$$-\frac{1}{40}(i-1)\sqrt{10\sqrt{5}-20}\log\left(100(i+1)x+100i\sqrt{\sqrt{5}-1}\right)+\frac{1}{40}(i-1)\sqrt{10\sqrt{5}-20}\log\left(100(i+1)x-100i\sqrt{\sqrt{5}-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/40*(i - 1)*sqrt(10*sqrt(5) - 20)*log(100*(i + 1)*x + 100*i*sqrt(sqrt(5)
- 1)) + 1/40*(i - 1)*sqrt(10*sqrt(5) - 20)*log(100*(i + 1)*x - 100*i*sqrt(s
qrt(5) - 1)) + 1/40*(i + 1)*sqrt(10*sqrt(5) - 20)*log(100*(i + 1)*x + 100*s
qrt(sqrt(5) - 1)) - 1/40*(i + 1)*sqrt(10*sqrt(5) - 20)*log(100*(i + 1)*x -
100*sqrt(sqrt(5) - 1)) + 1/40*(i - 1)*sqrt(10*sqrt(5) + 20)*log(20*(i + 1)*
x + 20*i*sqrt(sqrt(5) + 1)) - 1/40*(i - 1)*sqrt(10*sqrt(5) + 20)*log(20*(i
+ 1)*x - 20*i*sqrt(sqrt(5) + 1)) - 1/40*(i + 1)*sqrt(10*sqrt(5) + 20)*log(2
0*(i + 1)*x + 20*sqrt(sqrt(5) + 1)) + 1/40*(i + 1)*sqrt(10*sqrt(5) + 20)*lo
g(20*(i + 1)*x - 20*sqrt(sqrt(5) + 1))
```

3.380 $\int \frac{x^4}{1+3x^4+x^8}\, dx$

Optimal. Leaf size=451

$$\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+}}{}$$

```
[Out] ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4
)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4
)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 +
Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(
3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*Lo
g[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*
Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[
5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]
) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1
/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rubi [A] time = 0.280081, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1374, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4
)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4
)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 +
Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(
3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*Lo
g[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*
Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[
5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]
) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1
/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = -\left(\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\left(\frac{1}{4}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)-\frac{1}{4}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{4}\sqrt{\frac{1}{5}\left(3+\cdots\right.}$$

$$= -\left(\frac{1}{4}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx\right)-\frac{1}{4}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\int \frac{\cdots}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\cdots}$$

$$= \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\cdots\right.}{4\ 2^{3/4}\sqrt{5}}$$

$$= \frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3\cdots}}{\cdots}$$

Mathematica [C] time = 0.0111006, size = 39, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4+3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1)/(3 + 2*#1^4) & ]/4
```

Maple [C] time = 0.006, size = 40, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^4\ln(x-\_R)}{2\_R^7+3\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] time = 1.86906, size = 2758, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(
-1/80*sqrt(10)*(7*sqrt(5)*x - 15*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3)
 + 1/80*sqrt(sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) + 6)^(1/4) + 10*x^2 + 5*
sqrt(2*sqrt(5) + 6))*(7*sqrt(5) - 15)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) +
3) + 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3
)) + 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*ar
ctan(-1/80*sqrt(10)*(7*sqrt(5)*x - 15*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5)
 + 3) + 1/80*sqrt(-sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) + 6)^(1/4) + 10*x^
2 + 5*sqrt(2*sqrt(5) + 6))*(7*sqrt(5) - 15)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt
(5) + 3) - 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(
```

```
5) + 3)) + 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^
(3/4)*arctan(1/80*sqrt(sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) +
10*x^2 + 5*sqrt(-2*sqrt(5) + 6))*(7*sqrt(5) + 15)*sqrt(-sqrt(5) + 3)*(-2*sq
rt(5) + 6)^(5/4) - 1/80*(sqrt(10)*(7*sqrt(5)*x + 15*x)*(-2*sqrt(5) + 6)^(5/
4) + 10*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) +
 3)) + 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4
)*arctan(1/80*sqrt(-sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 10*
x^2 + 5*sqrt(-2*sqrt(5) + 6))*(7*sqrt(5) + 15)*sqrt(-sqrt(5) + 3)*(-2*sqrt(
5) + 6)^(5/4) - 1/80*(sqrt(10)*(7*sqrt(5)*x + 15*x)*(-2*sqrt(5) + 6)^(5/4)
- 10*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)
) + 1/80*sqrt(10)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(sqrt(10)*sqrt(5)*sqrt(2
)*x*(2*sqrt(5) + 6)^(1/4) + 10*x^2 + 5*sqrt(2*sqrt(5) + 6)) - 1/80*sqrt(10)
*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(-sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) +
 6)^(1/4) + 10*x^2 + 5*sqrt(2*sqrt(5) + 6)) - 1/80*sqrt(10)*sqrt(2)*(-2*sqr
t(5) + 6)^(1/4)*log(sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 10*
x^2 + 5*sqrt(-2*sqrt(5) + 6)) + 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4
)*log(-sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 10*x^2 + 5*sqrt(
-2*sqrt(5) + 6))
```

Sympy [A] time = 1.14065, size = 24, normalized size = 0.05

$$\operatorname{RootSum}\left(40960000t^8 + 19200t^4 + 1, \left(t \mapsto t\log\left(-51200t^5 - 12t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-51200*_t**5 -
12*_t + x)))
```

Giac [A] time = 1.33872, size = 342, normalized size = 0.76

$$-\frac{1}{40}(i+1)\sqrt{5\sqrt{5}-5}\log\left(65(i+1)x + 65i\sqrt{\sqrt{5}-1}\right) + \frac{1}{40}(i+1)\sqrt{5\sqrt{5}-5}\log\left(65(i+1)x - 65i\sqrt{\sqrt{5}-1}\right) + \frac{1}{40}(i-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/40*(i + 1)*sqrt(5*sqrt(5) - 5)*log(65*(i + 1)*x + 65*i*sqrt(sqrt(5) - 1)
) + 1/40*(i + 1)*sqrt(5*sqrt(5) - 5)*log(65*(i + 1)*x - 65*i*sqrt(sqrt(5) -
 1)) + 1/40*(i - 1)*sqrt(5*sqrt(5) - 5)*log(65*(i + 1)*x + 65*sqrt(sqrt(5)
- 1)) - 1/40*(i - 1)*sqrt(5*sqrt(5) - 5)*log(65*(i + 1)*x - 65*sqrt(sqrt(5)
 - 1)) + 1/40*(i + 1)*sqrt(5*sqrt(5) + 5)*log(25*(i + 1)*x + 25*i*sqrt(sqrt
(5) + 1)) - 1/40*(i + 1)*sqrt(5*sqrt(5) + 5)*log(25*(i + 1)*x - 25*i*sqrt(s
qrt(5) + 1)) - 1/40*(i - 1)*sqrt(5*sqrt(5) + 5)*log(25*(i + 1)*x + 25*sqrt(
sqrt(5) + 1)) + 1/40*(i - 1)*sqrt(5*sqrt(5) + 5)*log(25*(i + 1)*x - 25*sqrt
(sqrt(5) + 1))
```

3.381 $\int \frac{x^2}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=427

$$\frac{\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}\right.}{4\sqrt{5}\sqrt[4]{2\left(3+\right.}}$$

```
[Out] -ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2*Sqrt[5]*(2*(3 - Sqrt[5]))^(
1/4)) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2*Sqrt[5]*(2*(3 - Sqrt
[5]))^(1/4)) + ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2*Sqrt[5]*(2*(3
 + Sqrt[5]))^(1/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2*Sqrt[5
]*(2*(3 + Sqrt[5]))^(1/4)) + Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5])
)^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 - Sqrt[5]))^(1/4)) - Log[Sqrt[2*(3 - Sq
rt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 - Sqrt[5]))
^(1/4)) - Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/
(4*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 +
 Sqrt[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4))
```

Rubi [A] time = 0.25911, antiderivative size = 431, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1375, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 3*x^4 + x^8),x]
```

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/
4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/
4)])/(2*2^(3/4)*Sqrt[5]) + ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2*S
qrt[5]*(2*(3 + Sqrt[5]))^(1/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4
)]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3
 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) -
((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*
x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqr
t[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) + Log[Sqrt[2*(3
 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sqrt
[5]))^(1/4))
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \frac{\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= -\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}$$

$$= \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3-\sqrt{5})}-\sqrt[4]{2(3-\sqrt{5})}x+x^2}\,dx}{4\sqrt{5}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3-\sqrt{5})}+\sqrt[4]{2(3-\sqrt{5})}x+x^2}\,dx}{4\sqrt{5}} - \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3+\sqrt{5})}-\sqrt[4]{2(3+\sqrt{5})}x+x^2}\,dx}{4\sqrt{5}} - \cdots$$

$$= \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\cdots\right.}\right.}{4\,2^{3/4}\sqrt{5}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} + \frac{\sqrt[4]{3+\cdots}}{\cdots}$$

Mathematica [C] time = 0.0104187, size = 40, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^5+3\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1 + 2*#1^5) & ]/4
```

Maple [C] time = 0.005, size = 40, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^2\ln(x-\_R)}{2\_R^7+3\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] time = 1.85963, size = 3101, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(
-1/40*sqrt(10)*(3*sqrt(5)*x - 5*x)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)
+ 1/80*sqrt(sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(3
/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3))*(3*sqrt(5) - 5)*(2*sqr
t(5) + 6)^(3/4)*sqrt(sqrt(5) + 3) + 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(
2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqr
t(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(-1/40*sqrt(10)*(3*sqrt(5)*x - 5*x)*(2*s
qrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3) + 1/80*sqrt(-sqrt(10)*(3*sqrt(5)*sqrt(2
)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(3/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*
(sqrt(5) - 3))*(3*sqrt(5) - 5)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3) - 1/
8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/
80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4)*arctan(
1/80*sqrt(sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(3/
4) + 40*x^2 + 10*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))*(3*sqrt(5) + 5)*sqrt(-
sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4) - 1/40*(sqrt(10)*(3*sqrt(5)*x + 5*x)*(-
2*sqrt(5) + 6)^(3/4) + 5*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6)
)*sqrt(-sqrt(5) + 3)) + 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*
sqrt(5) + 6)^(3/4)*arctan(1/80*sqrt(-sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt
(2)*x)*(-2*sqrt(5) + 6)^(3/4) + 40*x^2 + 10*(sqrt(5) + 3)*sqrt(-2*sqrt(5) +
 6))*(3*sqrt(5) + 5)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4) - 1/40*(sqrt
(10)*(3*sqrt(5)*x + 5*x)*(-2*sqrt(5) + 6)^(3/4) - 5*(sqrt(5)*sqrt(2) + 3*sq
rt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)) - 1/80*sqrt(10)*sqrt(2)*(2
*sqrt(5) + 6)^(1/4)*log(sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqr
t(5) + 6)^(3/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3)) + 1/80*sqr
t(10)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(-sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*
sqrt(2)*x)*(2*sqrt(5) + 6)^(3/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*(sqrt(5)
 - 3)) + 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*log(sqrt(10)*(3*sqrt(
5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(3/4) + 40*x^2 + 10*(sqrt(5) +
 3)*sqrt(-2*sqrt(5) + 6)) - 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*lo
g(-sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(3/4) + 40
*x^2 + 10*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))
```

Sympy [A] time = 1.14028, size = 26, normalized size = 0.06

$$\operatorname{RootSum}\left(40960000t^8 + 19200t^4 + 1, \left(t \mapsto t\log\left(-6144000t^7 - 2240t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-6144000*_t**7
- 2240*_t**3 + x)))
```

Giac [A] time = 1.36071, size = 342, normalized size = 0.8

$$-\frac{1}{40}(i-1)\sqrt{5\sqrt{5}-5}\log\left(130(i+1)x+130i\sqrt{\sqrt{5}+1}\right)+\frac{1}{40}(i-1)\sqrt{5\sqrt{5}-5}\log\left(130(i+1)x-130i\sqrt{\sqrt{5}+1}\right)+\frac{}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/40*(i - 1)*sqrt(5*sqrt(5) - 5)*log(130*(i + 1)*x + 130*i*sqrt(sqrt(5) +
1)) + 1/40*(i - 1)*sqrt(5*sqrt(5) - 5)*log(130*(i + 1)*x - 130*i*sqrt(sqrt(
5) + 1)) + 1/40*(i + 1)*sqrt(5*sqrt(5) - 5)*log(130*(i + 1)*x + 130*sqrt(sq
rt(5) + 1)) - 1/40*(i + 1)*sqrt(5*sqrt(5) - 5)*log(130*(i + 1)*x - 130*sqrt
(sqrt(5) + 1)) + 1/40*(i - 1)*sqrt(5*sqrt(5) + 5)*log(50*(i + 1)*x + 50*i*s
qrt(sqrt(5) - 1)) - 1/40*(i - 1)*sqrt(5*sqrt(5) + 5)*log(50*(i + 1)*x - 50*
i*sqrt(sqrt(5) - 1)) - 1/40*(i + 1)*sqrt(5*sqrt(5) + 5)*log(50*(i + 1)*x +
50*sqrt(sqrt(5) - 1)) + 1/40*(i + 1)*sqrt(5*sqrt(5) + 5)*log(50*(i + 1)*x -
 50*sqrt(sqrt(5) - 1))
```

3.382 $\int \frac{1}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=414

$$-\frac{\sqrt[4]{9+4\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9+4\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\log\left(\right.}{}$$

```
[Out] -((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*Sqr
t[10]) + ((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
)/(2*Sqrt[10]) + ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*(2*(3
 + Sqrt[5]))^(3/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*
(2*(3 + Sqrt[5]))^(3/4)) - ((9 + 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]
 - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9 + 4*Sqrt[5])^(1
/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sq
rt[10]) + Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/
(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4)) - Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 +
 Sqrt[5]))^(1/4)*x + 2*x^2]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4))
```

Rubi [A] time = 0.256883, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {1347, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{9+4\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9+4\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x^4 + x^8)^(-1),x]
```


```
[Out] -((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*Sqr
t[10]) + ((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
)/(2*Sqrt[10]) + ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*(2*(3
 + Sqrt[5]))^(3/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*
(2*(3 + Sqrt[5]))^(3/4)) - ((9 + 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]
 - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9 + 4*Sqrt[5])^(1
/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sq
rt[10]) + Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/
(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4)) - Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 +
 Sqrt[5]))^(1/4)*x + 2*x^2]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4))
```


Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1+3x^4+x^8}\,dx = \frac{\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= \frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3-\sqrt{5}\right)}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3+\sqrt{5}\right)}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3+\sqrt{5}\right)}}$$

$$= \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{2\sqrt{10\left(3-\sqrt{5}\right)}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{2\sqrt{10\left(3-\sqrt{5}\right)}} + \frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} + \cdots$$

$$= -\frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\cdots\right.}\right.}{4\sqrt{10}}\cdots$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3-\sqrt{5}\right)\right)^{3/4}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3-\sqrt{5}\right)\right)^{3/4}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\sqrt[4]{9+4\cdots}}{\cdots}$$

Mathematica [C] time = 0.0114351, size = 42, normalized size = 0.1

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x^4 + x^8)^(-1),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.006, size = 37, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\ln(x-\_R)}{2\_R^7+3\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(1/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] time = 1.77359, size = 2304, normalized size = 5.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*sqrt(2)*(4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 - sqrt(4*s
qrt(5) + 9)*(3*sqrt(5) - 7) + (sqrt(5)*sqrt(2)*x - 3*sqrt(2)*x)*(4*sqrt(5)
+ 9)^(1/4))*(4*sqrt(5) + 9)^(3/4)*(3*sqrt(5) - 7) - 1/2*(3*sqrt(5)*sqrt(2)*
x - 7*sqrt(2)*x)*(4*sqrt(5) + 9)^(3/4) - 1) + 1/10*sqrt(5)*sqrt(2)*(4*sqrt(
5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 - sqrt(4*sqrt(5) + 9)*(3*sqrt(5) - 7) -
 (sqrt(5)*sqrt(2)*x - 3*sqrt(2)*x)*(4*sqrt(5) + 9)^(1/4))*(4*sqrt(5) + 9)^(
3/4)*(3*sqrt(5) - 7) - 1/2*(3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*x)*(4*sqrt(5) +
 9)^(3/4) + 1) + 1/10*sqrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqr
t(2*x^2 + (3*sqrt(5) + 7)*sqrt(-4*sqrt(5) + 9) + (sqrt(5)*sqrt(2)*x + 3*sqr
t(2)*x)*(-4*sqrt(5) + 9)^(1/4))*(3*sqrt(5) + 7)*(-4*sqrt(5) + 9)^(3/4) - 1/
2*(3*sqrt(5)*sqrt(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) - 1) + 1/10*sq
rt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 + (3*sqrt(5) + 7
)*sqrt(-4*sqrt(5) + 9) - (sqrt(5)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)
^(1/4))*(3*sqrt(5) + 7)*(-4*sqrt(5) + 9)^(3/4) - 1/2*(3*sqrt(5)*sqrt(2)*x +
 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + 1) - 1/40*sqrt(5)*sqrt(2)*(4*sqrt(5)
 + 9)^(1/4)*log(2*x^2 - sqrt(4*sqrt(5) + 9)*(3*sqrt(5) - 7) + (sqrt(5)*sqrt
(2)*x - 3*sqrt(2)*x)*(4*sqrt(5) + 9)^(1/4)) + 1/40*sqrt(5)*sqrt(2)*(4*sqrt(
5) + 9)^(1/4)*log(2*x^2 - sqrt(4*sqrt(5) + 9)*(3*sqrt(5) - 7) - (sqrt(5)*sq
rt(2)*x - 3*sqrt(2)*x)*(4*sqrt(5) + 9)^(1/4)) - 1/40*sqrt(5)*sqrt(2)*(-4*sq
rt(5) + 9)^(1/4)*log(2*x^2 + (3*sqrt(5) + 7)*sqrt(-4*sqrt(5) + 9) + (sqrt(5
)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)^(1/4)) + 1/40*sqrt(5)*sqrt(2)*(
-4*sqrt(5) + 9)^(1/4)*log(2*x^2 + (3*sqrt(5) + 7)*sqrt(-4*sqrt(5) + 9) - (s
qrt(5)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)^(1/4))
```

Sympy [A] time = 1.10694, size = 26, normalized size = 0.06

$$\operatorname{RootSum}\left(40960000t^{8}+115200t^{4}+1,\left(t\mapsto t\log\left(-9600t^{5}-\frac{47t}{2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-9600*_t**5 -
47*_t/2 + x)))
```

Giac [A] time = 1.24306, size = 342, normalized size = 0.83

$$-\frac{1}{40}(i+1)\sqrt{10\sqrt{5}-20}\log\left(100(i+1)x+100i\sqrt{\sqrt{5}+1}\right)+\frac{1}{40}(i+1)\sqrt{10\sqrt{5}-20}\log\left(100(i+1)x-100i\sqrt{\sqrt{5}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/40*(i + 1)*sqrt(10*sqrt(5) - 20)*log(100*(i + 1)*x + 100*i*sqrt(sqrt(5)
+ 1)) + 1/40*(i + 1)*sqrt(10*sqrt(5) - 20)*log(100*(i + 1)*x - 100*i*sqrt(s
qrt(5) + 1)) + 1/40*(i - 1)*sqrt(10*sqrt(5) - 20)*log(100*(i + 1)*x + 100*s
qrt(sqrt(5) + 1)) - 1/40*(i - 1)*sqrt(10*sqrt(5) - 20)*log(100*(i + 1)*x -
100*sqrt(sqrt(5) + 1)) + 1/40*(i + 1)*sqrt(10*sqrt(5) + 20)*log(20*(i + 1)*
x + 20*i*sqrt(sqrt(5) - 1)) - 1/40*(i + 1)*sqrt(10*sqrt(5) + 20)*log(20*(i
+ 1)*x - 20*i*sqrt(sqrt(5) - 1)) - 1/40*(i - 1)*sqrt(10*sqrt(5) + 20)*log(2
0*(i + 1)*x + 20*sqrt(sqrt(5) - 1)) + 1/40*(i - 1)*sqrt(10*sqrt(5) + 20)*lo
g(20*(i + 1)*x - 20*sqrt(sqrt(5) - 1))
```

3.383 $\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=416

$$-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\ 2^{3/4}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\ 2^{3/4}\sqrt{5}}+$$

```
[Out] -x^(-1) + ((3 + Sqrt[5])^(5/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqr
t[5])^(1/4)])/(4*2^(3/4)*Sqrt[5]) - ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan[1 -
 (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/20 + ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan
[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/20 - ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2
*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5])
 + ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/
4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5]) + ((6150 - 2750*Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/40 - ((6150 - 2750
*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2
*x^2])/40
```

Rubi [A] time = 0.285476, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1368, 1510, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\ 2^{3/4}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\ 2^{3/4}\sqrt{5}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + 3*x^4 + x^8)),x]
```


```
[Out] -x^(-1) + ((3 + Sqrt[5])^(5/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqr
t[5])^(1/4)])/(4*2^(3/4)*Sqrt[5]) - ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan[1 -
 (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/20 + ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan
[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/20 - ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2
*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5])
 + ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/
4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5]) + ((6150 - 2750*Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/40 - ((6150 - 2750
*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2
*x^2])/40
```


Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+3x^4+x^8\right)}\,dx = -\frac{1}{x}+\int \frac{x^2\left(-3-x^4\right)}{1+3x^4+x^8}\,dx$$

$$= -\frac{1}{x}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\frac{1}{x}-\frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}+\frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}+\frac{\left(3+\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} \cdots$$

$$= -\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)^{5/4}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{8\;2^{3/4}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{8\;2^{3/4}\sqrt{5}} \cdots$$

$$= -\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{8\;2^{3/4}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(\sqrt{2\left(3-\cdots\right.}\right.}{8\;2\cdots}$$

$$= -\frac{1}{x}+\frac{\sqrt[4]{246+110\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}}-\frac{\sqrt[4]{246+110\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}}-\frac{1}{20}\sqrt[4]{615\cdots}$$

Mathematica [C] time = 0.0166801, size = 61, normalized size = 0.15

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+3\log(x-\#1)}{2\#1^5+3\#1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^4)/
(3*#1 + 2*#1^5) & ]/4
```

Maple [C] time = 0.009, size = 52, normalized size = 0.1

$$-\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(\_R^6+3\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+3*x^4+1),x)
```

```
[Out] -1/4*sum((_R^6+3*_R^2)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))-1
/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x}-\int \frac{x^6+3\,x^2}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^6 + 3*x^2)/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] time = 1.85182, size = 3545, normalized size = 8.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/80*(sqrt(10)*(55*sqrt(5)*x - 123*x)*(110*sqrt(5) + 246)^(3/4)*sqrt(55*sq
rt(5) + 123)*arctan(-1/20*sqrt(10)*(161*sqrt(5)*x - 360*x)*(110*sqrt(5) + 2
46)^(3/4)*sqrt(55*sqrt(5) + 123) + 1/40*sqrt(sqrt(10)*(47*sqrt(5)*sqrt(2)*x
 - 105*sqrt(2)*x)*(110*sqrt(5) + 246)^(3/4) + 40*x^2 - 20*sqrt(110*sqrt(5)
+ 246)*(4*sqrt(5) - 9))*(161*sqrt(5) - 360)*(110*sqrt(5) + 246)^(3/4)*sqrt(
55*sqrt(5) + 123) + 1/8*(55*sqrt(5)*sqrt(2) - 123*sqrt(2))*sqrt(110*sqrt(5)
 + 246)*sqrt(55*sqrt(5) + 123)) + sqrt(10)*(55*sqrt(5)*x - 123*x)*(110*sqrt
(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123)*arctan(-1/20*sqrt(10)*(161*sqrt(5)*
x - 360*x)*(110*sqrt(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123) + 1/40*sqrt(-sq
rt(10)*(47*sqrt(5)*sqrt(2)*x - 105*sqrt(2)*x)*(110*sqrt(5) + 246)^(3/4) + 4
0*x^2 - 20*sqrt(110*sqrt(5) + 246)*(4*sqrt(5) - 9))*(161*sqrt(5) - 360)*(11
0*sqrt(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123) - 1/8*(55*sqrt(5)*sqrt(2) - 1
23*sqrt(2))*sqrt(110*sqrt(5) + 246)*sqrt(55*sqrt(5) + 123)) + sqrt(10)*(55*
sqrt(5)*x + 123*x)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(3/4)*arcta
n(1/40*sqrt(sqrt(10)*(47*sqrt(5)*sqrt(2)*x + 105*sqrt(2)*x)*(-110*sqrt(5) +
 246)^(3/4) + 40*x^2 + 20*(4*sqrt(5) + 9)*sqrt(-110*sqrt(5) + 246))*(161*sq
rt(5) + 360)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(3/4) - 1/40*(2*s
qrt(10)*(161*sqrt(5)*x + 360*x)*(-110*sqrt(5) + 246)^(3/4) + 5*(55*sqrt(5)*
sqrt(2) + 123*sqrt(2))*sqrt(-110*sqrt(5) + 246))*sqrt(-55*sqrt(5) + 123)) +
 sqrt(10)*(55*sqrt(5)*x + 123*x)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 24
6)^(3/4)*arctan(1/40*sqrt(-sqrt(10)*(47*sqrt(5)*sqrt(2)*x + 105*sqrt(2)*x)*
(-110*sqrt(5) + 246)^(3/4) + 40*x^2 + 20*(4*sqrt(5) + 9)*sqrt(-110*sqrt(5)
+ 246))*(161*sqrt(5) + 360)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(3
/4) - 1/40*(2*sqrt(10)*(161*sqrt(5)*x + 360*x)*(-110*sqrt(5) + 246)^(3/4) -
 5*(55*sqrt(5)*sqrt(2) + 123*sqrt(2))*sqrt(-110*sqrt(5) + 246))*sqrt(-55*sq
rt(5) + 123)) - sqrt(10)*sqrt(2)*x*(110*sqrt(5) + 246)^(1/4)*log(sqrt(10)*(
47*sqrt(5)*sqrt(2)*x - 105*sqrt(2)*x)*(110*sqrt(5) + 246)^(3/4) + 40*x^2 -
20*sqrt(110*sqrt(5) + 246)*(4*sqrt(5) - 9)) + sqrt(10)*sqrt(2)*x*(110*sqrt(
5) + 246)^(1/4)*log(-sqrt(10)*(47*sqrt(5)*sqrt(2)*x - 105*sqrt(2)*x)*(110*s
qrt(5) + 246)^(3/4) + 40*x^2 - 20*sqrt(110*sqrt(5) + 246)*(4*sqrt(5) - 9))
+ sqrt(10)*sqrt(2)*x*(-110*sqrt(5) + 246)^(1/4)*log(sqrt(10)*(47*sqrt(5)*sq
rt(2)*x + 105*sqrt(2)*x)*(-110*sqrt(5) + 246)^(3/4) + 40*x^2 + 20*(4*sqrt(5
) + 9)*sqrt(-110*sqrt(5) + 246)) - sqrt(10)*sqrt(2)*x*(-110*sqrt(5) + 246)^
(1/4)*log(-sqrt(10)*(47*sqrt(5)*sqrt(2)*x + 105*sqrt(2)*x)*(-110*sqrt(5) +
246)^(3/4) + 40*x^2 + 20*(4*sqrt(5) + 9)*sqrt(-110*sqrt(5) + 246)) + 80)/x
```

Sympy [A] time = 1.24525, size = 32, normalized size = 0.08

$$\operatorname{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t \log\left(\frac{19251200t^7}{11} + \frac{369792t^3}{11} + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(19251200*_t**7
/11 + 369792*_t**3/11 + x))) - 1/x
```

Giac [A] time = 1.32929, size = 348, normalized size = 0.84

$$\frac{1}{40}(i-1)\sqrt{25\sqrt{5}-55}\log\left(865(i+1)x+865i\sqrt{\sqrt{5}+1}\right)-\frac{1}{40}(i-1)\sqrt{25\sqrt{5}-55}\log\left(865(i+1)x-865i\sqrt{\sqrt{5}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/40*(i - 1)*sqrt(25*sqrt(5) - 55)*log(865*(i + 1)*x + 865*i*sqrt(sqrt(5) +
 1)) - 1/40*(i - 1)*sqrt(25*sqrt(5) - 55)*log(865*(i + 1)*x - 865*i*sqrt(sq
rt(5) + 1)) - 1/40*(i + 1)*sqrt(25*sqrt(5) - 55)*log(865*(i + 1)*x + 865*sq
rt(sqrt(5) + 1)) + 1/40*(i + 1)*sqrt(25*sqrt(5) - 55)*log(865*(i + 1)*x - 8
65*sqrt(sqrt(5) + 1)) - 1/40*(i - 1)*sqrt(25*sqrt(5) + 55)*log(425*(i + 1)*
x + 425*i*sqrt(sqrt(5) - 1)) + 1/40*(i - 1)*sqrt(25*sqrt(5) + 55)*log(425*(
i + 1)*x - 425*i*sqrt(sqrt(5) - 1)) + 1/40*(i + 1)*sqrt(25*sqrt(5) + 55)*lo
g(425*(i + 1)*x + 425*sqrt(sqrt(5) - 1)) - 1/40*(i + 1)*sqrt(25*sqrt(5) + 5
5)*log(425*(i + 1)*x - 425*sqrt(sqrt(5) - 1)) - 1/x
```

3.384 $\int \frac{1}{x^4(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=466

$$-\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(\right.}\right.}{4\,2^{3/4}\sqrt{5}}$$

```
[Out] -1/(3*x^3) + ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5]
)^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3
/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/
4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((843
 - 377*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/
4)*Sqrt[5]) + ((843 + 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(
3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^
(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2
*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) + ((843 - 377*Sqrt
[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]
)/(4*2^(3/4)*Sqrt[5])
```

Rubi [A] time = 0.367145, antiderivative size = 466, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1368, 1422, 211, 1165, 628, 1162, 617, 204}

$$-\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(\right.}\right.}{4\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + 3*x^4 + x^8)),x]
```


```
[Out] -1/(3*x^3) + ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5]
)^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3
/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/
4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((843
 - 377*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/
4)*Sqrt[5]) + ((843 + 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(
3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^
(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2
*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) + ((843 - 377*Sqrt
[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]
)/(4*2^(3/4)*Sqrt[5])
```


Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(1+3x^4+x^8\right)}\,dx = -\frac{1}{3x^3}+\frac{1}{3}\int \frac{-9-3x^4}{1+3x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\frac{1}{3x^3}-\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{8\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{8\sqrt{5}}+\frac{\left(-5+3\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}-}{\frac{3}{2}+\frac{\sqrt{5}}{2}+}}{20\sqrt{3+\sqrt{5}}}$$

$$= -\frac{1}{3x^3}-\frac{\sqrt[4]{843-377\sqrt{5}}\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}}}{4\,2^{3/4}\sqrt{5}}$$

$$= -\frac{1}{3x^3}+\frac{\left(3+\sqrt{5}\right)^{7/4}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{16\sqrt[4]{2}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{7/4}\log\left(\sqrt{2\left(3-\sqrt{\right.}}\right.}{16\sqrt[4]{}}$$

$$= -\frac{1}{3x^3}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\,\mathrm{ta}}{2\,2^{3/}}$$

Mathematica [C] time = 0.0152448, size = 65, normalized size = 0.14

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+3\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^
4)/(3*#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.007, size = 50, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(-\_R^4-3\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4-3)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))-1/3/x^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3x^3}-\int \frac{x^4+3}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate((x^4 + 3)/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] time = 2.17148, size = 3754, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/240*(3*sqrt(10)*sqrt(2)*x^3*(754*sqrt(5) + 1686)^(1/4)*log(20*x^2 + sqrt
(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sq
rt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47)) - 3*sqrt(10)*sqrt(2)*x^3*(754*sqr
t(5) + 1686)^(1/4)*log(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*
x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47
)) - 3*sqrt(10)*sqrt(2)*x^3*(-754*sqrt(5) + 1686)^(1/4)*log(20*x^2 + sqrt(1
0)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21
*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1686)) + 3*sqrt(10)*sqrt(2)*x^3*(-754*sq
rt(5) + 1686)^(1/4)*log(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)
*x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1
686)) - 3*sqrt(10)*(377*sqrt(5)*x^3 - 843*x^3)*(754*sqrt(5) + 1686)^(3/4)*s
qrt(377*sqrt(5) + 843)*arctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(7*sqrt(
5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5
) + 1686)*(21*sqrt(5) - 47))*(23184*sqrt(5) - 51841)*(754*sqrt(5) + 1686)^(
5/4)*sqrt(377*sqrt(5) + 843) + 1/40*sqrt(10)*(51841*sqrt(5)*x - 115920*x)*(
754*sqrt(5) + 1686)^(5/4)*sqrt(377*sqrt(5) + 843) - 1/8*(377*sqrt(5)*sqrt(2
) - 843*sqrt(2))*sqrt(754*sqrt(5) + 1686)*sqrt(377*sqrt(5) + 843)) - 3*sqrt
(10)*(377*sqrt(5)*x^3 - 843*x^3)*(754*sqrt(5) + 1686)^(3/4)*sqrt(377*sqrt(5
) + 843)*arctan(1/80*sqrt(10)*sqrt(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x -
 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*
sqrt(5) - 47))*(23184*sqrt(5) - 51841)*(754*sqrt(5) + 1686)^(5/4)*sqrt(377*
sqrt(5) + 843) + 1/40*sqrt(10)*(51841*sqrt(5)*x - 115920*x)*(754*sqrt(5) +
1686)^(5/4)*sqrt(377*sqrt(5) + 843) + 1/8*(377*sqrt(5)*sqrt(2) - 843*sqrt(2
))*sqrt(754*sqrt(5) + 1686)*sqrt(377*sqrt(5) + 843)) + 3*sqrt(10)*(377*sqrt
(5)*x^3 + 843*x^3)*sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(3/4)*arc
tan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*
x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 16
86))*(23184*sqrt(5) + 51841)*sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)
^(5/4) - 1/40*(sqrt(10)*(51841*sqrt(5)*x + 115920*x)*(-754*sqrt(5) + 1686)^
(5/4) + 5*(377*sqrt(5)*sqrt(2) + 843*sqrt(2))*sqrt(-754*sqrt(5) + 1686))*sq
rt(-377*sqrt(5) + 843)) + 3*sqrt(10)*(377*sqrt(5)*x^3 + 843*x^3)*sqrt(-377*
sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2
 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 1686)^(1/4
) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1686))*(23184*sqrt(5) + 51841)*
sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(5/4) - 1/40*(sqrt(10)*(5184
1*sqrt(5)*x + 115920*x)*(-754*sqrt(5) + 1686)^(5/4) - 5*(377*sqrt(5)*sqrt(2
) + 843*sqrt(2))*sqrt(-754*sqrt(5) + 1686))*sqrt(-377*sqrt(5) + 843)) + 80)
/x^3
```

Sympy [A] time = 1.23233, size = 34, normalized size = 0.07

$$\mathrm{RootSum}\left(40960000t^8+5395200t^4+1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}+\frac{23112t}{377}+x\right)\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 5395200*_t**4 + 1, Lambda(_t, _t*log(179200*_t**5/
377 + 23112*_t/377 + x))) - 1/(3*x**3)
```

Giac [A] time = 1.3757, size = 348, normalized size = 0.75

$$\frac{1}{40}(i+1)\sqrt{65\sqrt{5}-145}\log\left(9650(i+1)x+9650i\sqrt{\sqrt{5}+1}\right)-\frac{1}{40}(i+1)\sqrt{65\sqrt{5}-145}\log\left(9650(i+1)x-9650i\sqrt{\sqrt{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/40*(i + 1)*sqrt(65*sqrt(5) - 145)*log(9650*(i + 1)*x + 9650*i*sqrt(sqrt(5
) + 1)) - 1/40*(i + 1)*sqrt(65*sqrt(5) - 145)*log(9650*(i + 1)*x - 9650*i*s
qrt(sqrt(5) + 1)) - 1/40*(i - 1)*sqrt(65*sqrt(5) - 145)*log(9650*(i + 1)*x
+ 9650*sqrt(sqrt(5) + 1)) + 1/40*(i - 1)*sqrt(65*sqrt(5) - 145)*log(9650*(i
 + 1)*x - 9650*sqrt(sqrt(5) + 1)) - 1/40*(i + 1)*sqrt(65*sqrt(5) + 145)*log
(7330*(i + 1)*x + 7330*i*sqrt(sqrt(5) - 1)) + 1/40*(i + 1)*sqrt(65*sqrt(5)
+ 145)*log(7330*(i + 1)*x - 7330*i*sqrt(sqrt(5) - 1)) + 1/40*(i - 1)*sqrt(6
5*sqrt(5) + 145)*log(7330*(i + 1)*x + 7330*sqrt(sqrt(5) - 1)) - 1/40*(i - 1
)*sqrt(65*sqrt(5) + 145)*log(7330*(i + 1)*x - 7330*sqrt(sqrt(5) - 1)) - 1/3
/x^3
```

3.385 $\int \frac{x^m}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=117

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(3 - Sqrt[5
])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1, (1
 + m)/4, (5 + m)/4, (2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1 + m))
```

Rubi [A] time = 0.0646287, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1375, 364}

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 - 3*x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(3 - Sqrt[5
])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1, (1
 + m)/4, (5 + m)/4, (2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1 + m))
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{1-3x^4+x^8}\,dx &= \frac{\int \frac{x^m}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}-\frac{\int \frac{x^m}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}\\ &= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}\end{aligned}$$

Mathematica [C] time = 0.171662, size = 191, normalized size = 1.63

$$\frac{1}{4}x^{m+1}\left(-\frac{x^2\text{RootSum}\left[\#1^4-\#1^2-1\&,\frac{{}_2F_1\left(1,m+3;m+4;\frac{x}{\#1}\right)}{\#1^2+2}\&\right]}{m+3}-\frac{x^2\text{RootSum}\left[\#1^4+\#1^2-1\&,\frac{{}_2F_1\left(1,m+3;m+4;\frac{x}{\#1}\right)}{\#1^2-2}\&\right]}{m+3}+\text{Roo}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(1 - 3*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*(RootSum[-1 - #1^2 + #1^4 & , Hypergeometric2F1[1, 1 + m, 2 + m,
 x/#1]/(2 + #1^2) & ]/(1 + m) - (x^2*RootSum[-1 - #1^2 + #1^4 & , Hypergeom
etric2F1[1, 3 + m, 4 + m, x/#1]/(2 + #1^2) & ])/(3 + m) - RootSum[-1 + #1^2
 + #1^4 & , Hypergeometric2F1[1, 1 + m, 2 + m, x/#1]/(-2 + #1^2) & ]/(1 + m
) - (x^2*RootSum[-1 + #1^2 + #1^4 & , Hypergeometric2F1[1, 3 + m, 4 + m, x/
#1]/(-2 + #1^2) & ])/(3 + m)))/4
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8-3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8-3*x^4+1),x)
```

```
[Out] int(x^m/(x^8-3*x^4+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8-3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{x^8-3x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 - 3*x^4 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(x^4-x^2-1\right)\left(x^4+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8-3*x**4+1),x)
```

```
[Out] Integral(x**m/((x**4 - x**2 - 1)*(x**4 + x**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

3.386 $\int \frac{x^{11}}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=62

$$\frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

```
[Out] x^4/4 + ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] - 2*x^4])/40
```

Rubi [A] time = 0.045602, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 703, 632, 31}

$$\frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 - 3*x^4 + x^8),x]
```

```
[Out] x^4/4 + ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] - 2*x^4])/40
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{4}\mathrm{Subst}\left(\int \frac{x^2}{1-3x+x^2}\,dx,x,x^4\right)$$
$$= \frac{x^4}{4}+\frac{1}{4}\mathrm{Subst}\left(\int \frac{-1+3x}{1-3x+x^2}\,dx,x,x^4\right)$$
$$= \frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\right.$$
$$= \frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

Mathematica [A] time = 0.0328176, size = 56, normalized size = 0.9

$$\frac{1}{40}\left(10x^4+\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 - 3*x^4 + x^8),x]
```

```
[Out] (10*x^4 + (15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] + (15 - 7*Sqrt[5])*Log[
-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.003, size = 38, normalized size = 0.6

$$\frac{x^4}{4}+\frac{3\ln\left(x^8-3x^4+1\right)}{8}-\frac{7\sqrt{5}}{20}\mathrm{Artanh}\left(\frac{\left(2x^4-3\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8-3*x^4+1),x)
```

```
[Out] 1/4*x^4+3/8*ln(x^8-3*x^4+1)-7/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Maxima [A] time = 1.46732, size = 68, normalized size = 1.1

$$\frac{1}{4}x^4+\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)+\frac{3}{8}\log\left(x^8-3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 3
/8*log(x^8 - 3*x^4 + 1)
```

Fricas [A] time = 1.71304, size = 157, normalized size = 2.53

$$\frac{1}{4}x^4+\frac{7}{40}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right)+\frac{3}{8}\log\left(x^8-3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 -
 3*x^4 + 1)) + 3/8*log(x^8 - 3*x^4 + 1)
```

Sympy [A] time = 0.136557, size = 58, normalized size = 0.94

$$\frac{x^4}{4} + \left(\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) + \left(\frac{3}{8} - \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8-3*x**4+1),x)
```

```
[Out] x**4/4 + (3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (3/8 - 7*sqrt(5
)/40)*log(x**4 - 3/2 + sqrt(5)/2)
```

Giac [A] time = 1.14956, size = 72, normalized size = 1.16

$$\frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{\left|2\,x^4 - \sqrt{5} - 3\right|}{\left|2\,x^4 + \sqrt{5} - 3\right|}\right) + \frac{3}{8}\,\log\left(\left|x^8 - 3\,x^4 + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3
)) + 3/8*log(abs(x^8 - 3*x^4 + 1))
```

3.387 $\int \frac{x^9}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=90

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (S
qrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi [A] time = 0.0720074, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1359, 1122, 1166, 207}

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (S
qrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{1-3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{x^2}{2} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1-3x^2}{1-3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{x^2}{2} - \frac{1}{20}\left(-15+7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right) + \frac{1}{20}\left(15+7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\right.$$
$$= \frac{x^2}{2} - \frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{20}\sqrt{180-80\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Mathematica [A] time = 0.0509876, size = 103, normalized size = 1.14

$$\frac{1}{20}\left(10x^2 + \left(2\sqrt{5}-5\right)\log\left(-2x^2+\sqrt{5}-1\right) + \left(5+2\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right) + \left(5-2\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-1\right) - \left(5+2\sqrt{5}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] (10*x^2 + (-5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[
1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + 2*S
qrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

Maple [A] time = 0.005, size = 67, normalized size = 0.7

$$\frac{x^2}{2} - \frac{\ln\left(x^4+x^2-1\right)}{4} - \frac{\sqrt{5}}{5}\operatorname{Artanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right) + \frac{\ln\left(x^4-x^2-1\right)}{4} - \frac{\sqrt{5}}{5}\operatorname{Artanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8-3*x^4+1),x)
```

```
[Out] 1/2*x^2-1/4*ln(x^4+x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))+1/4*ln
(x^4-x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

Maxima [A] time = 1.46521, size = 124, normalized size = 1.38

$$\frac{1}{2}x^2 + \frac{1}{10}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right) + \frac{1}{10}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right) - \frac{1}{4}\log\left(x^4+x^2-1\right) + \frac{1}{4}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1
/10*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/4*log(x^4
+ x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Fricas [B] time = 1.71002, size = 290, normalized size = 3.22

$$\frac{1}{2}x^2 + \frac{1}{10}\sqrt{5}\log\left(\frac{2x^4 + 2x^2 - \sqrt{5}\left(2x^2 + 1\right) + 3}{x^4 + x^2 - 1}\right) + \frac{1}{10}\sqrt{5}\log\left(\frac{2x^4 - 2x^2 - \sqrt{5}\left(2x^2 - 1\right) + 3}{x^4 - x^2 - 1}\right) - \frac{1}{4}\log\left(x^4 + x^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 +
 x^2 - 1)) + 1/10*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^
4 - x^2 - 1)) - 1/4*log(x^4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Sympy [B] time = 0.462109, size = 170, normalized size = 1.89

$$\frac{x^2}{2} + \left(-\frac{1}{4} - \frac{\sqrt{5}}{10}\right)\log\left(x^2 - \frac{47}{8} - \frac{47\sqrt{5}}{20} - 120\left(-\frac{1}{4} - \frac{\sqrt{5}}{10}\right)^3\right) + \left(-\frac{1}{4} + \frac{\sqrt{5}}{10}\right)\log\left(x^2 - \frac{47}{8} - 120\left(-\frac{1}{4} + \frac{\sqrt{5}}{10}\right)^3 + \frac{47\sqrt{5}}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8-3*x**4+1),x)
```

```
[Out] x**2/2 + (-1/4 - sqrt(5)/10)*log(x**2 - 47/8 - 47*sqrt(5)/20 - 120*(-1/4 -
sqrt(5)/10)**3) + (-1/4 + sqrt(5)/10)*log(x**2 - 47/8 - 120*(-1/4 + sqrt(5)
/10)**3 + 47*sqrt(5)/20) + (1/4 - sqrt(5)/10)*log(x**2 - 47*sqrt(5)/20 - 12
0*(1/4 - sqrt(5)/10)**3 + 47/8) + (sqrt(5)/10 + 1/4)*log(x**2 - 120*(sqrt(5
)/10 + 1/4)**3 + 47*sqrt(5)/20 + 47/8)
```

Giac [A] time = 1.17847, size = 131, normalized size = 1.46

$$\frac{1}{2}x^2 + \frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2 - \sqrt{5} + 1\right|}{2x^2 + \sqrt{5} + 1}\right) + \frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2 - \sqrt{5} - 1\right|}{\left|2x^2 + \sqrt{5} - 1\right|}\right) - \frac{1}{4}\log\left(\left|x^4 + x^2 - 1\right|\right) + \frac{1}{4}\log\left(\left|x^4 - x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1))
+ 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/4
*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

3.388 $\int \frac{x^7}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=55

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] - 2*x^4])/40
```

Rubi [A] time = 0.0318653, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 632, 31}

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] - 2*x^4])/40
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1-3x^4+x^8} \, dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1-3x+x^2} \, dx, x, x^4\right)\\
&= \frac{1}{40}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x} \, dx, x, x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x} \, dx, x, x^4\right)\\
&= \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0208405, size = 53, normalized size = 0.96

$$\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((5 - 3*Sqrt[5])*Log[-3 + S
qrt[5] + 2*x^4])/40
```

Maple [A] time = 0.002, size = 33, normalized size = 0.6

$$\frac{\ln\left(x^8-3x^4+1\right)}{8}-\frac{3\sqrt{5}}{20}\text{Artanh}\left(\frac{\left(2x^4-3\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8-3*x^4+1),x)
```

```
[Out] 1/8*ln(x^8-3*x^4+1)-3/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Maxima [A] time = 1.47837, size = 61, normalized size = 1.11

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)+\frac{1}{8}\log\left(x^8-3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 1/8*log(x^8
 - 3*x^4 + 1)
```

Fricas [A] time = 1.72496, size = 143, normalized size = 2.6

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right)+\frac{1}{8}\log\left(x^8-3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
)) + 1/8*log(x^8 - 3*x^4 + 1)
```

Sympy [A] time = 0.137365, size = 53, normalized size = 0.96

$$\left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(\frac{1}{8}-\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8-3*x**4+1),x)
```

```
[Out] (1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (1/8 - 3*sqrt(5)/40)*log
(x**4 - 3/2 + sqrt(5)/2)
```

Giac [A] time = 1.14867, size = 65, normalized size = 1.18

$$\frac{3}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)+\frac{1}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/8*l
og(abs(x^8 - 3*x^4 + 1))
```

3.389 $\int \frac{x^5}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=81

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)$$

```
[Out] -(Sqrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sqrt[(3 -
 Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi [A] time = 0.0532177, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1359, 1130, 207}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(Sqrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sqrt[(3 -
 Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{1-3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{1}{20}\left(5-3\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right) + \frac{1}{20}\left(5+3\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)$$
$$= -\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Mathematica [A] time = 0.03512, size = 91, normalized size = 1.12

$$\frac{1}{40}\left(\left(\sqrt{5}-5\right)\log\left(-2x^2+\sqrt{5}-1\right)+\left(5+\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(2x^2+\sqrt{5}-1\right)-\left(5+\sqrt{5}\right)\log\left(2x^2+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + Sqrt[5])*Log[1 + Sqrt[5] -
 2*x^2] - (-5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + Sqrt[5])*Log[1 +
Sqrt[5] + 2*x^2])/40
```

Maple [A] time = 0.003, size = 62, normalized size = 0.8

$$-\frac{\ln\left(x^4+x^2-1\right)}{8}-\frac{\sqrt{5}}{20}\operatorname{Artanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)+\frac{\ln\left(x^4-x^2-1\right)}{8}-\frac{\sqrt{5}}{20}\operatorname{Artanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8-3*x^4+1),x)
```

```
[Out] -1/8*ln(x^4+x^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))+1/8*ln(x^4-x
^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

Maxima [B] time = 1.50073, size = 117, normalized size = 1.44

$$\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqrt(5
)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 - 1)
 + 1/8*log(x^4 - x^2 - 1)
```

Fricas [B] time = 1.77738, size = 277, normalized size = 3.42

$$\frac{1}{40}\sqrt{5}\log\left(\frac{2x^4+2x^2-\sqrt{5}(2x^2+1)+3}{x^4+x^2-1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{2x^4-2x^2-\sqrt{5}(2x^2-1)+3}{x^4-x^2-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1))
 + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 -
1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

Sympy [B] time = 0.447933, size = 165, normalized size = 2.04

$$\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-\frac{3\sqrt{5}}{10}-640\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3\right)+\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-640\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{3\sqrt{5}}{10}\right)+\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8-3*x**4+1),x)
```

```
[Out] (-1/8 - sqrt(5)/40)*log(x**2 - 3/2 - 3*sqrt(5)/10 - 640*(-1/8 - sqrt(5)/40)
**3) + (-1/8 + sqrt(5)/40)*log(x**2 - 3/2 - 640*(-1/8 + sqrt(5)/40)**3 + 3*
sqrt(5)/10) + (1/8 - sqrt(5)/40)*log(x**2 - 3*sqrt(5)/10 - 640*(1/8 - sqrt(
5)/40)**3 + 3/2) + (sqrt(5)/40 + 1/8)*log(x**2 - 640*(sqrt(5)/40 + 1/8)**3
+ 3*sqrt(5)/10 + 3/2)
```

Giac [B] time = 1.16845, size = 124, normalized size = 1.53

$$\frac{1}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{1}{8}\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{8}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqr
t(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs(x
^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

3.390 $\int \frac{x^3}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] ArcTanh[(3 - 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rubi [A] time = 0.0269306, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTanh[(3 - 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{1-3x^4+x^8} \, dx &= \frac{1}{4} \text{Subst}\left(\int \frac{1}{1-3x+x^2} \, dx, x, x^4\right)\\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{5-x^2} \, dx, x, -3+2x^4\right)\right)\\
&= \frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.009951, size = 38, normalized size = 1.65

$$\frac{\log\left(-2x^4+\sqrt{5}+3\right)-\log\left(2x^4+\sqrt{5}-3\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - 3*x^4 + x^8),x]
```

```
[Out] (Log[3 + Sqrt[5] - 2*x^4] - Log[-3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

Maple [A] time = 0.002, size = 19, normalized size = 0.8

$$-\frac{\sqrt{5}}{10}\text{Artanh}\left(\frac{\left(2x^4-3\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-3*x^4+1),x)
```

```
[Out] -1/10*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Maxima [A] time = 1.5021, size = 42, normalized size = 1.83

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3))
```

Fricas [B] time = 1.64281, size = 107, normalized size = 4.65

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
))
```

Sympy [A] time = 0.118755, size = 42, normalized size = 1.83

$$\frac{\sqrt{5}\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}{20}-\frac{\sqrt{5}\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-3*x**4+1),x)
```

```
[Out] sqrt(5)*log(x**4 - 3/2 - sqrt(5)/2)/20 - sqrt(5)*log(x**4 - 3/2 + sqrt(5)/2
)/20
```

Giac [A] time = 1.16324, size = 45, normalized size = 1.96

$$\frac{1}{20}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3))
```

3.391 $\int \frac{x}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

```
[Out] -(ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + S
qrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi [A] time = 0.0377546, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1359, 1093, 207}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + S
qrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{1-3x^4+x^8}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}}$$
$$= -\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}} + \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Mathematica [A] time = 0.0292294, size = 91, normalized size = 1.21

$$\frac{1}{40}\left(-\left(5+\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}-1\right)-\left(\sqrt{5}-5\right)\log\left(-2x^2+\sqrt{5}+1\right)+\left(5+\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-1\right)+\left(\sqrt{5}-5\right)\log\left(2x^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] (-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2]) - (-5 + Sqrt[5])*Log[1 + Sqrt[5
] - 2*x^2] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + Sqrt[5])*Log[1
 + Sqrt[5] + 2*x^2])/40
```

Maple [A] time = 0.003, size = 62, normalized size = 0.8

$$-\frac{\ln\left(x^4+x^2-1\right)}{8}+\frac{\sqrt{5}}{20}\text{Artanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)+\frac{\ln\left(x^4-x^2-1\right)}{8}+\frac{\sqrt{5}}{20}\text{Artanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8-3*x^4+1),x)
```

```
[Out] -1/8*ln(x^4+x^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))+1/8*ln(x^4-x
^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

Maxima [B] time = 1.4865, size = 117, normalized size = 1.56

$$-\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*sqrt(
5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 - 1
) + 1/8*log(x^4 - x^2 - 1)
```

Fricas [B] time = 1.74332, size = 277, normalized size = 3.69

$$\frac{1}{40}\sqrt{5}\log\left(\frac{2x^4+2x^2+\sqrt{5}(2x^2+1)+3}{x^4+x^2-1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{2x^4-2x^2+\sqrt{5}(2x^2-1)+3}{x^4-x^2-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1))
 + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 -
1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

Sympy [B] time = 0.437, size = 165, normalized size = 2.2

$$\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-\frac{7}{2}-\frac{7\sqrt{5}}{10}+960\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3\right)+\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7}{2}+960\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}\right)+\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8-3*x**4+1),x)
```

```
[Out] (sqrt(5)/40 + 1/8)*log(x**2 - 7/2 - 7*sqrt(5)/10 + 960*(sqrt(5)/40 + 1/8)**
3) + (1/8 - sqrt(5)/40)*log(x**2 - 7/2 + 960*(1/8 - sqrt(5)/40)**3 + 7*sqrt
(5)/10) + (-1/8 + sqrt(5)/40)*log(x**2 - 7*sqrt(5)/10 + 960*(-1/8 + sqrt(5)
/40)**3 + 7/2) + (-1/8 - sqrt(5)/40)*log(x**2 + 960*(-1/8 - sqrt(5)/40)**3
+ 7*sqrt(5)/10 + 7/2)
```

Giac [B] time = 1.14187, size = 124, normalized size = 1.65

$$-\frac{1}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)-\frac{1}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{1}{8}\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{8}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*sq
rt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs(
x^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

3.392 $\int \frac{1}{x(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\log(x)$$

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((5 - 3*Sqrt[5])*L
og[3 + Sqrt[5] - 2*x^4])/40
```

Rubi [A] time = 0.0302591, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 705, 29, 632, 31}

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((5 - 3*Sqrt[5])*L
og[3 + Sqrt[5] - 2*x^4])/40
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1-3x^4+x^8)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1-3x+x^2)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)+\frac{1}{4}\text{Subst}\left(\int \frac{3-x}{1-3x+x^2}\,dx,x,x^4\right)\\
&= \log(x)+\frac{1}{40}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)-\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\ldots}\right.\\
&= \log(x)-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0312623, size = 55, normalized size = 0.96

$$\frac{1}{40}\left(3\sqrt{5}-5\right)\log\left(-2x^4+\sqrt{5}+3\right)+\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] + ((-5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((-5 - 3*Sqrt[5])
*Log[-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.01, size = 64, normalized size = 1.1

$$-\frac{\ln\left(x^4-x^2-1\right)}{8}-\frac{3\sqrt{5}}{20}\text{Artanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)+\ln(x)-\frac{\ln\left(x^4+x^2-1\right)}{8}+\frac{3\sqrt{5}}{20}\text{Artanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8-3*x^4+1),x)
```

```
[Out] -1/8*ln(x^4-x^2-1)-3/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))+ln(x)-1/8*ln
(x^4+x^2-1)+3/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Maxima [A] time = 1.4845, size = 69, normalized size = 1.21

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)-\frac{1}{8}\log\left(x^8-3x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/8*log(x^8
 - 3*x^4 + 1) + 1/4*log(x^4)
```

Fricas [A] time = 1.77523, size = 155, normalized size = 2.72

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right)-\frac{1}{8}\log\left(x^8-3x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
)) - 1/8*log(x^8 - 3*x^4 + 1) + log(x)
```

Sympy [A] time = 0.156253, size = 58, normalized size = 1.02

$$\log(x)+\left(-\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(-\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8-3*x**4+1),x)
```

```
[Out] log(x) + (-1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-3*sqrt(5)/40
 - 1/8)*log(x**4 - 3/2 + sqrt(5)/2)
```

Giac [A] time = 1.16524, size = 73, normalized size = 1.28

$$\frac{3}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)+\frac{1}{4}\log\left(x^4\right)-\frac{1}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/4*l
og(x^4) - 1/8*log(abs(x^8 - 3*x^4 + 1))
```

3.393 $\int \frac{1}{x^3(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

```
[Out] -1/(2*x^2) - (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2
 + ((3 + Sqrt[5])^(3/2)*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])
```

Rubi [A] time = 0.0576259, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1359, 1123, 1166, 207}

$$-\frac{1}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2
 + ((3 + Sqrt[5])^(3/2)*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(1-3x^4+x^8\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-3x^2+x^4\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{3-x^2}{1-3x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}+\frac{1}{20}\left(-5+3\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)-\frac{1}{20}\left(5+3\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}}\right.$$
$$= -\frac{1}{2x^2}-\frac{1}{10}\sqrt{45-20\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

Mathematica [A] time = 0.0602255, size = 103, normalized size = 1.16

$$\frac{1}{20}\left(-\frac{10}{x^2}-\left(5+2\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}-1\right)+\left(5-2\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)+\left(5+2\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-1\right)+\left(2\sqrt{5}-5\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^2 - (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[
1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + 2*
Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

Maple [A] time = 0.009, size = 67, normalized size = 0.8

$$\frac{\ln\left(x^4-x^2-1\right)}{4}+\frac{\sqrt{5}}{5}\operatorname{Artanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)-\frac{\ln\left(x^4+x^2-1\right)}{4}+\frac{\sqrt{5}}{5}\operatorname{Artanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8-3*x^4+1),x)
```

```
[Out] 1/4*ln(x^4-x^2-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/4*ln(x^4+x^2
-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))-1/2/x^2
```

Maxima [A] time = 1.47979, size = 124, normalized size = 1.39

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{1}{2x^2}-\frac{1}{4}\log\left(x^4+x^2-1\right)+\frac{1}{4}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sqrt(
5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/4*log(x^4
```

```
+ x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Fricas [B] time = 1.75756, size = 306, normalized size = 3.44

$$\frac{2\sqrt{5}x^2\log\left(\frac{2x^4+2x^2+\sqrt{5}(2x^2+1)+3}{x^4+x^2-1}\right)+2\sqrt{5}x^2\log\left(\frac{2x^4-2x^2+\sqrt{5}(2x^2-1)+3}{x^4-x^2-1}\right)-5x^2\log\left(x^4+x^2-1\right)+5x^2\log\left(x^4-x^2-1\right)}{20x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*(2*sqrt(5)*x^2*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^
2 - 1)) + 2*sqrt(5)*x^2*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4
- x^2 - 1)) - 5*x^2*log(x^4 + x^2 - 1) + 5*x^2*log(x^4 - x^2 - 1) - 10)/x^2
```

Sympy [B] time = 0.476674, size = 172, normalized size = 1.93

$$\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-\frac{123}{8}-\frac{123\sqrt{5}}{20}+280\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3\right)+\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123}{8}+280\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}\right)+\left(-\frac{1}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8-3*x**4+1),x)
```

```
[Out] (sqrt(5)/10 + 1/4)*log(x**2 - 123/8 - 123*sqrt(5)/20 + 280*(sqrt(5)/10 + 1/
4)**3) + (1/4 - sqrt(5)/10)*log(x**2 - 123/8 + 280*(1/4 - sqrt(5)/10)**3 +
123*sqrt(5)/20) + (-1/4 + sqrt(5)/10)*log(x**2 - 123*sqrt(5)/20 + 280*(-1/4
 + sqrt(5)/10)**3 + 123/8) + (-1/4 - sqrt(5)/10)*log(x**2 + 280*(-1/4 - sqr
t(5)/10)**3 + 123*sqrt(5)/20 + 123/8) - 1/(2*x**2)
```

Giac [A] time = 1.16964, size = 131, normalized size = 1.47

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{1}{2x^2}-\frac{1}{4}\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{4}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sq
rt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/
4*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

3.394 $\int \frac{1}{x^5(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{4x^4}-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+3\log(x)$$

```
[Out] -1/(4*x^4) + 3*Log[x] - ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((
15 - 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40
```

Rubi [A] time = 0.0626971, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 709, 800, 632, 31}

$$-\frac{1}{4x^4}-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) + 3*Log[x] - ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((
15 - 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(1-3x^4+x^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-3x+x^2\right)}\,dx, x, x^4\right)\\ &= -\frac{1}{4x^4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{3-x}{x\left(1-3x+x^2\right)}\,dx, x, x^4\right)\\ &= -\frac{1}{4x^4}+\frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{3}{x}+\frac{8-3x}{1-3x+x^2}\right)dx, x, x^4\right)\\ &= -\frac{1}{4x^4}+3\log(x)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{8-3x}{1-3x+x^2}\,dx, x, x^4\right)\\ &= -\frac{1}{4x^4}+3\log(x)+\frac{1}{40}\left(-15+7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\operatorname{Sub}\\ &= -\frac{1}{4x^4}+3\log(x)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)\end{aligned}$$

Mathematica [A] time = 0.034366, size = 61, normalized size = 0.92

$$\frac{1}{40}\left(-\frac{10}{x^4}+\left(7\sqrt{5}-15\right)\log\left(-2x^4+\sqrt{5}+3\right)-\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)+120\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^4 + 120*Log[x] + (-15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] - (15 +
7*Sqrt[5])*Log[-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.008, size = 71, normalized size = 1.1

$$-\frac{3\ln\left(x^4-x^2-1\right)}{8}-\frac{7\sqrt{5}}{20}\operatorname{Artanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)-\frac{1}{4x^4}+3\ln(x)-\frac{3\ln\left(x^4+x^2-1\right)}{8}+\frac{7\sqrt{5}}{20}\operatorname{Artanh}\left(\frac{\left(2x^2+1\right)}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8-3*x^4+1),x)
```

```
[Out] -3/8*ln(x^4-x^2-1)-7/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/4/x^4+3*ln
(x)-3/8*ln(x^4+x^2-1)+7/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Maxima [A] time = 1.50719, size = 76, normalized size = 1.15

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)-\frac{1}{4x^4}-\frac{3}{8}\log\left(x^8-3x^4+1\right)+\frac{3}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/4/x^4 - 3
/8*log(x^8 - 3*x^4 + 1) + 3/4*log(x^4)
```

Fricas [A] time = 1.70473, size = 193, normalized size = 2.92

$$\frac{7\sqrt{5}x^4\log\left(\frac{2x^8-6x^4-\sqrt{5}(2x^4-3)+7}{x^8-3x^4+1}\right)-15x^4\log\left(x^8-3x^4+1\right)+120x^4\log(x)-10}{40x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(7*sqrt(5)*x^4*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*
x^4 + 1)) - 15*x^4*log(x^8 - 3*x^4 + 1) + 120*x^4*log(x) - 10)/x^4
```

Sympy [A] time = 0.195624, size = 66, normalized size = 1.

$$3\log(x)+\left(-\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(-\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8-3*x**4+1),x)
```

```
[Out] 3*log(x) + (-3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-7*sqrt(5)/
40 - 3/8)*log(x**4 - 3/2 + sqrt(5)/2) - 1/(4*x**4)
```

Giac [A] time = 1.14576, size = 89, normalized size = 1.35

$$\frac{7}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)-\frac{3x^4+1}{4x^4}+\frac{3}{4}\log\left(x^4\right)-\frac{3}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) - 1/4*(
3*x^4 + 1)/x^4 + 3/4*log(x^4) - 3/8*log(abs(x^8 - 3*x^4 + 1))
```

3.395 $\int \frac{1}{x^7(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=97

$$-\frac{3}{2x^2}-\frac{1}{6x^6}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] -1/(6*x^6) - 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + S
qrt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/
2]*x^2])/2
```

Rubi [A] time = 0.0904821, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1359, 1123, 1281, 1166, 207}

$$-\frac{3}{2x^2}-\frac{1}{6x^6}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) - 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + S
qrt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/
2]*x^2])/2
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(1-3x^4+x^8\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4\left(1-3x^2+x^4\right)}\,dx,x,x^2\right)\\ &= -\frac{1}{6x^6}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{9-3x^2}{x^2\left(1-3x^2+x^4\right)}\,dx,x,x^2\right)\\ &= -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{6}\operatorname{Subst}\left(\int \frac{-24+9x^2}{1-3x^2+x^4}\,dx,x,x^2\right)\\ &= -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{20}\left(15-7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)-\frac{1}{20}\left(15+7\sqrt{5}\right)\operatorname{Subst}\left(\int -\right.\\ &= -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{20}\sqrt{1230+550\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{1}{2}}\right.\end{aligned}$$

Mathematica [A] time = 0.0739215, size = 111, normalized size = 1.14

$$\frac{1}{120}\left(-\frac{180}{x^2}-\frac{20}{x^6}-3\left(25+11\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}-1\right)+3\left(25-11\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)+3\left(25+11\sqrt{5}\right)\log\left(2x^2+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-20/x^6 - 180/x^2 - 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + 3*(25
- 11*Sqrt[5])*Log[1 + Sqrt[5] - 2*x^2] + 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[
5] + 2*x^2] + 3*(-25 + 11*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/120
```

Maple [A] time = 0.01, size = 72, normalized size = 0.7

$$\frac{5\ln\left(x^4-x^2-1\right)}{8}+\frac{11\sqrt{5}}{20}\operatorname{Artanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)-\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{5\ln\left(x^4+x^2-1\right)}{8}+\frac{11\sqrt{5}}{20}\operatorname{Artanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8-3*x^4+1),x)
```

```
[Out] 5/8*ln(x^4-x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/6/x^6-3/2/
x^2-5/8*ln(x^4+x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Maxima [A] time = 1.47745, size = 134, normalized size = 1.38

$$-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{9x^4+1}{6x^6}-\frac{5}{8}\log\left(x^4+x^2-1\right)+\frac{5}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -11/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40*sqr
t(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x^4 + 1)/x^6
 - 5/8*log(x^4 + x^2 - 1) + 5/8*log(x^4 - x^2 - 1)
```

Fricas [B] time = 1.71408, size = 327, normalized size = 3.37

$$\frac{33\sqrt{5}x^6\log\left(\frac{2x^4+2x^2+\sqrt{5}(2x^2+1)+3}{x^4+x^2-1}\right)+33\sqrt{5}x^6\log\left(\frac{2x^4-2x^2+\sqrt{5}(2x^2-1)+3}{x^4-x^2-1}\right)-75x^6\log\left(x^4+x^2-1\right)+75x^6\log\left(x^4-x^2\right.}{120x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(33*sqrt(5)*x^6*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 +
x^2 - 1)) + 33*sqrt(5)*x^6*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x
^4 - x^2 - 1)) - 75*x^6*log(x^4 + x^2 - 1) + 75*x^6*log(x^4 - x^2 - 1) - 18
0*x^4 - 20)/x^6
```

Sympy [B] time = 0.516308, size = 197, normalized size = 2.03

$$\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\log\left(x^2-\frac{2207}{22}-\frac{2207\sqrt{5}}{50}+\frac{1152\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)^3}{11}\right)+\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207}{22}+\frac{1152\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{22}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8-3*x**4+1),x)
```

```
[Out] (11*sqrt(5)/40 + 5/8)*log(x**2 - 2207/22 - 2207*sqrt(5)/50 + 1152*(11*sqrt(
5)/40 + 5/8)**3/11) + (5/8 - 11*sqrt(5)/40)*log(x**2 - 2207/22 + 1152*(5/8
- 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)/50) + (-5/8 + 11*sqrt(5)/40)*log(x**2
 - 2207*sqrt(5)/50 + 1152*(-5/8 + 11*sqrt(5)/40)**3/11 + 2207/22) + (-5/8 -
 11*sqrt(5)/40)*log(x**2 + 1152*(-5/8 - 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)
/50 + 2207/22) - (9*x**4 + 1)/(6*x**6)
```

Giac [A] time = 1.16999, size = 140, normalized size = 1.44

$$-\frac{11}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{9x^4+1}{6x^6}-\frac{5}{8}\log\left(\left|x^4+x^2-1\right|\right)+\frac{5}{8}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -11/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40*
sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x^4
 + 1)/x^6 - 5/8*log(abs(x^4 + x^2 - 1)) + 5/8*log(abs(x^4 - x^2 - 1))
```

3.396 $\int \frac{x^8}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=170

$$x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+}}\right.}{2\sqrt{5}}$$

```
[Out] x - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqr
t[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sq
rt[5]) - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/
(2*Sqrt[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]
)/(4*Sqrt[5])
```

Rubi [A] time = 0.114406, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1367, 1422, 212, 206, 203}

$$x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+}}\right.}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] x - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqr
t[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sq
rt[5]) - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/
(2*Sqrt[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]
)/(4*Sqrt[5])
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{1-3x^4+x^8}\,dx &= x-\int \frac{1-3x^4}{1-3x^4+x^8}\,dx\\ &= x-\frac{1}{10}\left(-15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\\ &= x+\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx+\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx-\frac{\left(-15-\right.}{}\\ &= x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123+\right.}}{}\end{aligned}$$

Mathematica [A] time = 0.274404, size = 160, normalized size = 0.94

$$x+\frac{\left(\sqrt{5}-2\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(2+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(\sqrt{5}-2\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(2+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\right.}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] x + ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5]
)] - ((2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5])]
 + ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5]
)] - ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5])
]
```

Maple [A] time = 0.053, size = 205, normalized size = 1.2

$$x-\frac{2\sqrt{5}}{5\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)-\frac{1}{\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)-\frac{2\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8-3*x^4+1),x)
```

```
[Out] x-2/5*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))-1/(2+2*5
^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))-2/5*5^(1/2)/(-2+2*5^(1/2))^(
1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2
+2*5^(1/2))^(1/2))-2/5*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/
2))^(1/2))+1/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))-2/5*5^(
1/2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/(2+2*5^(1/2))^(1
/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x + \frac{1}{2} \int \frac{2x^2+1}{x^4-x^2-1}\,dx - \frac{1}{2} \int \frac{2x^2-1}{x^4+x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] x + 1/2*integrate((2*x^2 + 1)/(x^4 - x^2 - 1), x) - 1/2*integrate((2*x^2 -
1)/(x^4 + x^2 - 1), x)
```

Fricas [B] time = 2.00951, size = 988, normalized size = 5.81

$$-\frac{1}{10}\sqrt{10}\sqrt{5\sqrt{5}+11}\arctan\left(\frac{1}{20}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}+1}\left(2\sqrt{5}\sqrt{2}-5\sqrt{2}\right)-2\sqrt{10}\left(2\sqrt{5}x-5x\right)\right)\sqrt{5\sqrt{5}+11}\right)-\frac{1}{10}\sqrt{1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(5*sqrt(5) + 11)*arctan(1/20*(sqrt(10)*sqrt(2*x^2 + sqrt
(5) + 1)*(2*sqrt(5)*sqrt(2) - 5*sqrt(2)) - 2*sqrt(10)*(2*sqrt(5)*x - 5*x))*
sqrt(5*sqrt(5) + 11)) - 1/10*sqrt(10)*sqrt(5*sqrt(5) - 11)*arctan(1/20*(sqr
t(10)*sqrt(2*x^2 + sqrt(5) - 1)*(2*sqrt(5)*sqrt(2) + 5*sqrt(2)) - 2*sqrt(10
)*(2*sqrt(5)*x + 5*x))*sqrt(5*sqrt(5) - 11)) + 1/40*sqrt(10)*sqrt(5*sqrt(5)
 - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(3*sqrt(5) + 5) + 20*x) - 1/40*sqr
t(10)*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt(5*sqrt(5) - 11)*(3*sqrt(5) +
5) + 20*x) - 1/40*sqrt(10)*sqrt(5*sqrt(5) + 11)*log(sqrt(10)*sqrt(5*sqrt(5)
 + 11)*(3*sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(5*sqrt(5) + 11)*log(-sq
rt(10)*sqrt(5*sqrt(5) + 11)*(3*sqrt(5) - 5) + 20*x) + x
```

Sympy [A] time = 0.921476, size = 58, normalized size = 0.34

$$x + \operatorname{RootSum}\left(6400t^4 - 880t^2 - 1, \left(t \mapsto t\log\left(-\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4 + 880t^2 - 1, \left(t \mapsto t\,\text{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-3*x**4+1),x)
```

```
[Out] x + RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(-15360*_t**5/11 +
 1288*_t/55 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*log(
-15360*_t**5/11 + 1288*_t/55 + x)))
```

Giac [A] time = 1.22195, size = 200, normalized size = 1.18

$$-\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(
50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5)
+ 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*
log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*log(abs
(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) - 110)*log(abs(x - sq
rt(1/2*sqrt(5) - 1/2))) + x
```

3.397 $\int \frac{x^6}{1-3x^4+x^8} dx$

Optimal. Leaf size=167

$$\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{144-64\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{144-64}}{}$$

```
[Out] ((3 + Sqrt[5])^(3/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt[5])
 - ((144 - 64*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqrt[5])
 - ((3 + Sqrt[5])^(3/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt
[5]) + ((144 - 64*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqr
t[5])
```

Rubi [A] time = 0.0783492, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1374, 298, 203, 206}

$$\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{144-64\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{144-64}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((3 + Sqrt[5])^(3/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt[5])
 - ((144 - 64*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqrt[5])
 - ((3 + Sqrt[5])^(3/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt
[5]) + ((144 - 64*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqr
t[5])
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx + \frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}} - \frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}} + \frac{\left(3+\sqrt{5}\right)\int}{2}$$

$$= \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\left(3-\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\right.}{2\,2^{3/4}\sqrt{5}}$$

Mathematica [A] time = 0.150366, size = 160, normalized size = 0.96

$$\frac{\frac{\left(\sqrt{5}-3\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{\sqrt{5}-1}} + \frac{\left(3+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}} - \frac{\left(\sqrt{5}-3\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{\sqrt{5}-1}} - \frac{\left(3+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] (((-3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] + ((3
 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] - ((-3 + Sqr
t[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((3 + Sqrt[5]
)*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

Maple [A] time = 0.024, size = 206, normalized size = 1.2

$$-\frac{1}{2\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right) - \frac{3\sqrt{5}}{10\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right) + \frac{1}{2\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8-3*x^4+1),x)
```

```
[Out] -1/2/(2+2*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))-3/10*5^(1/2)/(2+2
*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))+1/2/(-2+2*5^(1/2))^(1/2)*a
rctan(2*x/(-2+2*5^(1/2))^(1/2))-3/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*
x/(-2+2*5^(1/2))^(1/2))-1/2/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))
^(1/2))+3/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))
+1/2/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))+3/10*5^(1/2)/(2+2*
5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{x^8 - 3x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 - 3*x^4 + 1), x)
```

Fricas [B] time = 1.96073, size = 844, normalized size = 5.05

$$\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}+2}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}+1}\left(\sqrt{5}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{5}+2}-\frac{1}{2}\left(\sqrt{5}x-3x\right)\sqrt{\sqrt{5}+2}\right)+\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}-2}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*sqrt(2*x^2 + sqrt(5) + 1)*(sqrt(5)
*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(5) + 2) - 1/2*(sqrt(5)*x - 3*x)*sqrt(sqrt(5
) + 2)) + 1/5*sqrt(5)*sqrt(sqrt(5) - 2)*arctan(1/4*sqrt(2*x^2 + sqrt(5) - 1
)*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(5) - 2) - 1/2*(sqrt(5)*x + 3*x)*s
qrt(sqrt(5) - 2)) - 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(sqrt(sqrt(5) + 2)*(s
qrt(5) - 1) + 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(-sqrt(sqrt(5) + 2)*
(sqrt(5) - 1) + 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log((sqrt(5) + 1)*sqr
t(sqrt(5) - 2) + 2*x) - 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(sqrt(5) + 1)*s
qrt(sqrt(5) - 2) + 2*x)
```

Sympy [A] time = 0.896756, size = 53, normalized size = 0.32

$$\mathrm{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right)+\mathrm{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(-1792000*_t**7 + 4920
*_t**3 + x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(-1792
000*_t**7 + 4920*_t**3 + x)))
```

Giac [A] time = 1.21738, size = 198, normalized size = 1.19

$$\frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(5*s
qrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) + 10)*
log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x
 - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x + sqrt(1
/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x - sqrt(1/2*sqrt(5
) - 1/2)))
```

3.398 $\int \frac{x^4}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=173

$$-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3\right.}}{}$$

```
[Out] -(((3 + Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) +
(((3 - Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (
((3 + Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (
((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rubi [A] time = 0.075094, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1374, 212, 206, 203}

$$-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(((3 + Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) +
(((3 - Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (
((3 + Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (
((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$
$$= \frac{1}{2}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx+\frac{1}{2}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx-\frac{1}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{5}\right.}$$
$$= -\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\right.}{2\sqrt{5}}$$

Mathematica [A] time = 0.196874, size = 132, normalized size = 0.76

$$\frac{\sqrt{\sqrt{5}-1}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-\sqrt{1+\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)+\sqrt{\sqrt{5}-1}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-\sqrt{1+\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] (Sqrt[-1 + Sqrt[5]]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[1 + Sqrt[5]]*Ar
cTan[Sqrt[2/(1 + Sqrt[5])]*x] + Sqrt[-1 + Sqrt[5]]*ArcTanh[Sqrt[2/(-1 + Sqr
t[5])]*x] - Sqrt[1 + Sqrt[5]]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10]
)
```

Maple [A] time = 0.034, size = 206, normalized size = 1.2

$$-\frac{1}{2\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)-\frac{\sqrt{5}}{10\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)+\frac{1}{2\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8-3*x^4+1),x)
```

```
[Out] -1/2/(2+2*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))-1/10*5^(1/2)/(2+2
*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))+1/2/(-2+2*5^(1/2))^(1/2)*a
rctan(2*x/(-2+2*5^(1/2))^(1/2))-1/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*
x/(-2+2*5^(1/2))^(1/2))+1/2/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))
^(1/2))-1/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))
-1/2/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/10*5^(1/2)/(2+2*
5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{x^8-3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 - 3*x^4 + 1), x)
```

Fricas [B] time = 1.9188, size = 932, normalized size = 5.39

$$-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2x^2+\sqrt{5}+1}\left(\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{5}+1}-\frac{1}{20}\sqrt{10}\left(\sqrt{5}x-5x\right)\sqrt{\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(1/40*sqrt(10)*sqrt(2*x^2 + sqrt(5)
+ 1)*(sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 1) - 1/20*sqrt(10)*(sqrt(
5)*x - 5*x)*sqrt(sqrt(5) + 1)) - 1/10*sqrt(10)*sqrt(sqrt(5) - 1)*arctan(1/4
0*sqrt(10)*sqrt(2*x^2 + sqrt(5) - 1)*(sqrt(5)*sqrt(2) + 5*sqrt(2))*sqrt(sqr
t(5) - 1) - 1/20*sqrt(10)*(sqrt(5)*x + 5*x)*sqrt(sqrt(5) - 1)) - 1/40*sqrt(
10)*sqrt(sqrt(5) + 1)*log(sqrt(10)*sqrt(5)*sqrt(sqrt(5) + 1) + 10*x) + 1/40
*sqrt(10)*sqrt(sqrt(5) + 1)*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(5) + 1) + 10*x)
 + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*sqrt(5)*sqrt(sqrt(5) - 1) +
 10*x) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(5)
 - 1) + 10*x)
```

Sympy [A] time = 0.883801, size = 49, normalized size = 0.28

$$\operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(-51200t^5+12t+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(-51200\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 + 12*_t +
 x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 +
 12*_t + x)))
```

Giac [A] time = 1.24809, size = 198, normalized size = 1.14

$$-\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(10*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(1
0*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5) +
10)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(
```

```
abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*log(abs(x +
sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) - 10)*log(abs(x - sqrt(1/2
*sqrt(5) - 1/2)))
```

3.399 $\int \frac{x^2}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=145

$$\frac{1}{20}\sqrt{10\sqrt{5}-10}\tan^{-1}\left(\frac{1}{2}\sqrt{2\sqrt{5}-2}x\right)-\frac{1}{20}\sqrt{10+10\sqrt{5}}\tan^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}x\right)-\frac{1}{20}\sqrt{10\sqrt{5}-10}\tanh^{-1}\left(\frac{1}{2}\sqrt{2\sqrt{5}-\right.$$

```
[Out] (Sqrt[-10 + 10*Sqrt[5]]*ArcTan[(Sqrt[-2 + 2*Sqrt[5]]*x)/2])/20 - (Sqrt[10 +
 10*Sqrt[5]]*ArcTan[(Sqrt[2 + 2*Sqrt[5]]*x)/2])/20 - (Sqrt[-10 + 10*Sqrt[5]
]*ArcTanh[(Sqrt[-2 + 2*Sqrt[5]]*x)/2])/20 + (Sqrt[10 + 10*Sqrt[5]]*ArcTanh[
(Sqrt[2 + 2*Sqrt[5]]*x)/2])/20
```

Rubi [A] time = 0.0599003, antiderivative size = 166, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1375, 298, 203, 206}

$$\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}+\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right.\right.}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^(1/4)) - (
((3 + Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - Ar
cTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^(1/4)) + ((
(3 + Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \frac{\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$
$$= \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{10}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{10}} - \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{10}} + \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{10}}$$
$$= \frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} + \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan$$

Mathematica [A] time = 0.0451395, size = 131, normalized size = 0.9

$$-\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])]) + ArcTan[Sqrt[2
/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*
x]/Sqrt[10*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 +
Sqrt[5])]
```

Maple [A] time = 0.02, size = 110, normalized size = 0.8

$$-\frac{\sqrt{5}}{5\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right) - \frac{\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right) + \frac{\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8-3*x^4+1),x)
```

```
[Out] -1/5*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))-1/5*5^(1/
2)/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))+1/5*5^(1/2)/(-2+2*
5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+1/5*5^(1/2)/(2+2*5^(1/2))^
(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 - 3*x^4 + 1), x)
```

Fricas [B] time = 1.92728, size = 903, normalized size = 6.23

$$\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}+1}-\frac{1}{10}\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}-1}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(1/20*sqrt(10)*sqrt(5)*sqrt(2)*sqrt(2
*x^2 + sqrt(5) - 1)*sqrt(sqrt(5) + 1) - 1/10*sqrt(10)*sqrt(5)*x*sqrt(sqrt(5
) + 1)) - 1/10*sqrt(10)*sqrt(sqrt(5) - 1)*arctan(1/20*sqrt(10)*sqrt(5)*sqrt
(2)*sqrt(2*x^2 + sqrt(5) + 1)*sqrt(sqrt(5) - 1) - 1/10*sqrt(10)*sqrt(5)*x*s
qrt(sqrt(5) - 1)) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*(sqrt(5) +
 5)*sqrt(sqrt(5) - 1) + 20*x) + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(1
0)*(sqrt(5) + 5)*sqrt(sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) + 1
)*log(sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt
(sqrt(5) + 1)*log(-sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x)
```

Sympy [A] time = 0.891353, size = 53, normalized size = 0.37

$$\operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(6144000t^7-2240t^3+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(614\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(6144000*_t**7 - 2240*_
t**3 + x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(6144000*
_t**7 - 2240*_t**3 + x)))
```

Giac [A] time = 1.21715, size = 198, normalized size = 1.37

$$\frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(10
*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5) - 1
0)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*log(a
bs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x + s
qrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x - sqrt(1/2*
sqrt(5) - 1/2)))
```

3.400 $\int \frac{1}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=169

$$-\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\,2^{3/4}\sqrt{5}}$$

```
[Out] -(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4)))
+ ((3 + Sqrt[5])^(3/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sqrt[5
]) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4
)) + ((3 + Sqrt[5])^(3/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sq
rt[5])
```

Rubi [A] time = 0.0596203, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1347, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^4 + x^8)^(-1),x]
```

```
[Out] -(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4)))
+ ((3 + Sqrt[5])^(3/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sqrt[5
]) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4
)) + ((3 + Sqrt[5])^(3/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sq
rt[5])
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1-3x^4+x^8}\,dx = \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{5\left(3-\sqrt{5}\right)}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{5\left(3+\sqrt{5}\right)}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{5\left(3+\sqrt{5}\right)}}$$

$$= -\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\,\mathrm{t}}{}$$

Mathematica [A] time = 0.17215, size = 160, normalized size = 0.95

$$\frac{\frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{\sqrt{5}-1}} - \frac{\left(\sqrt{5}-1\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}} + \frac{\left(1+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{\sqrt{5}-1}} - \frac{\left(\sqrt{5}-1\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x^4 + x^8)^(-1),x]
```

```
[Out] (((1 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-1
 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] + ((1 + Sqrt
[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-1 + Sqrt[5]
)*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

Maple [A] time = 0.021, size = 206, normalized size = 1.2

$$-\frac{1}{2\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right) + \frac{\sqrt{5}}{10\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right) + \frac{1}{2\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-3*x^4+1),x)
```

```
[Out] -1/2/(2+2*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))+1/10*5^(1/2)/(2+2
*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))+1/2/(-2+2*5^(1/2))^(1/2)*a
rctan(2*x/(-2+2*5^(1/2))^(1/2))+1/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*
x/(-2+2*5^(1/2))^(1/2))+1/2/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))
^(1/2))+1/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))
-1/2/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))+1/10*5^(1/2)/(2+2*
5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^8 - 3x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - 3*x^4 + 1), x)
```

Fricas [B] time = 1.87717, size = 834, normalized size = 4.93

$$-\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}+2}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}-1}\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+2}-\frac{1}{2}\left(\sqrt{5}x-x\right)\sqrt{\sqrt{5}+2}\right)+\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}-2}\arctan\left(\frac{1}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*sqrt(2*x^2 + sqrt(5) - 1)*(sqrt(5
)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 2) - 1/2*(sqrt(5)*x - x)*sqrt(sqrt(5) +
 2)) + 1/5*sqrt(5)*sqrt(sqrt(5) - 2)*arctan(1/4*sqrt(2*x^2 + sqrt(5) + 1)*(
sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 2) - 1/2*(sqrt(5)*x + x)*sqrt(sqr
t(5) - 2)) - 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log((sqrt(5) + 3)*sqrt(sqrt(5)
- 2) + 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(sqrt(5) + 3)*sqrt(sqrt(5
) - 2) + 2*x) - 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(sqrt(sqrt(5) + 2)*(sqrt(
5) - 3) + 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(-sqrt(sqrt(5) + 2)*(sqr
t(5) - 3) + 2*x)
```

Sympy [A] time = 0.90778, size = 53, normalized size = 0.31

$$\operatorname{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(9600t^5-\frac{47t}{2}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(9600t^5-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5 - 47*_t/2
+ x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5 -
 47*_t/2 + x)))
```

Giac [A] time = 1.20588, size = 198, normalized size = 1.17

$$-\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/10*sqrt(5*
sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) - 10)
*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(abs(
x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x + sqrt(
1/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x - sqrt(1/2*sqrt(
5) - 1/2)))
```

3.401 $\int \frac{1}{x^2(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=172

$$-\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{984-440\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}}+\frac{(3}{}$$

```
[Out] -x^(-1) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(4*
Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*2^(1/
4)*Sqrt[5]) - ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]
)/(4*Sqrt[5]) + ((3 + Sqrt[5])^(5/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4
*2^(1/4)*Sqrt[5])
```

Rubi [A] time = 0.0887916, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1368, 1510, 298, 203, 206}

$$-\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{984-440\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}}+\frac{(3}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(4*
Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*2^(1/
4)*Sqrt[5]) - ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]
)/(4*Sqrt[5]) + ((3 + Sqrt[5])^(5/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4
*2^(1/4)*Sqrt[5])
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(1-3x^4+x^8\right)}\,dx &= -\frac{1}{x}+\int \frac{x^2\left(3-x^4\right)}{1-3x^4+x^8}\,dx\\ &= -\frac{1}{x}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx\\ &= -\frac{1}{x}-\frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(3+\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}\\ &= -\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{984-44}}{}\end{aligned}$$

Mathematica [A] time = 0.270837, size = 174, normalized size = 1.01

$$-\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(\sqrt{5}-3\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(3+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\left(\sqrt{5}-3\right)\tanh^{-1}\left(\sqrt{\right.}\right.}{2\sqrt{10\left(1+\sqrt{5}\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) - ((3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[10*(-1 +
 Sqrt[5])]) - ((-3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(
1 + Sqrt[5])]) + ((3 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[
10*(-1 + Sqrt[5])]) + ((-3 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*
Sqrt[10*(1 + Sqrt[5])])
```

Maple [A] time = 0.024, size = 211, normalized size = 1.2

$$\frac{1}{2\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)-\frac{3\sqrt{5}}{10\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)-\frac{1}{2\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8-3*x^4+1),x)
```

```
[Out] 1/2/(2+2*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))-3/10*5^(1/2)/(2+2*
5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))-1/2/(-2+2*5^(1/2))^(1/2)*ar
ctan(2*x/(-2+2*5^(1/2))^(1/2))-3/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*x
/(-2+2*5^(1/2))^(1/2))+1/2/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^
(1/2))+3/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))-
1/2/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))+3/10*5^(1/2)/(2+2*5
^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x}-\frac{1}{2}\int\frac{x^2+2}{x^4+x^2-1}\,dx-\frac{1}{2}\int\frac{x^2-2}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - 1/2*integrate((x^2 + 2)/(x^4 + x^2 - 1), x) - 1/2*integrate((x^2 - 2
)/(x^4 - x^2 - 1), x)
```

Fricas [B] time = 1.93412, size = 981, normalized size = 5.7

$$\frac{4\sqrt{10}x\sqrt{5\sqrt{5}+11}\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}-1}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)-2\sqrt{10}\left(3\sqrt{5}x-5x\right)\right)\sqrt{5\sqrt{5}+11}\right)-4\sqrt{10}x\sqrt{5\sqrt{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(4*sqrt(10)*x*sqrt(5*sqrt(5) + 11)*arctan(1/40*(sqrt(10)*sqrt(2*x^2 +
sqrt(5) - 1)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2)) - 2*sqrt(10)*(3*sqrt(5)*x - 5*
x))*sqrt(5*sqrt(5) + 11)) - 4*sqrt(10)*x*sqrt(5*sqrt(5) - 11)*arctan(1/40*(
sqrt(10)*sqrt(2*x^2 + sqrt(5) + 1)*(3*sqrt(5)*sqrt(2) + 5*sqrt(2)) - 2*sqrt
(10)*(3*sqrt(5)*x + 5*x))*sqrt(5*sqrt(5) - 11)) - sqrt(10)*x*sqrt(5*sqrt(5)
 - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(2*sqrt(5) + 5) + 10*x) + sqrt(10)
*x*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt(5*sqrt(5) - 11)*(2*sqrt(5) + 5)
+ 10*x) - sqrt(10)*x*sqrt(5*sqrt(5) + 11)*log(sqrt(10)*sqrt(5*sqrt(5) + 11)
*(2*sqrt(5) - 5) + 10*x) + sqrt(10)*x*sqrt(5*sqrt(5) + 11)*log(-sqrt(10)*sq
rt(5*sqrt(5) + 11)*(2*sqrt(5) - 5) + 10*x) - 40)/x
```

Sympy [A] time = 0.94756, size = 63, normalized size = 0.37

$$\operatorname{RootSum}\left(6400t^4-880t^2-1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+880t^2-1,\left(t\mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(19251200*_t**7/11 - 3
69792*_t**3/11 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*l
```

```
og(19251200*_t**7/11 - 369792*_t**3/11 + x))) - 1/x
```

Giac [A] time = 1.23243, size = 205, normalized size = 1.19

$$\frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x+\sqrt{\frac{1}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(5
0*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5) -
 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*l
og(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*log(abs(
x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) + 110)*log(abs(x - sqr
t(1/2*sqrt(5) - 1/2))) - 1/x
```

3.402 $\int \frac{1}{x^4(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=182

$$-\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\cdots}}\right.}{2\sqrt{5}}$$

```
[Out] -1/(3*x^3) - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*
x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(
4*2^(3/4)*Sqrt[5]) - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]
))^(1/4)*x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTanh[((3 + Sqrt[5])/2)^(
1/4)*x])/(4*2^(3/4)*Sqrt[5])
```

Rubi [A] time = 0.117854, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1368, 1422, 212, 206, 203}

$$-\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\cdots}}\right.}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*
x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(
4*2^(3/4)*Sqrt[5]) - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]
))^(1/4)*x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTanh[((3 + Sqrt[5])/2)^(
1/4)*x])/(4*2^(3/4)*Sqrt[5])
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx &= -\frac{1}{3x^3}+\frac{1}{3}\int \frac{9-3x^4}{1-3x^4+x^8}\,dx\\ &= -\frac{1}{3x^3}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx\\ &= -\frac{1}{3x^3}+\frac{\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}+\frac{\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}+\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{1}{\sqrt{3-\sqrt{5}-}}}{4\sqrt{5}}\\ &= -\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(843+377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.267259, size = 166, normalized size = 0.91

$$-\frac{1}{3x^3}+\frac{\left(2+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(\sqrt{5}-2\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(2+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(\sqrt{5}-2\right)\tanh^{-1}\left(\right.}{\sqrt{10\left(1+\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) + ((2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 +
 Sqrt[5])] - ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 +
Sqrt[5])] + ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 +
 Sqrt[5])] - ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 +
 Sqrt[5])]
```

Maple [A] time = 0.029, size = 209, normalized size = 1.2

$$\frac{2\sqrt{5}}{5\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)-\frac{1}{\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)+\frac{2\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8-3*x^4+1),x)
```

```
[Out] 2/5*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))-1/(2+2*5^(
1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))+2/5*5^(1/2)/(-2+2*5^(1/2))^(1/
2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2+2
*5^(1/2))^(1/2))+2/5*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2)
)^(1/2))+1/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+2/5*5^(1/
2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/(2+2*5^(1/2))^(1/2
)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/3/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3\,x^3}-\frac{1}{2}\int \frac{2\,x^2+3}{x^4+x^2-1}\,dx+\frac{1}{2}\int \frac{2\,x^2-3}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - 1/2*integrate((2*x^2 + 3)/(x^4 + x^2 - 1), x) + 1/2*integrate((2
*x^2 - 3)/(x^4 - x^2 - 1), x)
```

Fricas [B] time = 1.90534, size = 1037, normalized size = 5.7

$$12\,\sqrt{10}x^3\sqrt{13\,\sqrt{5}+29}\,\arctan\left(\frac{1}{20}\left(\sqrt{10}\sqrt{2\,x^2+\sqrt{5}-1}\left(2\,\sqrt{5}\sqrt{2}-5\,\sqrt{2}\right)-2\,\sqrt{10}\left(2\,\sqrt{5}x-5\,x\right)\right)\sqrt{13\,\sqrt{5}+29}\right)+12\,\sqrt{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(12*sqrt(10)*x^3*sqrt(13*sqrt(5) + 29)*arctan(1/20*(sqrt(10)*sqrt(2*x
^2 + sqrt(5) - 1)*(2*sqrt(5)*sqrt(2) - 5*sqrt(2)) - 2*sqrt(10)*(2*sqrt(5)*x
 - 5*x))*sqrt(13*sqrt(5) + 29)) + 12*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29)*arc
tan(1/20*(sqrt(10)*sqrt(2*x^2 + sqrt(5) + 1)*(2*sqrt(5)*sqrt(2) + 5*sqrt(2)
) - 2*sqrt(10)*(2*sqrt(5)*x + 5*x))*sqrt(13*sqrt(5) - 29)) - 3*sqrt(10)*x^3
*sqrt(13*sqrt(5) - 29)*log(sqrt(10)*sqrt(13*sqrt(5) - 29)*(7*sqrt(5) + 15)
+ 20*x) + 3*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29)*log(-sqrt(10)*sqrt(13*sqrt(5
) - 29)*(7*sqrt(5) + 15) + 20*x) + 3*sqrt(10)*x^3*sqrt(13*sqrt(5) + 29)*log
(sqrt(10)*sqrt(13*sqrt(5) + 29)*(7*sqrt(5) - 15) + 20*x) - 3*sqrt(10)*x^3*s
qrt(13*sqrt(5) + 29)*log(-sqrt(10)*sqrt(13*sqrt(5) + 29)*(7*sqrt(5) - 15) +
 20*x) - 40)/x^3
```

Sympy [A] time = 0.974728, size = 63, normalized size = 0.35

$$\operatorname{RootSum}\left(6400t^4-2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+2320t^2-1,\left(t\mapsto t\log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 2320*_t**2 - 1, Lambda(_t, _t*log(179200*_t**5/377 - 2
3112*_t/377 + x))) + RootSum(6400*_t**4 + 2320*_t**2 - 1, Lambda(_t, _t*log
(179200*_t**5/377 - 23112*_t/377 + x))) - 1/(3*x**3)
```

Giac [A] time = 1.22524, size = 205, normalized size = 1.13

$$-\frac{1}{20}\sqrt{130\sqrt{5}-290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{130\sqrt{5}+290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{130\sqrt{5}-290}\log\left(\left|x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(130*sqrt(5) - 290)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt
(130*sqrt(5) + 290)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(130*sqrt(
5) - 290)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5) - 2
90)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5) + 290)*lo
g(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(130*sqrt(5) + 290)*log(abs(
x - sqrt(1/2*sqrt(5) - 1/2))) - 1/3/x^3
```

3.403 $\int \frac{1}{x^6(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=173

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{2889-1292\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889+1292\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889-1292\sqrt{5}}\tanh^{-1}}{2\sqrt{5}}$$

```
[Out] -1/(5*x^5) - 3/x + ((2889 - 1292*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1
/4)*x])/(2*Sqrt[5]) - ((2889 + 1292*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)
^(1/4)*x])/(2*Sqrt[5]) - ((2889 - 1292*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[
5]))^(1/4)*x])/(2*Sqrt[5]) + ((2889 + 1292*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqr
t[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rubi [A] time = 0.14026, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1368, 1504, 1510, 298, 203, 206}

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{2889-1292\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889+1292\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889-1292\sqrt{5}}\tanh^{-1}}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - 3*x^4 + x^8)),x]
```


```
[Out] -1/(5*x^5) - 3/x + ((2889 - 1292*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1
/4)*x])/(2*Sqrt[5]) - ((2889 + 1292*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)
^(1/4)*x])/(2*Sqrt[5]) - ((2889 - 1292*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[
5]))^(1/4)*x])/(2*Sqrt[5]) + ((2889 + 1292*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqr
t[5])/2)^(1/4)*x])/(2*Sqrt[5])
```


Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
```

```
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(1-3x^4+x^8\right)}\,dx &= -\frac{1}{5x^5}+\frac{1}{5}\int \frac{15-5x^4}{x^2\left(1-3x^4+x^8\right)}\,dx\\
&= -\frac{1}{5x^5}-\frac{3}{x}-\frac{1}{5}\int \frac{x^2\left(-40+15x^4\right)}{1-3x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}-\frac{3}{x}-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx\\
&= -\frac{1}{5x^5}-\frac{3}{x}-\frac{\left(7-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(7-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(7+3\sqrt{5}\right)\int \frac{}{\sqrt{3-}}}{2\sqrt{10}}\\
&= -\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{46224-20672\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{46224+20672\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\right.}\right.}{4\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.293554, size = 189, normalized size = 1.09

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\left(-7-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\left(7-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\left(-7-3\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(7-3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - 3/x + ((-7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*S
qrt[10*(-1 + Sqrt[5])]) + ((7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])
/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5
])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(1 +
Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

Maple [A] time = 0.033, size = 216, normalized size = 1.3

$$-\frac{7\sqrt{5}}{10\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)+\frac{3}{2\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)-\frac{7\sqrt{5}}{10\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8-3*x^4+1),x)
```

```
[Out] -7/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))+3/2/(2+2
*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))-7/10*5^(1/2)/(-2+2*5^(1/2)
)^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))-3/2/(-2+2*5^(1/2))^(1/2)*arctan(2*
x/(-2+2*5^(1/2))^(1/2))-1/5/x^5-3/x+7/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arcta
nh(2*x/(-2+2*5^(1/2))^(1/2))+3/2/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(
1/2))^(1/2))+7/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2
))-3/2/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{15\,x^4+1}{5\,x^5}-\frac{1}{2}\int\frac{3\,x^2+5}{x^4+x^2-1}\,dx-\frac{1}{2}\int\frac{3\,x^2-5}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*(15*x^4 + 1)/x^5 - 1/2*integrate((3*x^2 + 5)/(x^4 + x^2 - 1), x) - 1/2
*integrate((3*x^2 - 5)/(x^4 - x^2 - 1), x)
```

Fricas [B] time = 1.94677, size = 913, normalized size = 5.28

$$-\frac{4\sqrt{5}x^5\sqrt{17\sqrt{5}+38}\arctan\left(\frac{1}{4}\left(\sqrt{2\,x^2+\sqrt{5}-1}\left(3\sqrt{5}\sqrt{2}-7\sqrt{2}\right)-6\sqrt{5}x+14\,x\right)\sqrt{17\sqrt{5}+38}\right)+4\sqrt{5}x^5\sqrt{17\sqrt{5}-38}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/20*(4*sqrt(5)*x^5*sqrt(17*sqrt(5) + 38)*arctan(1/4*(sqrt(2*x^2 + sqrt(5)
 - 1)*(3*sqrt(5)*sqrt(2) - 7*sqrt(2)) - 6*sqrt(5)*x + 14*x)*sqrt(17*sqrt(5)
 + 38)) + 4*sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*arctan(1/4*(sqrt(2*x^2 + sqrt
(5) + 1)*(3*sqrt(5)*sqrt(2) + 7*sqrt(2)) - 6*sqrt(5)*x - 14*x)*sqrt(17*sqrt
(5) - 38)) + sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*log(sqrt(17*sqrt(5) - 38)*(5
*sqrt(5) + 11) + 2*x) - sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*log(-sqrt(17*sqrt
(5) - 38)*(5*sqrt(5) + 11) + 2*x) - sqrt(5)*x^5*sqrt(17*sqrt(5) + 38)*log(s
qrt(17*sqrt(5) + 38)*(5*sqrt(5) - 11) + 2*x) + sqrt(5)*x^5*sqrt(17*sqrt(5)
+ 38)*log(-sqrt(17*sqrt(5) + 38)*(5*sqrt(5) - 11) + 2*x) + 60*x^4 + 4)/x^5
```

Sympy [A] time = 0.98265, size = 71, normalized size = 0.41

$$\operatorname{RootSum}\left(6400t^4-6080t^2-1,\left(t\mapsto t\log\left(\frac{215808000t^7}{323}-\frac{194833880t^3}{323}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+6080t^2-1,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 6080*_t**2 - 1, Lambda(_t, _t*log(215808000*_t**7/323
- 194833880*_t**3/323 + x))) + RootSum(6400*_t**4 + 6080*_t**2 - 1, Lambda(
_t, _t*log(215808000*_t**7/323 - 194833880*_t**3/323 + x))) - (15*x**4 + 1)
/(5*x**5)
```

Giac [A] time = 1.22727, size = 215, normalized size = 1.24

$$\frac{1}{10}\sqrt{85\sqrt{5}-190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{10}\sqrt{85\sqrt{5}+190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{85\sqrt{5}-190}\log\left(\left|x+\sqrt{\frac{1}{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(85*sqrt(5) - 190)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(8
5*sqrt(5) + 190)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(85*sqrt(5) -
 190)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) - 190)*l
og(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) + 190)*log(abs(
x + sqrt(1/2*sqrt(5) - 1/2))) - 1/20*sqrt(85*sqrt(5) + 190)*log(abs(x - sqr
t(1/2*sqrt(5) - 1/2))) - 1/5*(15*x^4 + 1)/x^5
```

3.404 $\int \frac{1}{x^8(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=189

$$-\frac{1}{x^3}-\frac{1}{7x^7}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}}(39603}{}$$

```
[Out] -1/(7*x^7) - x^(-3) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqr
t[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])/2)^(1/4)*ArcTan[((
3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*
ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])
/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rubi [A] time = 0.162722, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1368, 1504, 1422, 212, 206, 203}

$$-\frac{1}{x^3}-\frac{1}{7x^7}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}}(39603}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) - x^(-3) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqr
t[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])/2)^(1/4)*ArcTan[((
3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*
ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])
/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
```

```
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8\left(1-3x^4+x^8\right)}\,dx &= -\frac{1}{7x^7}+\frac{1}{7}\int \frac{21-7x^4}{x^4\left(1-3x^4+x^8\right)}\,dx\\
&= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{1}{21}\int \frac{-168+63x^4}{1-3x^4+x^8}\,dx\\
&= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx\\
&= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\left(-15+7\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}-\frac{\left(-15+7\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}+\frac{1}{2}\sqrt{\frac{1}{5}}\,(123\\
&= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tan^{-1}\left(\right.}{2\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.275646, size = 189, normalized size = 1.

$$-\frac{1}{x^3}-\frac{1}{7x^7}+\frac{\left(11+5\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\left(11-5\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\left(-11-5\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{(5\sqrt{5}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) - x^(-3) + ((11 + 5*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(
2*Sqrt[10*(-1 + Sqrt[5])]) + ((11 - 5*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]
*x])/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-11 - 5*Sqrt[5])*ArcTanh[Sqrt[2/(-1 + S
qrt[5])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((-11 + 5*Sqrt[5])*ArcTanh[Sqrt[
```

```
2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

Maple [A] time = 0.036, size = 216, normalized size = 1.1

$$\frac{11\sqrt{5}}{10\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)-\frac{5}{2\sqrt{2+2\sqrt{5}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)+\frac{11\sqrt{5}}{10\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8-3*x^4+1),x)
```

```
[Out] 11/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))-5/2/(2+2
*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))+11/10*5^(1/2)/(-2+2*5^(1/2
))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))+5/2/(-2+2*5^(1/2))^(1/2)*arctan(2
*x/(-2+2*5^(1/2))^(1/2))-1/7/x^7-1/x^3+11/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*a
rctanh(2*x/(-2+2*5^(1/2))^(1/2))+5/2/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2
*5^(1/2))^(1/2))+11/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))
^(1/2))-5/2/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{7x^4+1}{7x^7}-\frac{1}{2}\int\frac{5x^2+8}{x^4+x^2-1}\,dx+\frac{1}{2}\int\frac{5x^2-8}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/7*(7*x^4 + 1)/x^7 - 1/2*integrate((5*x^2 + 8)/(x^4 + x^2 - 1), x) + 1/2*
integrate((5*x^2 - 8)/(x^4 - x^2 - 1), x)
```

Fricas [B] time = 1.95542, size = 1077, normalized size = 5.7

$$28\sqrt{10}x^7\sqrt{89\sqrt{5}+199}\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}-1}\left(11\sqrt{5}\sqrt{2}-25\sqrt{2}\right)-2\sqrt{10}\left(11\sqrt{5}x-25x\right)\right)\sqrt{89\sqrt{5}+199}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/280*(28*sqrt(10)*x^7*sqrt(89*sqrt(5) + 199)*arctan(1/40*(sqrt(10)*sqrt(2*
x^2 + sqrt(5) - 1)*(11*sqrt(5)*sqrt(2) - 25*sqrt(2)) - 2*sqrt(10)*(11*sqrt(
5)*x - 25*x))*sqrt(89*sqrt(5) + 199)) + 28*sqrt(10)*x^7*sqrt(89*sqrt(5) - 1
99)*arctan(1/40*(sqrt(10)*sqrt(2*x^2 + sqrt(5) + 1)*(11*sqrt(5)*sqrt(2) + 2
5*sqrt(2)) - 2*sqrt(10)*(11*sqrt(5)*x + 25*x))*sqrt(89*sqrt(5) - 199)) - 7*
sqrt(10)*x^7*sqrt(89*sqrt(5) - 199)*log(sqrt(10)*sqrt(89*sqrt(5) - 199)*(9*
sqrt(5) + 20) + 10*x) + 7*sqrt(10)*x^7*sqrt(89*sqrt(5) - 199)*log(-sqrt(10)
*sqrt(89*sqrt(5) - 199)*(9*sqrt(5) + 20) + 10*x) + 7*sqrt(10)*x^7*sqrt(89*s
qrt(5) + 199)*log(sqrt(10)*sqrt(89*sqrt(5) + 199)*(9*sqrt(5) - 20) + 10*x)
- 7*sqrt(10)*x^7*sqrt(89*sqrt(5) + 199)*log(-sqrt(10)*sqrt(89*sqrt(5) + 199
```

```
)*(9*sqrt(5) - 20) + 10*x) - 280*x^4 - 40)/x^7
```

Sympy [A] time = 0.996892, size = 68, normalized size = 0.36

$$\operatorname{RootSum}\left(6400t^4 - 15920t^2 - 1, \left(t \mapsto t\log\left(\frac{460800t^5}{17711} - \frac{2842588t}{17711} + x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4 + 15920t^2 - 1, \left(t \mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 15920*_t**2 - 1, Lambda(_t, _t*log(460800*_t**5/17711
- 2842588*_t/17711 + x))) + RootSum(6400*_t**4 + 15920*_t**2 - 1, Lambda(_t
, _t*log(460800*_t**5/17711 - 2842588*_t/17711 + x))) - (7*x**4 + 1)/(7*x**
7)
```

Giac [A] time = 1.20188, size = 215, normalized size = 1.14

$$-\frac{1}{20}\sqrt{890\sqrt{5} - 1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5} + \frac{1}{2}}}\right) + \frac{1}{20}\sqrt{890\sqrt{5} + 1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5} - \frac{1}{2}}}\right) - \frac{1}{40}\sqrt{890\sqrt{5} - 1990}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(890*sqrt(5) - 1990)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqr
t(890*sqrt(5) + 1990)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(890*sqr
t(5) - 1990)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(5)
- 1990)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(5) + 199
0)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(890*sqrt(5) + 1990)*lo
g(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/7*(7*x^4 + 1)/x^7
```

3.405 $\int \frac{x^3}{2+3x^4+x^8} \, dx$

Optimal. Leaf size=21

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

```
[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Rubi [A] time = 0.0127365, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 616, 31}

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 + 3*x^4 + x^8),x]

[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{2+3x^4+x^8} \, dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{2+3x+x^2} \, dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, x^4\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{2+x} \, dx, x, x^4\right)\\ &= \frac{1}{4}\log\left(1+x^4\right)-\frac{1}{4}\log\left(2+x^4\right)\end{aligned}$$

Mathematica [A] time = 0.0042694, size = 21, normalized size = 1.

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 + 3*x^4 + x^8),x]
```

```
[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Maple [A] time = 0.006, size = 18, normalized size = 0.9

$$\frac{\ln\left(x^4+1\right)}{4}-\frac{\ln\left(x^4+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+3*x^4+2),x)
```

```
[Out] 1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

Maxima [A] time = 0.969577, size = 23, normalized size = 1.1

$$-\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Fricas [A] time = 1.69313, size = 51, normalized size = 2.43

$$-\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Sympy [A] time = 0.113925, size = 15, normalized size = 0.71

$$\frac{\log\left(x^4+1\right)}{4}-\frac{\log\left(x^4+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+3*x**4+2),x)
```

```
[Out] log(x**4 + 1)/4 - log(x**4 + 2)/4
```

Giac [A] time = 1.1405, size = 23, normalized size = 1.1

$$-\frac{1}{4}\,\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

3.406 $\int \frac{x^{11}}{2+3x^4+x^8} \, dx$

Optimal. Leaf size=26

$$\frac{x^4}{4} + \frac{1}{4}\log\left(x^4+1\right) - \log\left(x^4+2\right)$$

```
[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

Rubi [A] time = 0.0189562, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 703, 632, 31}

$$\frac{x^4}{4} + \frac{1}{4}\log\left(x^4+1\right) - \log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(2 + 3*x^4 + x^8),x]
```

```
[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{2+3x^4+x^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{2+3x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{-2-3x}{2+3x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^4\right)-\operatorname{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{4}\log\left(1+x^4\right)-\log\left(2+x^4\right)\end{aligned}$$

Mathematica [A] time = 0.0046948, size = 26, normalized size = 1.

$$\frac{x^4}{4}+\frac{1}{4}\log\left(x^4+1\right)-\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(2 + 3*x^4 + x^8),x]
```

```
[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

Maple [A] time = 0.006, size = 23, normalized size = 0.9

$$\frac{x^4}{4}+\frac{\ln\left(x^4+1\right)}{4}-\ln\left(x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+3*x^4+2),x)
```

```
[Out] 1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)
```

Maxima [A] time = 1.0044, size = 30, normalized size = 1.15

$$\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Fricas [A] time = 1.6815, size = 58, normalized size = 2.23

$$\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Sympy [A] time = 0.125721, size = 19, normalized size = 0.73

$$\frac{x^4}{4}+\frac{\log\left(x^4+1\right)}{4}-\log\left(x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+3*x**4+2),x)
```

```
[Out] x**4/4 + log(x**4 + 1)/4 - log(x**4 + 2)
```

Giac [A] time = 1.10572, size = 30, normalized size = 1.15

$$\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

3.407 $\int \frac{x^9}{2+x^5+x^{10}} dx$

Optimal. Leaf size=37

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[7]]/(5*Sqrt[7]) + Log[2 + x^5 + x^10]/10
```

Rubi [A] time = 0.0342108, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(2 + x^5 + x^10),x]
```

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[7]]/(5*Sqrt[7]) + Log[2 + x^5 + x^10]/10
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{2+x^5+x^{10}}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{x}{2+x+x^2}\,dx, x, x^5\right)\\
&= -\left(\frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{2+x+x^2}\,dx, x, x^5\right)\right)+\frac{1}{10}\operatorname{Subst}\left(\int \frac{1+2x}{2+x+x^2}\,dx, x, x^5\right)\\
&= \frac{1}{10}\log\left(2+x^5+x^{10}\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, 1+2x^5\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}+\frac{1}{10}\log\left(2+x^5+x^{10}\right)
\end{aligned}$$

Mathematica [A] time = 0.0120461, size = 37, normalized size = 1.

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(2 + x^5 + x^10),x]
```

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[7]]/(5*Sqrt[7]) + Log[2 + x^5 + x^10]/10
```

Maple [A] time = 0.002, size = 31, normalized size = 0.8

$$\frac{\ln\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}}{35}\arctan\left(\frac{\left(2x^5+1\right)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^10+x^5+2),x)
```

```
[Out] 1/10*ln(x^10+x^5+2)-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Maxima [A] time = 1.47833, size = 41, normalized size = 1.11

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="maxima")
```

```
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

Fricas [A] time = 1.66923, size = 100, normalized size = 2.7

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="fricas")
```

```
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

Sympy [A] time = 0.140561, size = 37, normalized size = 1.

$$\frac{\log\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**10+x**5+2),x)
```

```
[Out] log(x**10 + x**5 + 2)/10 - sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

Giac [A] time = 2.65546, size = 41, normalized size = 1.11

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="giac")
```

```
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

3.408 $\int \frac{x^4}{2+x^5+x^{10}} \, dx$

Optimal. Leaf size=23

$$\frac{2\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

```
[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Rubi [A] time = 0.0218787, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1352, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + x^5 + x^10),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{2+x^5+x^{10}} \, dx &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{2+x+x^2} \, dx, x, x^5\right)\\ &= -\left(\frac{2}{5}\text{Subst}\left(\int \frac{1}{-7-x^2} \, dx, x, 1+2x^5\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}\end{aligned}$$

Mathematica [A] time = 0.0060291, size = 23, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + x^5 + x^10),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Maple [A] time = 0.001, size = 19, normalized size = 0.8

$$\frac{2\sqrt{7}}{35}\arctan\left(\frac{(2x^5+1)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^10+x^5+2),x)
```

```
[Out] 2/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Maxima [A] time = 1.51551, size = 24, normalized size = 1.04

$$\frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^5+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="maxima")
```

```
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Fricas [A] time = 1.68778, size = 62, normalized size = 2.7

$$\frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^5+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="fricas")
```

```
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Sympy [A] time = 0.12709, size = 27, normalized size = 1.17

$$\frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**10+x**5+2),x)
```

```
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

Giac [A] time = 2.6525, size = 24, normalized size = 1.04

$$\frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="giac")
```

```
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

3.409 $\int \frac{1}{x(1+x^5+x^{10})} dx$

Optimal. Leaf size=39

$$-\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)$$

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 + x^10]/10
```

Rubi [A] time = 0.0353901, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^5 + x^10)),x]
```

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 + x^10]/10
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x\left(1+x+x^2\right)}\,dx, x, x^5\right)\\
&= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^5\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, x^5\right)\\
&= \log(x)-\frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^5\right)-\frac{1}{10}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^5\right)\\
&= \log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^5\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)
\end{aligned}$$

Mathematica [C] time = 0.0362244, size = 197, normalized size = 5.05

$$-\frac{1}{5}\operatorname{RootSum}\left[\#1^8-\#1^7+\#1^5-\#1^4+\#1^3-\#1+1\&, \frac{4\#1^7\log(x-\#1)-3\#1^6\log(x-\#1)-\#1^5\log(x-\#1)+3\#1^4 \cdots}{8\#1^7-7\#1^6+5 \cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^5 + x^10)),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Root
Sum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*Lo
g[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^5 -
 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 -
7*#1^6 + 8*#1^7) & ]/5
```

Maple [B] time = 0.037, size = 66, normalized size = 1.7

$$-\frac{\ln\left(x^2+x+1\right)}{10}-\frac{\ln\left(4x^8-4x^7+4x^5-4x^4+4x^3-4x+4\right)}{10}-\frac{\sqrt{3}}{15}\arctan\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^10+x^5+1),x)
```

```
[Out] -1/10*ln(x^2+x+1)-1/10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)-1/15*3^(1/2)
*arctan(2/3*3^(1/2)*x^5+1/3*3^(1/2))+ln(x)
```

Maxima [A] time = 1.51733, size = 49, normalized size = 1.26

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\frac{1}{5}\log\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
1/5*log(x^5)
```

Fricas [A] time = 1.70077, size = 112, normalized size = 2.87

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(x)
```

Sympy [A] time = 0.164685, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**10+x**5+1),x)
```

```
[Out] log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3)
/3)/15
```

Giac [A] time = 1.11495, size = 45, normalized size = 1.15

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(abs(x))
```

3.410 $\int \frac{1}{x^6\left(1+x^5+x^{10}\right)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\log(x)$$

```
[Out] -1/(5*x^5) - ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) - Log[x] + Log[1 + x^5
 + x^10]/10
```

Rubi [A] time = 0.0512029, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] -1/(5*x^5) - ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) - Log[x] + Log[1 + x^5
 + x^10]/10
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(1+x^5+x^{10}\right)}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x^2\left(1+x+x^2\right)}\,dx, x, x^5\right)\\
&= -\frac{1}{5x^5}+\frac{1}{5}\,\text{Subst}\left(\int \frac{-1-x}{x\left(1+x+x^2\right)}\,dx, x, x^5\right)\\
&= -\frac{1}{5x^5}+\frac{1}{5}\,\text{Subst}\left(\int \left(-\frac{1}{x}+\frac{x}{1+x+x^2}\right)dx, x, x^5\right)\\
&= -\frac{1}{5x^5}-\log(x)+\frac{1}{5}\,\text{Subst}\left(\int \frac{x}{1+x+x^2}\,dx, x, x^5\right)\\
&= -\frac{1}{5x^5}-\log(x)-\frac{1}{10}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^5\right)+\frac{1}{10}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^5\right)\\
&= -\frac{1}{5x^5}-\log(x)+\frac{1}{10}\log\left(1+x^5+x^{10}\right)+\frac{1}{5}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^5\right)\\
&= -\frac{1}{5x^5}-\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}-\log(x)+\frac{1}{10}\log\left(1+x^5+x^{10}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0412759, size = 208, normalized size = 4.33

$$\frac{1}{30}\left(6\text{RootSum}\left[\#1^8-\#1^7+\#1^5-\#1^4+\#1^3-\#1+1\&,\frac{4\#1^7\log(x-\#1)-4\#1^6\log(x-\#1)+\#1^5\log(x-\#1)+2\#1^4}{8\#1^7-7\#}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] (-6/x^5 + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 30*Log[x] + 3*Log[1 + x + x
^2] + 6*RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-Log[x - #1]
 + Log[x - #1]*#1 + Log[x - #1]*#1^2 - 3*Log[x - #1]*#1^3 + 2*Log[x - #1]*#
1^4 + Log[x - #1]*#1^5 - 4*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#
1^2 - 4*#1^3 + 5*#1^4 - 7*#1^6 + 8*#1^7) & ])/30
```

Maple [A] time = 0.021, size = 73, normalized size = 1.5

$$\frac{\ln\left(x^{2}+x+1\right)}{10}+\frac{\ln\left(4x^{8}-4x^{7}+4x^{5}-4x^{4}+4x^{3}-4x+4\right)}{10}-\frac{\sqrt{3}}{15}\arctan\left(\frac{2\sqrt{3}x^{5}}{3}+\frac{\sqrt{3}}{3}\right)-\frac{1}{5x^{5}}-\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^10+x^5+1),x)
```

```
[Out] 1/10*ln(x^2+x+1)+1/10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)-1/15*3^(1/2)*
arctan(2/3*3^(1/2)*x^5+1/3*3^(1/2))-1/5/x^5-ln(x)
```

Maxima [A] time = 1.49093, size = 55, normalized size = 1.15

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{5}+1\right)\right)-\frac{1}{5x^{5}}+\frac{1}{10}\log\left(x^{10}+x^{5}+1\right)-\frac{1}{5}\log\left(x^{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/5/x^5 + 1/10*log(x^10 + x
^5 + 1) - 1/5*log(x^5)
```

Fricas [A] time = 1.59873, size = 144, normalized size = 3.

$$-\frac{2\sqrt{3}x^{5}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{5}+1\right)\right)-3x^{5}\log\left(x^{10}+x^{5}+1\right)+30x^{5}\log\left(x\right)+6}{30x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="fricas")
```

```
[Out] -1/30*(2*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 3*x^5*log(x^10 + x^5
 + 1) + 30*x^5*log(x) + 6)/x^5
```

Sympy [A] time = 0.219102, size = 48, normalized size = 1.

$$-\log\left(x\right)+\frac{\log\left(x^{10}+x^{5}+1\right)}{10}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^{5}}{3}+\frac{\sqrt{3}}{3}\right)}{15}-\frac{1}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**10+x**5+1),x)
```

```
[Out] -log(x) + log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3
)/3)/15 - 1/(5*x**5)
```

Giac [A] time = 1.11508, size = 61, normalized size = 1.27

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)+\frac{x^5-1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) + 1/5*(x^5 - 1)/x^5 + 1/10*lo
g(x^10 + x^5 + 1) - log(abs(x))
```

3.411 $\int \frac{1}{x+x^6+x^{11}}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)$$

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 + x^10]/10
```

Rubi [A] time = 0.0356537, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {1594, 1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^6 + x^11)^(-1),x]
```

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 + x^10]/10
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x+x^6+x^{11}}\,dx &= \int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx\\ &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x\left(1+x+x^2\right)}\,dx,x,x^5\right)\\ &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^5\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx,x,x^5\right)\\ &= \log(x)-\frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^5\right)-\frac{1}{10}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^5\right)\\ &= \log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^5\right)\\ &= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)\end{aligned}$$

Mathematica [C] time = 0.0188937, size = 197, normalized size = 5.05

$$-\frac{1}{5}\text{RootSum}\left[\#1^8-\#1^7+\#1^5-\#1^4+\#1^3-\#1+1\&,\frac{4\#1^7\log(x-\#1)-3\#1^6\log(x-\#1)-\#1^5\log(x-\#1)+3\#1^4\log}{8\#1^7-7\#1^6+5\#1^4}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^6 + x^11)^(-1),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Root
Sum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*Lo
g[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^5 -
 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 -
7*#1^6 + 8*#1^7) & ]/5
```

Maple [B] time = 0.02, size = 66, normalized size = 1.7

$$-\frac{\ln\left(x^2+x+1\right)}{10}-\frac{\ln\left(4x^8-4x^7+4x^5-4x^4+4x^3-4x+4\right)}{10}-\frac{\sqrt{3}}{15}\arctan\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11+x^6+x),x)
```

```
[Out] -1/10*ln(x^2+x+1)-1/10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)-1/15*3^(1/2)
*arctan(2/3*3^(1/2)*x^5+1/3*3^(1/2))+ln(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{5}\int\frac{4x^7-3x^6-x^5+3x^4-x^3+2x^2-x}{x^8-x^7+x^5-x^4+x^3-x+1}\,dx-\frac{1}{10}\log\left(x^2+x+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="maxima")
```

```
[Out] 1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/5*integrate((4*x^7 - 3*x^6 -
 x^5 + 3*x^4 - x^3 + 2*x^2 - x)/(x^8 - x^7 + x^5 - x^4 + x^3 - x + 1), x) -
 1/10*log(x^2 + x + 1) + log(x)
```

Fricas [A] time = 1.6487, size = 112, normalized size = 2.87

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(x)
```

Sympy [A] time = 0.166106, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**11+x**6+x),x)
```

```
[Out] log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3)
/3)/15
```

Giac [A] time = 1.10021, size = 45, normalized size = 1.15

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(abs(x))
```

3.412 $\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=147

$$\frac{(a^2c^2-3ab^2c+b^4)\log(a+bx+cx^2)}{2c^5}+\frac{b(5a^2c^2-5ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}+\frac{x^2(b^2-ac)}{2c^3}-\frac{bx(b^2-2ac)}{c^4}-\frac{bx}{3c}$$

```
[Out] -((b*(b^2 - 2*a*c)*x)/c^4) + ((b^2 - a*c)*x^2)/(2*c^3) - (b*x^3)/(3*c^2) +
x^4/(4*c) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + b*
x + c*x^2])/(2*c^5)
```

Rubi [A] time = 0.136513, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1354, 701, 634, 618, 206, 628}

$$\frac{(a^2c^2-3ab^2c+b^4)\log(a+bx+cx^2)}{2c^5}+\frac{b(5a^2c^2-5ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}+\frac{x^2(b^2-ac)}{2c^3}-\frac{bx(b^2-2ac)}{c^4}-\frac{bx}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(c + a/x^2 + b/x),x]
```


```
[Out] -((b*(b^2 - 2*a*c)*x)/c^4) + ((b^2 - a*c)*x^2)/(2*c^3) - (b*x^3)/(3*c^2) +
x^4/(4*c) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + b*
x + c*x^2])/(2*c^5)
```


Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \int \frac{x^5}{a+bx+cx^2}\,dx$$

$$= \int \left(-\frac{b\left(b^2-2ac\right)}{c^4}+\frac{\left(b^2-ac\right)x}{c^3}-\frac{bx^2}{c^2}+\frac{x^3}{c}+\frac{ab\left(b^2-2ac\right)+\left(b^4-3ab^2c+a^2c^2\right)x}{c^4\left(a+bx+cx^2\right)}\right)dx$$

$$= -\frac{b\left(b^2-2ac\right)x}{c^4}+\frac{\left(b^2-ac\right)x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{\int \frac{ab\left(b^2-2ac\right)+\left(b^4-3ab^2c+a^2c^2\right)x}{a+bx+cx^2}\,dx}{c^4}$$

$$= -\frac{b\left(b^2-2ac\right)x}{c^4}+\frac{\left(b^2-ac\right)x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{\left(b^4-3ab^2c+a^2c^2\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^5}-\frac{\left(b\left(b^4-5ab^2c+5a\right.\right.}{2c}$$

$$= -\frac{b\left(b^2-2ac\right)x}{c^4}+\frac{\left(b^2-ac\right)x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{\left(b^4-3ab^2c+a^2c^2\right)\log\left(a+bx+cx^2\right)}{2c^5}+\frac{\left(b\left(b^4-5ab^2c\right.\right.}{}$$

$$= -\frac{b\left(b^2-2ac\right)x}{c^4}+\frac{\left(b^2-ac\right)x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{b\left(b^4-5ab^2c+5a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}+\frac{\left(b^4-3ab^2c\right.}{}$$

Mathematica [A] time = 0.121937, size = 140, normalized size = 0.95

$$\frac{6\left(a^2c^2-3ab^2c+b^4\right)\log(a+x(b+cx))-\frac{12b\left(5a^2c^2-5ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx\left(-4bc\left(cx^2-6a\right)+3c^2x\left(cx^2-2a\right)+6b^2cx\right.}{12c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(-12*b^3 + 6*b^2*c*x - 4*b*c*(-6*a + c*x^2) + 3*c^2*x*(-2*a + c*x^2))
- (12*b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/Sqrt[-b^2 + 4*a*c] + 6*(b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + x*(b + c*x)])
/(12*c^5)
```

Maple [A] time = 0.006, size = 236, normalized size = 1.6

$$\frac{x^4}{4c}-\frac{bx^3}{3c^2}-\frac{x^2a}{2c^2}+\frac{x^2b^2}{2c^3}+2\frac{abx}{c^3}-\frac{b^3x}{c^4}+\frac{\ln\left(cx^2+bx+a\right)a^2}{2c^3}-\frac{3\ln\left(cx^2+bx+a\right)ab^2}{2c^4}+\frac{\ln\left(cx^2+bx+a\right)b^4}{2c^5}-5\frac{}{c^3\sqrt{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c+a/x^2+b/x),x)
```

```
[Out] 1/4*x^4/c-1/3*b*x^3/c^2-1/2/c^2*x^2*a+1/2/c^3*x^2*b^2+2/c^3*a*b*x-1/c^4*b^3
*x+1/2/c^3*ln(c*x^2+b*x+a)*a^2-3/2/c^4*ln(c*x^2+b*x+a)*a*b^2+1/2/c^5*ln(c*x
^2+b*x+a)*b^4-5/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a
^2*b+5/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3-1/c^
5/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8236, size = 1006, normalized size = 6.84

$$\left[\frac{3\left(b^2c^4 - 4ac^5\right)x^4 - 4\left(b^3c^3 - 4abc^4\right)x^3 + 6\left(b^4c^2 - 5ab^2c^3 + 4a^2c^4\right)x^2 + 6\left(b^5 - 5ab^3c + 5a^2bc^2\right)\sqrt{b^2 - 4ac}\log\left(\frac{2c^2\cdots}{\cdots}\right.}{12\left(b^2c^5\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3 + 6*(b^4*c^2
 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 6*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
x + b))/(c*x^2 + b*x + a)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(
b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b^2*c^
5 - 4*a*c^6), 1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3
 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 12*(b^5 - 5*a*b^3*c + 5*a^2*
b*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(b^6 - 7*a*b^4*c + 13*a
^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6)]
```

Sympy [B] time = 1.04048, size = 600, normalized size = 4.08

$$-\frac{bx^3}{3c^2} + \left(-\frac{b\sqrt{-4ac + b^2}\left(5a^2c^2 - 5ab^2c + b^4\right)}{2c^5\left(4ac - b^2\right)} + \frac{a^2c^2 - 3ab^2c + b^4}{2c^5}\right)\log\left(x + \frac{2a^3c^2 - 4a^2b^2c + ab^4 - 4ac^5\left(-\frac{b\sqrt{-4ac+b^2}(\cdots}{2c^5(\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c+a/x**2+b/x),x)
```

```
[Out] -b*x**3/(3*c**2) + (-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4
)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5))*log(x
 + (2*a**3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c**5*(-b*sqrt(-4*a*c + b**2)
```

```
*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3
*a*b**2*c + b**4)/(2*c**5)) + b**2*c**4*(-b*sqrt(-4*a*c + b**2)*(5*a**2*c**
2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c +
b**4)/(2*c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**5)) + (b*sqrt(-4*a*c + b*
*2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2
- 3*a*b**2*c + b**4)/(2*c**5))*log(x + (2*a**3*c**2 - 4*a**2*b**2*c + a*b**
4 - 4*a*c**5*(b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c*
*5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)) + b**2*c**4*
(b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b
**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)))/(5*a**2*b*c**2 - 5*a*b**
3*c + b**5)) + x**4/(4*c) - x**2*(a*c - b**2)/(2*c**3) + x*(2*a*b*c - b**3)
/c**4
```

Giac [A] time = 1.10676, size = 196, normalized size = 1.33

$$\frac{3\,c^3x^4 - 4\,bc^2x^3 + 6\,b^2cx^2 - 6\,ac^2x^2 - 12\,b^3x + 24\,abcx}{12\,c^4} + \frac{\left(b^4 - 3\,ab^2c + a^2c^2\right)\log\left(cx^2 + bx + a\right)}{2\,c^5} - \frac{\left(b^5 - 5\,ab^3c + 5\,a^2bc^2\right.}{\sqrt{-b^2 +}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3*x^4 - 4*b*c^2*x^3 + 6*b^2*c*x^2 - 6*a*c^2*x^2 - 12*b^3*x + 24*a
*b*c*x)/c^4 + 1/2*(b^4 - 3*a*b^2*c + a^2*c^2)*log(c*x^2 + b*x + a)/c^5 - (b
^5 - 5*a*b^3*c + 5*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(
-b^2 + 4*a*c)*c^5)
```

3.413 $\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx$

Optimal. Leaf size=118

$$-\frac{(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2c^4}+\frac{x(b^2-ac)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

```
[Out] ((b^2 - a*c)*x)/c^3 - (b*x^2)/(2*c^2) + x^3/(3*c) - ((b^4 - 4*a*b^2*c + 2*a
^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) - (
b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.104779, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1354, 701, 634, 618, 206, 628}

$$-\frac{(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2c^4}+\frac{x(b^2-ac)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] ((b^2 - a*c)*x)/c^3 - (b*x^2)/(2*c^2) + x^3/(3*c) - ((b^4 - 4*a*b^2*c + 2*a
^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) - (
b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \int \frac{x^4}{a+bx+cx^2}\,dx$$
$$= \int \left(\frac{b^2-ac}{c^3} - \frac{bx}{c^2} + \frac{x^2}{c} - \frac{a\left(b^2-ac\right)+b\left(b^2-2ac\right)x}{c^3\left(a+bx+cx^2\right)}\right) dx$$
$$= \frac{\left(b^2-ac\right)x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{\int \frac{a\left(b^2-ac\right)+b\left(b^2-2ac\right)x}{a+bx+cx^2}\,dx}{c^3}$$
$$= \frac{\left(b^2-ac\right)x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{\left(b\left(b^2-2ac\right)\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^4} + \frac{\left(b^4-4ab^2c+2a^2c^2\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^4}$$
$$= \frac{\left(b^2-ac\right)x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4} - \frac{\left(b^4-4ab^2c+2a^2c^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,d$$
$$= \frac{\left(b^2-ac\right)x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{\left(b^4-4ab^2c+2a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}} - \frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}$$

Mathematica [A] time = 0.084465, size = 112, normalized size = 0.95

$$\frac{\frac{6\left(2a^2c^2-4ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + cx\left(-6ac+6b^2-3bcx+2c^2x^2\right) - 3\left(b^3-2abc\right)\log(a+x(b+cx))}{6c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(6*b^2 - 6*a*c - 3*b*c*x + 2*c^2*x^2) + (6*(b^4 - 4*a*b^2*c + 2*a^2*c^
2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 3*(b^3 - 2*
a*b*c)*Log[a + x*(b + c*x)])/(6*c^4)
```

Maple [A] time = 0.004, size = 190, normalized size = 1.6

$$\frac{x^3}{3\,c} - \frac{bx^2}{2\,c^2} - \frac{ax}{c^2} + \frac{b^2x}{c^3} + \frac{\ln\left(cx^2+bx+a\right)ab}{c^3} - \frac{\ln\left(cx^2+bx+a\right)b^3}{2\,c^4} + 2\,\frac{a^2}{c^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right) - 4\,\frac{ab^2}{c^3\sqrt{4\,ac-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c+a/x^2+b/x),x)
```

```
[Out] 1/3*x^3/c-1/2*b*x^2/c^2-1/c^2*a*x+1/c^3*b^2*x+1/c^3*ln(c*x^2+b*x+a)*a*b-1/2
/c^4*ln(c*x^2+b*x+a)*b^3+2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
```

```
2)^(1/2))*a^2-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a
*b^2+1/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82093, size = 829, normalized size = 7.03

$$\left[\frac{2\left(b^{2} c^{3}-4 a c^{4}\right) x^{3}-3\left(b^{3} c^{2}-4 a b c^{3}\right) x^{2}+3\left(b^{4}-4 a b^{2} c+2 a^{2} c^{2}\right) \sqrt{b^{2}-4 a c} \log \left(\frac{2 c^{2} x^{2}+2 b c x+b^{2}-2 a c-\sqrt{b^{2}-4 a c}(2 c x+b)}{c x^{2}+b x+a}\right)+6}{6\left(b^{2} c^{4}-4 a c^{5}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="fricas")

[Out] [1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b*c^3)*x^2 + 3*(b^4 - 4*
a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4*c - 5*a*b^2
*c^2 + 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a
))/(b^2*c^4 - 4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b
*c^3)*x^2 - 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt
(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^4*c - 5*a*b^2*c^2 + 4*a^2*
c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a))/(b^2*c^4 -
 4*a*c^5)]
```

Sympy [B] time = 0.963526, size = 496, normalized size = 4.2

$$-\frac{b x^{2}}{2 c^{2}}+\left(\frac{b\left(2 a c-b^{2}\right)}{2 c^{4}}-\frac{\sqrt{-4 a c+b^{2}}\left(2 a^{2} c^{2}-4 a b^{2} c+b^{4}\right)}{2 c^{4}\left(4 a c-b^{2}\right)}\right) \log \left(x+\frac{-3 a^{2} b c+a b^{3}+4 a c^{4}\left(\frac{b\left(2 a c-b^{2}\right)}{2 c^{4}}-\frac{\sqrt{-4 a c+b^{2}}\left(2 a^{2} c^{2}-4 a b\right.}{2 c^{4}\left(4 a c-b^{2}\right)}\right.}{2 a^{2} c^{2}-4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c+a/x**2+b/x),x)

[Out] -b*x**2/(2*c**2) + (b*(2*a*c - b**2)/(2*c**4) - sqrt(-4*a*c + b**2)*(2*a**2
*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)))*log(x + (-3*a**2*b*c +
a*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) - sqrt(-4*a*c + b**2)*(2*a**2*
c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))) - b**2*c**3*(b*(2*a*c -
b**2)/(2*c**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c
**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c + b**4)) + (b*(2*a*c - b**2
)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*
```

```
(4*a*c - b**2)))*log(x + (-3*a**2*b*c + a*b**3 + 4*a*c**4*(b*(2*a*c - b**2)
/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(
4*a*c - b**2))) - b**2*c**3*(b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2
)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))))/(2*a**2*c**2
- 4*a*b**2*c + b**4)) + x**3/(3*c) - x*(a*c - b**2)/c**3
```

Giac [A] time = 1.12624, size = 153, normalized size = 1.3

$$\frac{2\,c^2x^3 - 3\,bcx^2 + 6\,b^2x - 6\,acx}{6\,c^3} - \frac{\left(b^3 - 2\,abc\right)\log\left(cx^2 + bx + a\right)}{2\,c^4} + \frac{\left(b^4 - 4\,ab^2c + 2\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*x^3 - 3*b*c*x^2 + 6*b^2*x - 6*a*c*x)/c^3 - 1/2*(b^3 - 2*a*b*c)*l
og(c*x^2 + b*x + a)/c^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/
sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

3.414 $\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=89

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^
3)
```

Rubi [A] time = 0.0861381, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1354, 701, 634, 618, 206, 628}

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x/(c + a/x^2 + b/x),x]
```

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^
3)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx &= \int \frac{x^3}{a+bx+cx^2}\,dx \\
&= \int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx}{c^2} \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}-\frac{\left(b\left(b^2-3ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^3}+\frac{\left(b^2-ac\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3} \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^3} \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}
\end{aligned}$$

Mathematica [A] time = 0.107658, size = 84, normalized size = 0.94

$$\frac{\left(b^2-ac\right)\log(a+x(b+cx))-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx(cx-2b)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

Maple [A] time = 0.004, size = 132, normalized size = 1.5

$$\frac{x^2}{2c}-\frac{bx}{c^2}-\frac{\ln\left(cx^2+bx+a\right)a}{2c^2}+\frac{\ln\left(cx^2+bx+a\right)b^2}{2c^3}+3\frac{ab}{c^2\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)-\frac{b^3}{c^3}\arctan\left((2cx+b)\frac{}{\sqrt{4a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c+a/x^2+b/x),x)
```

```
[Out] 1/2*x^2/c-b*x/c^2-1/2/c^2*ln(c*x^2+b*x+a)*a+1/2/c^3*ln(c*x^2+b*x+a)*b^2+3/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b-1/c^3/(4*a*c-b
```

```
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72491, size = 653, normalized size = 7.34

$$\left[\frac{\left(b^2c^2-4\,ac^3\right)x^2-\left(b^3-3\,abc\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\left(b^3c-4\,abc^2\right)x+\left(b^4-5\,ab^2c+4\right.}{2\left(b^2c^3-4\,ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2
+ b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3 - 3*
a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 +
b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] time = 0.786597, size = 381, normalized size = 4.28

$$-\frac{bx}{c^2}+\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{2c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{2c^3}\right)\log\left(x+\frac{2a^2c-ab^2+4ac^3\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(-\frac{b\sqrt{-4ac+b}}{2c^3(4a}\right.}{3abc-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x**2+b/x),x)
```

```
[Out] -b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
- (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-4*
a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))
 - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*
c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*
c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c
- b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2)*(3*a*c
 - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)
```

```
) + x**2/(2*c)
```

Giac [A] time = 1.14168, size = 116, normalized size = 1.3

$$\frac{cx^2 - 2\,bx}{2\,c^2} + \frac{\left(b^2 - ac\right)\log\left(cx^2 + bx + a\right)}{2\,c^3} - \frac{\left(b^3 - 3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3 -
 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.415 $\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=70

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
- 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.0490922, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1340, 703, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^2 + b/x)^(-1),x]
```

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
- 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx &= \int \frac{x^2}{a+bx+cx^2}\,dx\\ &= \frac{x}{c}+\frac{\int \frac{-a-bx}{a+bx+cx^2}\,dx}{c}\\ &= \frac{x}{c}-\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{\left(b^2-2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2}\\ &= \frac{x}{c}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2}\\ &= \frac{x}{c}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.0645854, size = 73, normalized size = 1.04

$$\frac{\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^2 + b/x)^(-1),x]
```

```
[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2
 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Maple [A] time = 0.001, size = 101, normalized size = 1.4

$$\frac{x}{c}-\frac{b\ln\left(cx^2+bx+a\right)}{2\,c^2}-2\,\frac{a}{c\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+\frac{b^2}{c^2}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x),x)
```

```
[Out] x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*a+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*
b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68884, size = 537, normalized size = 7.67

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\left(b^2c-4\,ac^2\right)x+\left(b^3-4\,abc\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c^2-4\,ac^3\right)},-\frac{2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c^2
)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2*(b
^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(
b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 0.638012, size = 306, normalized size = 4.37

$$\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{2c^2(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{2c^2(4ac-b^2)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x),x)
```

```
[Out] (-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*
log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c*
*2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)
)) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*
c - b**2))))/(2*a*c - b**2)) + x/c
```

Giac [A] time = 1.15291, size = 90, normalized size = 1.29

$$\frac{x}{c}-\frac{b\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.416 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx$

Optimal. Leaf size=56

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
b*x + c*x^2]/(2*c)
```

Rubi [A] time = 0.0362539, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
b*x + c*x^2]/(2*c)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx=\int \frac{x}{a+bx+cx^2}\,dx$$
$$=\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c}-\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2c}$$
$$=\frac{\log\left(a+bx+cx^2\right)}{2c}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c}$$
$$=\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Mathematica [A] time = 0.0314967, size = 57, normalized size = 1.02

$$\frac{\log(a+x(b+cx))-\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a +
 x*(b + c*x)])/(2*c)
```

Maple [A] time = 0.003, size = 56, normalized size = 1.

$$\frac{\ln\left(cx^2+bx+a\right)}{2c}-\frac{b}{c}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x,x)
```

```
[Out] 1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69683, size = 427, normalized size = 7.62

$$\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c-4\,ac^2\right)},\ \frac{2\sqrt{-b^2+4\,ac}b\arctan\left(-\frac{\sqrt{-b^2+4\,a\ldots}}{b^2-\ldots}\right.}{2\left(b\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c
 - 4*a*c^2)]
```

Sympy [B] time = 0.311239, size = 216, normalized size = 3.86

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)\log\left(x+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-b
*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqrt(
-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*
c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*
c - b**2)) + 1/(2*c)))/b)
```

Giac [A] time = 1.15931, size = 74, normalized size = 1.32

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c}+\frac{\log\left(cx^2+bx+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log(
c*x^2 + b*x + a)/c
```

3.417 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx$

Optimal. Leaf size=36

$$\frac{2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] (2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.0324579, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 618, 206}

$$\frac{2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^2),x]
```

```
[Out] (2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx &= -\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx, x, \frac{1}{x}\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\frac{2a}{x}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.0065326, size = 38, normalized size = 1.06

$$\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^2),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0., size = 35, normalized size = 1.

$$2\frac{1}{\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^2,x)
```

```
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7764, size = 277, normalized size = 7.69

$$\left[\frac{\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4ac}}, -\frac{2\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)}{b^2-4ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="fricas")
```

```
[Out] [log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] time = 0.202506, size = 124, normalized size = 3.44

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

Giac [A] time = 1.12915, size = 46, normalized size = 1.28

$$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.418 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$

Optimal. Leaf size=62

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a
 - Log[a + b*x + c*x^2]/(2*a)
```

Rubi [A] time = 0.0476562, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a
 - Log[a + b*x + c*x^2]/(2*a)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx &= \int \frac{1}{x\left(a+bx+cx^2\right)}\,dx\\
&= \frac{\int \frac{1}{x}\,dx}{a}+\frac{\int \frac{-b-cx}{a+bx+cx^2}\,dx}{a}\\
&= \frac{\log(x)}{a}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a}-\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0680658, size = 61, normalized size = 0.98

$$-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\log(a+x(b+cx))-2\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] -((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*Log[x
] + Log[a + x*(b + c*x)])/(2*a)
```

Maple [A] time = 0.005, size = 62, normalized size = 1.

$$\frac{\ln(x)}{a}-\frac{\ln\left(cx^2+bx+a\right)}{2a}-\frac{b}{a}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^3,x)
```

```
[Out] ln(x)/a-1/2*ln(c*x^2+b*x+a)/a-1/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89301, size = 494, normalized size = 7.97

$$\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)+2\left(b^2-4\,ac\right)\log\left(x\right)}{2\left(ab^2-4\,a^2c\right)}, \frac{2\sqrt{-b^2+4}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*c)*
b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log
(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] time = 1.90599, size = 564, normalized size = 9.1

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)^2-12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)+\cdots}{9abc^2-2b\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**3,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**
2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a
*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4
```

```
*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
 b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b*
*3*c)) + log(x)/a
```

Giac [A] time = 1.13397, size = 84, normalized size = 1.35

$$-\frac{b \arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a} - \frac{\log\left(cx^2+bx+a\right)}{2\,a} + \frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log(
c*x^2 + b*x + a)/a + log(abs(x))/a
```

3.419 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sq
rt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)
```

Rubi [A] time = 0.102417, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sq
rt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx &= \int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx\\
&= -\frac{1}{ax}+\frac{\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx}{a}\\
&= -\frac{1}{ax}+\frac{\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx}{a}\\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx}{a^2}\\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2}+\frac{\left(b^2-2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2}\\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a^2}\\
&= -\frac{1}{ax}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0792688, size = 77, normalized size = 0.95

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+b\log(a+x(b+cx))-\frac{2a}{x}-2b\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-
b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [A] time = 0.008, size = 112, normalized size = 1.4

$$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(cx^2+bx+a\right)}{2a^2}-2\frac{c}{a\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)+\frac{b^2}{a^2}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^4,x)
```

```
[Out] -1/a/x-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2-2/a/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*c+1/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82571, size = 626, normalized size = 7.73

$$\left[-\frac{\left(b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}x\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}(2\,cx+b)}{cx^{2}+bx+a}\right)+2\,ab^{2}-8\,a^{2}c-\left(b^{3}-4\,abc\right)x\log\left(cx^{2}+bx+a\right)+2\left(\right.}{2\left(a^{2}b^{2}-4\,a^{3}c\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^2*
c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((
a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4
*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*
a^3*c)*x)]
```

Sympy [B] time = 3.49527, size = 862, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**4,x)
```

```
[Out] (b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))*l
og(x + (-28*a**6*b*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2
*a**2*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b/(2*a**2) - sqrt(-4*a*c + b**2
)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 4*a**5*c**3*(b/(2*a**2) - sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 2*a**4*b**5*(b/
(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 -
```

```
 3*a**4*b**2*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*
(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b
**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*a**2*b**3*c**2 - 14*a*b*
*5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3 - 12*a*b**4*c**2 + 2*b**6*c
)) + (b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2
)))*log(x + (-28*a**6*b*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(2*a**2*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b/(2*a**2) + sqrt(-4*a*c +
 b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 4*a**5*c**3*(b/(2*a**2)
 + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 2*a**4*b**
5*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))
**2 - 3*a**4*b**2*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*
a**2*(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*
c - b**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*a**2*b**3*c**2 - 14
*a*b**5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3 - 12*a*b**4*c**2 + 2*b
**6*c)) - 1/(a*x) - b*log(x)/a**2
```

Giac [A] time = 1.11534, size = 107, normalized size = 1.32

$$\frac{b\log\left(cx^2+bx+a\right)}{2\,a^2}-\frac{b\log\left(|x|\right)}{a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

3.420 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$

Optimal. Leaf size=104

$$-\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c)*
Log[a + b*x + c*x^2])/(2*a^3)
```

Rubi [A] time = 0.152542, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^5),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c)*
Log[a + b*x + c*x^2])/(2*a^3)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx &= \int \frac{1}{x^3\left(a+bx+cx^2\right)}\,dx \\
&= -\frac{1}{2ax^2}+\frac{\int \frac{-b-cx}{x^2\left(a+bx+cx^2\right)}\,dx}{a} \\
&= -\frac{1}{2ax^2}+\frac{\int \left(-\frac{b}{ax^2}+\frac{b^2-ac}{a^2x}+\frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a^2\left(a+bx+cx^2\right)}\right)dx}{a} \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{\left(b^2-ac\right)\log(x)}{a^3}+\frac{\int \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a+bx+cx^2}\,dx}{a^3} \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b\left(b^2-3ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^3}-\frac{\left(b^2-ac\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^3} \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-\cdots}\right.}{a^3} \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.136071, size = 102, normalized size = 0.98

$$\frac{-\frac{a^2}{x^2}+2\log(x)\left(b^2-ac\right)+\left(ac-b^2\right)\log(a+x(b+cx))-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{2ab}{x}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^5),x]
```

```
[Out] (-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a +
x*(b + c*x)])/(2*a^3)
```

Maple [A] time = 0.007, size = 150, normalized size = 1.4

$$-\frac{1}{2\,a x^{2}}-\frac{\ln \left(x \right) c}{a^{2}}+\frac{\ln \left(x \right) b^{2}}{a^{3}}+\frac{b}{x a^{2}}+\frac{c \ln \left(c x^{2}+b x +a \right)}{2\,a^{2}}-\frac{\ln \left(c x^{2}+b x +a \right) b^{2}}{2\,a^{3}}+3\,\frac{b c}{a^{2}\sqrt{4\,a c -b^{2}}}\arctan \left(\frac{2\,c x +b}{\sqrt{4\,a c -b^{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^5,x)
```

```
[Out] -1/2/a/x^2-1/a^2*ln(x)*c+1/a^3*ln(x)*b^2+b/a^2/x+1/2/a^2*c*ln(c*x^2+b*x+a)-
1/2/a^3*ln(c*x^2+b*x+a)*b^2+3/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b*c-1/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11755, size = 798, normalized size = 7.67

$$\left[-\frac{\left(b^{3}-3\,a b c\right)\sqrt{b^{2}-4\,a c}x^{2}\log \left(\frac{2\,c^{2}x^{2}+2\,b c x+b^{2}-2\,a c-\sqrt{b^{2}-4\,a c}(2\,c x+b)}{c x^{2}+b x+a}\right)+a^{2}b^{2}-4\,a^{3}c+\left(b^{4}-5\,a b^{2}c+4\,a^{2}c^{2}\right)x^{2}\log \left(c x^{2}+b\right.}{2\,\left(a^{3}b^{2}-4\,a^{4}c\right)x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2
 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 - 4*
a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4 - 5
*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a
^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a*b^2*c
 + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^
2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

Sympy [B] time = 5.23101, size = 1525, normalized size = 14.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**5,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**
2)/(2*a**3))*log(x + (24*a**9*c**3*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(
2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 - 42*a**8*b**2*c**2*(-b*
sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(
2*a**3))**2 + 17*a**7*b**4*c*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3
*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 + 12*a**7*c**4*(-b*sqrt(-4*a*c
 + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) -
2*a**6*b**6*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2))
+ (a*c - b**2)/(2*a**3))**2 - 15*a**6*b**2*c**3*(-b*sqrt(-4*a*c + b**2)*(3*
a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) + 7*a**5*b**4*
c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c
- b**2)/(2*a**3)) - 12*a**5*c**5 - a**4*b**6*c*(-b*sqrt(-4*a*c + b**2)*(3*a
*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) + 63*a**4*b**2*
c**4 - 103*a**3*b**4*c**3 + 70*a**2*b**6*c**2 - 20*a*b**8*c + 2*b**10)/(27*
a**4*b*c**5 - 63*a**3*b**3*c**4 + 54*a**2*b**5*c**3 - 18*a*b**7*c**2 + 2*b*
*9*c)) + (b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a
*c - b**2)/(2*a**3))*log(x + (24*a**9*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c -
b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 - 42*a**8*b**2*c*
*2*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b
**2)/(2*a**3))**2 + 17*a**7*b**4*c*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2
*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 + 12*a**7*c**4*(b*sqrt(-4
*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)
) - 2*a**6*b**6*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2
)) + (a*c - b**2)/(2*a**3))**2 - 15*a**6*b**2*c**3*(b*sqrt(-4*a*c + b**2)*(
3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) + 7*a**5*b**
4*c**2*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c
 - b**2)/(2*a**3)) - 12*a**5*c**5 - a**4*b**6*c*(b*sqrt(-4*a*c + b**2)*(3*a
*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) + 63*a**4*b**2*
c**4 - 103*a**3*b**4*c**3 + 70*a**2*b**6*c**2 - 20*a*b**8*c + 2*b**10)/(27*
a**4*b*c**5 - 63*a**3*b**3*c**4 + 54*a**2*b**5*c**3 - 18*a*b**7*c**2 + 2*b*
*9*c)) + (-a + 2*b*x)/(2*a**2*x**2) - (a*c - b**2)*log(x + (-12*a**5*c**5 +
 63*a**4*b**2*c**4 - 12*a**4*c**4*(a*c - b**2) - 103*a**3*b**4*c**3 + 15*a*
*3*b**2*c**3*(a*c - b**2) + 24*a**3*c**3*(a*c - b**2)**2 + 70*a**2*b**6*c**
2 - 7*a**2*b**4*c**2*(a*c - b**2) - 42*a**2*b**2*c**2*(a*c - b**2)**2 - 20*
a*b**8*c + a*b**6*c*(a*c - b**2) + 17*a*b**4*c*(a*c - b**2)**2 + 2*b**10 -
2*b**6*(a*c - b**2)**2)/(27*a**4*b*c**5 - 63*a**3*b**3*c**4 + 54*a**2*b**5*
c**3 - 18*a*b**7*c**2 + 2*b**9*c))/a**3
```

Giac [A] time = 1.12457, size = 142, normalized size = 1.37

$$-\frac{\left(b^2-ac\right)\log\left(cx^2+bx+a\right)}{2\,a^3}+\frac{\left(b^2-ac\right)\log\left(|x|\right)}{a^3}-\frac{\left(b^3-3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="giac")
```

```
[Out] -1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 - (
b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a
^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

3.421 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$

Optimal. Leaf size=137

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}+\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{b^2-ac}{a^3x}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}+\frac{b}{2a^2x^2}-$$

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) -
(b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^
4)
```

Rubi [A] time = 0.1928, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}+\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{b^2-ac}{a^3x}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}+\frac{b}{2a^2x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) -
(b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^
4)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \int \frac{1}{x^4\left(a+bx+cx^2\right)}\,dx$$
$$= -\frac{1}{3ax^3}+\frac{\int \frac{-b-cx}{x^3\left(a+bx+cx^2\right)}\,dx}{a}$$
$$= -\frac{1}{3ax^3}+\frac{\int \left(-\frac{b}{ax^3}+\frac{b^2-ac}{a^2x^2}+\frac{-b^3+2abc}{a^3x}+\frac{b^4-3ab^2c+a^2c^2+bc\left(b^2-2ac\right)x}{a^3\left(a+bx+cx^2\right)}\right)dx}{a}$$
$$= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b\left(b^2-2ac\right)\log(x)}{a^4}+\frac{\int \frac{b^4-3ab^2c+a^2c^2+bc\left(b^2-2ac\right)x}{a+bx+cx^2}\,dx}{a^4}$$
$$= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b\left(b^2-2ac\right)\log(x)}{a^4}+\frac{\left(b\left(b^2-2ac\right)\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^4}+\frac{\left(b^4-4ab^2c+2\right.}{}$$
$$= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b\left(b^2-2ac\right)\log(x)}{a^4}+\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{\left(b^4-4ab^2c\right.}{}$$
$$= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{\left(b^4-4ab^2c+2a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{b\left(b^2-2ac\right)\log(x)}{a^4}+\frac{b\left(b^2\right.}{}$$

Mathematica [A] time = 0.100566, size = 131, normalized size = 0.96

$$\frac{\frac{6\left(2a^2c^2-4ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{3a^2b}{x^2}-\frac{2a^3}{x^3}+\frac{6a\left(ac-b^2\right)}{x}-6\log(x)\left(b^3-2abc\right)+3\left(b^3-2abc\right)\log(a+x(b+cx))}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
+ 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6
```

```
*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4)
```

Maple [A] time = 0.008, size = 214, normalized size = 1.6

$$-\frac{1}{3\,ax^3}+\frac{c}{xa^2}-\frac{b^2}{a^3x}+2\,\frac{b\ln(x)\,c}{a^3}-\frac{b^3\ln(x)}{a^4}+\frac{b}{2\,a^2x^2}-\frac{c\ln\left(cx^2+bx+a\right)b}{a^3}+\frac{\ln\left(cx^2+bx+a\right)b^3}{2\,a^4}+2\,\frac{c^2}{a^2\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^6,x)
```

```
[Out] -1/3/a/x^3+1/a^2/x*c-1/a^3/x*b^2+2*b/a^3*ln(x)*c-b^3/a^4*ln(x)+1/2*b/a^2/x^
2-1/a^3*c*ln(c*x^2+b*x+a)*b+1/2/a^4*ln(c*x^2+b*x+a)*b^3+2/a^2/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2-4/a^3/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c+1/a^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.26154, size = 979, normalized size = 7.15

$$\left[\frac{3\left(b^4-4\,ab^2c+2\,a^2c^2\right)\sqrt{b^2-4\,ac}x^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,a^3b^2+8\,a^4c+3\left(b^5-6\,ab^3c+8\,a^2bc^2\right)}{6\left(a^4b^2-4\,a^5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
 - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 +
b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*a^2*
b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^
3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c - 3*(b^5
- 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*b^3*c +
8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 - 3*(a^2*
b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

Sympy [B] time = 8.59681, size = 2105, normalized size = 15.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**6,x)
```

```
[Out] (-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c
 + b**4)/(2*a**4*(4*a*c - b**2)))*log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b*
*2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**
4*(4*a*c - b**2)))**2 + 57*a**10*b**3*c**2*(-b*(2*a*c - b**2)/(2*a**4) - sq
rt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))
)**2 - 19*a**9*b**5*c*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*
a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))**2 + 4*a**9*c**5*(-
b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c +
 b**4)/(2*a**4*(4*a*c - b**2))) + 2*a**8*b**7*(-b*(2*a*c - b**2)/(2*a**4) -
 sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**
2)))**2 + 23*a**8*b**2*c**4*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**
2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) - 26*a**7*b**
4*c**3*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a
*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 9*a**6*b**6*c**2*(-b*(2*a*c - b*
*2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**
4*(4*a*c - b**2))) - 8*a**6*b*c**6 - a**5*b**8*c*(-b*(2*a*c - b**2)/(2*a**4
) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c -
b**2))) + 166*a**5*b**3*c**5 - 361*a**4*b**5*c**4 + 312*a**3*b**7*c**3 - 13
0*a**2*b**9*c**2 + 26*a*b**11*c - 2*b**13)/(2*a**6*c**7 + 60*a**5*b**2*c**6
 - 207*a**4*b**4*c**5 + 224*a**3*b**6*c**4 - 108*a**2*b**8*c**3 + 24*a*b**1
0*c**2 - 2*b**12*c)) + (-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2
*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*log(x + (-52*a**11
*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*
a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))**2 + 57*a**10*b**3*c**2*(-b*(2*a*
c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/
(2*a**4*(4*a*c - b**2)))**2 - 19*a**9*b**5*c*(-b*(2*a*c - b**2)/(2*a**4) +
sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2
)))**2 + 4*a**9*c**5*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a
**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 2*a**8*b**7*(-b*(2
*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**
4)/(2*a**4*(4*a*c - b**2)))**2 + 23*a**8*b**2*c**4*(-b*(2*a*c - b**2)/(2*a*
*4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c
- b**2))) - 26*a**7*b**4*c**3*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b
**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 9*a**6*b*
*6*c**2*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*
a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) - 8*a**6*b*c**6 - a**5*b**8*c*(-b
*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c +
b**4)/(2*a**4*(4*a*c - b**2))) + 166*a**5*b**3*c**5 - 361*a**4*b**5*c**4 +
312*a**3*b**7*c**3 - 130*a**2*b**9*c**2 + 26*a*b**11*c - 2*b**13)/(2*a**6*c
**7 + 60*a**5*b**2*c**6 - 207*a**4*b**4*c**5 + 224*a**3*b**6*c**4 - 108*a**
2*b**8*c**3 + 24*a*b**10*c**2 - 2*b**12*c)) + (-2*a**2 + 3*a*b*x + x**2*(6*
a*c - 6*b**2))/(6*a**3*x**3) + b*(2*a*c - b**2)*log(x + (-8*a**6*b*c**6 + 1
66*a**5*b**3*c**5 + 4*a**5*b*c**5*(2*a*c - b**2) - 361*a**4*b**5*c**4 + 23*
a**4*b**3*c**4*(2*a*c - b**2) + 312*a**3*b**7*c**3 - 26*a**3*b**5*c**3*(2*a
*c - b**2) - 52*a**3*b**3*c**3*(2*a*c - b**2)**2 - 130*a**2*b**9*c**2 + 9*a
**2*b**7*c**2*(2*a*c - b**2) + 57*a**2*b**5*c**2*(2*a*c - b**2)**2 + 26*a*b
**11*c - a*b**9*c*(2*a*c - b**2) - 19*a*b**7*c*(2*a*c - b**2)**2 - 2*b**13
+ 2*b**9*(2*a*c - b**2)**2)/(2*a**6*c**7 + 60*a**5*b**2*c**6 - 207*a**4*b**
4*c**5 + 224*a**3*b**6*c**4 - 108*a**2*b**8*c**3 + 24*a*b**10*c**2 - 2*b**1
2*c))/a**4
```

Giac [A] time = 1.15574, size = 184, normalized size = 1.34

$$\frac{\left(b^3 - 2\,abc\right)\log\left(cx^2 + bx + a\right)}{2\,a^4} - \frac{\left(b^3 - 2\,abc\right)\log\left(|x|\right)}{a^4} + \frac{\left(b^4 - 4\,ab^2c + 2\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}a^4} + \frac{3\,a^2bx - 2\,a^3 - 6}{6\,a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))/
a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^2)/
(a^4*x^3)
```

3.422 $\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} dx$

Optimal. Leaf size=196

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(3b^2-8ac\right)}{2c^2\left(b^2-4ac\right)}+\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}-\frac{bx\left(3b^2-11ac\right)}{c^3\left(b^2-4ac\right)}+\frac{}{\left(b^2-\right.}$$

```
[Out] -((b*(3*b^2 - 11*a*c)*x)/(c^3*(b^2 - 4*a*c))) + ((3*b^2 - 8*a*c)*x^2)/(2*c^
2*(b^2 - 4*a*c)) - (b*x^3)/(c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x))/((b^2 - 4*
a*c)*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Lo
g[a + b*x + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.204786, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1354, 738, 800, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(3b^2-8ac\right)}{2c^2\left(b^2-4ac\right)}+\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}-\frac{bx\left(3b^2-11ac\right)}{c^3\left(b^2-4ac\right)}+\frac{}{\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/(c + a/x^2 + b/x)^2,x]
```


```
[Out] -((b*(3*b^2 - 11*a*c)*x)/(c^3*(b^2 - 4*a*c))) + ((3*b^2 - 8*a*c)*x^2)/(2*c^
2*(b^2 - 4*a*c)) - (b*x^3)/(c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x))/((b^2 - 4*
a*c)*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Lo
g[a + b*x + c*x^2])/(2*c^4)
```


Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx=\int \frac{x^5}{\left(a+bx+cx^2\right)^2}\,dx$$

$$=\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{x^3(8a+3bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$=\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \left(\frac{b\left(3b^2-11ac\right)}{c^3}-\frac{\left(3b^2-8ac\right)x}{c^2}+\frac{3bx^2}{c}-\frac{ab\left(3b^2-11ac\right)+\left(b^2-4ac\right)\left(3b^2-2ac\right)x}{c^3\left(a+bx+cx^2\right)}\right)dx}{-b^2+4ac}$$

$$=-\frac{b\left(3b^2-11ac\right)x}{c^3\left(b^2-4ac\right)}+\frac{\left(3b^2-8ac\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{c\left(b^2-4ac\right)}+\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{ab\left(3b^2-11ac\right)+\ldots}{a\ldots}}{c^3(\ldots}$$

$$=-\frac{b\left(3b^2-11ac\right)x}{c^3\left(b^2-4ac\right)}+\frac{\left(3b^2-8ac\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{c\left(b^2-4ac\right)}+\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\left(3b^2-2ac\right)\int\ldots}{2c^{\ldots}}$$

$$=-\frac{b\left(3b^2-11ac\right)x}{c^3\left(b^2-4ac\right)}+\frac{\left(3b^2-8ac\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{c\left(b^2-4ac\right)}+\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\left(3b^2-2ac\right)\mathrm{lo}\ldots}{\ldots}$$

$$=-\frac{b\left(3b^2-11ac\right)x}{c^3\left(b^2-4ac\right)}+\frac{\left(3b^2-8ac\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{c\left(b^2-4ac\right)}+\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b\left(3b^4-20ab\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.22912, size = 163, normalized size = 0.83

$$\frac{\frac{2\left(a^2bc(5cx-4b)+2a^3c^2+ab^3(b-5cx)+b^5x\right)}{\left(b^2-4ac\right)(a+x(b+cx))}+\frac{2b\left(30a^2c^2-20ab^2c+3b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\left(3b^2-2ac\right)\log(a+x(b+cx))-4bcx+c^2x^2}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(c + a/x^2 + b/x)^2,x]
```

```
[Out] (-4*b*c*x + c^2*x^2 + (2*(2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(
-4*b + 5*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*
c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2
) + (3*b^2 - 2*a*c)*Log[a + x*(b + c*x)])/(2*c^4)
```

Maple [B] time = 0.01, size = 434, normalized size = 2.2

$$\frac{x^2}{2c^2} - 2\frac{bx}{c^3} - 5\frac{bxa^2}{c^2\left(cx^2+bx+a\right)\left(4ac-b^2\right)} + 5\frac{b^3xa}{c^3\left(cx^2+bx+a\right)\left(4ac-b^2\right)} - \frac{b^5x}{c^4\left(cx^2+bx+a\right)\left(4ac-b^2\right)} - 2\frac{}{c^2\left(cx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c+a/x^2+b/x)^2,x)
```

```
[Out] 1/2/c^2*x^2-2/c^3*b*x-5/c^2/(c*x^2+b*x+a)*b/(4*a*c-b^2)*x*a^2+5/c^3/(c*x^2+
b*x+a)*b^3/(4*a*c-b^2)*x*a-1/c^4/(c*x^2+b*x+a)*b^5/(4*a*c-b^2)*x-2/c^2/(c*x
^2+b*x+a)*a^3/(4*a*c-b^2)+4/c^3/(c*x^2+b*x+a)*a^2/(4*a*c-b^2)*b^2-1/c^4/(c*
x^2+b*x+a)*a/(4*a*c-b^2)*b^4-4/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a^2+7/c^3/(4
*a*c-b^2)*ln(c*x^2+b*x+a)*a*b^2-3/2/c^4/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^4+30/
c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b-20/c^3/(4*a
*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3+3/c^4/(4*a*c-b^2)^(
3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.79405, size = 2201, normalized size = 11.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3 - 8*a
*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 -
 (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 - (3*a*b^5 - 20
*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^2 + (
3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 +
2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
```

```
 2*(b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2
*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2
*c^3 - 32*a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^
3)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6
)*x), 1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3
- 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*
x^3 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 + 2*(3*a*b
^5 - 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*
x^2 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^7 - 11*a*b^5*c + 41*a^2
*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*
a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^2 + (3*b
^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x)*log(c*x^2 + b*x + a))/(
a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x)]
```

Sympy [B] time = 2.09912, size = 1012, normalized size = 5.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x**2+b/x)**2,x)
```

```
[Out] -2*b*x/c**3 + (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*
b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2
*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**2 - 17*a**2*b**2*c + 16*a**2*c
**5*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c
**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b
**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*
a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + b**4*c**3*(-b*sqrt(-
(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))
)/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) + (b*sqrt(-(4*a*c - b**2)**3)*(3
0*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**
2 - 17*a**2*b**2*c + 16*a**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2
 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(b*s
qrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c
**4)) + b**4*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c +
 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) -
 (2*a*c - 3*b**2)/(2*c**4)))/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) - (2*
a**3*c**2 - 4*a**2*b**2*c + a*b**4 + x*(5*a**2*b*c**2 - 5*a*b**3*c + b**5))
/(4*a**2*c**5 - a*b**2*c**4 + x**2*(4*a*c**6 - b**2*c**5) + x*(4*a*b*c**5 -
 b**3*c**4)) + x**2/(2*c**2)
```

Giac [A] time = 1.12732, size = 254, normalized size = 1.3

$$-\frac{\left(3\,b^5-20\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^4-4\,ac^5\right)\sqrt{-b^2+4\,ac}}+\frac{\left(3\,b^2-2\,ac\right)\log\left(cx^2+bx+a\right)}{2\,c^4}+\frac{c^2x^2-4\,bcx}{2\,c^4}+\frac{ab^4-4\,a^2b^2c+2}{\left(cx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/c^4 + 1/2*(c^2*x^2 - 4*b*c*x)/c^4 + (a*b^4 - 4*a^2*b^2*c + 2*a^3*
c^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c
^4)
```

3.423 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{2x\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}-\frac{b\log\left(a+bx\right.}{c^3}$$

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*
(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^
2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) -
(b*Log[a + b*x + c*x^2])/c^3
```

Rubi [A] time = 0.148108, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1340, 738, 800, 634, 618, 206, 628}

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{2x\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}-\frac{b\log\left(a+bx\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^2 + b/x)^(-2),x]
```

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*
(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^
2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) -
(b*Log[a + b*x + c*x^2])/c^3
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \int \frac{x^4}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \left(-\frac{2\left(b^2-3ac\right)}{c^2}+\frac{2bx}{c}+\frac{2\left(a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x\right)}{c^2\left(a+bx+cx^2\right)}\right)dx}{-b^2+4ac}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{2\int \frac{a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx}{c^2\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{c^3} + \frac{\left(b^4-6ab^2c+6a^2c^2\right)\cdots}{c^3\left(b^2-4\cdots\right.}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{b\log\left(a+bx+cx^2\right)}{c^3} - \frac{\left(2\left(b^4-6ab^2c\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{2\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4a\cdots}}\right.}{c^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.184312, size = 132, normalized size = 0.88

$$\frac{-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{a^2c(3b-2cx)-ab^2(b-4cx)+b^4(-x)}{\left(b^2-4ac\right)(a+x(b+cx))}-b\log(a+x(b+cx))+cx}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^2 + b/x)^(-2),x]
```

```
[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

Maple [B] time = 0.01, size = 352, normalized size = 2.4

$$\frac{x}{c^2}+2\,\frac{xa^2}{c\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}-4\,\frac{xab^2}{c^2\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}+\frac{xb^4}{c^3\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}-3\,\frac{a^2 \ldots}{c^2\left(cx^2+bx+ \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2,x)
```

```
[Out] x/c^2+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2-4/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x
*a*b^2+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4-3/c^2/(c*x^2+b*x+a)*b*a^2/(4*a
*c-b^2)+1/c^3/(c*x^2+b*x+a)*b^3*a/(4*a*c-b^2)-4/c^2/(4*a*c-b^2)*ln(c*x^2+b*
x+a)*a*b+1/c^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3-12/c/(4*a*c-b^2)^(3/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2+12/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*a*b^2-2/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.83224, size = 1760, normalized size = 11.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(
b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a^2*b^
2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x
^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), -(a*b^5
 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^3 -
```

```
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c
^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2
)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^
3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [B] time = 1.60346, size = 842, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2,x)
```

```
[Out] (-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**
2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a
*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2
*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**
2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*
(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a
*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - b**
4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4
)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(12*a**2
*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 + x*(2*a**2*c**2 - 4
*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*a*c**5 - b**2*c**4)
 + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

Giac [A] time = 1.10734, size = 217, normalized size = 1.45

$$\frac{2\left(b^{4}-6\,ab^{2}c+6\,a^{2}c^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{2}c^{3}-4\,ac^{4}\right)\sqrt{-b^{2}+4\,ac}}+\frac{x}{c^{2}}-\frac{b\log\left(cx^{2}+bx+a\right)}{c^{3}}-\frac{\frac{\left(b^{4}-4\,ab^{2}c+2\,a^{2}c^{2}\right)x}{c}+\frac{ab^{3}-3\,a^{2}bc}{c}}{\left(cx^{2}+bx+a\right)\left(b^{2}-4\,ac\right)c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^
2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3 -
 ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b*x
+ a)*(b^2 - 4*a*c)*c^2)
```

3.424 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x} dx$

Optimal. Leaf size=114

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2
- 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)
```

Rubi [A] time = 0.10029, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 738, 773, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x),x]
```

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2
- 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx &= \int \frac{x^3}{\left(a+bx+cx^2\right)^2}\,dx\\
&= \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}\\
&= -\frac{bx}{c\left(b^2-4ac\right)}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{-ab+\left(-b^2+4ac\right)x}{a+bx+cx^2}\,dx}{c\left(b^2-4ac\right)}\\
&= -\frac{bx}{c\left(b^2-4ac\right)}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}-\frac{\left(b\left(b^2-6ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2\left(b^2-4ac\right)}\\
&= -\frac{bx}{c\left(b^2-4ac\right)}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}+\frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac}\right.}{c^2\left(b^2-4ac\right)}\\
&= -\frac{bx}{c\left(b^2-4ac\right)}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}
$$

Mathematica [A] time = 0.148453, size = 109, normalized size = 0.96

$$
\frac{\frac{2\left(-2a^2c+ab(b-3cx)+b^3x\right)}{\left(b^2-4ac\right)(a+x(b+cx))}+\frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\log(a+x(b+cx))}{2c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x),x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x)))
 + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c
```

```
)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

Maple [A] time = 0.007, size = 209, normalized size = 1.8

$$\frac{1}{cx^2+bx+a}\left(\frac{b\left(3\,ac-b^2\right)x}{c^2\left(4\,ac-b^2\right)}+\frac{a\left(2\,ac-b^2\right)}{c^2\left(4\,ac-b^2\right)}\right)+2\,\frac{\ln\left(cx^2+bx+a\right)a}{c\left(4\,ac-b^2\right)}-\frac{\ln\left(cx^2+bx+a\right)b^2}{2\,c^2\left(4\,ac-b^2\right)}-6\,\frac{ab}{c\left(4\,ac-b^2\right)^{3/2}}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x,x)
```

```
[Out] (b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+
a)+2/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a-1/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^
2-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b+1/c^2/(4*a*
c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.75188, size = 1374, normalized size = 12.05

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+\left(ab^3-6\,a^2bc+\left(b^3c-6\,abc^2\right)x^2+\left(b^4-6\,ab^2c\right)x\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4}}{cx^2+bx+a}\right.}{2\left(ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4+\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x),
1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^
2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^
5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

Sympy [B] time = 1.23636, size = 729, normalized size = 6.39

$$\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{2c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{2c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqrt(-(4*
a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*a*b*c - b**3))/(
4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b
**3*c**2))
```

Giac [A] time = 1.1207, size = 169, normalized size = 1.48

$$-\frac{\left(b^3-6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2-4\,ac^3\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{ab^2-2\,a^2c+\left(b^3-3\,abc\right)x}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3
)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c + (
b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

3.425 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2} dx$

Optimal. Leaf size=71

$$\frac{\frac{2a}{x}+b}{\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (b + (2*a)/x)/((b^2 - 4*a*c)*(c + a/x^2 + b/x)) - (4*a*ArcTanh[(b + (2*a)/x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0421379, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1352, 614, 618, 206}

$$\frac{\frac{2a}{x}+b}{\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^2),x]
```

```
[Out] (b + (2*a)/x)/((b^2 - 4*a*c)*(c + a/x^2 + b/x)) - (4*a*ArcTanh[(b + (2*a)/x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx = -\text{Subst}\left(\int \frac{1}{(c+bx+ax^2)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{b+\frac{2a}{x}}{(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx, x, \frac{1}{x}\right)}{b^2-4ac}$$

$$= \frac{b+\frac{2a}{x}}{(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} - \frac{(4a)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\frac{2a}{x}\right)}{b^2-4ac}$$

$$= \frac{b+\frac{2a}{x}}{(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} - \frac{4a\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Mathematica [A] time = 0.0969133, size = 81, normalized size = 1.14

$$\frac{a(b-2cx)+b^2x}{c(4ac-b^2)(a+x(b+cx))} + \frac{4a\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^2),x]
```

```
[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.007, size = 97, normalized size = 1.4

$$\frac{1}{cx^2+bx+a}\left(-\frac{(2ac-b^2)x}{c(4ac-b^2)}+\frac{ab}{c(4ac-b^2)}\right)+4\frac{a}{(4ac-b^2)^{3/2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^2,x)
```

```
[Out] (-(2*a*c-b^2)/c/(4*a*c-b^2)*x+a*b/c/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.80585, size = 826, normalized size = 11.63

$$\left[-\frac{ab^3 - 4a^2bc + 2\left(ac^2x^2 + abcx + a^2c\right)\sqrt{b^2 - 4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right) + \left(b^4 - 6ab^2c + 8a^2c^2\right)x}{ab^4c - 8a^2b^2c^2 + 16a^3c^3 + \left(b^4c^2 - 8ab^2c^3 + 16a^2c^4\right)x^2 + \left(b^5c - 8ab^3c^2 + 16a^2bc^3\right)x}, - \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*lo
g((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 +
 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x + a^2*c
)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
+ (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 +
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x)]
```

Sympy [B] time = 0.752775, size = 280, normalized size = 3.94

$$-2a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x + \frac{-32a^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} + 16a^2b^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} - 2ab^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} + 2ab}{4ac}\right) + 2a\sqrt{-\frac{1}{\left(4ac-b^2\right.}} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**2,x)
```

```
[Out] -2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*a*
c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (3
2*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c - b*
*2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) - (-a*b + x
*(2*a*c - b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*
(4*a*b*c**2 - b**3*c))
```

Giac [A] time = 1.11492, size = 119, normalized size = 1.68

$$-\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\left(b^2-4ac\right)\sqrt{-b^2+4ac}} - \frac{b^2x - 2acx + ab}{\left(b^2c - 4ac^2\right)\left(cx^2 + bx + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="giac")
```

```
[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

3.426 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3} dx$

Optimal. Leaf size=66

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.033888, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1354, 638, 618, 206}

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^3),x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx = \int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$

$$= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac}$$

$$= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.0665612, size = 69, normalized size = 1.05

$$\frac{2a+bx}{\left(b^2-4ac\right)(a+x(b+cx))} - \frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^3),x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.003, size = 70, normalized size = 1.1

$$\frac{-bx-2\,a}{\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)} - 2\,\frac{b}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^3,x)
```

```
[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.70397, size = 740, normalized size = 11.21

$$\left[\frac{2\,ab^2-8\,a^2c-\left(bcx^2+b^2x+ab\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+\left(b^3-4\,abc\right)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x}, \frac{2\,ab^2-8\,a^2c-2\,(bcx}{ab^4-8\,a^2b^2c+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="fricas")
```

```
[Out] [(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b^2 -
 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*x)]
```

Sympy [B] time = 0.714274, size = 252, normalized size = 3.82

$$b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)-b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**3,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2
)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**2)
**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*c**
2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5
*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) - (2*a + b*x)/(4*a**2*c - a*b*
*2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Giac [A] time = 1.164, size = 103, normalized size = 1.56

$$\frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}+\frac{bx+2\,a}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="giac")
```

```
[Out] 2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.427 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} dx$

Optimal. Leaf size=66

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.032642, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1354, 614, 618, 206}

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{(2c)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$

$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{(4c)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac}$$

$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.0798264, size = 70, normalized size = 1.06

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{b+2cx}{a+x(b+cx)}}{b^2-4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Maple [A] time = 0.003, size = 68, normalized size = 1.

$$\frac{2\,cx+b}{\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)} + 4\,\frac{c}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^4,x)
```

```
[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.77133, size = 745, normalized size = 11.29

$$\left[-\frac{b^3-4\,abc+2\left(c^2x^2+bcx+ac\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\left(b^2c-4\,ac^2\right)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x},-\frac{b^3-4\,abc-4}{ab^4-8\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="fricas")
```

```
[Out] [-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 -
 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] time = 0.800024, size = 265, normalized size = 4.02

$$-2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+16ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-2b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2bc}{4c^2}\right)+2c\sqrt{-\frac{1}{\left(4ac-b^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**4,x)
```

```
[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*a*
c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (
32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*
c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Giac [A] time = 1.12242, size = 103, normalized size = 1.56

$$-\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,cx+b}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="giac")
```

```
[Out] -4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.428 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x
]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rubi [A] time = 0.144734, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^5),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x
]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx &= \int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{-b^2+4ac-bcx}{x\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \left(\frac{-b^2+4ac}{ax}+\frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a\left(a+bx+cx^2\right)}\right)dx}{a\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^2}-\frac{\int \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx}{a^2\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^2}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2}-\frac{\left(b\left(b^2-6ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-\cdots}\right.}{a^2\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.188197, size = 107, normalized size = 0.99

$$
\frac{\frac{2a\left(-2ac+b^2+bcx\right)}{\left(b^2-4ac\right)(a+x(b+cx))}+\frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}-\log(a+x(b+cx))+2\log(x)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^5),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
- 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*L
og[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [B] time = 0.012, size = 237, normalized size = 2.2

$$\frac{\ln(x)}{a^2} - \frac{bcx}{a\left(cx^2+bx+a\right)\left(4ac-b^2\right)} + 2\frac{c}{\left(cx^2+bx+a\right)\left(4ac-b^2\right)} - \frac{b^2}{a\left(cx^2+bx+a\right)\left(4ac-b^2\right)} - 2\frac{c\ln\left(cx^2+bx+a\right)}{a\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^5,x)
```

```
[Out] ln(x)/a^2-1/a/(c*x^2+b*x+a)*b*c/(4*a*c-b^2)*x+2/(c*x^2+b*x+a)/(4*a*c-b^2)*c
-1/a/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2-2/a/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)+1/2/a^
2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*b*c+1/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28047, size = 1685, normalized size = 15.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2
 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^
3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
```

```
 - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2
+ (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

Sympy [B] time = 7.44203, size = 2236, normalized size = 20.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**5,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))*log(x + (1536*a**9*c**5*(-b
*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b*
*2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 2112*a**8*b**2*c**4*(-b*s
qrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 1136*a**7*b**4*c**3*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c
**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 768*a**7*c**5*(-b*sqrt(-(4*a*
c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) - 1/(2*a**2)) - 300*a**6*b**6*c**2*(-b*sqrt(-(4*a*c - b*
*2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)) - 1/(2*a**2))**2 + 624*a**6*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) - 1/(2*a**2)) + 39*a**5*b**8*c*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*
c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))
- 1/(2*a**2))**2 - 184*a**5*b**4*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c -
 b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1
/(2*a**2)) - 768*a**5*c**5 - 2*a**4*b**10*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a
*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))
 - 1/(2*a**2))**2 + 23*a**4*b**6*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c -
 b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1
/(2*a**2)) + 1488*a**4*b**2*c**4 - a**3*b**8*c*(-b*sqrt(-(4*a*c - b**2)**3)
*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b
**6)) - 1/(2*a**2)) - 952*a**3*b**4*c**3 + 277*a**2*b**6*c**2 - 38*a*b**8*c
 + 2*b**10)/(864*a**4*b*c**5 - 738*a**3*b**3*c**4 + 243*a**2*b**5*c**3 - 36
*a*b**7*c**2 + 2*b**9*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a
**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))*
log(x + (1536*a**9*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 -
 2112*a**8*b**2*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 11
36*a**7*b**4*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a*
*3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 768*a
**7*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 -
 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 300*a**6*b**6*c**
2*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 624*a**6*b**2*c**4*(b
*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b*
*2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) + 39*a**5*b**8*c*(b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 184*a**5*b**4*c**3*(b*sqrt(-(4*a*c -
 b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*
b**4*c - b**6)) - 1/(2*a**2)) - 768*a**5*c**5 - 2*a**4*b**10*(b*sqrt(-(4*a*
```

```
c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 23*a**4*b**6*c**2*(b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) - 1/(2*a**2)) + 1488*a**4*b**2*c**4 - a**3*b**8*c*(b*sqrt(-(4
*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 952*a**3*b**4*c**3 + 277*a**2*b**6*c*
*2 - 38*a*b**8*c + 2*b**10)/(864*a**4*b*c**5 - 738*a**3*b**3*c**4 + 243*a**
2*b**5*c**3 - 36*a*b**7*c**2 + 2*b**9*c)) - (-2*a*c + b**2 + b*c*x)/(4*a**3
*c - a**2*b**2 + x**2*(4*a**2*c**2 - a*b**2*c) + x*(4*a**2*b*c - a*b**3)) +
 log(x)/a**2
```

Giac [A] time = 1.13712, size = 170, normalized size = 1.57

$$-\frac{\left(b^3-6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}-\frac{\log\left(cx^2+bx+a\right)}{2\,a^2}+\frac{\log\left(|x|\right)}{a^2}+\frac{abcx+ab^2-2\,a^2c}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c
)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 + (a
*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

3.429 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx$

Optimal. Leaf size=148

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{2\left(b^2-3ac\right)}{a^2x\left(b^2-4ac\right)}+\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2+bc}{ax\left(b^2-4ac\right)\left(a+b\right.}$$

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 -
4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 +
(b*Log[a + b*x + c*x^2])/a^3
```

Rubi [A] time = 0.182122, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 740, 800, 634, 618, 206, 628}

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{2\left(b^2-3ac\right)}{a^2x\left(b^2-4ac\right)}+\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2+bc}{ax\left(b^2-4ac\right)\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^6),x]
```

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 -
4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 +
(b*Log[a + b*x + c*x^2])/a^3
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx &= \int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)}-\frac{\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)}-\frac{\int \left(\frac{2\left(-b^2+3ac\right)}{ax^2}-\frac{2b\left(-b^2+4ac\right)}{a^2x}+\frac{2\left(-b^4+5ab^2c-3a^2c^2-bc\left(b^2-4ac\right)x\right)}{a^2\left(a+bx+cx^2\right)}\right)dx}{a\left(b^2-4ac\right)}\\
&= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)}-\frac{2b\log(x)}{a^3}-\frac{2\int \frac{-b^4+5ab^2c-3a^2c^2-bc\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx}{a^3\left(b^2-4ac\right)}\\
&= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{a^3}+\frac{\left(b^4-6ab^2c+\cdots\right.}{a^3\left(\cdots\right.}\\
&= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{\left(2\left(b^4-\cdots\right.\right.}{\cdots}\\
&= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)}-\frac{2\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-2\cdots
\end{aligned}
$$

Mathematica [A] time = 0.293125, size = 131, normalized size = 0.89

$$
-\frac{\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{a\left(-3abc-2ac^2x+b^2cx+b^3\right)}{\left(b^2-4ac\right)(a+x(b+cx))}-b\log(a+x(b+cx))+\frac{a}{x}+2b\log(x)}{a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^6),x]
```

```
[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(b
 + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^3)
```

Maple [B] time = 0.014, size = 328, normalized size = 2.2

$$-\frac{1}{xa^2}-2\,\frac{b\ln(x)}{a^3}-2\,\frac{c^2x}{a\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}+\frac{cxb^2}{a^2\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}-3\,\frac{bc}{a\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}+\frac{}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^6,x)
```

```
[Out] -1/a^2/x-2*b*ln(x)/a^3-2/a/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x+1/a^2/(c*x^2+b*x
+a)*c/(4*a*c-b^2)*x*b^2-3/a/(c*x^2+b*x+a)*b/(4*a*c-b^2)*c+1/a^2/(c*x^2+b*x+
a)*b^3/(4*a*c-b^2)+4/a^2/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*b-1/a^3/(4*a*c-b^2)*
ln(c*x^2+b*x+a)*b^3-12/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*c^2+12/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c-
2/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.8916, size = 2049, normalized size = 13.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="fricas")
```

```
[Out] [-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3
*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a
^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*log(
(2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2
+ b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5
*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c
```

```
- 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^
3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)
) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^
3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5*b^2*c + 1
6*a^6*c^2)*x)]
```

Sympy [B] time = 11.4878, size = 2672, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**6,x)
```

```
[Out] (b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-1728*a*
*11*b*c**5*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b
**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 2
256*a**10*b**3*c**4*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b
**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
))**2 - 1172*a**9*b**5*c**3*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2
 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)))**2 - 288*a**9*c**6*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c*
*2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4
*c - b**6))) + 303*a**8*b**7*c**2*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**
2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*
b**4*c - b**6)))**2 - 432*a**8*b**2*c**5*(b/a**3 - sqrt(-(4*a*c - b**2)**3)
*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6))) - 39*a**7*b**9*c*(b/a**3 - sqrt(-(4*a*c - b**2)**3)
*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)))**2 + 558*a**7*b**4*c**4*(b/a**3 - sqrt(-(4*a*c - b*
*2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6))) + 2*a**6*b**11*(b/a**3 - sqrt(-(4*a*c - b**2
)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*
c**2 + 12*a*b**4*c - b**6)))**2 - 212*a**6*b**6*c**3*(b/a**3 - sqrt(-(4*a*c
 - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6))) - 576*a**6*b*c**6 + 34*a**5*b**8*c**2*(
b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(
64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 6048*a**5*b**3*c
**5 - 2*a**4*b**10*c*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*
b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
))) - 7908*a**4*b**5*c**4 + 4264*a**3*b**7*c**3 - 1144*a**2*b**9*c**2 + 152
*a*b**11*c - 8*b**13)/(216*a**6*c**7 + 2808*a**5*b**2*c**6 - 5292*a**4*b**4
*c**5 + 3384*a**3*b**6*c**4 - 1008*a**2*b**8*c**3 + 144*a*b**10*c**2 - 8*b*
*12*c)) + (b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b*
*4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x +
 (-1728*a**11*b*c**5*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*
b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
)))**2 + 2256*a**10*b**3*c**4*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c*
*2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4
*c - b**6)))**2 - 1172*a**9*b**5*c**3*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6
*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
```

```
2*a*b**4*c - b**6)))**2 - 288*a**9*c**6*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*
(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
 12*a*b**4*c - b**6))) + 303*a**8*b**7*c**2*(b/a**3 + sqrt(-(4*a*c - b**2)*
*3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6)))**2 - 432*a**8*b**2*c**5*(b/a**3 + sqrt(-(4*a*c -
 b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6))) - 39*a**7*b**9*c*(b/a**3 + sqrt(-(4*a*c -
 b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 558*a**7*b**4*c**4*(b/a**3 + sqrt(-(
4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 4
8*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a**6*b**11*(b/a**3 + sqrt(-(4*
a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 212*a**6*b**6*c**3*(b/a**3 + sq
rt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**
3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 576*a**6*b*c**6 + 34*a**5*b
**8*c**2*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**
4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 6048*a
**5*b**3*c**5 - 2*a**4*b**10*c*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c
**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6))) - 7908*a**4*b**5*c**4 + 4264*a**3*b**7*c**3 - 1144*a**2*b**9*
c**2 + 152*a*b**11*c - 8*b**13)/(216*a**6*c**7 + 2808*a**5*b**2*c**6 - 5292
*a**4*b**4*c**5 + 3384*a**3*b**6*c**4 - 1008*a**2*b**8*c**3 + 144*a*b**10*c
**2 - 8*b**12*c)) - (4*a**2*c - a*b**2 + x**2*(6*a*c**2 - 2*b**2*c) + x*(7*
a*b*c - 2*b**3))/(x**3*(4*a**3*c**2 - a**2*b**2*c) + x**2*(4*a**3*b*c - a**
2*b**3) + x*(4*a**4*c - a**3*b**2)) - 2*b*log(x)/a**3
```

Giac [A] time = 1.13286, size = 231, normalized size = 1.56

$$\frac{2\left(b^4-6\,ab^2c+6\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^2-4\,a^4c\right)\sqrt{-b^2+4\,ac}}-\frac{2\,b^2cx^2-6\,ac^2x^2+2\,b^3x-7\,abcx+ab^2-4\,a^2c}{\left(a^2b^2-4\,a^3c\right)\left(cx^3+bx^2+ax\right)}+\frac{b\log\left(cx^2+bx+a\right)}{a^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3*x
 - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*x))
 + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

3.430 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7} dx$

Optimal. Leaf size=202

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{3/2}}-\frac{3b^2-8ac}{2a^2x^2\left(b^2-4ac\right)}-\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}+\frac{b\left(3b^2-11ac\right)}{a^3x\left(b^2-4ac\right)}+\log$$

```
[Out] -(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b^2
 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)
) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 -
2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rubi [A] time = 0.238815, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{3/2}}-\frac{3b^2-8ac}{2a^2x^2\left(b^2-4ac\right)}-\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}+\frac{b\left(3b^2-11ac\right)}{a^3x\left(b^2-4ac\right)}+\log$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^7),x]
```


```
[Out] -(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b^2
 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)
) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 -
2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)
```


Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = \int \frac{1}{x^3\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} - \frac{\int \frac{-3b^2+8ac-3bcx}{x^3\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-3b^2+8ac}{ax^3} + \frac{3b^3-11abc}{a^2x^2} + \frac{\left(b^2-4ac\right)\left(-3b^2+2ac\right)}{a^3x} + \frac{b\left(3b^4-17ab^2c+19a^2 \ldots}{a^3\left(\ldots\right.}\right.}{a\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2ac\right)\log(x)}{a^4} - \ldots$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2ac\right)\log(x)}{a^4} - \ldots$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2ac\right)\log(x)}{a^4} - \ldots$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{b\left(3b^4-20ab^2c+3 \ldots\right.}{a^4\left(b^2 \ldots\right.}$$

Mathematica [A] time = 0.367868, size = 175, normalized size = 0.87

$$\frac{\frac{2a\left(2a^2c^2-4ab^2c-3abc^2x+b^3cx+b^4\right)}{\left(b^2-4ac\right)(a+x(b+cx))} + \frac{2b\left(30a^2c^2-20ab^2c+3b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} - \frac{a^2}{x^2} + 2\log(x)\left(3b^2-2ac\right) + \left(2ac-3b^2\right)\log(a+x(b+ \ldots}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^7),x]
```

```
[Out] (-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a
*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 3
0*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4)
```

Maple [B] time = 0.018, size = 418, normalized size = 2.1

$$-\frac{1}{2a^2x^2}-2\frac{\ln(x)c}{a^3}+3\frac{\ln(x)b^2}{a^4}+2\frac{b}{a^3x}+3\frac{bc^2x}{a^2(cx^2+bx+a)(4ac-b^2)}-\frac{b^3cx}{a^3(cx^2+bx+a)(4ac-b^2)}-2\frac{}{a(cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^7,x)
```

```
[Out] -1/2/a^2/x^2-2/a^3*ln(x)*c+3/a^4*ln(x)*b^2+2/a^3*b/x+3/a^2/(c*x^2+b*x+a)*b*
c^2/(4*a*c-b^2)*x-1/a^3/(c*x^2+b*x+a)*b^3*c/(4*a*c-b^2)*x-2/a/(c*x^2+b*x+a)
/(4*a*c-b^2)*c^2+4/a^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c-1/a^3/(c*x^2+b*x+a)/
(4*a*c-b^2)*b^4+4/a^2/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)-7/a^3/(4*a*c-b^2)*c*l
n(c*x^2+b*x+a)*b^2+3/2/a^4/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^4+30/a^2/(4*a*c-b^
2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2-20/a^3/(4*a*c-b^2)^(3/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c+3/a^4/(4*a*c-b^2)^(3/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.63456, size = 2606, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 +
 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3
)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c
+ 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c
*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x +
```

```
((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a
*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*
a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^
4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3
*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^
5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*
c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a
*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)
*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64
*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*
a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2
)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a
^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3
*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^
2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2)]
```

Sympy [B] time = 18.6671, size = 4083, normalized size = 20.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**7,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)
/(2*a**4))*log(x + (3072*a**14*c**6*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c
**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 9408*a**13*b**2*c**5*(-
b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(6
4*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(
2*a**4))**2 + 9040*a**12*b**4*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c*
*2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a
*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4116*a**11*b**6*c**3*(-b
*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2
*a**4))**2 + 3072*a**11*c**7*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 2
0*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 987*a**10*b**8*c**2*(-b*sqrt(-(4*
a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2
 - 7536*a**10*b**2*c**6*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b
**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b
**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 121*a**9*b**10*c*(-b*sqrt(-(4*a*c - b*
*2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a*
*2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 + 8152*
a**9*b**4*c**5*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3
*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (
2*a*c - 3*b**2)/(2*a**4)) + 6*a**8*b**12*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a
**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4343*a**8*b**6*c**
4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**
```

4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 6144*a**8*c**8 + 1198*a**7*b**8*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 50208*a**7*b**2*c**7 - 165*a**6*b**10*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 137792*a**6*b**4*c**6 + 9*a**5*b**12*c*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 176474*a**5*b**6*c**5 - 119275*a**4*b**8*c**4 + 45448*a**3*b**10*c**3 - 9846*a**2*b**12*c**2 + 1134*a*b**14*c - 54*b**16)/(17280*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 112428*a**5*b**5*c**6 - 88605*a**4*b**7*c**5 + 37600*a**3*b**9*c**4 - 8820*a**2*b**11*c**3 + 1080*a*b**13*c**2 - 54*b**15*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))*log(x + (3072*a**14*c**6*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 9408*a**13*b**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 + 9040*a**12*b**4*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4116*a**11*b**6*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 + 3072*a**11*c**7*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 987*a**10*b**8*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 7536*a**10*b**2*c**6*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 121*a**9*b**10*c*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 + 8152*a**9*b**4*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 6*a**8*b**12*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4343*a**8*b**6*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 6144*a**8*c**8 + 1198*a**7*b**8*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 50208*a**7*b**2*c**7 - 165*a**6*b**10*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 137792*a**6*b**4*c**6 + 9*a**5*b**12*c*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 176474*a**5*b**6*c**5 - 119275*a**4*b**8*c**4 + 45448*a**3*b**10*c**3 - 9846*a**2*b**12*c**2 + 1134*a*b**14*c - 54*b**16)/(17280*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 112428*a**5*b**5*c**6 - 88605*a**4*b**7*c**5 + 37600*a**3*b**9*c**4 - 8820*a**2*b**11*c**3 + 1080*a*b**13*c**2 - 54*b**15*c)) + (-4*a**3*c + a**2*b**2 + x**3*(22*a*b*c**2 - 6*b**3*c) + x**2*(-8*a**2*c**2 + 25*a*b**2*c - 6*b**4) + x*(12*a**2*b*c - 3*a*b**3))/(x**4*(8*a**4*c**2 - 2*a**3*b**2*c) + x**3*(8*a**4*b*c - 2*a**3*b**3) + x**2*(8*a**5*c - 2*a**4*b**2)) - (2*a*c - 3*b**2)*log(x + (-6144*a**8*c**8 + 50208*a**7*b**2*c**7 - 3072*a**7*c**7*(2*a*c - 3*b**2) - 137792*a**6*b**4*c**

```
6 + 7536*a**6*b**2*c**6*(2*a*c - 3*b**2) + 3072*a**6*c**6*(2*a*c - 3*b**2)*
*2 + 176474*a**5*b**6*c**5 - 8152*a**5*b**4*c**5*(2*a*c - 3*b**2) - 9408*a*
*5*b**2*c**5*(2*a*c - 3*b**2)**2 - 119275*a**4*b**8*c**4 + 4343*a**4*b**6*c
**4*(2*a*c - 3*b**2) + 9040*a**4*b**4*c**4*(2*a*c - 3*b**2)**2 + 45448*a**3
*b**10*c**3 - 1198*a**3*b**8*c**3*(2*a*c - 3*b**2) - 4116*a**3*b**6*c**3*(2
*a*c - 3*b**2)**2 - 9846*a**2*b**12*c**2 + 165*a**2*b**10*c**2*(2*a*c - 3*b
**2) + 987*a**2*b**8*c**2*(2*a*c - 3*b**2)**2 + 1134*a*b**14*c - 9*a*b**12*
c*(2*a*c - 3*b**2) - 121*a*b**10*c*(2*a*c - 3*b**2)**2 - 54*b**16 + 6*b**12
*(2*a*c - 3*b**2)**2)/(17280*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 112428*a*
*5*b**5*c**6 - 88605*a**4*b**7*c**5 + 37600*a**3*b**9*c**4 - 8820*a**2*b**1
1*c**3 + 1080*a*b**13*c**2 - 54*b**15*c))/a**4
```

Giac [A] time = 1.15794, size = 309, normalized size = 1.53

$$-\frac{\left(3\,b^5-20\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^4b^2-4\,a^5c\right)\sqrt{-b^2+4\,ac}}-\frac{\left(3\,b^2-2\,ac\right)\log\left(cx^2+bx+a\right)}{2\,a^4}+\frac{\left(3\,b^2-2\,ac\right)\log\left(|x|\right)}{a^4}-\frac{a^3b^2-4\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c -
2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2
 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)
```

3.431 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx$

Optimal. Leaf size=238

$$\frac{3x\left(10a^2c^2-7ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^2}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}}-\frac{3bx^2\left(b^2-6ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+b}{2\left(b^2-4ac\right)(a+}$$

```
[Out] (3*(b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^2) - (3*b*(b^2 - 6*
a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a +
 b*x + c*x^2)^2) + (x^3*(a*(b^2 - 10*a*c) + b*(b^2 - 7*a*c)*x))/(c*(b^2 - 4
*a*c)^2*(a + b*x + c*x^2)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3
*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(5/2)) - (
3*b*Log[a + b*x + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.291033, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1340, 738, 818, 800, 634, 618, 206, 628}

$$\frac{3x\left(10a^2c^2-7ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^2}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}}-\frac{3bx^2\left(b^2-6ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+b}{2\left(b^2-4ac\right)(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^2 + b/x)^(-3),x]
```

```
[Out] (3*(b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^2) - (3*b*(b^2 - 6*
a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a +
 b*x + c*x^2)^2) + (x^3*(a*(b^2 - 10*a*c) + b*(b^2 - 7*a*c)*x))/(c*(b^2 - 4
*a*c)^2*(a + b*x + c*x^2)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3
*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(5/2)) - (
3*b*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
```

```
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx=\int \frac{x^6}{\left(a+bx+cx^2\right)^3}\,dx$$

$$=\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int \frac{x^4(10a+2bx)}{\left(a+bx+cx^2\right)^2}dx}{2\left(b^2-4ac\right)}$$

$$=\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2-10ac\right)+b\left(b^2-7ac\right)x\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{\int \frac{x^2\left(6a\left(b^2-10ac\right)+6b\left(b^2-6ac\right)x\right)}{a+bx+cx^2}dx}{2c\left(b^2-4ac\right)^2}$$

$$=\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2-10ac\right)+b\left(b^2-7ac\right)x\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{\int\left(-\frac{6\left(b^4-7ab^2c+10a^2c^2\right)}{c^2}+\frac{6b\left(b^2-6\right.}{c}\right.}{2c}$$

$$=\frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2}-\frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2-10ac\right)+\right.}{c\left(b^2-4ac\right)^2\left(a\right.}$$

$$=\frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2}-\frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2-10ac\right)+\right.}{c\left(b^2-4ac\right)^2\left(a\right.}$$

$$=\frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2}-\frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2-10ac\right)+\right.}{c\left(b^2-4ac\right)^2\left(a\right.}$$

$$=\frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2}-\frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2-10ac\right)+\right.}{c\left(b^2-4ac\right)^2\left(a\right.}$$

Mathematica [A] time = 0.401403, size = 260, normalized size = 1.09

$$\frac{\frac{a^2b^2c(5b-9cx)+a^3c^2(2cx-5b)-ab^4(b-6cx)+b^6(-x)}{\left(b^2-4ac\right)(a+x(b+cx))^2}+\frac{-102a^2b^2c^3x+61a^2b^3c^2-78a^3bc^3+36a^3c^4x+48ab^4c^2x-14ab^5c-6b^6cx+b^7}{\left(b^2-4ac\right)^2(a+x(b+cx))}+\frac{6c\left(30a^2b^2c^2-20a^3c^3-10ab^4c+\right.}{\left(4ac-b^2\right)^{5/}}}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^2 + b/x)^(-3),x]
```

```
[Out] (2*c^2*x + (b^7 - 14*a*b^5*c + 61*a^2*b^3*c^2 - 78*a^3*b*c^3 - 6*b^6*c*x +
48*a*b^4*c^2*x - 102*a^2*b^2*c^3*x + 36*a^3*c^4*x)/((b^2 - 4*a*c)^2*(a + x*
(b + c*x))) + (-(b^6*x) + a^2*b^2*c*(5*b - 9*c*x) - a*b^4*(b - 6*c*x) + a^3
*c^2*(-5*b + 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (6*c*(b^6 - 10*a
*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/(-b^2 + 4*a*c)^(5/2) - 3*b*c*Log[a + x*(b + c*x)])/(2*c^5)
```

Maple [B] time = 0.016, size = 1040, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3,x)
```

```
[Out] x/c^3+18/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a^3-51/c/(c*x^2+b*x
+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a^2*b^2+24/c^2/(c*x^2+b*x+a)^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^3*a*b^4-3/c^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^3*b^6-21/c/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^3-41/2/
c^2/(c*x^2+b*x+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^2+17/c^3/(c*x^2+b*
x+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a-5/2/c^4/(c*x^2+b*x+a)^2*b^7/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^2+14/c/(c*x^2+b*x+a)^2*a^4/(16*a^2*c^2-8*a*b^2*c+
b^4)*x-71/c^2/(c*x^2+b*x+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2+38/c^3/(
c*x^2+b*x+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4-5/c^4/(c*x^2+b*x+a)^2*a
/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^6-29/c^2/(c*x^2+b*x+a)^2*b*a^4/(16*a^2*c^2-
8*a*b^2*c+b^4)+18/c^3/(c*x^2+b*x+a)^2*b^3*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)-5/
2/c^4/(c*x^2+b*x+a)^2*b^5*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)-24/c^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a^2*b+12/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(
c*x^2+b*x+a)*a*b^3-3/2/c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^5-6
0/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*a^3+90/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^2-30/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^4+3/c^4/(16*a^2*c^2-8*a
*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02883, size = 4120, normalized size = 17.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(5*a^2*b^7 - 56*a^3*b^5*c + 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*
c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b
^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 1
23*a^2*b^4*c^3 - 254*a^3*b^2*c^4 + 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c +
 225*a^2*b^5*c^2 - 314*a^3*b^3*c^3 + 88*a^4*b*c^4)*x^2 + 3*(a^2*b^6 - 10*a^
3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3
*b*c^4)*x^3 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c
^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*
c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^4*c^2
- 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3
*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c
^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^3 +
(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^2 +
2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x^2 + b
*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6
```

```
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 - 12*a*
b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^5 + 24
*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*a^2*b^
5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x), -1/2*(5*a^2*b^7 - 56*a^3*b^5*c +
 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c
^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*
b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 123*a^2*b^4*c^3 - 254*a^3*b^2*c^4
+ 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c + 225*a^2*b^5*c^2 - 314*a^3*b^3*c^
3 + 88*a^4*b*c^4)*x^2 + 6*(a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5
*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*
c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^3 + (b^8 - 8*a*b^6*c +
10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c
 + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^
4*c^2 - 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a
^4*b^3*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a
^3*b*c^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*
x^3 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*
x^2 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x
^2 + b*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7
+ (b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 -
 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^
5 + 24*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*
a^2*b^5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x)]
```

Sympy [B] time = 4.28044, size = 1714, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3,x)
```

```
[Out] (-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66
*a**3*b*c**2 - 64*a**3*c**6*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20
*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**
5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b*
*8*c - b**10))) + 27*a**2*b**3*c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) - 3*sqr
t(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**
6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160
*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))))/(60*a**3*c**3 - 9
0*a**2*b**2*c**2 + 30*a*b**4*c - 3*b**6)) + (-3*b/(2*c**4) + 3*sqrt(-(4*a*c
 - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**
4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**
6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66*a**3*b*c**2 - 64*a**3*c**6*(-3
*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2
+ 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a
**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 27*a**2*b**3*
c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*
```

```
c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 12
80*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
 b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10))))/(60*a**3*c**3 - 90*a**2*b**2*c**2 + 30*a*b**4*c
- 3*b**6)) + (-58*a**4*b*c**2 + 36*a**3*b**3*c - 5*a**2*b**5 + x**3*(36*a**
3*c**4 - 102*a**2*b**2*c**3 + 48*a*b**4*c**2 - 6*b**6*c) + x**2*(-42*a**3*b
*c**3 - 41*a**2*b**3*c**2 + 34*a*b**5*c - 5*b**7) + x*(28*a**4*c**3 - 142*a
**3*b**2*c**2 + 76*a**2*b**4*c - 10*a*b**6))/(32*a**4*c**6 - 16*a**3*b**2*c
**5 + 2*a**2*b**4*c**4 + x**4*(32*a**2*c**8 - 16*a*b**2*c**7 + 2*b**4*c**6)
 + x**3*(64*a**2*b*c**7 - 32*a*b**3*c**6 + 4*b**5*c**5) + x**2*(64*a**3*c**
7 - 12*a*b**4*c**5 + 2*b**6*c**4) + x*(64*a**3*b*c**6 - 32*a**2*b**3*c**5 +
 4*a*b**5*c**4)) + x/c**3
```

Giac [A] time = 1.15774, size = 381, normalized size = 1.6

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}c^{4}-8\,ab^{2}c^{5}+16\,a^{2}c^{6}\right)\sqrt{-b^{2}+4\,ac}}+\frac{x}{c^{3}}-\frac{3\,b\log\left(cx^{2}+bx+a\right)}{2\,c^{4}}-\frac{5\,a^{2}b^{5}-36\,a^{3}b^{3}c+58\,a^{4}bc^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(
-b^2 + 4*a*c))/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) +
x/c^3 - 3/2*b*log(c*x^2 + b*x + a)/c^4 - 1/2*(5*a^2*b^5 - 36*a^3*b^3*c + 58
*a^4*b*c^2 + 6*(b^6*c - 8*a*b^4*c^2 + 17*a^2*b^2*c^3 - 6*a^3*c^4)*x^3 + (5*
b^7 - 34*a*b^5*c + 41*a^2*b^3*c^2 + 42*a^3*b*c^3)*x^2 + 2*(5*a*b^6 - 38*a^2
*b^4*c + 71*a^3*b^2*c^2 - 14*a^4*c^3)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)
^2*c^4)
```

3.432 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$

Optimal. Leaf size=190

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx\left(b^2-7ac\right)}{c^2\left(b^2-4ac\right)^2}+\frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-\right.\right.}{2c\left(b^2-4ac\right)^2\left(a+bx+\right.}$$

```
[Out] -((b*(b^2 - 7*a*c)*x)/(c^2*(b^2 - 4*a*c)^2)) + (x^4*(2*a + b*x))/(2*(b^2 -
4*a*c)*(a + b*x + c*x^2)^2) + (x^2*(a*(b^2 - 16*a*c) + b*(b^2 - 10*a*c)*x))
/(2*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^
2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + Log[
a + b*x + c*x^2]/(2*c^3)
```

Rubi [A] time = 0.27942, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1354, 738, 818, 773, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx\left(b^2-7ac\right)}{c^2\left(b^2-4ac\right)^2}+\frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-\right.\right.}{2c\left(b^2-4ac\right)^2\left(a+bx+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] -((b*(b^2 - 7*a*c)*x)/(c^2*(b^2 - 4*a*c)^2)) + (x^4*(2*a + b*x))/(2*(b^2 -
4*a*c)*(a + b*x + c*x^2)^2) + (x^2*(a*(b^2 - 16*a*c) + b*(b^2 - 10*a*c)*x))
/(2*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^
2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + Log[
a + b*x + c*x^2]/(2*c^3)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
```

```
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \int \frac{x^5}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{x^3(8a+bx)}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\int \frac{x\left(2a\left(b^2-16ac\right)+2b\left(b^2-7ac\right)x\right)}{a+bx+cx^2}\,d}{2c\left(b^2-4ac\right)^2}$$

$$= -\frac{b\left(b^2-7ac\right)x}{c^2\left(b^2-4ac\right)^2} + \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \int \frac{-2ab\left(b^2\right.}{}$$

$$= -\frac{b\left(b^2-7ac\right)x}{c^2\left(b^2-4ac\right)^2} + \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\int \frac{b+2cx}{a+bx+c}}{2c^3}$$

$$= -\frac{b\left(b^2-7ac\right)x}{c^2\left(b^2-4ac\right)^2} + \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\log\left(a+\right.}{2}$$

$$= -\frac{b\left(b^2-7ac\right)x}{c^2\left(b^2-4ac\right)^2} + \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{b\left(b^4-1\right.}{}$$

Mathematica [A] time = 0.337448, size = 221, normalized size = 1.16

$$\frac{\frac{-39a^2b^2c^2+50a^2bc^3x+32a^3c^3-30ab^3c^2x+11ab^4c+4b^5cx-b^6}{\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{a^2bc(5cx-4b)+2a^3c^2+ab^3(b-5cx)+b^5x}{\left(b^2-4ac\right)(a+x(b+cx))^2} - \frac{2bc\left(30a^2c^2-10ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + c\log(a}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] ((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x - 30*a*b^3*c^
2*x + 50*a^2*b*c^3*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (2*a^3*c^2 + b^
5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(-4*b + 5*c*x))/((b^2 - 4*a*c)*(a + x*(b
+ c*x))^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt
[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + x*(b + c*x)])/(2*c^4)
```

Maple [B] time = 0.013, size = 530, normalized size = 2.8

$$\frac{1}{\left(cx^2+bx+a\right)^2}\left(\frac{b\left(25a^2c^2-15ab^2c+2b^4\right)x^3}{c^2\left(16a^2c^2-8ab^2c+b^4\right)} + \frac{\left(32a^3c^3+11a^2b^2c^2-19ab^4c+3b^6\right)x^2}{2c^3\left(16a^2c^2-8ab^2c+b^4\right)} + \frac{ab\left(31a^2c^2-22ab^2c+3b^4\right.}{c^3\left(16a^2c^2-8ab^2c+b^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x,x)
```

```
[Out] (1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*(
32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^2+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+3/2*
```

```
a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)
^2+8/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a^2-4/c^2/(16*a^2*c^2-8*a
*b^2*c+b^4)*ln(c*x^2+b*x+a)*a*b^2+1/2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x
^2+b*x+a)*b^4-30/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*a^2*b+10/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3-1/c^3/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05508, size = 3429, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="fricas")
```

```
[Out] [1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
 - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 31*a*b^6*c
 + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^
3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c
 - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 +
 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2
 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x
 + b))/(c*x^2 + b*x + a)) + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 12
4*a^4*b*c^3)*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b
^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a
^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b
^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^
3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 -
128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*
b*c^6)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2
*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 3
1*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a^2*b^5
 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^
4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a
^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x + (a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^
```

```
3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5
- 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 +
 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3
- 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*
b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x)]
```

Sympy [B] time = 2.83448, size = 1510, normalized size = 7.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1
024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(-b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10)) + 1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(-b*sqrt(-
(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c
**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*
b**8*c - b**10)) + 1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(-b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10)) + 1/(2*c**3)) + a*b**4 + b**6*c**2*(-b*sqrt(-(4*a*c - b**2)
**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**
4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10
)) + 1/(2*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (b*sqrt(-(4*a*c
- b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1
280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
- b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(b*sqrt(-(4*a*c - b**2)**5)*
(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**
2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) +
1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(b*sqrt(-(4*a*c - b**2)**5)*
(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**
2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) +
1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(b*sqrt(-(4*a*c - b**2)**5)*(3
0*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*
c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/
(2*c**3)) + a*b**4 + b**6*c**2*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 -
10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**
3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)))/(30
*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (24*a**4*c**2 - 21*a**3*b**2*c + 3*a*
*2*b**4 + x**3*(50*a**2*b*c**3 - 30*a*b**3*c**2 + 4*b**5*c) + x**2*(32*a**3
*c**3 + 11*a**2*b**2*c**2 - 19*a*b**4*c + 3*b**6) + x*(62*a**3*b*c**2 - 44*
a**2*b**3*c + 6*a*b**5))/(32*a**4*c**5 - 16*a**3*b**2*c**4 + 2*a**2*b**4*c*
*3 + x**4*(32*a**2*c**7 - 16*a*b**2*c**6 + 2*b**4*c**5) + x**3*(64*a**2*b*c
**6 - 32*a*b**3*c**5 + 4*b**5*c**4) + x**2*(64*a**3*c**6 - 12*a*b**4*c**4 +
 2*b**6*c**3) + x*(64*a**3*b*c**5 - 32*a**2*b**3*c**4 + 4*a*b**5*c**3))
```

Giac [A] time = 1.13125, size = 331, normalized size = 1.74

$$-\frac{\left(b^5 - 10\,ab^3c + 30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5\right)\sqrt{-b^2+4\,ac}} + \frac{\log\left(cx^2+bx+a\right)}{2\,c^3} + \frac{3\,a^2b^4 - 21\,a^3b^2c + 24\,a^4c^2 + 2\left(2\,b^5c - 15\,ab^3c^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="giac")
```

```
[Out] -(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 +
b*x + a)/c^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 2*(2*b^5*c - 15
*a*b^3*c^2 + 25*a^2*b*c^3)*x^3 + (3*b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 + 32*
a^3*c^3)*x^2 + 2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2)*x)/((c*x^2 + b*x +
 a)^2*(b^2 - 4*a*c)^2*c^3)
```

3.433 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2} dx$

Optimal. Leaf size=111

$$\frac{12a^2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{3a\left(\frac{2a}{x}+b\right)}{\left(b^2-4ac\right)^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

```
[Out] (b + (2*a)/x)/(2*(b^2 - 4*a*c)*(c + a/x^2 + b/x)^2) - (3*a*(b + (2*a)/x))/(
(b^2 - 4*a*c)^2*(c + a/x^2 + b/x)) + (12*a^2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.0665031, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1352, 614, 618, 206}

$$\frac{12a^2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{3a\left(\frac{2a}{x}+b\right)}{\left(b^2-4ac\right)^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] (b + (2*a)/x)/(2*(b^2 - 4*a*c)*(c + a/x^2 + b/x)^2) - (3*a*(b + (2*a)/x))/(
(b^2 - 4*a*c)^2*(c + a/x^2 + b/x)) + (12*a^2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = -\operatorname{Subst}\left(\int \frac{1}{\left(c+bx+ax^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} + \frac{(3a)\operatorname{Subst}\left(\int \frac{1}{\left(c+bx+ax^2\right)^2}\,dx, x, \frac{1}{x}\right)}{b^2-4ac}$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} - \frac{\left(6a^2\right)\operatorname{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx, x, \frac{1}{x}\right)}{\left(b^2-4ac\right)^2}$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{\left(12a^2\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\right.}{\left(b^2-4ac\right)^2}$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{12a^2\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.182434, size = 174, normalized size = 1.57

$$\frac{1}{2}\left(\frac{a^2c(2cx-3b)+ab^2(b-4cx)+b^4x}{c^3\left(4ac-b^2\right)(a+x(b+cx))^2} + \frac{22a^2bc^2-20a^2c^3x+16ab^2c^2x-8ab^3c-2b^4cx+b^5}{c^3\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{24a^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] ((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x + 16*a*b^2*c^2*x - 20*a^2*c^3*
x)/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (b^4*x + a*b^2*(b - 4*c*x) + a
^2*c*(-3*b + 2*c*x))/(c^3*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + (24*a^2*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [B] time = 0.012, size = 260, normalized size = 2.3

$$\frac{1}{\left(cx^2+bx+a\right)^2}\left(-\frac{\left(10\,a^2c^2-8\,ab^2c+b^4\right)x^3}{c\left(16\,a^2c^2-8\,ab^2c+b^4\right)} + \frac{b\left(2\,a^2c^2+8\,ab^2c-b^4\right)x^2}{2\,c^2\left(16\,a^2c^2-8\,ab^2c+b^4\right)} - \frac{a\left(6\,a^2c^2-10\,ab^2c+b^4\right)x}{c^2\left(16\,a^2c^2-8\,ab^2c+b^4\right)} + \frac{a^2b\,(1}{2\,c^2\left(16\,a^2c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^2,x)
```

```
[Out] (-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*b*(2*a^
2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-a*(6*a^2*c^2-10*a*b
^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x+1/2*a^2*b*(10*a*c-b^2)/c^2/(16*a
```

```
^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+12*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05225, size = 2006, normalized size = 18.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 42
*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3
*b*c^3)*x^2 - 12*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x + a^4*c^2 +
 (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^
6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^
4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*
c^6 - 64*a^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3
*b*c^6)*x^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 1
28*a^4*c^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b
*c^5)*x), -1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4
*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^
2 + 8*a^3*b*c^3)*x^2 + 24*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x +
a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b
^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64
*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^4 + 2*(
b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^3 + (b^8*c^2 - 10
*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*x^2 + 2*(a*b^7*
c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x)]
```

Sympy [B] time = 1.7504, size = 547, normalized size = 4.93

$$-6a^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-384a^5c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+288a^4b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-72a^3b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+6a^2b^6\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\cdots}{12a^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**2,x)
```

```
[Out] -6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**5*c**3*sqrt(-1/(4*a*c -
 b**2)**5) + 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 72*a**3*b**4*c
*sqrt(-1/(4*a*c - b**2)**5) + 6*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5) + 6*a*
*2*b)/(12*a**2*c)) + 6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**5*c*
*3*sqrt(-1/(4*a*c - b**2)**5) - 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**
5) + 72*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 6*a**2*b**6*sqrt(-1/(4*a*c
 - b**2)**5) + 6*a**2*b)/(12*a**2*c)) - (-10*a**3*b*c + a**2*b**3 + x**3*(2
0*a**2*c**3 - 16*a*b**2*c**2 + 2*b**4*c) + x**2*(-2*a**2*b*c**2 - 8*a*b**3*
c + b**5) + x*(12*a**3*c**2 - 20*a**2*b**2*c + 2*a*b**4))/(32*a**4*c**4 - 1
6*a**3*b**2*c**3 + 2*a**2*b**4*c**2 + x**4*(32*a**2*c**6 - 16*a*b**2*c**5 +
 2*b**4*c**4) + x**3*(64*a**2*b*c**5 - 32*a*b**3*c**4 + 4*b**5*c**3) + x**2
*(64*a**3*c**5 - 12*a*b**4*c**3 + 2*b**6*c**2) + x*(64*a**3*b*c**4 - 32*a**
2*b**3*c**3 + 4*a*b**5*c**2))
```

Giac [A] time = 1.15053, size = 273, normalized size = 2.46

$$\frac{12\,a^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{2\,b^4cx^3-16\,ab^2c^2x^3+20\,a^2c^3x^3+b^5x^2-8\,ab^3cx^2-2\,a^2bc^2x^2+2\,ab^4x-20\,a^2}{2\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="giac")
```

```
[Out] 12*a^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^4*c*x^3 - 16*a*b^2*c^2*x^3 + 20*a^2*c^3*x
^3 + b^5*x^2 - 8*a*b^3*c*x^2 - 2*a^2*b*c^2*x^2 + 2*a*b^4*x - 20*a^2*b^2*c*x
 + 12*a^3*c^2*x + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*(c*x^2 + b*x + a)^2)
```

3.434 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx$

Optimal. Leaf size=107

$$-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] -(x^3*(b + 2*c*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*x*(2*a + b*
x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*a*b*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.0500256, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 728, 722, 618, 206}

$$-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] -(x^3*(b + 2*c*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*x*(2*a + b*
x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*a*b*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 728

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] + Dist[(m*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx = \int \frac{x^3}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{(3b)\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{(3ab)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{(6ab)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,\right.}{\left(b^2-4ac\right)^2}$$

$$= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.213499, size = 126, normalized size = 1.18

$$-\frac{a^2\left(b^2+10bcx+16c^2x^2\right)+8a^3c+abx\left(2b^2+bcx+6c^2x^2\right)+b^4x^2}{2c\left(b^2-4ac\right)^2(a+x(b+cx))^2} - \frac{6ab\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] -(8*a^3*c + b^4*x^2 + a*b*x*(2*b^2 + b*c*x + 6*c^2*x^2) + a^2*(b^2 + 10*b*c
*x + 16*c^2*x^2))/(2*c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (6*a*b*ArcTan
[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

Maple [B] time = 0.009, size = 223, normalized size = 2.1

$$\frac{1}{\left(cx^2+bx+a\right)^2}\left(-3\frac{abcx^3}{16a^2c^2-8ab^2c+b^4} - \frac{\left(16a^2c^2+ab^2c+b^4\right)x^2}{2c\left(16a^2c^2-8ab^2c+b^4\right)} - \frac{\left(5ac+b^2\right)abx}{c\left(16a^2c^2-8ab^2c+b^4\right)} - \frac{a^2\left(8ac+b\right.}{2c\left(16a^2c^2-8ab\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^3,x)
```

```
[Out] (-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^2-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x-
1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2-6*a*b/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.83539, size = 1848, normalized size = 17.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^3
 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^2 - 6*(a*b*c^3*x^4 + 2
*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^2)*sqr
t(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(
2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x)/(a
^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b
^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*
a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 +
32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^
3*c^3 - 64*a^4*b*c^4)*x), -1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b
^3*c^2 - 4*a^2*b*c^3)*x^3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)
*x^2 - 12*(a*b*c^3*x^4 + 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3
*c + 2*a^2*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x
 + b)/(b^2 - 4*a*c)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x)/(a^2*b^6*c -
 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^
4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2
*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64
*a^4*b*c^4)*x)]
```

Sympy [B] time = 1.51743, size = 510, normalized size = 4.77

$$3ab\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-192a^4bc^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36a^2b^5c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^7\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+}{6abc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**3,x)
```

```
[Out] 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**4*b*c**3*sqrt(-1/(4*a*c -
 b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b**5*c
*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**
2)/(6*a*b*c)) - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**4*b*c**3*s
qrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) +
 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*c - b**2
)**5) + 3*a*b**2)/(6*a*b*c)) - (8*a**3*c + a**2*b**2 + 6*a*b*c**2*x**3 + x*
*2*(16*a**2*c**2 + a*b**2*c + b**4) + x*(10*a**2*b*c + 2*a*b**3))/(32*a**4*
c**3 - 16*a**3*b**2*c**2 + 2*a**2*b**4*c + x**4*(32*a**2*c**5 - 16*a*b**2*c
**4 + 2*b**4*c**3) + x**3*(64*a**2*b*c**4 - 32*a*b**3*c**3 + 4*b**5*c**2) +
 x**2*(64*a**3*c**4 - 12*a*b**4*c**2 + 2*b**6*c) + x*(64*a**3*b*c**3 - 32*a
**2*b**3*c**2 + 4*a*b**5*c))
```

Giac [A] time = 1.12866, size = 220, normalized size = 2.06

$$-\frac{6\,ab\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,abc^2x^3+b^4x^2+ab^2cx^2+16\,a^2c^2x^2+2\,ab^3x+10\,a^2bcx+a^2b^2+8\,a^3c}{2\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="giac")
```

```
[Out] -6*a*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*a*b*c^2*x^3 + b^4*x^2 + a*b^2*c*x^2 + 16*a^
2*c^2*x^2 + 2*a*b^3*x + 10*a^2*b*c*x + a^2*b^2 + 8*a^3*c)/((b^4*c - 8*a*b^2
*c^2 + 16*a^2*c^3)*(c*x^2 + b*x + a)^2)
```

3.435 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4} dx$

Optimal. Leaf size=115

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(2ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] (x*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*a*b + (b^2 + 2*a
*c)*x)/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (2*(b^2 + 2*a*c)*ArcTanh[(b +
2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.0682724, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 738, 638, 618, 206}

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(2ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] (x*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*a*b + (b^2 + 2*a
*c)*x)/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (2*(b^2 + 2*a*c)*ArcTanh[(b +
2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \int \frac{x^2}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{2a-2bx}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\left(b^2+2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\left(2\left(b^2+2ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\right.}{\left(b^2-4ac\right)^2}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{2\left(b^2+2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.14782, size = 131, normalized size = 1.14

$$\frac{1}{2}\left(\frac{\left(2ac+b^2\right)(b+2cx)}{c\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{a(b-2cx)+b^2x}{c\left(4ac-b^2\right)(a+x(b+cx))^2} + \frac{4\left(2ac+b^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] ((b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + ((b^2 + 2
*a*c)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (4*(b^2 + 2*a*c)
*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [B] time = 0.009, size = 262, normalized size = 2.3

$$\frac{1}{\left(cx^2+bx+a\right)^2}\left(\frac{c\left(2\,ac+b^2\right)x^3}{16\,a^2c^2-8\,ab^2c+b^4} + \frac{3\,b\left(2\,ac+b^2\right)x^2}{32\,a^2c^2-16\,ab^2c+2\,b^4} - \frac{a\left(2\,ac-5\,b^2\right)x}{16\,a^2c^2-8\,ab^2c+b^4} + 3\,\frac{a^2b}{16\,a^2c^2-8\,ab^2c+b^4}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^4,x)
```

```
[Out] (c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(2*a*c+b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^2-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+3*a^2*b/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c+2/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.79526, size = 1893, normalized size = 16.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*
(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2
+ 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 +
 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2
- 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^4 -
22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), 1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*
(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x
^2 - 4*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2 + 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)
*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 + 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(5*a*
b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
+ 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*
a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] time = 1.4503, size = 570, normalized size = 4.96

$$-\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)\log\left(x+\frac{-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)-12ab^4c\sqrt{-\frac{1}{(4ac-}}}{4ac^2+2b^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**4,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (-64*a**3*c**3*sqrt(-1/(
4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)
 + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/(4*a*c*
*2 + 2*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (64*a**
3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**2*sqrt(-
1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5
)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)
 + b**3)/(4*a*c**2 + 2*b**2*c)) + (6*a**2*b + x**3*(4*a*c**2 + 2*b**2*c) +
x**2*(6*a*b*c + 3*b**3) + x*(-4*a**2*c + 10*a*b**2))/(32*a**4*c**2 - 16*a**
3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2)
 + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 -
 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Giac [A] time = 1.15729, size = 208, normalized size = 1.81

$$\frac{2\left(b^2+2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{2\,b^2cx^3+4\,ac^2x^3+3\,b^3x^2+6\,abcx^2+10\,ab^2x-4\,a^2cx+6\,a^2b}{2\left(b^4-8\,ab^2c+16\,a^2c^2\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="giac")
```

```
[Out] 2*(b^2 + 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*b^2*c*x^3 + 4*a*c^2*x^3 + 3*b^3*x^
2 + 6*a*b*c*x^2 + 10*a*b^2*x - 4*a^2*c*x + 6*a^2*b)/((b^4 - 8*a*b^2*c + 16*
a^2*c^2)*(c*x^2 + b*x + a)^2)
```

3.436 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} \, dx$

Optimal. Leaf size=103

$$\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] (2*a + b*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*b*(b + 2*c*x))/(2*(b
^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*b*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.0412466, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 638, 614, 618, 206}

$$\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^5),x]
```

```
[Out] (2*a + b*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*b*(b + 2*c*x))/(2*(b
^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*b*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = \int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{(3b)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{(3bc)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{(6bc)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, \right.}{\left(b^2-4ac\right)^2}$$

$$= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.103252, size = 102, normalized size = 0.99

$$\frac{\frac{\left(b^2-4ac\right)(2a+bx)}{(a+x(b+cx))^2} - \frac{12bc\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{3b(b+2cx)}{a+x(b+cx)}}{2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^5),x]
```

```
[Out] (((b^2 - 4*a*c)*(2*a + b*x))/(a + x*(b + c*x))^2 - (3*b*(b + 2*c*x))/(a + x
*(b + c*x)) - (12*b*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c])/(2*(b^2 - 4*a*c)^2)
```

Maple [A] time = 0.003, size = 130, normalized size = 1.3

$$\frac{-bx-2a}{\left(8ac-2b^2\right)\left(cx^2+bx+a\right)^2} - 3\frac{bcx}{\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)} - \frac{3b^2}{2\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)} - 6\frac{bc}{\left(4ac-b^2\right)^{5/2}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^5,x)
```

```
[Out] 1/2*(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)^2-3*b/(4*a*c-b^2)^2/(c*x^2+b*x+a)*
c*x-3/2*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)-6*b/(4*a*c-b^2)^(5/2)*c*arctan((2*c
```

```
*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.79401, size = 1692, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(
b^4*c - 4*a*b^2*c^2)*x^2 - 6*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2
*b*c + (b^3*c + 2*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^5
+ a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(a*b^4 + 4*a^2*b^2*c - 32*a
^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(b^4*c - 4*a*b^2*c^2)*x^2 - 12*(b*
c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2*b*c + (b^3*c + 2*a*b*c^2)*x^2)*
sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) +
2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] time = 1.35848, size = 479, normalized size = 4.65

$$3bc\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-192a^3bc^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36ab^5c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3b^7c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+\cdots}{6bc^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**5,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**3*b*c**4*sqrt(-1/(4*a*c -
 b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c**2
*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*
```

```
c)/(6*b*c**2)) - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**3*b*c**4*
sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5)
+ 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c - b**
2)**5) + 3*b**2*c)/(6*b*c**2)) - (8*a**2*c + a*b**2 + 9*b**2*c*x**2 + 6*b*c
**2*x**3 + x*(10*a*b*c + 2*b**3))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a**2*b
**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**2*b*
c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c + 2*b*
*6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Giac [A] time = 1.14007, size = 182, normalized size = 1.77

$$-\frac{6\,bc\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2x^3+9\,b^2cx^2+2\,b^3x+10\,abcx+ab^2+8\,a^2c}{2\left(b^4-8\,ab^2c+16\,a^2c^2\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="giac")
```

```
[Out] -6*b*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*b*c^2*x^3 + 9*b^2*c*x^2 + 2*b^3*x + 10*a*b*
c*x + a*b^2 + 8*a^2*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

3.437 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx$

Optimal. Leaf size=101

$$-\frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

```
[Out] -(b + 2*c*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*c*(b + 2*c*x))/((b^
2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*c^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.0412279, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1354, 614, 618, 206}

$$-\frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] -(b + 2*c*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*c*(b + 2*c*x))/((b^
2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*c^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{(3c)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{b^2-4ac}$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\left(6c^2\right)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\left(12c^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x\right.}{\left(b^2-4ac\right)^2}$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.101889, size = 97, normalized size = 0.96

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{(b+2cx)\left(-2c\left(5a+3cx^2\right)+b^2-6bcx\right)}{(a+x(b+cx))^2}}{2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] (-(((b + 2*c*x)*(b^2 - 6*b*c*x - 2*c*(5*a + 3*c*x^2)))/(a + x*(b + c*x))^2)
 + (24*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(
b^2 - 4*a*c)^2)
```

Maple [A] time = 0.004, size = 129, normalized size = 1.3

$$\frac{2\,cx+b}{\left(8\,ac-2\,b^2\right)\left(cx^2+bx+a\right)^2} + 6\,\frac{c^2x}{\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)} + 3\,\frac{bc}{\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)} + 12\,\frac{c^2}{\left(4\,ac-b^2\right)^{5/2}}\,\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^6,x)
```

```
[Out] 1/2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+6*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)
*x+3*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b+12*c^2/(4*a*c-b^2)^(5/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.90834, size = 1685, normalized size = 16.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b
^3*c^2 - 4*a*b*c^3)*x^2 - 12*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2
 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 4*(b^4*c +
a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*
c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b^3*c^2 - 4*a*b*c^3)*x^2 + 24*(c^4*x
^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 4*(b^4*
c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*
(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^
6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] time = 1.37773, size = 474, normalized size = 4.69

$$-6c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-384a^3c^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+288a^2b^2c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-72ab^4c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+6b^6c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+\cdots}{12c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**6,x)
```

```
[Out] -6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**3*c**5*sqrt(-1/(4*a*c -
 b**2)**5) + 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 72*a*b**4*c**3
*sqrt(-1/(4*a*c - b**2)**5) + 6*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 6*b*
c**2)/(12*c**3)) + 6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**3*c**5
*sqrt(-1/(4*a*c - b**2)**5) - 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5)
 + 72*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 6*b**6*c**2*sqrt(-1/(4*a*c -
 b**2)**5) + 6*b*c**2)/(12*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**2 + 12*
c**3*x**3 + x*(20*a*c**2 + 4*b**2*c))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a*
*2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**
2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c +
2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Giac [A] time = 1.16977, size = 184, normalized size = 1.82

$$\frac{12\,c^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx+20\,ac^2x-b^3+10\,abc}{2\left(b^4-8\,ab^2c+16\,a^2c^2\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="giac")
```

```
[Out] 12*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x + 20*a*
c^2*x - b^3 + 10*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2
)
```

3.438 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx$

Optimal. Leaf size=185

$$\frac{16a^2c^2+2bcx\left(b^2-7ac\right)-15ab^2c+2b^4}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\cdots$$

```
[Out] (b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (2*b^4 - 15
*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*(a +
b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a + b*x + c*x^
2]/(2*a^3)
```

Rubi [A] time = 0.220315, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1354, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bcx\left(b^2-7ac\right)-15ab^2c+2b^4}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^7),x]
```


```
[Out] (b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (2*b^4 - 15
*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*(a +
b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a + b*x + c*x^
2]/(2*a^3)
```


Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
```

```
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{-2\left(b^2-4ac\right)-3bcx}{x\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\int \frac{2\left(b^2-4ac\right)^2+2bc\left(b^2-7\right.}{x\left(a+bx+cx^2\right)}}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\int\left(\frac{2\left(-b^2+4ac\right)^2}{ax}+\frac{2(-}{}\right.}{2}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^3} + \int \frac{-b\left(b^4-9a\right.}{}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^3} - \frac{\int \frac{b+2cx}{a+bx+cx^2}}{2a^3}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^3} - \frac{\log\left(a+b\right.}{2a}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{b\left(b^4-10ab^2c+30\right.}{a^3\left(b^2\right.}$$

Mathematica [A] time = 0.365448, size = 178, normalized size = 0.96

$$\frac{\frac{a\left(16a^2c^2-15ab^2c-14abc^2x+2b^3cx+2b^4\right)}{\left(b^2-4ac\right)^2(a+x(b+cx))} - \frac{2b\left(30a^2c^2-10ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{a^2\left(-2ac+b^2+bcx\right)}{\left(b^2-4ac\right)(a+x(b+cx))^2} - \log(a+x(b+cx)) + 2\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^7),x]
```

```
[Out] ((a^2*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (a*(2*b^
4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x - 14*a*b*c^2*x))/((b^2 - 4*a*c)^2*(
a + x*(b + c*x))) - (2*b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + 2*Log[x] - Log[a + x*(b + c*x)
])/(2*a^3)
```

Maple [B] time = 0.016, size = 781, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^7,x)
```

```
[Out] ln(x)/a^3-7/a/(c*x^2+b*x+a)^2*b*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/a^2/(c
*x^2+b*x+a)^2*b^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+8/(c*x^2+b*x+a)^2*c^3/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-29/2/a/(c*x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^2*b^2+2/a^2/(c*x^2+b*x+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^
4-1/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^2-6/a/(c*x^2+b*x+a)^2*
b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c+1/a^2/(c*x^2+b*x+a)^2*b^5/(16*a^2*c^2-8*
a*b^2*c+b^4)*x+12*a/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2-21/2/(c*
x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c+3/2/a/(c*x^2+b*x+a)^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*b^4-8/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^2+b*x+a)+
4/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^2+b*x+a)*b^2-1/2/a^3/(16*a^2*c^2-
8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^4-30/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2+10/a^2/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c-1/a^
3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.5515, size = 4228, normalized size = 22.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="fricas")
```

```
[Out] [1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^
2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*
a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b
^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^
2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*
(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2
 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)
) + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^2*b^6 -
12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x
)*log(c*x^2 + b*x + a) + 2*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(x))/(a^5*b^6 - 12*a^6*b^4*c + 4
8*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4
 - 64*a^6*c^5)*x^4 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^
6*b*c^4)*x^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 -
128*a^7*c^4)*x^2 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^
```

```
3)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*
b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2
+ 132*a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + 2*(a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*
c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2
 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*
x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(-b^2 + 4*a*c)*arcta
n(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^7 - 10*a^2*b^5*c
+ 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a) + 2*(
a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^
2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^
4*b*c^3)*x)*log(x))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 +
 (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^4 + 2*(a^3*
b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^3 + (a^3*b^8 - 10
*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^2 + 2*(a^4*b^
7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*x)]
```

Sympy [B] time = 21.887, size = 4862, normalized size = 26.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**7,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1
024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))*log(x + (98304*a**14*c**8*(-b*sqrt
(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5
*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*
a*b**8*c - b**10)) - 1/(2*a**3))**2 - 211968*a**13*b**2*c**7*(-b*sqrt(-(4*a
*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5
- 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8
*c - b**10)) - 1/(2*a**3))**2 + 196352*a**12*b**4*c**6*(-b*sqrt(-(4*a*c - b
**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280
*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b
**10)) - 1/(2*a**3))**2 - 102528*a**11*b**6*c**5*(-b*sqrt(-(4*a*c - b**2)**
5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*
b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))
 - 1/(2*a**3))**2 - 49152*a**11*c**8*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*
c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3
)) + 33120*a**10*b**8*c**4*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*
a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b
**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2 + 68
544*a**10*b**2*c**7*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*
c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**
3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) - 6796*a**9*b*
*10*c**3*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(
2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**
2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2 - 41296*a**9*b**4*c**6
*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(
```

```
1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c
**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) + 867*a**8*b**12*c**2*(-b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10)) - 1/(2*a**3))**2 + 14036*a**8*b**6*c**5*(-b*sqrt(-(4*a*c -
b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 128
0*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
b**10)) - 1/(2*a**3)) - 49152*a**8*c**8 - 63*a**7*b**14*c*(-b*sqrt(-(4*a*c
- b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1
280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
- b**10)) - 1/(2*a**3))**2 - 2935*a**7*b**8*c**4*(-b*sqrt(-(4*a*c - b**2)**
5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*
b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))
 - 1/(2*a**3)) + 143424*a**7*b**2*c**7 + 2*a**6*b**16*(-b*sqrt(-(4*a*c - b*
*2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*
a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b*
*10)) - 1/(2*a**3))**2 + 382*a**6*b**10*c**3*(-b*sqrt(-(4*a*c - b**2)**5)*(
30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2
*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1
/(2*a**3)) - 155056*a**6*b**4*c**6 - 29*a**5*b**12*c**2*(-b*sqrt(-(4*a*c -
b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 128
0*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
b**10)) - 1/(2*a**3)) + 88492*a**5*b**6*c**5 + a**4*b**14*c*(-b*sqrt(-(4*a*
c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 -
 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*
c - b**10)) - 1/(2*a**3)) - 30185*a**4*b**8*c**4 + 6414*a**3*b**10*c**3 - 8
38*a**2*b**12*c**2 + 62*a*b**14*c - 2*b**16)/(69120*a**7*b*c**8 - 102690*a*
*6*b**3*c**7 + 67554*a**5*b**5*c**6 - 25155*a**4*b**7*c**5 + 5690*a**3*b**9
*c**4 - 780*a**2*b**11*c**3 + 60*a*b**13*c**2 - 2*b**15*c)) + (b*sqrt(-(4*a
*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5
- 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8
*c - b**10)) - 1/(2*a**3))*log(x + (98304*a**14*c**8*(b*sqrt(-(4*a*c - b**2
)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a*
*4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**1
0)) - 1/(2*a**3))**2 - 211968*a**13*b**2*c**7*(b*sqrt(-(4*a*c - b**2)**5)*(
30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2
*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1
/(2*a**3))**2 + 196352*a**12*b**4*c**6*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2
*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**
3))**2 - 102528*a**11*b**6*c**5*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 -
 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a*
*3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2
- 49152*a**11*c**8*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c
+ b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3
- 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) + 33120*a**10*b*
*8*c**4*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*
a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*
b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2 + 68544*a**10*b**2*c**7*
(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(10
24*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**
2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) - 6796*a**9*b**10*c**3*(b*sqrt(-(4*
a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5
 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**
8*c - b**10)) - 1/(2*a**3))**2 - 41296*a**9*b**4*c**6*(b*sqrt(-(4*a*c - b**
2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a
**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**
10)) - 1/(2*a**3)) + 867*a**8*b**12*c**2*(b*sqrt(-(4*a*c - b**2)**5)*(30*a*
*2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4
```

```
 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a
**3))**2 + 14036*a**8*b**6*c**5*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 -
 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a*
*3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) - 4
9152*a**8*c**8 - 63*a**7*b**14*c*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2
- 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a
**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2
 - 2935*a**7*b**8*c**4*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**
2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c
**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) + 143424*a**
7*b**2*c**7 + 2*a**6*b**16*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a
*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b*
*4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2 + 382
*a**6*b**10*c**3*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c +
b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 -
160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) - 155056*a**6*b**4
*c**6 - 29*a**5*b**12*c**2*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a
*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b*
*4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) + 88492*
a**5*b**6*c**5 + a**4*b**14*c*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 1
0*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) - 301
85*a**4*b**8*c**4 + 6414*a**3*b**10*c**3 - 838*a**2*b**12*c**2 + 62*a*b**14
*c - 2*b**16)/(69120*a**7*b*c**8 - 102690*a**6*b**3*c**7 + 67554*a**5*b**5*
c**6 - 25155*a**4*b**7*c**5 + 5690*a**3*b**9*c**4 - 780*a**2*b**11*c**3 + 6
0*a*b**13*c**2 - 2*b**15*c)) - (-24*a**3*c**2 + 21*a**2*b**2*c - 3*a*b**4 +
 x**3*(14*a*b*c**3 - 2*b**3*c**2) + x**2*(-16*a**2*c**3 + 29*a*b**2*c**2 -
4*b**4*c) + x*(2*a**2*b*c**2 + 12*a*b**3*c - 2*b**5))/(32*a**6*c**2 - 16*a*
*5*b**2*c + 2*a**4*b**4 + x**4*(32*a**4*c**4 - 16*a**3*b**2*c**3 + 2*a**2*b
**4*c**2) + x**3*(64*a**4*b*c**3 - 32*a**3*b**3*c**2 + 4*a**2*b**5*c) + x**
2*(64*a**5*c**3 - 12*a**3*b**4*c + 2*a**2*b**6) + x*(64*a**5*b*c**2 - 32*a*
*4*b**3*c + 4*a**3*b**5)) + log(x)/a**3
```

Giac [A] time = 1.15702, size = 323, normalized size = 1.75

$$-\frac{\left(b^5-10\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^4-8\,a^4b^2c+16\,a^5c^2\right)\sqrt{-b^2+4\,ac}}-\frac{\log\left(cx^2+bx+a\right)}{2\,a^3}+\frac{\log\left(|x|\right)}{a^3}+\frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\left(ab^3c^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="giac")
```

```
[Out] -(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 +
b*x + a)/a^3 + log(abs(x))/a^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2
 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*x^3 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c
^3)*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2
- 4*a*c)^2*a^3)
```

3.439 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8} \, dx$

Optimal. Leaf size=239

$$\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{2a^2x\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{5/2}}-\frac{3\left(b^2-5ac\right)\left(b^2-\right.}{a^3x\left(b^2-4ac\right.}$$

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*c + b
*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + (3*b^4 - 20*a*b^2*c + 20*
a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*x*(a + b*x + c*x^2)
) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3*b*Lo
g[a + b*x + c*x^2])/(2*a^4)
```

Rubi [A] time = 0.277369, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1354, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{2a^2x\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{5/2}}-\frac{3\left(b^2-5ac\right)\left(b^2-\right.}{a^3x\left(b^2-4ac\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*c + b
*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + (3*b^4 - 20*a*b^2*c + 20*
a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*x*(a + b*x + c*x^2)
) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3*b*Lo
g[a + b*x + c*x^2])/(2*a^4)
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} - \frac{\int \frac{-3b^2+10ac-4bcx}{x^2\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)} + \frac{\int \frac{6\left(b^2-5ac\right)\left(b^2-\right.}{x^2\left(a-\right.}}{2a^2\left(b\right.}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)} + \frac{\int \left(\frac{6\left(b^2-5ac\right)\left(b^2\right.}{ax^2}\right.}{}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b\right.}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b\right.}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b\right.}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b\right.}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+\right.}$$

Mathematica [A] time = 0.471998, size = 221, normalized size = 0.92

$$\frac{\frac{a^2\left(-3abc-2ac^2x+b^2cx+b^3\right)}{\left(4ac-b^2\right)(a+x(b+cx))^2} - \frac{a\left(46a^2bc^2+28a^2c^3x-26ab^2c^2x-29ab^3c+4b^4cx+4b^5\right)}{\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{6\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + 3b\log(a+\ldots}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] ((-2*a)/x + (a^2*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((-b^2 + 4*a*c)*(a
+ x*(b + c*x))^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x - 26*
a*b^2*c^2*x + 28*a^2*c^3*x))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (6*(b^6
- 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/(-b^2 + 4*a*c)^(5/2) - 6*b*Log[x] + 3*b*Log[a + x*(b + c*x)])/(2*a
^4)
```

Maple [B] time = 0.02, size = 954, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^8,x)
```

```
[Out] -1/a^3/x-3*b*ln(x)/a^4-14/a/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^3+13/a^2/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2-2/a^3/(c*
x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4-37/a/(c*x^2+b*x+a)^2*b*
c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+55/2/a^2/(c*x^2+b*x+a)^2*b^3*c^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^2-4/a^3/(c*x^2+b*x+a)^2*b^5*c/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^2-18/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3-7/a/(c*x^2+b*x+
a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2+12/a^2/(c*x^2+b*x+a)^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*x*b^4*c-2/a^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x
*b^6-29/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2+18/a/(c*x^2+b*x+a)
^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c-5/2/a^2/(c*x^2+b*x+a)^2*b^5/(16*a^2*c^2
-8*a*b^2*c+b^4)+24/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^2+b*x+a)*b-12/
a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^2+b*x+a)*b^3+3/2/a^4/(16*a^2*c^2-8*
a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^5-60/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3+90/a^2/(16*a^2*c^2-8*a*b^2*
c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2-30/a^3
/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*b^4*c+3/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.04604, size = 4849, normalized size = 20.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c
^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a*b^7*c - 45*
a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8 - 30*a^2*b^
6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 3*((b^6*c^2 - 10
*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 3
0*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*
a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 -
20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*
x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*
a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4
*b^3*c^2 - 488*a^5*b*c^3)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 -
 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*
c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*
c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 +
 (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*
x + a) + 6*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 +
2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*
a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 -
```

```
12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^
5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*
c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48
*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2
 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b
^3*c^2 - 64*a^8*b*c^3)*x^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*x), -1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3
+ 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a
*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8
 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 6*((
b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a
*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b
^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^
3*b^3*c^2 - 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 -
20*a^5*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b
^2 - 4*a*c)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3
)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(
b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b
^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12
*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c
 + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*x + a) + 6*((b^7*c^2 - 1
2*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2
+ 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2
 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b
^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6
4*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 6
4*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*
c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*
a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x
^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x)]
```

Sympy [B] time = 52.6089, size = 5722, normalized size = 23.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**8,x)
```

```
[Out] (3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**
2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640
*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-108
544*a**16*b*c**8*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 -
 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**
4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10
)))**2 + 224768*a**15*b**3*c**7*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*
(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*
c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a
*b**8*c - b**10)))**2 - 202752*a**14*b**5*c**6*(3*b/(2*a**4) - 3*sqrt(-(4*a
*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a
**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b
**6*c**2 + 20*a*b**8*c - b**10)))**2 + 104128*a**13*b**7*c**5*(3*b/(2*a**4)
 - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4
*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c*
*3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 19200*a**13*c**9*(3*b
/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 +
10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**
```

3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 33320*a**12*b**
9*c**4*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b
2*c2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**
4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 4
4736*a**12*b**2*c**8*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c*
*3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280
*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b
10))) + 6806*a11*b**11*c**3*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*
(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*
c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a
*b**8*c - b**10)))**2 + 101232*a**11*b**4*c**7*(3*b/(2*a**4) - 3*sqrt(-(4*a
*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a
4*(1024*a5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b
6*c2 + 20*a*b**8*c - b**10))) - 867*a**10*b**13*c**2*(3*b/(2*a**4) - 3*
sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c -
b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 -
160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 77268*a**10*b**6*c**6*(3*b
/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 +
10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**
3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 63*a**9*b**15*c
*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 + 31368*a
9*b8*c**5*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30
*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b
2*c4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))
 - 57600*a**9*b*c**9 - 2*a**8*b**17*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)*
5)(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a
5*c5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 +
20*a*b**8*c - b**10)))**2 - 7545*a**8*b**10*c**4*(3*b/(2*a**4) - 3*sqrt(-(4
*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2
*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2
*b**6*c**2 + 20*a*b**8*c - b**10))) + 842688*a**8*b**3*c**8 + 1086*a**7*b**
12*c**3*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*
b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c*
*4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 171
9216*a**7*b**5*c**7 - 87*a**6*b**14*c**2*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b
2)5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1
024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10))) + 1592964*a**6*b**7*c**6 + 3*a**5*b**16*c*(3*b/
(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 843048*a**5*b**9
*c**5 + 277245*a**4*b**11*c**4 - 57996*a**3*b**13*c**3 + 7542*a**2*b**15*c*
*2 - 558*a*b**17*c + 18*b**19)/(18000*a**9*c**10 + 333720*a**8*b**2*c**9 -
991980*a**7*b**4*c**8 + 1099710*a**6*b**6*c**7 - 651186*a**5*b**8*c**6 + 23
1795*a**4*b**10*c**5 - 51480*a**3*b**12*c**4 + 7020*a**2*b**14*c**3 - 540*a
*b**16*c**2 + 18*b**18*c)) + (3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20
*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**
5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b*
*8*c - b**10)))*log(x + (-108544*a**16*b*c**8*(3*b/(2*a**4) + 3*sqrt(-(4*a*
c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a*
4(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b*
*6*c**2 + 20*a*b**8*c - b**10)))**2 + 224768*a**15*b**3*c**7*(3*b/(2*a**4)
+ 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*
c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**
3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 202752*a**14*b**5*c**6
*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64

0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 + 104128*
a**13*b**7*c**5*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 -
30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4
*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)
))**2 - 19200*a**13*c**9*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**
3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 -
1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
 - b**10))) - 33320*a**12*b**9*c**4*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)*
5)(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a
5*c5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 +
20*a*b**8*c - b**10)))**2 - 44736*a**12*b**2*c**8*(3*b/(2*a**4) + 3*sqrt(-(
4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(
2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**
2*b**6*c**2 + 20*a*b**8*c - b**10))) + 6806*a**11*b**11*c**3*(3*b/(2*a**4)
+ 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*
c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**
3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 + 101232*a**11*b**4*c**7
*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 867*a**10*
b**13*c**2*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a*
*2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2
*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2
 - 77268*a**10*b**6*c**6*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**
3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 -
1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
 - b**10))) + 63*a**9*b**15*c*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(2
0*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
8*c - b10)))**2 + 31368*a**9*b**8*c**5*(3*b/(2*a**4) + 3*sqrt(-(4*a*c -
 b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*
(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*
c**2 + 20*a*b**8*c - b**10))) - 57600*a**9*b*c**9 - 2*a**8*b**17*(3*b/(2*a*
*4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b
4*c - b6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4
*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 7545*a**8*b**10*c*
4(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*
c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 842688*a
8*b3*c**8 + 1086*a**7*b**12*c**3*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)
5)*(20*a3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*
a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 +
 20*a*b**8*c - b**10))) - 1719216*a**7*b**5*c**7 - 87*a**6*b**14*c**2*(3*b/
(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 1592964*a**6*b**
7*c**6 + 3*a**5*b**16*c*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3
*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1
280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
- b**10))) - 843048*a**5*b**9*c**5 + 277245*a**4*b**11*c**4 - 57996*a**3*b*
*13*c**3 + 7542*a**2*b**15*c**2 - 558*a*b**17*c + 18*b**19)/(18000*a**9*c**
10 + 333720*a**8*b**2*c**9 - 991980*a**7*b**4*c**8 + 1099710*a**6*b**6*c**7
 - 651186*a**5*b**8*c**6 + 231795*a**4*b**10*c**5 - 51480*a**3*b**12*c**4 +
 7020*a**2*b**14*c**3 - 540*a*b**16*c**2 + 18*b**18*c)) - (32*a**4*c**2 - 1
6*a**3*b**2*c + 2*a**2*b**4 + x**4*(60*a**2*c**4 - 42*a*b**2*c**3 + 6*b**4*
c**2) + x**3*(138*a**2*b*c**3 - 87*a*b**3*c**2 + 12*b**5*c) + x**2*(100*a**
3*c**3 + 14*a**2*b**2*c**2 - 36*a*b**4*c + 6*b**6) + x*(122*a**3*b*c**2 - 6
8*a**2*b**3*c + 9*a*b**5))/(x**5*(32*a**5*c**4 - 16*a**4*b**2*c**3 + 2*a**3
*b**4*c**2) + x**4*(64*a**5*b*c**3 - 32*a**4*b**3*c**2 + 4*a**3*b**5*c) + x

```
**3*(64*a**6*c**3 - 12*a**4*b**4*c + 2*a**3*b**6) + x**2*(64*a**6*b*c**2 -
32*a**5*b**3*c + 4*a**4*b**5) + x*(32*a**7*c**2 - 16*a**6*b**2*c + 2*a**5*b
**4)) - 3*b*log(x)/a**4
```

Giac [A] time = 1.14143, size = 417, normalized size = 1.74

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{3\,b\log\left(cx^{2}+bx+a\right)}{2\,a^{4}}-\frac{3\,b\log\left(|x|\right)}{a^{4}}-\frac{2\,a^{3}b^{4}-16\,a^{4}b^{2}c+32}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(
-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) +
3/2*b*log(c*x^2 + b*x + a)/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(2*a^3*b^4 - 16*
a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*x^4 + 3
*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*x^3 + 2*(3*a*b^6 - 18*a^2*b^4*
c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b
*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*a^4*x)
```

3.440 $\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx$

Optimal. Leaf size=40

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rubi [A] time = 0.0223199, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1354, 701, 632, 31}

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \int \frac{x^4}{2+13x+15x^2}\,dx$$
$$= \int \left(\frac{139}{3375}-\frac{13x}{225}+\frac{x^2}{15}-\frac{278+1417x}{3375(2+13x+15x^2)}\right)dx$$
$$= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{\int \frac{278+1417x}{2+13x+15x^2}\,dx}{3375}$$
$$= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}+\frac{3}{875}\int \frac{1}{3+15x}\,dx-\frac{80}{189}\int \frac{1}{10+15x}\,dx$$
$$= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

Mathematica [A] time = 0.0055139, size = 40, normalized size = 1.

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Maple [A] time = 0.006, size = 31, normalized size = 0.8

$$\frac{139\,x}{3375}-\frac{13\,x^2}{450}+\frac{x^3}{45}-\frac{16\ln(2+3\,x)}{567}+\frac{\ln(1+5\,x)}{4375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(15+2/x^2+13/x),x)
```

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Maxima [A] time = 1.05625, size = 41, normalized size = 1.02

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log(5\,x+1)-\frac{16}{567}\,\log(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Fricas [A] time = 1.66718, size = 108, normalized size = 2.7

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log(5\,x+1)-\frac{16}{567}\,\log(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Sympy [A] time = 0.151797, size = 34, normalized size = 0.85

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x + \frac{1}{5}\right)}{4375} - \frac{16\log\left(x + \frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(15+2/x**2+13/x),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/56
7
```

Giac [A] time = 1.19157, size = 43, normalized size = 1.08

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(|5x + 1|) - \frac{16}{567}\log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(
abs(3*x + 2))
```

3.441 $\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rubi [A] time = 0.0195103, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1354, 701, 632, 31}

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx &= \int \frac{x^3}{2+13x+15x^2}\,dx\\
&= \int \left(-\frac{13}{225}+\frac{x}{15}+\frac{26+139x}{225\left(2+13x+15x^2\right)}\right)dx\\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{1}{225}\int \frac{26+139x}{2+13x+15x^2}\,dx\\
&= -\frac{13x}{225}+\frac{x^2}{30}-\frac{3}{175}\int \frac{1}{3+15x}\,dx+\frac{40}{63}\int \frac{1}{10+15x}\,dx\\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)
\end{aligned}
$$

Mathematica [A] time = 0.004231, size = 33, normalized size = 1.

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Maple [A] time = 0.006, size = 26, normalized size = 0.8

$$-\frac{13\,x}{225}+\frac{x^2}{30}+\frac{8\,\ln(2+3\,x)}{189}-\frac{\ln(1+5\,x)}{875}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(15+2/x^2+13/x),x)
```

```
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Maxima [A] time = 1.08109, size = 34, normalized size = 1.03

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log(5\,x+1)+\frac{8}{189}\,\log(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="maxima")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Fricas [A] time = 1.71385, size = 85, normalized size = 2.58

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log(5\,x+1)+\frac{8}{189}\,\log(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="fricas")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A] time = 0.145308, size = 27, normalized size = 0.82

$$\frac{x^2}{30} - \frac{13x}{225} - \frac{\log\left(x + \frac{1}{5}\right)}{875} + \frac{8\log\left(x + \frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x**2+13/x),x)
```

```
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

Giac [A] time = 1.20236, size = 36, normalized size = 1.09

$$\frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(|5\,x + 1|\right) + \frac{8}{189}\,\log\left(|3\,x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="giac")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

3.442 $\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}} dx$

Optimal. Leaf size=26

$$\frac{x}{15} - \frac{4}{63}\log(3x+2) + \frac{1}{175}\log(5x+1)$$

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rubi [A] time = 0.0147211, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1340, 703, 632, 31}

$$\frac{x}{15} - \frac{4}{63}\log(3x+2) + \frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(15 + 2/x^2 + 13/x)^(-1),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx &= \int \frac{x^2}{2+13x+15x^2}\,dx\\
&= \frac{x}{15}+\frac{1}{15}\int \frac{-2-13x}{2+13x+15x^2}\,dx\\
&= \frac{x}{15}+\frac{3}{35}\int \frac{1}{3+15x}\,dx-\frac{20}{21}\int \frac{1}{10+15x}\,dx\\
&= \frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)
\end{aligned}
$$

Mathematica [A] time = 0.0036947, size = 26, normalized size = 1.

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 + 2/x^2 + 13/x)^(-1),x]

[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Maple [A] time = 0.005, size = 21, normalized size = 0.8

$$\frac{x}{15}-\frac{4\ln(2+3x)}{63}+\frac{\ln(1+5x)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x),x)

[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Maxima [A] time = 1.06458, size = 27, normalized size = 1.04

$$\frac{1}{15}x+\frac{1}{175}\log(5x+1)-\frac{4}{63}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="maxima")

[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Fricas [A] time = 1.66489, size = 66, normalized size = 2.54

$$\frac{1}{15}x+\frac{1}{175}\log(5x+1)-\frac{4}{63}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="fricas")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A] time = 0.142313, size = 20, normalized size = 0.77

$$\frac{x}{15} + \frac{\log\left(x + \frac{1}{5}\right)}{175} - \frac{4\log\left(x + \frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x),x)
```

```
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

Giac [A] time = 1.19542, size = 30, normalized size = 1.15

$$\frac{1}{15}\,x + \frac{1}{175}\,\log\left(|5\,x + 1|\right) - \frac{4}{63}\,\log\left(|3\,x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="giac")
```

```
[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

3.443 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx$

Optimal. Leaf size=21

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rubi [A] time = 0.0112666, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1354, 632, 31}

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx &= \int \frac{x}{2+13x+15x^2}\,dx\\ &= -\left(\frac{3}{7}\int \frac{1}{3+15x}\,dx\right)+\frac{10}{7}\int \frac{1}{10+15x}\,dx\\ &= \frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0029912, size = 21, normalized size = 1.

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Maple [A] time = 0.003, size = 18, normalized size = 0.9

$$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x,x)
```

```
[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Maxima [A] time = 1.07219, size = 23, normalized size = 1.1

$$-\frac{1}{35}\log(5x+1)+\frac{2}{21}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="maxima")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Fricas [A] time = 1.64476, size = 54, normalized size = 2.57

$$-\frac{1}{35}\log(5x+1)+\frac{2}{21}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="fricas")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A] time = 0.106166, size = 17, normalized size = 0.81

$$-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x,x)
```

```
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

Giac [A] time = 1.11624, size = 26, normalized size = 1.24

$$-\frac{1}{35}\log\left(|5x+1|\right)+\frac{2}{21}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="giac")
```

```
[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

3.444 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{7}\log\left(\frac{1}{x}+5\right)-\frac{1}{7}\log\left(\frac{2}{x}+3\right)$$

```
[Out] Log[5 + x^(-1)]/7 - Log[3 + 2/x]/7
```

Rubi [A] time = 0.0141307, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 616, 31}

$$\frac{1}{7}\log\left(\frac{1}{x}+5\right)-\frac{1}{7}\log\left(\frac{2}{x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

```
[Out] Log[5 + x^(-1)]/7 - Log[3 + 2/x]/7
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx &= -\text{Subst}\left(\int \frac{1}{15+13x+2x^2}\,dx,x,\frac{1}{x}\right)\\
&= -\left(\frac{2}{7}\text{Subst}\left(\int \frac{1}{3+2x}\,dx,x,\frac{1}{x}\right)\right)+\frac{2}{7}\text{Subst}\left(\int \frac{1}{10+2x}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{7}\log\left(5+\frac{1}{x}\right)-\frac{1}{7}\log\left(3+\frac{2}{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0031588, size = 21, normalized size = 0.91

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Maple [A] time = 0.004, size = 18, normalized size = 0.8

$$-\frac{\ln(2+3x)}{7}+\frac{\ln(1+5x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^2,x)
```

```
[Out] -1/7*ln(2+3*x)+1/7*ln(1+5*x)
```

Maxima [A] time = 1.0112, size = 23, normalized size = 1.

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Fricas [A] time = 1.67949, size = 50, normalized size = 2.17

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A] time = 0.147482, size = 15, normalized size = 0.65

$$\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**2,x)
```

```
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

Giac [A] time = 1.09891, size = 26, normalized size = 1.13

$$\frac{1}{7}\log\left(|5x+1|\right)-\frac{1}{7}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="giac")
```

```
[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

3.445 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0168466, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 705, 29, 632, 31}

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^3),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx &= \int \frac{1}{x\left(2+13x+15x^2\right)}\,dx \\
&= \frac{1}{2}\int \frac{1}{x}\,dx + \frac{1}{2}\int \frac{-13-15x}{2+13x+15x^2}\,dx \\
&= \frac{\log(x)}{2} + \frac{45}{14}\int \frac{1}{10+15x}\,dx - \frac{75}{7}\int \frac{1}{3+15x}\,dx \\
&= \frac{\log(x)}{2} + \frac{3}{14}\log(2+3x) - \frac{5}{7}\log(1+5x)
\end{aligned}
$$

Mathematica [A] time = 0.004636, size = 27, normalized size = 1.

$$\frac{\log(x)}{2} + \frac{3}{14}\log(3x+2) - \frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^3),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Maple [A] time = 0.007, size = 22, normalized size = 0.8

$$\frac{\ln(x)}{2} + \frac{3\ln(2+3x)}{14} - \frac{5\ln(1+5x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^3,x)
```

```
[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Maxima [A] time = 1.10679, size = 28, normalized size = 1.04

$$-\frac{5}{7}\log(5x+1) + \frac{3}{14}\log(3x+2) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="maxima")
```

```
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Fricas [A] time = 1.76418, size = 70, normalized size = 2.59

$$-\frac{5}{7}\log(5x+1) + \frac{3}{14}\log(3x+2) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="fricas")
```

```
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] time = 0.187052, size = 24, normalized size = 0.89

$$\frac{\log(x)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**3,x)
```

```
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

Giac [A] time = 1.11762, size = 32, normalized size = 1.19

$$-\frac{5}{7}\log(|5x+1|)+\frac{3}{14}\log(|3x+2|)+\frac{1}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="giac")
```

```
[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

3.446 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0314264, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1354, 709, 800}

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^4),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx &= \int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx \\
&= -\frac{1}{2x}+\frac{1}{2}\int \frac{-13-15x}{x\left(2+13x+15x^2\right)}\,dx \\
&= -\frac{1}{2x}+\frac{1}{2}\int \left(-\frac{13}{2x}-\frac{27}{14(2+3x)}+\frac{250}{7(1+5x)}\right)dx \\
&= -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)
\end{aligned}
$$

Mathematica [A] time = 0.0040947, size = 34, normalized size = 1.

$$
-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^4),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Maple [A] time = 0.008, size = 27, normalized size = 0.8

$$
-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(2+3x)}{28}+\frac{25\ln(1+5x)}{7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^4,x)
```

```
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Maxima [A] time = 1.07449, size = 35, normalized size = 1.03

$$
-\frac{1}{2x}+\frac{25}{7}\log(5x+1)-\frac{9}{28}\log(3x+2)-\frac{13}{4}\log(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

Fricas [A] time = 1.71471, size = 90, normalized size = 2.65

$$
\frac{100\,x\log(5x+1)-9\,x\log(3x+2)-91\,x\log(x)-14}{28\,x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

Sympy [A] time = 0.220308, size = 31, normalized size = 0.91

$$-\frac{13\log(x)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**4,x)
```

```
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

Giac [A] time = 1.09537, size = 39, normalized size = 1.15

$$-\frac{1}{2x}+\frac{25}{7}\log(|5x+1|)-\frac{9}{28}\log(|3x+2|)-\frac{13}{4}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="giac")
```

```
[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))
```

3.447 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rubi [A] time = 0.0348749, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1354, 709, 800}

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = \int \frac{1}{x^3\left(2+13x+15x^2\right)}\,dx$$
$$= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{-13-15x}{x^2\left(2+13x+15x^2\right)}\,dx$$
$$= -\frac{1}{4x^2}+\frac{1}{2}\int \left(-\frac{13}{2x^2}+\frac{139}{4x}+\frac{81}{28(2+3x)}-\frac{1250}{7(1+5x)}\right)dx$$
$$= -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

Mathematica [A] time = 0.0048711, size = 41, normalized size = 1.

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Maple [A] time = 0.008, size = 32, normalized size = 0.8

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\ln(x)}{8}+\frac{27\ln(2+3x)}{56}-\frac{125\ln(1+5x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^5,x)
```

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Maxima [A] time = 1.08034, size = 42, normalized size = 1.02

$$\frac{13x-1}{4x^2}-\frac{125}{7}\log(5x+1)+\frac{27}{56}\log(3x+2)+\frac{139}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)
```

Fricas [A] time = 1.7276, size = 117, normalized size = 2.85

$$-\frac{1000x^2\log(5x+1)-27x^2\log(3x+2)-973x^2\log(x)-182x+14}{56x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x
 + 14)/x^2
```

Sympy [A] time = 0.160573, size = 36, normalized size = 0.88

$$\frac{139\log(x)}{8} - \frac{125\log\left(x+\frac{1}{5}\right)}{7} + \frac{27\log\left(x+\frac{2}{3}\right)}{56} + \frac{13x-1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**5,x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)
```

Giac [A] time = 1.12561, size = 46, normalized size = 1.12

$$\frac{13\,x-1}{4\,x^2} - \frac{125}{7}\log\left(|5\,x+1|\right) + \frac{27}{56}\log\left(|3\,x+2|\right) + \frac{139}{8}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 13
9/8*log(abs(x))
```

3.448 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx$

Optimal. Leaf size=48

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0423328, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1354, 709, 800}

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = \int \frac{1}{x^4\left(2+13x+15x^2\right)}\,dx$$
$$= -\frac{1}{6x^3}+\frac{1}{2}\int \frac{-13-15x}{x^3\left(2+13x+15x^2\right)}\,dx$$
$$= -\frac{1}{6x^3}+\frac{1}{2}\int \left(-\frac{13}{2x^3}+\frac{139}{4x^2}-\frac{1417}{8x}-\frac{243}{56(2+3x)}+\frac{6250}{7(1+5x)}\right)dx$$
$$= -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

Mathematica [A] time = 0.0047954, size = 48, normalized size = 1.

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Maple [A] time = 0.009, size = 37, normalized size = 0.8

$$-\frac{1}{6\,x^3}+\frac{13}{8\,x^2}-\frac{139}{8\,x}-\frac{1417\,\ln(x)}{16}-\frac{81\,\ln(2+3\,x)}{112}+\frac{625\,\ln(1+5\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^6,x)
```

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Maxima [A] time = 1.08354, size = 49, normalized size = 1.02

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log(5\,x+1)-\frac{81}{112}\log(3\,x+2)-\frac{1417}{16}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) -
 1417/16*log(x)
```

Fricas [A] time = 1.75177, size = 138, normalized size = 2.88

$$\frac{30000\,x^3\log(5\,x+1)-243\,x^3\log(3\,x+2)-29757\,x^3\log(x)-5838\,x^2+546\,x-56}{336\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5
838*x^2 + 546*x - 56)/x^3
```

Sympy [A] time = 0.245562, size = 41, normalized size = 0.85

$$-\frac{1417\log(x)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}-\frac{417x^2-39x+4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**6,x)
```

```
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 - (417*x**2 - 39
*x + 4)/(24*x**3)
```

Giac [A] time = 1.13612, size = 53, normalized size = 1.1

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log\left(|5\,x+1|\right)-\frac{81}{112}\log\left(|3\,x+2|\right)-\frac{1417}{16}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3
*x + 2)) - 1417/16*log(abs(x))
```

3.449 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx$

Optimal. Leaf size=204

$$\frac{5\left(-48a^2c^2-24ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{128c^{3/2}}+\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)-\frac{5\left(\frac{2c(12ac+b^2)}{x}+b\left(44ac+b^2\right)\right)\sqrt{}}{64c}$$

```
[Out] (-5*(a + c/x^2 + b/x)^(3/2)*(7*b + (6*c)/x))/24 - (5*Sqrt[a + c/x^2 + b/x]*
(b*(b^2 + 44*a*c) + (2*c*(b^2 + 12*a*c))/x))/(64*c) + (a + c/x^2 + b/x)^(5/
2)*x + (5*a^(3/2)*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])
/2 + (5*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sq
rt[a + c/x^2 + b/x])])/(128*c^(3/2))
```

Rubi [A] time = 0.230679, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1342, 732, 814, 843, 621, 206, 724}

$$\frac{5\left(-48a^2c^2-24ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{128c^{3/2}}+\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)-\frac{5\left(\frac{2c(12ac+b^2)}{x}+b\left(44ac+b^2\right)\right)\sqrt{}}{64c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(5/2),x]
```

```
[Out] (-5*(a + c/x^2 + b/x)^(3/2)*(7*b + (6*c)/x))/24 - (5*Sqrt[a + c/x^2 + b/x]*
(b*(b^2 + 44*a*c) + (2*c*(b^2 + 12*a*c))/x))/(64*c) + (a + c/x^2 + b/x)^(5/
2)*x + (5*a^(3/2)*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])
/2 + (5*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sq
rt[a + c/x^2 + b/x])])/(128*c^(3/2))
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
```

```
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = -\text{Subst}\left(\int \frac{(a+bx+cx^2)^{5/2}}{x^2} dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} x - \frac{5}{2}\text{Subst}\left(\int \frac{(b+2cx)(a+bx+cx^2)^{3/2}}{x} dx, x, \frac{1}{x}\right)$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} x + \frac{5\,\text{Subst}\left(\int \frac{(-8abc-c(b^2+12ac)x)\sqrt{a+bx+cx^2}}{x} dx\right.}{16c}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)-\frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right)}{64c}+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)-\frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right)}{64c}+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)-\frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right)}{64c}+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)-\frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right)}{64c}+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}$$

Mathematica [A] time = 0.538654, size = 213, normalized size = 1.04

$$\frac{\sqrt{a+\frac{bx+c}{x^2}}\left(-2\sqrt{c}\sqrt{x(ax+b)+c}\left(2cx^2\left(-96a^2x^2+278abx+59b^2\right)+8c^2x(27ax+17b)+15b^3x^3+48c^3\right)+15x^4\left(-48a^2c\right.\right.}{384c^{3/2}x^3\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + (c + b*x)/x^2]*(-2*Sqrt[c]*Sqrt[c + x*(b + a*x)]*(48*c^3 + 15*b^3
*x^3 + 8*c^2*x*(17*b + 27*a*x) + 2*c*x^2*(59*b^2 + 278*a*b*x - 96*a^2*x^2))
 + 960*a^(3/2)*b*c^(3/2)*x^4*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b +
 a*x)])] + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*x^4*ArcTanh[(2*c + b*x)/(2*Sq
rt[c]*Sqrt[c + x*(b + a*x)])]))/(384*c^(3/2)*x^3*Sqrt[c + x*(b + a*x)])
```

Maple [B] time = 0.013, size = 701, normalized size = 3.4

$$\frac{x}{384\,c^4}\left(\frac{ax^2+bx+c}{x^2}\right)^{\frac{5}{2}}\left(-96\left(ax^2+bx+c\right)^{7/2}c^3a^{3/2}-30\,a^{3/2}\sqrt{ax^2+bx+c}x^4b^4c^2+4\,a^{3/2}\left(ax^2+bx+c\right)^{7/2}x^2b^2c-10\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(5/2),x)
```

```
[Out] 1/384*((a*x^2+b*x+c)/x^2)^(5/2)*x*(-96*(a*x^2+b*x+c)^(7/2)*c^3*a^(3/2)-30*a
^(3/2)*(a*x^2+b*x+c)^(1/2)*x^4*b^4*c^2+4*a^(3/2)*(a*x^2+b*x+c)^(7/2)*x^2*b^
2*c-10*a^(3/2)*(a*x^2+b*x+c)^(3/2)*x^4*b^4*c+16*a^(3/2)*(a*x^2+b*x+c)^(7/2)
*x*b*c^2+660*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^4*b^2*c^3+600*a^(7/2)*(a*x^2+b*x
+c)^(1/2)*x^5*b*c^3-30*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^5*b^3*c^2+960*ln(1/2*(
2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*x^4*b*c^4+15*ln((2*c+b*
x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(5/2)*a^(3/2)*x^4*b^4+260*a^(5/2)*(a*
x^2+b*x+c)^(3/2)*x^4*b^2*c^2+280*a^(7/2)*(a*x^2+b*x+c)^(3/2)*x^5*b*c^2-10*a
^(5/2)*(a*x^2+b*x+c)^(3/2)*x^5*b^3*c-152*a^(5/2)*(a*x^2+b*x+c)^(7/2)*x^3*b*
c+148*a^(5/2)*(a*x^2+b*x+c)^(5/2)*x^4*b^2*c+152*a^(7/2)*(a*x^2+b*x+c)^(5/2)
*x^5*b*c-360*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(7/2)*a^(5/2)*
x^4*b^2+720*a^(7/2)*(a*x^2+b*x+c)^(1/2)*x^4*c^4-6*a^(5/2)*(a*x^2+b*x+c)^(5/
2)*x^5*b^3+144*a^(7/2)*(a*x^2+b*x+c)^(5/2)*x^4*c^2-144*a^(5/2)*(a*x^2+b*x+c
)^(7/2)*x^2*c^2+240*a^(7/2)*(a*x^2+b*x+c)^(3/2)*x^4*c^3+6*a^(3/2)*(a*x^2+b*
x+c)^(7/2)*x^3*b^3-6*a^(3/2)*(a*x^2+b*x+c)^(5/2)*x^4*b^4-720*ln((2*c+b*x+2*
c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(9/2)*a^(7/2)*x^4)/(a*x^2+b*x+c)^(5/2)/c^
4/a^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(5/2), x)
```

Fricas [A] time = 3.94398, size = 2272, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(960*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2
*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 15*(b^4 - 24*a*b^2*c -
 48*a^2*c^2)*sqrt(c)*x^3*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x
^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 4*(192*a^2*c^2*x^4
- 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108
*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/768*(1920*sqrt(-a)*
a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)
/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(c)*x^3*
log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt(
(a*x^2 + b*x + c)/x^2))/x^2) - 4*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 -
(15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2
+ b*x + c)/x^2))/(c^2*x^3), 1/384*(480*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8
*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2
)) - 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*
c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(192
*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*
b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/384*(9
```

```
60*sqrt(-a)*a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b
*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*
sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2
)/(a*c*x^2 + b*c*x + c^2)) - 2*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (1
5*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 +
b*x + c)/x^2))/(c^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(5/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(5/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.450 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx$

Optimal. Leaf size=145

$$-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{8\sqrt{c}}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{4}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{3}{2}\sqrt{a}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\cdots}}\right.$$

```
[Out] (-3*Sqrt[a + c/x^2 + b/x]*(3*b + (2*c)/x))/4 + (a + c/x^2 + b/x)^(3/2)*x +
(3*Sqrt[a]*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/2 - (3
*(b^2 + 4*a*c)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/(8
*Sqrt[c])
```

Rubi [A] time = 0.134451, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1342, 732, 814, 843, 621, 206, 724}

$$-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{8\sqrt{c}}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{4}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{3}{2}\sqrt{a}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (-3*Sqrt[a + c/x^2 + b/x]*(3*b + (2*c)/x))/4 + (a + c/x^2 + b/x)^(3/2)*x +
(3*Sqrt[a]*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/2 - (3
*(b^2 + 4*a*c)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/(8
*Sqrt[c])
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
```

```
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = -\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2} dx, x, \frac{1}{x}\right)$$
$$= \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x - \frac{3}{2}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x} dx, x, \frac{1}{x}\right)$$
$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + \frac{3\,\text{Subst}\left(\int \frac{-4abc-c\left(b^2+4ac\right)x}{x\sqrt{a+bx+cx^2}} dx, x, \frac{1}{x}\right)}{8c}$$
$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, \frac{1}{x}\right. \ldots$$
$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + (3ab)\,\text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\ldots}}\right. \ldots$$
$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + \frac{3}{2}\sqrt{a}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) - \frac{3\left(b^2 \ldots\right.}{\ldots}$$

Mathematica [A] time = 0.258137, size = 163, normalized size = 1.12

$$\frac{\sqrt{a+\frac{bx+c}{x^2}}\left(-3x^2\left(4ac+b^2\right)\tanh^{-1}\left(\frac{bx+2c}{2\sqrt{c}\sqrt{x(ax+b)+c}}\right)+12\sqrt{a}b\sqrt{c}x^2\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)-2\sqrt{c}(x(5b-4ax)+2c)\sqrt{x(\ldots}\right.}{8\sqrt{c}x\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + (c + b*x)/x^2]*(-2*Sqrt[c]*(2*c + x*(5*b - 4*a*x))*Sqrt[c + x*(b
+ a*x)] + 12*Sqrt[a]*b*Sqrt[c]*x^2*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c +
x*(b + a*x)])] - 3*(b^2 + 4*a*c)*x^2*ArcTanh[(2*c + b*x)/(2*Sqrt[c]*Sqrt[c
+ x*(b + a*x)])]))/(8*Sqrt[c]*x*Sqrt[c + x*(b + a*x)])
```

Maple [B] time = 0.006, size = 334, normalized size = 2.3

$$-\frac{x}{8c^2}\left(\frac{ax^2+bx+c}{x^2}\right)^{\frac{3}{2}}\left(12a^{5/2}c^{5/2}\ln\left(\frac{2c+bx+2\sqrt{c}\sqrt{ax^2+bx+c}}{x}\right)x^2-2a^{5/2}\left(ax^2+bx+c\right)^{3/2}x^3b-4a^{5/2}\left(ax^2+b\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(3/2),x)
```

```
[Out] -1/8*((a*x^2+b*x+c)/x^2)^(3/2)*x*(12*a^(5/2)*c^(5/2)*ln((2*c+b*x+2*c^(1/2)*
(a*x^2+b*x+c)^(1/2))/x)*x^2-2*a^(5/2)*(a*x^2+b*x+c)^(3/2)*x^3*b-4*a^(5/2)*(
a*x^2+b*x+c)^(3/2)*x^2*c-6*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^3*b*c+3*a^(3/2)*c^
(3/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*x^2*b^2-12*a^(5/2)*(a*x
^2+b*x+c)^(1/2)*x^2*c^2+2*a^(3/2)*(a*x^2+b*x+c)^(5/2)*x*b-2*a^(3/2)*(a*x^2+
b*x+c)^(3/2)*x^2*b^2+4*(a*x^2+b*x+c)^(5/2)*c*a^(3/2)-6*a^(3/2)*(a*x^2+b*x+c
)^(1/2)*x^2*b^2*c-12*a^2*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(
```

```
1/2))*x^2*b*c^2)/(a*x^2+b*x+c)^(3/2)/c^2/a^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(3/2), x)
```

Fricas [A] time = 2.57067, size = 1689, normalized size = 11.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(12*sqrt(a)*b*c*x*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2
 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 3*(b^2 + 4*a*c)*sqrt(c)*x*lo
g(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a
*x^2 + b*x + c)/x^2))/x^2) + 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2)*sqrt((a*x^2 +
b*x + c)/x^2))/(c*x), -1/16*(24*sqrt(-a)*b*c*x*arctan(1/2*(2*a*x^2 + b*x)*s
qrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 3*(b^2 + 4*a
*c)*sqrt(c)*x*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)
*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) - 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2
)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), 1/8*(6*sqrt(a)*b*c*x*log(-8*a^2*x^2 -
 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x
^2)) + 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt(
(a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(4*a*c*x^2 - 5*b*c*x -
2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), -1/8*(12*sqrt(-a)*b*c*x*arctan(1
/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x +
a*c)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt
((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) - 2*(4*a*c*x^2 - 5*b*c*x -
 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(3/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(3/2), x)
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.451 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=105

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\sqrt{a}}-\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

```
[Out] Sqrt[a + c/x^2 + b/x]*x + (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2
+ b/x])])/(2*Sqrt[a]) - Sqrt[c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c
/x^2 + b/x])]
```

Rubi [A] time = 0.0827327, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1342, 732, 843, 621, 206, 724}

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\sqrt{a}}-\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] Sqrt[a + c/x^2 + b/x]*x + (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2
+ b/x])])/(2*Sqrt[a]) - Sqrt[c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c
/x^2 + b/x])]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x - \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right) - c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x + b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) - (2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+\frac{2c}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)$$

$$= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x + \frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2\sqrt{a}} - \sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)$$

Mathematica [A] time = 0.0850176, size = 128, normalized size = 1.22

$$\frac{x\sqrt{a+\frac{bx+c}{x^2}}\left(b\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)+2\sqrt{a}\left(\sqrt{x(ax+b)+c}-\sqrt{c}\tanh^{-1}\left(\frac{bx+2c}{2\sqrt{c}\sqrt{x(ax+b)+c}}\right)\right)\right)}{2\sqrt{a}\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*(b*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b
+ a*x)])] + 2*Sqrt[a]*(Sqrt[c + x*(b + a*x)] - Sqrt[c]*ArcTanh[(2*c + b*x)/
(2*Sqrt[c]*Sqrt[c + x*(b + a*x)])])))/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])
```

Maple [A] time = 0.005, size = 121, normalized size = 1.2

$$\frac{x}{2}\sqrt{\frac{ax^2+bx+c}{x^2}}\left(-2\sqrt{c}\ln\left(\frac{2c+bx+2\sqrt{c}\sqrt{ax^2+bx+c}}{x}\right)\sqrt{a}+b\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx+c}\sqrt{a}+2ax+b\right)\frac{1}{\sqrt{a}}\right)+2\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x^2+b*x+c)/x^2)^(1/2)*x*(-2*c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*
x+c)^(1/2))/x)*a^(1/2)+b*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/(a*x^2+b*x+c)^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x + c/x^2), x)
```

Fricas [A] time = 2.18527, size = 1407, normalized size = 13.4

$$\left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}} + \sqrt{a}b\log\left(-8\,a^2x^2 - 8\,abx - b^2 - 4\,ac - 4\left(2\,ax^2 + bx\right)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right) + 2\,a\sqrt{c}\log\left(-\frac{8\,bcx+\left(b^2+4\,ac\right)x^2+8\,c^2-4\cdots}{x^2}\cdots\right.}{4\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*
x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) +
2*a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*s
qrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x +
 c)/x^2) - sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x
 + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*
c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2
))/a, 1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + 4*a*sqrt(-c)*arctan(1/2*(b*x
^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) +
 sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(
a)*sqrt((a*x^2 + b*x + c)/x^2)))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2)
- sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2
)/(a^2*x^2 + a*b*x + a*c)) + 2*a*sqrt(-c)*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-
c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x + c/x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.452 $\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx$

Optimal. Leaf size=67

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}$$

```
[Out] (Sqrt[a + c/x^2 + b/x]*x)/a - (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/
x^2 + b/x])])/(2*a^(3/2))
```

Rubi [A] time = 0.0405132, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1342, 730, 724, 206}

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (Sqrt[a + c/x^2 + b/x]*x)/a - (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/
x^2 + b/x])])/(2*a^(3/2))
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{2a}\\ &= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a} - \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{a}\\ &= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a} - \frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0544175, size = 89, normalized size = 1.33

$$\frac{2\sqrt{a}(x(ax+b)+c)-b\sqrt{x(ax+b)+c}\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)}{2a^{3/2}x\sqrt{a+\frac{bx+c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (2*Sqrt[a]*(c + x*(b + a*x)) - b*Sqrt[c + x*(b + a*x)]*ArcTanh[(b + 2*a*x)/
(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])])/(2*a^(3/2)*x*Sqrt[a + (c + b*x)/x^2])
```

Maple [A] time = 0.006, size = 88, normalized size = 1.3

$$\frac{1}{2x}\sqrt{ax^2+bx+c}\left(2\sqrt{ax^2+bx+c}a^{3/2}-b\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx+c}\sqrt{a}+2ax+b\right)\frac{1}{\sqrt{a}}\right)a\right)\frac{1}{\sqrt{\frac{ax^2+bx+c}{x^2}}}a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)^(1/2),x)
```

```
[Out] 1/2*(a*x^2+b*x+c)^(1/2)*(2*(a*x^2+b*x+c)^(1/2)*a^(3/2)-b*ln(1/2*(2*(a*x^2+b
*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a)/((a*x^2+b*x+c)/x^2)^(1/2)/x/a^(5/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a + b/x + c/x^2), x)
```

Fricas [A] time = 1.85355, size = 410, normalized size = 6.12

$$\left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{a}b\log\left(-8\,a^2x^2-8\,abx-b^2-4\,ac+4\left(2\,ax^2+bx\right)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)}{4\,a^2}, \frac{2\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{-a}b\arctan\left(\ldots\right.}{2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*
x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)))/a
^2, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(-a)*b*arctan(1/2*(2*a*x^2
 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b/x + c/x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.453 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=133

$$\frac{x\left(3b^2-8ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a^2\left(b^2-4ac\right)}-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{5/2}}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

```
[Out] ((3*b^2 - 8*a*c)*Sqrt[a + c/x^2 + b/x]*x)/(a^2*(b^2 - 4*a*c)) - (2*(b^2 - 2
*a*c + (b*c)/x)*x)/(a*(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/x]) - (3*b*ArcTanh[(
2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(5/2))
```

Rubi [A] time = 0.0992594, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1342, 740, 806, 724, 206}

$$\frac{x\left(3b^2-8ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a^2\left(b^2-4ac\right)}-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{5/2}}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(-3/2),x]
```

```
[Out] ((3*b^2 - 8*a*c)*Sqrt[a + c/x^2 + b/x]*x)/(a^2*(b^2 - 4*a*c)) - (2*(b^2 - 2
*a*c + (b*c)/x)*x)/(a*(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/x]) - (3*b*ArcTanh[(
2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(5/2))
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
```

```
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}+\frac{2\operatorname{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{a\left(b^2-4ac\right)}\\
&= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2\left(b^2-4ac\right)}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}+\frac{(3b)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2\left(b^2-4ac\right)}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}-\frac{(3b)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{a^2}\\
&= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2\left(b^2-4ac\right)}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.174029, size = 138, normalized size = 1.04

$$
-\frac{2\sqrt{a}\left(-b^2\left(ax^2+3c\right)+10abcx+4ac\left(ax^2+2c\right)-3b^3x\right)+3b\left(b^2-4ac\right)\sqrt{x(ax+b)+c}\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)}{2a^{5/2}x\left(b^2-4ac\right)\sqrt{a+\frac{bx+c}{x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(-3/2),x]
```

```
[Out] -(2*Sqrt[a]*(-3*b^3*x + 10*a*b*c*x + 4*a*c*(2*c + a*x^2) - b^2*(3*c + a*x^2
)) + 3*b*(b^2 - 4*a*c)*Sqrt[c + x*(b + a*x)]*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]
*Sqrt[c + x*(b + a*x)])])/(2*a^(5/2)*(b^2 - 4*a*c)*x*Sqrt[a + (c + b*x)/x^2
```

])

Maple [A] time = 0.009, size = 197, normalized size = 1.5

$$\frac{ax^{2}+bx+c}{2\,x^{3}\left(4\,ac-b^{2}\right)}\left(8\,a^{7/2}x^{2}c-2\,a^{5/2}x^{2}b^{2}+20\,a^{5/2}xbc-6\,a^{3/2}xb^{3}+16\,a^{5/2}c^{2}-6\,a^{3/2}b^{2}c-12\,\ln\left(1/2\,\frac{2\,\sqrt{ax^{2}+bx+c}\sqrt{a}+}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)^(3/2),x)
```

```
[Out] 1/2*(a*x^2+b*x+c)/a^(7/2)*(8*a^(7/2)*x^2*c-2*a^(5/2)*x^2*b^2+20*a^(5/2)*x*b
*c-6*a^(3/2)*x*b^3+16*a^(5/2)*c^2-6*a^(3/2)*b^2*c-12*ln(1/2*(2*(a*x^2+b*x+c
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a^2*b*c+3*ln(1/2*(2*(
a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a*b^3)/((a
*x^2+b*x+c)/x^2)^(3/2)/x^3/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^{2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(-3/2), x)
```

Fricas [A] time = 2.30451, size = 1002, normalized size = 7.53

$$\left[\frac{3\left(b^{3}c-4\,abc^{2}+\left(ab^{3}-4\,a^{2}bc\right)x^{2}+\left(b^{4}-4\,ab^{2}c\right)x\right)\sqrt{a}\log\left(-8\,a^{2}x^{2}-8\,abx-b^{2}-4\,ac+4\left(2\,ax^{2}+bx\right)\sqrt{a}\sqrt{\frac{ax^{2}+bx+c}{x^{2}}}\right.}{4\left(a^{3}b^{2}c-4\,a^{4}c^{2}+\left(a^{4}b^{2}-4\,a^{5}c\right)x^{2}+\left(a^{3}b^{3}-4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)
*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)
*sqrt((a*x^2 + b*x + c)/x^2)) + 4*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*
a^2*b*c)*x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3
*b^2*c - 4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x), 1/
2*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)*sq
rt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2
*x^2 + a*b*x + a*c)) + 2*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*a^2*b*c)*
x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3*b^2*c -
```

```
4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(3/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.454 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{2x\left(32a^2c^2+\frac{bc(5b^2-28ac)}{x}-32ab^2c+5b^4\right)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{3a^3\left(b^2-4ac\right)^2}-\frac{5b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{7/2}}$$

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a + c/x^2 + b/x]*x)/(3*a^3*(b^2
- 4*a*c)^2) - (2*(b^2 - 2*a*c + (b*c)/x)*x)/(3*a*(b^2 - 4*a*c)*(a + c/x^2 +
 b/x)^(3/2)) - (2*(5*b^4 - 32*a*b^2*c + 32*a^2*c^2 + (b*c*(5*b^2 - 28*a*c))
/x)*x)/(3*a^2*(b^2 - 4*a*c)^2*Sqrt[a + c/x^2 + b/x]) - (5*b*ArcTanh[(2*a +
b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(7/2))
```

Rubi [A] time = 0.193687, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1342, 740, 822, 806, 724, 206}

$$-\frac{2x\left(32a^2c^2+\frac{bc(5b^2-28ac)}{x}-32ab^2c+5b^4\right)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{3a^3\left(b^2-4ac\right)^2}-\frac{5b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a + c/x^2 + b/x]*x)/(3*a^3*(b^2
- 4*a*c)^2) - (2*(b^2 - 2*a*c + (b*c)/x)*x)/(3*a*(b^2 - 4*a*c)*(a + c/x^2 +
 b/x)^(3/2)) - (2*(5*b^4 - 32*a*b^2*c + 32*a^2*c^2 + (b*c*(5*b^2 - 28*a*c))
/x)*x)/(3*a^2*(b^2 - 4*a*c)^2*Sqrt[a + c/x^2 + b/x]) - (5*b*ArcTanh[(2*a +
b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(7/2))
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}dx = -\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{5/2}}dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5b^2+16ac\right)-3bcx}{x^2\left(a+bx+cx^2\right)^{3/2}}dx, x, \frac{1}{x}\right)}{3a\left(b^2-4ac\right)}$$

$$= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32ab^2c+32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}\right)x}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{4\,\text{Subst}\left(\int \frac{\frac{1}{4}(1\!\!5}{}\right.}{}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32ab^2c\right.}{3a^2\left(b^2-\right.}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32ab^2c\right.}{3a^2\left(b^2-\right.}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32ab^2c\right.}{3a^2\left(b^2-\right.}$$

Mathematica [A] time = 0.400968, size = 256, normalized size = 1.16

$$\frac{2\sqrt{a}\left(3b^4\left(a^2x^4-30acx^2+5c^2\right)-4ab^2c\left(6a^2x^4-12acx^2+25c^2\right)+8a^2bc^2x\left(32ax^2+39c\right)+16a^2c^2\left(3a^2x^4+12acx^2\right.\right.}{6a^{7/2}x\left(b^2-4ac\right)^2(x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] (2*Sqrt[a]*(15*b^6*x^2 + 8*a^2*b*c^2*x*(39*c + 32*a*x^2) - 2*a*b^3*c*x*(105
*c + 74*a*x^2) + 10*b^5*(3*c*x + 2*a*x^3) + 3*b^4*(5*c^2 - 30*a*c*x^2 + a^2
*x^4) + 16*a^2*c^2*(8*c^2 + 12*a*c*x^2 + 3*a^2*x^4) - 4*a*b^2*c*(25*c^2 - 1
2*a*c*x^2 + 6*a^2*x^4)) - 15*b*(b^2 - 4*a*c)^2*(c + x*(b + a*x))^(3/2)*ArcT
anh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])])/(6*a^(7/2)*(b^2 - 4*a*c
)^2*x*(c + x*(b + a*x))*Sqrt[a + (c + b*x)/x^2])
```

Maple [A] time = 0.01, size = 376, normalized size = 1.7

$$\frac{ax^2+bx+c}{6x^5\left(4ac-b^2\right)^2}\left(96a^{13/2}x^4c^2-48a^{11/2}x^4b^2c+512a^{11/2}x^3bc^2+6a^{9/2}x^4b^4+384a^{11/2}x^2c^3-296a^{9/2}x^3b^3c+96a^{9/2}x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)^(5/2),x)
```

```
[Out] 1/6*(a*x^2+b*x+c)*(96*a^(13/2)*x^4*c^2-48*a^(11/2)*x^4*b^2*c+512*a^(11/2)*x
^3*b*c^2+6*a^(9/2)*x^4*b^4+384*a^(11/2)*x^2*c^3-296*a^(9/2)*x^3*b^3*c+96*a^
(9/2)*x^2*b^2*c^2+40*a^(7/2)*x^3*b^5+624*a^(9/2)*x*b*c^3-180*a^(7/2)*x^2*b^
4*c+256*a^(9/2)*c^4-420*a^(7/2)*x*b^3*c^2+30*a^(5/2)*x^2*b^6-200*a^(7/2)*b^
2*c^3+60*a^(5/2)*x*b^5*c+30*a^(5/2)*b^4*c^2-240*ln(1/2*(2*(a*x^2+b*x+c)^(1/
2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(3/2)*a^4*b*c^2+120*ln(1/2*(2*(a
*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(3/2)*a^3*b^3*c-1
5*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(3/
2)*a^2*b^5)/a^(11/2)/((a*x^2+b*x+c)/x^2)^(5/2)/x^5/(4*a*c-b^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(-5/2), x)
```

Fricas [B] time = 4.32664, size = 2286, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^2*b^5 - 8*a^3*b^3*c +
16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x^3 + (b^7 - 6
*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x)*
sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*
sqrt((a*x^2 + b*x + c)/x^2)) + 4*(3*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*x^
5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*a*b^6 - 30*a^2*b
^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35*a^2*b^3*c^2 + 5
2*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^3*c^4)*x)*sqrt((
a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4 + (a^6*b^4
 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)
*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^5*c - 8*a^5*b^3*
c^2 + 16*a^6*b*c^3)*x), 1/6*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b
^2*c^2)*x^3 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2
 + 16*a^2*b^2*c^3)*x)*sqrt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*
x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 2*(3*(a^3*b^4 - 8*a^4*b^2*c
+ 16*a^5*c^2)*x^5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*
a*b^6 - 30*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35
*a^2*b^3*c^2 + 52*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^
3*c^4)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^
6*c^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c
 + 16*a^7*b*c^2)*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^
```

```
5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(5/2),x)

[Out] Integral((a + b/x + c/x**2)**(-5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.455 $\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx$

Optimal. Leaf size=73

$$\frac{ax\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}-\frac{b\log\left(\frac{1}{x}\right)\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}$$

```
[Out] (a*Sqrt[a^2 + b^2/x^2 + (2*a*b)/x]*x)/(a + b/x) - (b*Sqrt[a^2 + b^2/x^2 + (
2*a*b)/x]*Log[x^(-1)])/(a + b/x)
```

Rubi [A] time = 0.0357841, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1342, 646, 43}

$$\frac{ax\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}-\frac{b\log\left(\frac{1}{x}\right)\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
```

```
[Out] (a*Sqrt[a^2 + b^2/x^2 + (2*a*b)/x]*x)/(a + b/x) - (b*Sqrt[a^2 + b^2/x^2 + (
2*a*b)/x]*Log[x^(-1)])/(a + b/x)
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\operatorname{Subst}\left(\int \frac{ab+b^2x}{x^2}\,dx, x, \frac{1}{x}\right)}{ab+\frac{b^2}{x}}$$
$$= -\frac{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\operatorname{Subst}\left(\int \left(\frac{ab}{x^2}+\frac{b^2}{x}\right)dx, x, \frac{1}{x}\right)}{ab+\frac{b^2}{x}}$$
$$= \frac{a\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,x}{a+\frac{b}{x}} + \frac{b\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\log(x)}{a+\frac{b}{x}}$$

Mathematica [A] time = 0.0196016, size = 32, normalized size = 0.44

$$\frac{x\sqrt{\frac{(ax+b)^2}{x^2}}(ax+b\log(x))}{ax+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
```

```
[Out] (x*Sqrt[(b + a*x)^2/x^2]*(a*x + b*Log[x]))/(b + a*x)
```

Maple [A] time = 0.009, size = 40, normalized size = 0.6

$$\frac{x(ax+b\ln(x))}{ax+b}\sqrt{\frac{a^2x^2+2\,abx+b^2}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^2+2*a*b/x)^(1/2),x)
```

```
[Out] ((a^2*x^2+2*a*b*x+b^2)/x^2)^(1/2)/(a*x+b)*x*(a*x+b*ln(x))
```

Maxima [A] time = 0.999886, size = 11, normalized size = 0.15

$$ax+b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] a*x + b*log(x)
```

Fricas [A] time = 2.02493, size = 22, normalized size = 0.3

$$ax+b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] a*x + b*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**2+2*a*b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b/x + b**2/x**2), x)
```

Giac [A] time = 1.1146, size = 39, normalized size = 0.53

$$ax\mathrm{sgn}\left(ax^2+bx\right)+b\log\left(|x|\right)\mathrm{sgn}\left(ax^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="giac")
```

```
[Out] a*x*sgn(a*x^2 + b*x) + b*log(abs(x))*sgn(a*x^2 + b*x)
```

3.456 $\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.291245, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1340, 1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \int \frac{x^4}{a+bx^2+cx^4}\,dx$$
$$= \frac{x}{c} - \frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.131076, size = 202, normalized size = 1.13

$$-\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.01, size = 343, normalized size = 1.9

$$\frac{x}{c} - \frac{\sqrt{2}b}{2\,c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}} + \sqrt{2}a\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{}{\sqrt{-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^4+b/x^2),x)
```

```
[Out] x/c-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/
2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+1/2/c*2^(1/2)/((-b+(-4*a*c+b^
```

```
2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b
+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [B] time = 2.10998, size = 2168, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b
*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2
*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

Sympy [A] time = 1.64967, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3abc^4-8t^3b^3c^3\cdot}{a^2\cdot}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**4+b/x**2),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

Giac [C] time = 2.32622, size = 4593, normalized size = 25.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="giac")
```

```
[Out] -2*(3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(3/4)
*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*(a*c^3)^(3/4)*b*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^
3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^2 - 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(3
/4)*b*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*c^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*c^2*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1
/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*b*c^2*abs(c) - (b^2
*c - 4*a*c^2)*c^2) - 2*(3*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
)))) - (a*c^3)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9
*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*b*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))) + 9*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2
```

*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3 + (a*c^3)^(3/4)*b*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
+ (a*c^3)^(1/4)*a*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3
)^(1/4)*a*c^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^
(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4
)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)
*b*c^2*abs(c) - (b^2*c - 4*a*c^2)*c^2) + ((a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3
)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*c
osh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2 - 9*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - (a*c^3)^(3/4)*b*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))) - (a*c^3)^(1/4)*a*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*b*c^2*abs(c) - (b^2*c - 4*a*c^2)*c^
2) + ((a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*
(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*
c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
)^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*(a*c^3)^(3/4)*b*cos(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)

```
))))^2 - (a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 +
 3*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*
c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*c^2*cos(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/4*pi +
 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a
*c)*b*c^2*abs(c) - (b^2*c - 4*a*c^2)*c^2) + x/c
```

3.457 $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=631

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-\dots}}\right.}{6\sqrt[3]{2}c^{4/3}\left(\dots\right.}$$

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 -
 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi [A] time = 1.16984, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {1340, 1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-\dots}}\right.}{6\sqrt[3]{2}c^{4/3}\left(\dots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^6 + b/x^3)^(-1),x]
```


```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 -
 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \int \frac{x^6}{a+bx^3+cx^6}\,dx$$

$$= \frac{x}{c} - \frac{\int \frac{a+bx^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \ldots$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\ldots\right.}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \ldots$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\ldots\right.}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \ldots$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\ldots\right.}{3\sqrt[3]{2}c^{4/3}\ldots} \ldots$$

Mathematica [C] time = 0.0367574, size = 70, normalized size = 0.11

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^3 b+\#1^6 c+a\&,\frac{\#1^3 b\log(x-\#1)+a\log(x-\#1)}{\#1^2 b+2\#1^5 c}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^6 + b/x^3)^(-1),x]
```

```
[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/
(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] time = 0.009, size = 59, normalized size = 0.1

$$\frac{x}{c} + \frac{1}{3c}\sum_{\_R=\text{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(-\_R^3b-a\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^6+b/x^3),x)
```

```
[Out] x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*
b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 6.32946, size = 11169, normalized size = 17.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*ar
ctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6
- 112*a^3*b^2*c^7 + 64*a^4*c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*
a^3*c^11)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 +
24*a^4*b*c^4)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 +
 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*
(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^
4*c^8)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(b^9 -
 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a
*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^1
1)))/(b^2*c^4 - 4*a*c^5))^(2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)
*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16
*a^4*c^4 - (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 -
 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c
^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4
*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c
^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c
+ 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8
- 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3
```

```
))/(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)) + 2*sqrt(3)*(a^3*b^4 - 4*a^4*b^2*c
+ 2*a^5*c^2))/(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)) - 4*sqrt(3)*(1/2)^(1/3)*
c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(s
qrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*
c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^9 -
 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*(-(b^3 - 2
*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^
3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c
^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(b^8*c^4 - 13*a*b^
6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8)*sqrt((b^8 - 8*a*b^6*
c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 +
48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^
2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c
^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(
2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 1
0*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 + (b^7*c^4 - 12*a*
b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9
 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/3
)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*
b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^
3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2
*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2
*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3))/(a^2*b^4 - 4*a^3*b^2*c +
 2*a^4*c^2)) - 2*sqrt(3)*(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2))/(a^3*b^4 - 4*
a^4*b^2*c + 2*a^5*c^2)) - (1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5
))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 -
 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12*
a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c
^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c
^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1
/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^
2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*
c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b
^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/3)*c*(-(b^3
 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a
^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*
c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3
+ 16*a^4*c^4 + (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8
- 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c
^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a
^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4
 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6
)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
```

```
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^
4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)
*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*
c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^
(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 -
 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b
^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*
(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2
*b^2*c^2 - 8*a^3*c^3 - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4
*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5
))^(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*
a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log
(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*
a^2*b^2*c^2 - 8*a^3*c^3 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8
- 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*
b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*
a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*
c^5))^(1/3)) + 6*x)/c
```

Sympy [A] time = 2.92855, size = 196, normalized size = 0.31

$$\operatorname{RootSum}\left(t^6\left(46656a^3c^7-34992a^2b^2c^6+8748ab^4c^5-729b^6c^4\right)+t^3\left(864a^3bc^3-864a^2b^3c^2+270ab^5c-27b^7\right)+a^4,\left(t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**6+b/x**3),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5*c
 - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*_t*
*4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**
2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)))) +
 x/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c+\frac{b}{x^3}+\frac{a}{x^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="giac")
```

```
[Out] integrate(1/(c + b/x^3 + a/x^6), x)
```

3.458 $\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

Optimal. Leaf size=376

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{(b-}{2}$$

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4
*a*c])^(3/4))
```

Rubi [A] time = 0.666361, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1340, 1367, 1422, 212, 208, 205}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{(b-}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^8 + b/x^4)^(-1),x]
```


```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4
*a*c])^(3/4))
```


Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
```

```
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \int \frac{x^8}{a+bx^4+cx^8}\,dx$$

$$= \frac{x}{c} - \frac{\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\cdots}{2c\sqrt{\cdots}}$$

$$= \frac{x}{c} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\cdots}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{\cdots}\right.}$$

Mathematica [C] time = 0.0443528, size = 70, normalized size = 0.19

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{\#1^4b\log(x-\#1)+a\log(x-\#1)}{\#1^3b+2\#1^7c}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^8 + b/x^4)^(-1),x]
```

```
[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] time = 0.002, size = 59, normalized size = 0.2

$$\frac{x}{c}+\frac{1}{4c}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(-\_R^4b-a\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^8+b/x^4),x)
```

```
[Out] x/c+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*
b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 5.52976, size = 10701, normalized size = 28.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/4*(2*sqrt(1/2)*((b^10*c^
5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128
*a^5*c^10)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^11 - 1
3*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c
^5)*x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*
c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^
6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
+ 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (
```

```
b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*sqrt((2*(a^2*b^4 - 3*a^3
*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^4*c^2 - 30*a^
3*b^2*c^3 + 8*a^4*c^4 + (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b
*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^
6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/
(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(a^4*b^
4 - 3*a^5*b^2*c + a^6*c^2)) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/
4*(2*sqrt(1/2)*((b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8
 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 6
4*a^3*c^13)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*
a^4*b^3*c^4 - 32*a^5*b*c^5)*x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 -
 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)) + (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c
^4 - 32*a^5*b*c^5 - (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4
*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*
a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*
b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*sqrt((2*(a^
2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^
4*c^2 - 30*a^3*b^2*c^3 + 8*a^4*c^4 - (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c
^7 - 64*a^3*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt
(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)))/(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))/(a^4*b^4 - 3*a^5*b^2*c + a
^6*c^2)) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5
 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3
*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*
a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3
```

```
*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6
*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log
((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2
- 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqr
t(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 -
7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*
c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6
*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqr
t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 +
16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c
 + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^
5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8
*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - 4*x)/c
```

Sympy [A] time = 20.0292, size = 218, normalized size = 0.58

$$\operatorname{RootSum}\left(t^8\left(16777216a^4c^9-16777216a^3b^2c^8+6291456a^2b^4c^7-1048576ab^6c^6+65536b^8c^5\right)+t^4\left(20480a^4bc^4-3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**8+b/x**4),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**9 - 16777216*a**3*b**2*c**8 + 6291456*a**2*
b**4*c**7 - 1048576*a*b**6*c**6 + 65536*b**8*c**5) + _t**4*(20480*a**4*b*c*
*4 - 30720*a**3*b**3*c**3 + 15616*a**2*b**5*c**2 - 3328*a*b**7*c + 256*b**9
) + a**5, Lambda(_t, _t*log(x + (16384*_t**5*a**2*b*c**7 - 8192*_t**5*a*b**
3*c**6 + 1024*_t**5*b**5*c**5 - 8*_t*a**3*c**3 + 36*_t*a**2*b**2*c**2 - 24*
_t*a*b**4*c + 4*_t*b**6)/(a**3*c**2 - 3*a**2*b**2*c + a*b**4)))) + x/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c+\frac{b}{x^4}+\frac{a}{x^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="giac")
```

```
[Out] integrate(1/(c + b/x^4 + a/x^8), x)
```

3.459 $\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$

Optimal. Leaf size=106

$$2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sqrt[a
]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*Sqr
t[a + b*Sqrt[x] + c*x])])/Sqrt[c]
```

Rubi [A] time = 0.0915753, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1357, 734, 843, 621, 206, 724}

$$2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sqrt[a
]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*Sqr
t[a + b*Sqrt[x] + c*x])])/Sqrt[c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{a+b\sqrt{x}+cx} - \text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{a+b\sqrt{x}+cx} + (2a)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \sqrt{x}\right) + b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x\right.\\
&= 2\sqrt{a+b\sqrt{x}+cx} - (4a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\sqrt{x}}{\sqrt{a+b\sqrt{x}+cx}}\right) + (2b)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x\right.\\
&= 2\sqrt{a+b\sqrt{x}+cx} - 2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.0577812, size = 106, normalized size = 1.

$$
2\sqrt{a+b\sqrt{x}+cx} - 2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sqrt[a
]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*Sqr
t[a + b*Sqrt[x] + c*x])])/Sqrt[c]
```

Maple [A] time = 0.007, size = 84, normalized size = 0.8

$$2\sqrt{a+cx+b\sqrt{x}}+b\ln\left(\left(\frac{b}{2}+c\sqrt{x}\right)\frac{1}{\sqrt{c}}+\sqrt{a+cx+b\sqrt{x}}\right)\frac{1}{\sqrt{c}}-2\sqrt{a}\ln\left(\frac{2a+b\sqrt{x}+2\sqrt{a}\sqrt{a+cx+b\sqrt{x}}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*x+b*x^(1/2))^(1/2)/x,x)
```

```
[Out] 2*(a+c*x+b*x^(1/2))^(1/2)+b*ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+c*x+b*x^(1/2))^
(1/2))/c^(1/2)-2*a^(1/2)*ln((2*a+b*x^(1/2)+2*a^(1/2)*(a+c*x+b*x^(1/2))^(1/2
))/x^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx+b\sqrt{x}+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x + b*sqrt(x) + a)/x, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(x) + c*x)/x, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.460 $\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx$

Optimal. Leaf size=40

$$\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}$$

```
[Out] -(b*(b + 2*c*Sqrt[x])^5)/(160*c^4) + (b + 2*c*Sqrt[x])^6/(192*c^4)
```

Rubi [A] time = 0.019456, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {28, 190, 43}

$$\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

```
[Out] -(b*(b + 2*c*Sqrt[x])^5)/(160*c^4) + (b + 2*c*Sqrt[x])^6/(192*c^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{\int \left(\frac{b}{2}+c\sqrt{x}\right)^4 dx}{c^2}$$
$$= \frac{2\,\text{Subst}\left(\int x\left(\frac{b}{2}+cx\right)^4 dx, x, \sqrt{x}\right)}{c^2}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{b\left(\frac{b}{2}+cx\right)^4}{2c}+\frac{\left(\frac{b}{2}+cx\right)^5}{c}\right) dx, x, \sqrt{x}\right)}{c^2}$$
$$= -\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}+\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}$$

Mathematica [A] time = 0.0257015, size = 29, normalized size = 0.72

$$-\frac{\left(b-10c\sqrt{x}\right)\left(b+2c\sqrt{x}\right)^5}{960c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

```
[Out] -((b - 10*c*Sqrt[x])*(b + 2*c*Sqrt[x])^5)/(960*c^4)
```

Maple [A] time = 0.003, size = 52, normalized size = 1.3

$$\frac{b^2x^2}{2}+\frac{b}{2\,c}\left(\frac{8\,c^2}{5}x^{\frac{5}{2}}+\frac{2\,b^2}{3}x^{\frac{3}{2}}\right)+\frac{1}{3\,c}\left(\frac{b^2}{4\,c}+cx\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*b^2/c+c*x+b*x^(1/2))^2,x)
```

```
[Out] 1/2*b^2*x^2+1/2*b/c*(8/5*c^2*x^(5/2)+2/3*x^(3/2)*b^2)+1/3*(1/4*b^2/c+c*x)^3
/c
```

Maxima [A] time = 1.0648, size = 73, normalized size = 1.82

$$\frac{1}{3}c^2x^3+\frac{4}{5}bcx^{\frac{5}{2}}+\frac{1}{2}b^2x^2+\frac{b^4x}{16\,c^2}+\frac{\left(3\,cx^2+4\,bx^{\frac{3}{2}}\right)b^2}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4/c*b^2+c*x+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 4/5*b*c*x^(5/2) + 1/2*b^2*x^2 + 1/16*b^4*x/c^2 + 1/12*(3*c*x^
2 + 4*b*x^(3/2))*b^2/c
```

Fricas [A] time = 1.99539, size = 126, normalized size = 3.15

$$\frac{80\,c^4x^3+180\,b^2c^2x^2+15\,b^4x+16\,\left(12\,bc^3x^2+5\,b^3cx\right)\sqrt{x}}{240\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4/c*b^2+c*x+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(80*c^4*x^3 + 180*b^2*c^2*x^2 + 15*b^4*x + 16*(12*b*c^3*x^2 + 5*b^3*c
*x)*sqrt(x))/c^2
```

Sympy [A] time = 0.411806, size = 51, normalized size = 1.27

$$\frac{b^4x}{16c^2}+\frac{b^3x^{\frac{3}{2}}}{3c}+\frac{3b^2x^2}{4}+\frac{4bcx^{\frac{5}{2}}}{5}+\frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4/c*b**2+c*x+b*x**(1/2))**2,x)
```

```
[Out] b**4*x/(16*c**2) + b**3*x**(3/2)/(3*c) + 3*b**2*x**2/4 + 4*b*c*x**(5/2)/5 +
 c**2*x**3/3
```

Giac [A] time = 1.11856, size = 66, normalized size = 1.65

$$\frac{80\,c^4x^3+192\,bc^3x^{\frac{5}{2}}+180\,b^2c^2x^2+80\,b^3cx^{\frac{3}{2}}+15\,b^4x}{240\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4/c*b^2+c*x+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/240*(80*c^4*x^3 + 192*b*c^3*x^(5/2) + 180*b^2*c^2*x^2 + 80*b^3*c*x^(3/2)
+ 15*b^4*x)/c^2
```

3.461 $\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

```
[Out] (2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])/b^2 - (2*a*(a + b*Sqrt[x])*Log[a + b*
Sqrt[x]])/(b^2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])
```

Rubi [A] time = 0.039791, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1341, 640, 608, 31}

$$\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

```
[Out] (2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])/b^2 - (2*a*(a + b*Sqrt[x])*Log[a + b*
Sqrt[x]])/(b^2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{\left(2a\left(a+b\sqrt{x}\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$
$$= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

Mathematica [A] time = 0.0299355, size = 50, normalized size = 0.67

$$\frac{2\left(a+b\sqrt{x}\right)\left(b\sqrt{x}-a\log\left(a+b\sqrt{x}\right)\right)}{b^2\sqrt{\left(a+b\sqrt{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

```
[Out] (2*(a + b*Sqrt[x])*(b*Sqrt[x] - a*Log[a + b*Sqrt[x]]))/(b^2*Sqrt[(a + b*Sqr
t[x])^2])
```

Maple [A] time = 0.01, size = 50, normalized size = 0.7

$$2\,\frac{\sqrt{a^2+b^2x+2\,ab\sqrt{x}}\left(b\sqrt{x}-a\ln\left(a+b\sqrt{x}\right)\right)}{\left(a+b\sqrt{x}\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x)
```

```
[Out] 2*(a^2+b^2*x+2*a*b*x^(1/2))^(1/2)*(b*x^(1/2)-a*ln(a+b*x^(1/2)))/(a+b*x^(1/2
))/b^2
```

Maxima [A] time = 1.03503, size = 31, normalized size = 0.41

$$-\frac{2\,a\log\left(b\sqrt{x}+a\right)}{b^2}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*log(b*sqrt(x) + a)/b^2 + 2*sqrt(x)/b
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2 + 2ab\sqrt{x} + b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2*x+2*a*b*x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b*sqrt(x) + b**2*x), x)
```

Giac [A] time = 1.16299, size = 74, normalized size = 0.99

$$-\frac{2\,|a|\log\left(\left|\sqrt{b^2x}\operatorname{sgn}(a)\operatorname{sgn}(b)+|a|\right|\right)}{b^2}+\frac{2\,|a|\log\left(|a|\right)}{b^2}+\frac{2\sqrt{b^2x}}{b^2\operatorname{sgn}(a)\operatorname{sgn}(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -2*abs(a)*log(abs(sqrt(b^2*x)*sgn(a)*sgn(b) + abs(a)))/b^2 + 2*abs(a)*log(a
bs(a))/b^2 + 2*sqrt(b^2*x)/(b^2*sgn(a)*sgn(b))
```

3.462 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx$

Optimal. Leaf size=137

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^9}{10b^3}-\frac{2a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^8}{3b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}$$

```
[Out] (3*a^2*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3) -
 (2*a*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(3*b^3) +
(3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(10*b^3)
```

Rubi [A] time = 0.0812464, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^9}{10b^3}-\frac{2a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^8}{3b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3) -
 (2*a*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(3*b^3) +
(3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(10*b^3)
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}dx = 3\,\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{7/2}dx,x,\sqrt[3]{x}\right)$$
$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^7dx,x,\sqrt[3]{x}\right)}{b^7\left(a+b\sqrt[3]{x}\right)}$$
$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2(ab+b^2x)^7}{b^2}-\frac{2a(ab+b^2x)^8}{b^3}+\frac{(ab+b^2x)^9}{b^4}\right)dx,x,\sqrt[3]{x}\right)}{b^7\left(a+b\sqrt[3]{x}\right)}$$
$$= \frac{3a^2\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{3b^3}+\frac{3(a}{}$$

Mathematica [A] time = 0.049556, size = 56, normalized size = 0.41

$$\frac{\left(a+b\sqrt[3]{x}\right)^7\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(a^2-8ab\sqrt[3]{x}+36b^2x^{2/3}\right)}{120b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] ((a + b*x^(1/3))^7*Sqrt[(a + b*x^(1/3))^2]*(a^2 - 8*a*b*x^(1/3) + 36*b^2*x^
(2/3)))/(120*b^3)
```

Maple [A] time = 0.012, size = 109, normalized size = 0.8

$$\frac{1}{120}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(36\,b^7x^{10/3}+945\,a^2b^5x^{8/3}+1800\,a^3b^4x^{7/3}+1512\,a^5b^2x^{5/3}+630\,a^6bx^{4/3}+280\,ab^6x^3+2100\,a^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x)
```

```
[Out] 1/120*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(36*b^7*x^(10/3)+945*a^2*b^5*x^
(8/3)+1800*a^3*b^4*x^(7/3)+1512*a^5*b^2*x^(5/3)+630*a^6*b*x^(4/3)+280*a*b^6
*x^3+2100*a^4*b^3*x^2+120*a^7*x)/(a+b*x^(1/3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13304, size = 197, normalized size = 1.44

$$\frac{7}{3}ab^6x^3 + \frac{35}{2}a^4b^3x^2 + a^7x + \frac{63}{40}\left(5a^2b^5x^2 + 8a^5b^2x\right)x^{\frac{2}{3}} + \frac{3}{20}\left(2b^7x^3 + 100a^3b^4x^2 + 35a^6bx\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")
```

```
[Out] 7/3*a*b^6*x^3 + 35/2*a^4*b^3*x^2 + a^7*x + 63/40*(5*a^2*b^5*x^2 + 8*a^5*b^2
*x)*x^(2/3) + 3/20*(2*b^7*x^3 + 100*a^3*b^4*x^2 + 35*a^6*b*x)*x^(1/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17033, size = 189, normalized size = 1.38

$$\frac{3}{10}b^7x^{\frac{10}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + \frac{7}{3}ab^6x^3\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + \frac{63}{8}a^2b^5x^{\frac{8}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + 15a^3b^4x^{\frac{7}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + \frac{35}{2}a^4b^3x^2\mathrm{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
```

```
[Out] 3/10*b^7*x^(10/3)*sgn(b*x^(1/3) + a) + 7/3*a*b^6*x^3*sgn(b*x^(1/3) + a) + 6
3/8*a^2*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15*a^3*b^4*x^(7/3)*sgn(b*x^(1/3) +
 a) + 35/2*a^4*b^3*x^2*sgn(b*x^(1/3) + a) + 63/5*a^5*b^2*x^(5/3)*sgn(b*x^(1
/3) + a) + 21/4*a^6*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^7*x*sgn(b*x^(1/3) + a)
```

3.463 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx$

Optimal. Leaf size=137

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}$$

```
[Out] (a^2*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3) - (
6*a*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(7*b^3) + (3
*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3)
```

Rubi [A] time = 0.0706979, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

```
[Out] (a^2*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3) - (
6*a*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(7*b^3) + (3
*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3)
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2} dx = 3\,\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^5 dx, x, \sqrt[3]{x}\right)}{b^5\left(a+b\sqrt[3]{x}\right)}$$

$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2(ab+b^2x)^5}{b^2}-\frac{2a(ab+b^2x)^6}{b^3}+\frac{(ab+b^2x)^7}{b^4}\right) dx, x, \sqrt[3]{x}\right.}{b^5\left(a+b\sqrt[3]{x}\right)}$$

$$= \frac{a^2\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3} - \frac{6a\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{7b^3} + \frac{3(}{}$$

Mathematica [A] time = 0.0365837, size = 56, normalized size = 0.41

$$\frac{\left(a+b\sqrt[3]{x}\right)^5\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(a^2-6ab\sqrt[3]{x}+21b^2x^{2/3}\right)}{56b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

```
[Out] ((a + b*x^(1/3))^5*Sqrt[(a + b*x^(1/3))^2]*(a^2 - 6*a*b*x^(1/3) + 21*b^2*x^
(2/3)))/(56*b^3)
```

Maple [A] time = 0.002, size = 87, normalized size = 0.6

$$\frac{1}{56}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(21\,b^5x^{8/3}+120\,ab^4x^{7/3}+336\,a^3b^2x^{5/3}+210\,a^4bx^{4/3}+280\,a^2b^3x^2+56\,a^5x\right)\left(a+b\sqrt[3]{x}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x)
```

```
[Out] 1/56*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(21*b^5*x^(8/3)+120*a*b^4*x^(7/3
)+336*a^3*b^2*x^(5/3)+210*a^4*b*x^(4/3)+280*a^2*b^3*x^2+56*a^5*x)/(a+b*x^(1
/3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03009, size = 140, normalized size = 1.02

$$5\,a^{2}b^{3}x^{2}+a^{5}x+\frac{3}{8}\left(b^{5}x^{2}+16\,a^{3}b^{2}x\right)x^{\frac{2}{3}}+\frac{15}{28}\left(4\,ab^{4}x^{2}+7\,a^{4}bx\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
```

```
[Out] 5*a^2*b^3*x^2 + a^5*x + 3/8*(b^5*x^2 + 16*a^3*b^2*x)*x^(2/3) + 15/28*(4*a*b
^4*x^2 + 7*a^4*b*x)*x^(1/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^{2}+2ab\sqrt[3]{x}+b^{2}x^{\frac{2}{3}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(5/2), x)
```

Giac [A] time = 1.13114, size = 138, normalized size = 1.01

$$\frac{3}{8}\,b^{5}x^{\frac{8}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{7}\,ab^{4}x^{\frac{7}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+5\,a^{2}b^{3}x^{2}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+6\,a^{3}b^{2}x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{4}\,a^{4}bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
```

```
[Out] 3/8*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15/7*a*b^4*x^(7/3)*sgn(b*x^(1/3) + a)
+ 5*a^2*b^3*x^2*sgn(b*x^(1/3) + a) + 6*a^3*b^2*x^(5/3)*sgn(b*x^(1/3) + a) +
 15/4*a^4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^5*x*sgn(b*x^(1/3) + a)
```

3.464 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx$

Optimal. Leaf size=137

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^4}{5b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^3}{4b^3}$$

```
[Out] (3*a^2*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(4*b^3) -
 (6*a*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(5*b^3) +
((a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3)
```

Rubi [A] time = 0.0558239, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1341, 645}

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^4}{5b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^3}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(4*b^3) -
 (6*a*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(5*b^3) +
((a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3)
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx = 3\,\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx,x,\sqrt[3]{x}\right)$$

$$=\frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^3}{b^2}-\frac{2a\left(ab+b^2x\right)^4}{b^3}+\frac{\left(ab+b^2x\right)^5}{b^4}\right)dx,x,\sqrt[3]{x}\right)}{b^3\left(a+b\sqrt[3]{x}\right)}$$

$$=\frac{3a^2\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{5b^3}+\frac{(\,}{}$$

Mathematica [A] time = 0.0328785, size = 65, normalized size = 0.47

$$\frac{x\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(45a^2b\sqrt[3]{x}+20a^3+36ab^2x^{2/3}+10b^3x\right)}{20\left(a+b\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^(1/3))^2]*x*(20*a^3 + 45*a^2*b*x^(1/3) + 36*a*b^2*x^(2/3) +
10*b^3*x))/(20*(a + b*x^(1/3)))
```

Maple [A] time = 0.003, size = 65, normalized size = 0.5

$$\frac{1}{20}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(36\,ab^2x^{5/3}+45\,a^2bx^{4/3}+10\,b^3x^2+20\,a^3x\right)\left(a+b\sqrt[3]{x}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x)
```

```
[Out] 1/20*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(36*a*b^2*x^(5/3)+45*a^2*b*x^(4/
3)+10*b^3*x^2+20*a^3*x)/(a+b*x^(1/3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95631, size = 82, normalized size = 0.6

$$\frac{1}{2}\,b^3x^2+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(3/2), x)
```

Giac [A] time = 1.1347, size = 86, normalized size = 0.63

$$\frac{1}{2}b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{9}{5}ab^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{9}{4}a^2bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^3x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x^(1/3) + a) + 9/5*a*b^2*x^(5/3)*sgn(b*x^(1/3) + a) + 9/4
*a^2*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^3*x*sgn(b*x^(1/3) + a)
```

3.465 $\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx$

Optimal. Leaf size=88

$$\frac{3bx^{4/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4\left(a+b\sqrt[3]{x}\right)}+\frac{ax\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{a+b\sqrt[3]{x}}$$

```
[Out] (a*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x)/(a + b*x^(1/3)) + (3*b*Sqrt[a
^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(4/3))/(4*(a + b*x^(1/3)))
```

Rubi [A] time = 0.0398336, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{3bx^{4/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4\left(a+b\sqrt[3]{x}\right)}+\frac{ax\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{a+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (a*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x)/(a + b*x^(1/3)) + (3*b*Sqrt[a
^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(4/3))/(4*(a + b*x^(1/3)))
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx &= 3\,\text{Subst}\left(\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx,x,\sqrt[3]{x}\right)\\ &= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)\,dx,x,\sqrt[3]{x}\right)}{b\left(a+b\sqrt[3]{x}\right)}\\ &= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int \left(abx^2+b^2x^3\right)\,dx,x,\sqrt[3]{x}\right)}{b\left(a+b\sqrt[3]{x}\right)}\\ &= \frac{a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x}{a+b\sqrt[3]{x}}+\frac{3b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x^{4/3}}{4\left(a+b\sqrt[3]{x}\right)}\end{aligned}$$

Mathematica [A] time = 0.0091801, size = 43, normalized size = 0.49

$$\frac{\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(4ax+3bx^{4/3}\right)}{4\left(a+b\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (Sqrt[(a + b*x^(1/3))^2]*(4*a*x + 3*b*x^(4/3)))/(4*(a + b*x^(1/3)))
```

Maple [A] time = 0.001, size = 43, normalized size = 0.5

$$\frac{1}{4}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(3\,bx^{4/3}+4\,ax\right)\left(a+b\sqrt[3]{x}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x)
```

```
[Out] 1/4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(3*b*x^(4/3)+4*a*x)/(a+b*x^(1/3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01913, size = 28, normalized size = 0.32

$$\frac{3}{4}\,bx^{\frac{4}{3}}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
```

```
[Out] 3/4*b*x^(4/3) + a*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), x)
```

Giac [A] time = 1.11565, size = 35, normalized size = 0.4

$$\frac{3}{4}\, bx^{\frac{4}{3}}\text{sgn}\left(bx^{\frac{1}{3}} + a\right) + ax\text{sgn}\left(bx^{\frac{1}{3}} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a*x*sgn(b*x^(1/3) + a)
```

3.466 $\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx$

Optimal. Leaf size=147

$$-\frac{3a\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3x^{2/3}\left(a+b\sqrt[3]{x}\right)}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] (-3*a*(a + b*x^(1/3))*x^(1/3))/(b^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]
) + (3*(a + b*x^(1/3))*x^(2/3))/(2*b*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)
]) + (3*a^2*(a + b*x^(1/3))*Log[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/
3) + b^2*x^(2/3)])
```

Rubi [A] time = 0.0703464, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3x^{2/3}\left(a+b\sqrt[3]{x}\right)}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (-3*a*(a + b*x^(1/3))*x^(1/3))/(b^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]
) + (3*(a + b*x^(1/3))*x^(2/3))/(2*b*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)
]) + (3*a^2*(a + b*x^(1/3))*Log[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/
3) + b^2*x^(2/3)])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3b\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+b^2x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= \frac{\left(3b\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(-\frac{a}{b^3}+\frac{x}{b^2}+\frac{a^2}{b^3(a+bx)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= -\frac{3a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3\left(a+b\sqrt[3]{x}\right)x^{2/3}}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Mathematica [A] time = 0.0380569, size = 65, normalized size = 0.44

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(2a^2\log\left(a+b\sqrt[3]{x}\right)+b\sqrt[3]{x}\left(b\sqrt[3]{x}-2a\right)\right)}{2b^3\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (3*(a + b*x^(1/3))*(b*(-2*a + b*x^(1/3))*x^(1/3) + 2*a^2*Log[a + b*x^(1/3)]
))/(2*b^3*Sqrt[(a + b*x^(1/3))^2])
```

Maple [A] time = 0.016, size = 103, normalized size = 0.7

$$\frac{1}{2\,b^3}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(3\,b^2x^{2/3}+2\,a^2\ln\left(b^3x+a^3\right)-2\,a^2\ln\left(b^2x^{2/3}-ab\sqrt[3]{x}+a^2\right)+4\,a^2\ln\left(a+b\sqrt[3]{x}\right)-6\,ab\sqrt[3]{x}\right)\left(a+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x)
```

```
[Out] 1/2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(3*b^2*x^(2/3)+2*a^2*ln(b^3*x+a^3
)-2*a^2*ln(b^2*x^(2/3)-a*b*x^(1/3)+a^2)+4*a^2*ln(a+b*x^(1/3))-6*a*b*x^(1/3)
)/(a+b*x^(1/3))/b^3
```

Maxima [A] time = 1.06445, size = 62, normalized size = 0.42

$$\frac{3\,a^2b^2\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{5}{2}}} - \frac{3\,abx^{\frac{1}{3}}}{\left(b^2\right)^{\frac{3}{2}}} + \frac{3\,x^{\frac{2}{3}}}{2\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
```

```
[Out] 3*a^2*b^2*log(x^(1/3) + a/b)/(b^2)^(5/2) - 3*a*b*x^(1/3)/(b^2)^(3/2) + 3/2*
x^(2/3)/sqrt(b^2)
```

Fricas [A] time = 2.06321, size = 89, normalized size = 0.61

$$\frac{3\left(2\,a^2\log\left(bx^{\frac{1}{3}}+a\right)+b^2x^{\frac{2}{3}}-2\,abx^{\frac{1}{3}}\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*(2*a^2*log(b*x^(1/3) + a) + b^2*x^(2/3) - 2*a*b*x^(1/3))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), x)
```

Giac [A] time = 1.13357, size = 82, normalized size = 0.56

$$\frac{3\left(bx^{\frac{2}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)-2\,ax^{\frac{1}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)\right)}{2\,b^2}+\frac{3\,a^2\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*(b*x^(2/3)*sgn(b*x^(1/3) + a) - 2*a*x^(1/3)*sgn(b*x^(1/3) + a))/b^2 + 3
*a^2*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a))
```

3.467 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] (6*a)/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - (3*a^2)/(2*b^3*(a + b
*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (3*(a + b*x^(1/3))*Log
[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rubi [A] time = 0.0720178, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```


```
[Out] (6*a)/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - (3*a^2)/(2*b^3*(a + b
*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (3*(a + b*x^(1/3))*Log
[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```


Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3b^3\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= \frac{\left(3b^3\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^5(a+bx)^3}-\frac{2a}{b^5(a+bx)^2}+\frac{1}{b^5(a+bx)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= \frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3\left(a+b\sqrt[3]{x}\right)\log}{b^3\sqrt{a^2+2ab\sqrt[3]{}}}$$

Mathematica [A] time = 0.0463178, size = 72, normalized size = 0.55

$$\frac{3a\left(3a+4b\sqrt[3]{x}\right)+6\left(a+b\sqrt[3]{x}\right)^2\log\left(a+b\sqrt[3]{x}\right)}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

```
[Out] (3*a*(3*a + 4*b*x^(1/3)) + 6*(a + b*x^(1/3))^2*Log[a + b*x^(1/3)])/(2*b^3*(
a + b*x^(1/3))*Sqrt[(a + b*x^(1/3))^2])
```

Maple [A] time = 0.009, size = 92, normalized size = 0.7

$$\frac{3}{2\,b^3}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(2\,\ln\left(a+b\sqrt[3]{x}\right)x^{2/3}b^2+4\,\ln\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}ab+2\,a^2\ln\left(a+b\sqrt[3]{x}\right)+4\,ab\sqrt[3]{x}+3\,a^2\right)\left(a+b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x)
```

```
[Out] 3/2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(2*ln(a+b*x^(1/3))*x^(2/3)*b^2+4*
ln(a+b*x^(1/3))*x^(1/3)*a*b+2*a^2*ln(a+b*x^(1/3))+4*a*b*x^(1/3)+3*a^2)/(a+b
*x^(1/3))^3/b^3
```

Maxima [A] time = 1.07779, size = 88, normalized size = 0.68

$$\frac{3\,\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{3}{2}}} + \frac{9\,a^2b^2}{2\left(b^2\right)^{\frac{7}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2} + \frac{6\,abx^{\frac{1}{3}}}{\left(b^2\right)^{\frac{5}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

```
[Out] 3*log(x^(1/3) + a/b)/(b^2)^(3/2) + 9/2*a^2*b^2/((b^2)^(7/2)*(x^(1/3) + a/b)
^2) + 6*a*b*x^(1/3)/((b^2)^(5/2)*(x^(1/3) + a/b)^2)
```

Fricas [A] time = 1.86619, size = 243, normalized size = 1.87

$$\frac{3\left(6\,a^3b^3x+3\,a^6+2\left(b^6x^2+2\,a^3b^3x+a^6\right)\log\left(bx^{\frac{1}{3}}+a\right)+\left(4\,ab^5x+a^4b^2\right)x^{\frac{2}{3}}-\left(5\,a^2b^4x+2\,a^5b\right)x^{\frac{1}{3}}\right)}{2\left(b^9x^2+2\,a^3b^6x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

```
[Out] 3/2*(6*a^3*b^3*x + 3*a^6 + 2*(b^6*x^2 + 2*a^3*b^3*x + a^6)*log(b*x^(1/3) +
a) + (4*a*b^5*x + a^4*b^2)*x^(2/3) - (5*a^2*b^4*x + 2*a^5*b)*x^(1/3))/(b^9*
x^2 + 2*a^3*b^6*x + a^6*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-3/2), x)
```

Giac [A] time = 1.17455, size = 86, normalized size = 0.66

$$\frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}+\frac{3\left(4\,ax^{\frac{1}{3}}+\frac{3\,a^2}{b}\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2b^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

```
[Out] 3*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a)) + 3/2*(4*a*x^(1/3) + 3*a
^2/b)/((b*x^(1/3) + a)^2*b^2*sgn(b*x^(1/3) + a))
```

3.468 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x}}$$

```
[Out] (-3*a^2)/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
+ (2*a)/(b^3*(a + b*x^(1/3))^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3
/(2*b^3*(a + b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rubi [A] time = 0.0746981, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

```
[Out] (-3*a^2)/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
+ (2*a)/(b^3*(a + b*x^(1/3))^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3
/(2*b^3*(a + b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3b^5\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= \frac{\left(3b^5\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^7(a+bx)^5}-\frac{2a}{b^7(a+bx)^4}+\frac{1}{b^7(a+bx)^3}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= -\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{}{2b^3}$$

Mathematica [A] time = 0.036853, size = 58, normalized size = 0.43

$$\frac{-a^2-4ab\sqrt[3]{x}-6b^2x^{2/3}}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

```
[Out] (-a^2 - 4*a*b*x^(1/3) - 6*b^2*x^(2/3))/(4*b^3*(a + b*x^(1/3))^3*Sqrt[(a + b
*x^(1/3))^2])
```

Maple [A] time = 0.006, size = 54, normalized size = 0.4

$$-\frac{1}{4\,b^3}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(6\,b^2x^{2/3}+4\,ab\sqrt[3]{x}+a^2\right)\left(a+b\sqrt[3]{x}\right)^{-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x)
```

```
[Out] -1/4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(6*b^2*x^(2/3)+4*a*b*x^(1/3)+a^2
)/(a+b*x^(1/3))^5/b^3
```

Maxima [A] time = 1.09819, size = 85, normalized size = 0.63

$$-\frac{3\,a^2b^2}{4\left(b^2\right)^{\frac{9}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}+\frac{2\,ab}{\left(b^2\right)^{\frac{7}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^3}-\frac{3}{2\left(b^2\right)^{\frac{5}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

```
[Out] -3/4*a^2*b^2/((b^2)^(9/2)*(x^(1/3) + a/b)^4) + 2*a*b/((b^2)^(7/2)*(x^(1/3)
+ a/b)^3) - 3/2/((b^2)^(5/2)*(x^(1/3) + a/b)^2)
```

Fricas [A] time = 1.87059, size = 285, normalized size = 2.11

$$\frac{20\,ab^9x^3 - 60\,a^4b^6x^2 - a^{10} - 9\left(5\,a^2b^8x^2 - 4\,a^5b^5x\right)x^{\frac{2}{3}} - 3\left(2\,b^{10}x^3 - 20\,a^3b^7x^2 + 5\,a^6b^4x\right)x^{\frac{1}{3}}}{4\left(b^{15}x^4 + 4\,a^3b^{12}x^3 + 6\,a^6b^9x^2 + 4\,a^9b^6x + a^{12}b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(20*a*b^9*x^3 - 60*a^4*b^6*x^2 - a^10 - 9*(5*a^2*b^8*x^2 - 4*a^5*b^5*x)
*x^(2/3) - 3*(2*b^10*x^3 - 20*a^3*b^7*x^2 + 5*a^6*b^4*x)*x^(1/3))/(b^15*x^4
 + 4*a^3*b^12*x^3 + 6*a^6*b^9*x^2 + 4*a^9*b^6*x + a^12*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
```

```
[Out] undef
```

3.469 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] -a^2/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (6
*a)/(5*b^3*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3/(
4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rubi [A] time = 0.0779012, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

```
[Out] -a^2/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (6
*a)/(5*b^3*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3/(
4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{7/2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3b^7\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^7}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= \frac{\left(3b^7\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^9(a+bx)^7}-\frac{2a}{b^9(a+bx)^6}+\frac{1}{b^9(a+bx)^5}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= -\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-$$

Mathematica [A] time = 0.0371616, size = 58, normalized size = 0.42

$$\frac{-a^2-6ab\sqrt[3]{x}-15b^2x^{2/3}}{20b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

```
[Out] (-a^2 - 6*a*b*x^(1/3) - 15*b^2*x^(2/3))/(20*b^3*(a + b*x^(1/3))^5*Sqrt[(a +
 b*x^(1/3))^2])
```

Maple [A] time = 0.007, size = 54, normalized size = 0.4

$$-\frac{1}{20\,b^3}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(15\,b^2x^{2/3}+6\,ab\sqrt[3]{x}+a^2\right)\left(a+b\sqrt[3]{x}\right)^{-7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x)
```

```
[Out] -1/20*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(15*b^2*x^(2/3)+6*a*b*x^(1/3)+a
^2)/(a+b*x^(1/3))^7/b^3
```

Maxima [A] time = 1.10424, size = 85, normalized size = 0.62

$$-\frac{a^2b^2}{2\left(b^2\right)^{\frac{11}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^6}+\frac{6\,ab}{5\left(b^2\right)^{\frac{9}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^5}-\frac{3}{4\left(b^2\right)^{\frac{7}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*b^2/((b^2)^(11/2)*(x^(1/3) + a/b)^6) + 6/5*a*b/((b^2)^(9/2)*(x^(1/
3) + a/b)^5) - 3/4/((b^2)^(7/2)*(x^(1/3) + a/b)^4)
```

Fricas [A] time = 1.93016, size = 477, normalized size = 3.48

$$-\frac{280\,a^{2}b^{12}x^{4}-1400\,a^{5}b^{9}x^{3}+735\,a^{8}b^{6}x^{2}-14\,a^{11}b^{3}x+a^{14}+3\left(5\,b^{14}x^{4}-210\,a^{3}b^{11}x^{3}+483\,a^{6}b^{8}x^{2}-112\,a^{9}b^{5}x\right)x^{\frac{2}{3}}-3\,(2\cdots}{20\left(b^{21}x^{6}+6\,a^{3}b^{18}x^{5}+15\,a^{6}b^{15}x^{4}+20\,a^{9}b^{12}x^{3}+15\,a^{12}b^{9}x^{2}+6\,a^{15}b^{6}x+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/20*(280*a^2*b^12*x^4 - 1400*a^5*b^9*x^3 + 735*a^8*b^6*x^2 - 14*a^11*b^3*
x + a^14 + 3*(5*b^14*x^4 - 210*a^3*b^11*x^3 + 483*a^6*b^8*x^2 - 112*a^9*b^5
*x)*x^(2/3) - 3*(28*a*b^13*x^4 - 357*a^4*b^10*x^3 + 390*a^7*b^7*x^2 - 35*a^
10*b^4*x)*x^(1/3))/(b^21*x^6 + 6*a^3*b^18*x^5 + 15*a^6*b^15*x^4 + 20*a^9*b^
12*x^3 + 15*a^12*b^9*x^2 + 6*a^15*b^6*x + a^18*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}+2ab\sqrt[3]{x}+b^{2}x^{\frac{2}{3}}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
```

```
[Out] undef
```

3.470 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b}}$$

```
[Out] (-3*a^2)/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
+ (6*a)/(7*b^3*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) -
 1/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rubi [A] time = 0.0787639, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

```
[Out] (-3*a^2)/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
+ (6*a)/(7*b^3*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) -
 1/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{9/2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3b^9\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^9}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= \frac{\left(3b^9\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^{11}(a+bx)^9}-\frac{2a}{b^{11}(a+bx)^8}+\frac{1}{b^{11}(a+bx)^7}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= -\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{}{2b}$$

Mathematica [A] time = 0.0371811, size = 58, normalized size = 0.42

$$\frac{-a^2-8ab\sqrt[3]{x}-28b^2x^{2/3}}{56b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

```
[Out] (-a^2 - 8*a*b*x^(1/3) - 28*b^2*x^(2/3))/(56*b^3*(a + b*x^(1/3))^7*Sqrt[(a +
 b*x^(1/3))^2])
```

Maple [A] time = 0.007, size = 54, normalized size = 0.4

$$-\frac{1}{56\,b^3}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(28\,b^2x^{2/3}+8\,ab\sqrt[3]{x}+a^2\right)\left(a+b\sqrt[3]{x}\right)^{-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x)
```

```
[Out] -1/56*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(28*b^2*x^(2/3)+8*a*b*x^(1/3)+a
^2)/(a+b*x^(1/3))^9/b^3
```

Maxima [A] time = 1.06896, size = 85, normalized size = 0.62

$$-\frac{3\,a^2b^2}{8\left(b^2\right)^{\frac{13}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^8}+\frac{6\,ab}{7\left(b^2\right)^{\frac{11}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^7}-\frac{1}{2\left(b^2\right)^{\frac{9}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="maxima")
```

```
[Out] -3/8*a^2*b^2/((b^2)^(13/2)*(x^(1/3) + a/b)^8) + 6/7*a*b/((b^2)^(11/2)*(x^(1
/3) + a/b)^7) - 1/2/((b^2)^(9/2)*(x^(1/3) + a/b)^6)
```

Fricas [B] time = 1.95698, size = 643, normalized size = 4.69

$$-\frac{28\,b^{18}x^{6} - 2856\,a^{3}b^{15}x^{5} + 18186\,a^{6}b^{12}x^{4} - 20608\,a^{9}b^{9}x^{3} + 4200\,a^{12}b^{6}x^{2} - 48\,a^{15}b^{3}x + a^{18} - 27\,\left(8\,ab^{17}x^{5} - 244\,a^{4}b^{1} \cdots\right.}{56\,\left(b^{27}x^{8} + 8\,a^{3}b^{24}x^{7} + 28\,a^{6}b^{21}x^{6} + 56\,a^{9}b^{18}x^{5} + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="fricas")
```

```
[Out] -1/56*(28*b^18*x^6 - 2856*a^3*b^15*x^5 + 18186*a^6*b^12*x^4 - 20608*a^9*b^9
*x^3 + 4200*a^12*b^6*x^2 - 48*a^15*b^3*x + a^18 - 27*(8*a*b^17*x^5 - 244*a^
4*b^14*x^4 + 840*a^7*b^11*x^3 - 553*a^10*b^8*x^2 + 56*a^13*b^5*x)*x^(2/3) +
 27*(35*a^2*b^16*x^5 - 448*a^5*b^13*x^4 + 876*a^8*b^10*x^3 - 328*a^11*b^7*x
^2 + 14*a^14*b^4*x)*x^(1/3))/(b^27*x^8 + 8*a^3*b^24*x^7 + 28*a^6*b^21*x^6 +
 56*a^9*b^18*x^5 + 70*a^12*b^15*x^4 + 56*a^15*b^12*x^3 + 28*a^18*b^9*x^2 +
8*a^21*b^6*x + a^24*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="giac")
```

```
[Out] undef
```

3.471 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] (-3*a^2)/(10*b^3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
 + (2*a)/(3*b^3*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
- 3/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rubi [A] time = 0.0780041, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

```
[Out] (-3*a^2)/(10*b^3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
 + (2*a)/(3*b^3*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
- 3/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{11/2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3b^{11}\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^{11}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= \frac{\left(3b^{11}\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^{13}(a+bx)^{11}}-\frac{2a}{b^{13}(a+bx)^{10}}+\frac{1}{b^{13}(a+bx)^9}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= -\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Mathematica [A] time = 0.038088, size = 58, normalized size = 0.42

$$\frac{-a^2-10ab\sqrt[3]{x}-45b^2x^{2/3}}{120b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

```
[Out] (-a^2 - 10*a*b*x^(1/3) - 45*b^2*x^(2/3))/(120*b^3*(a + b*x^(1/3))^9*Sqrt[(a
 + b*x^(1/3))^2])
```

Maple [A] time = 0.007, size = 54, normalized size = 0.4

$$-\frac{1}{120\,b^3}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(45\,b^2x^{2/3}+10\,ab\sqrt[3]{x}+a^2\right)\left(a+b\sqrt[3]{x}\right)^{-11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x)
```

```
[Out] -1/120*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(45*b^2*x^(2/3)+10*a*b*x^(1/3)
+a^2)/(a+b*x^(1/3))^11/b^3
```

Maxima [A] time = 1.01626, size = 85, normalized size = 0.62

$$-\frac{3\,a^2b^2}{10\left(b^2\right)^{\frac{15}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{10}}+\frac{2\,ab}{3\left(b^2\right)^{\frac{13}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{9}}-\frac{3}{8\left(b^2\right)^{\frac{11}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="maxima")
```

```
[Out] -3/10*a^2*b^2/((b^2)^(15/2)*(x^(1/3) + a/b)^10) + 2/3*a*b/((b^2)^(13/2)*(x^
(1/3) + a/b)^9) - 3/8/((b^2)^(11/2)*(x^(1/3) + a/b)^8)
```

Fricas [B] time = 2.04098, size = 842, normalized size = 6.15

$$\frac{440\,ab^{21}x^7-25630\,a^4b^{18}x^6+186252\,a^7b^{15}x^5-326150\,a^{10}b^{12}x^4+154000\,a^{13}b^9x^3-16005\,a^{16}b^6x^2+110\,a^{19}b^3x-a^{22}-}{120\left(b^{33}x^{10}+10\,a^3b^{30}x^9+45\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/120*(440*a*b^21*x^7 - 25630*a^4*b^18*x^6 + 186252*a^7*b^15*x^5 - 326150*a
^10*b^12*x^4 + 154000*a^13*b^9*x^3 - 16005*a^16*b^6*x^2 + 110*a^19*b^3*x -
a^22 - 27*(88*a^2*b^20*x^6 - 2200*a^5*b^17*x^5 + 9625*a^8*b^14*x^4 - 10910*
a^11*b^11*x^3 + 3245*a^14*b^8*x^2 - 176*a^17*b^5*x)*x^(2/3) - 9*(5*b^22*x^7
 - 990*a^3*b^19*x^6 + 12705*a^6*b^16*x^5 - 34760*a^9*b^13*x^4 + 25542*a^12*
b^10*x^3 - 4620*a^15*b^7*x^2 + 110*a^18*b^4*x)*x^(1/3))/(b^33*x^10 + 10*a^3
*b^30*x^9 + 45*a^6*b^27*x^8 + 120*a^9*b^24*x^7 + 210*a^12*b^21*x^6 + 252*a^
15*b^18*x^5 + 210*a^18*b^15*x^4 + 120*a^21*b^12*x^3 + 45*a^24*b^9*x^2 + 10*
a^27*b^6*x + a^30*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="giac")
```

```
[Out] undef
```

3.472 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx$

Optimal. Leaf size=77

$$\frac{x(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(3(m+1),-2p;3m+4;-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

```
[Out] ((a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x*(d*x)^m*Hypergeometric2F1[3*(1 + m
), -2*p, 4 + 3*m, -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

Rubi [A] time = 0.0367421, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1356, 343, 341, 64}

$$\frac{x(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(3(m+1),-2p;3m+4;-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

```
[Out] ((a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x*(d*x)^m*Hypergeometric2F1[3*(1 + m
), -2*p, 4 + 3*m, -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int \left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}(dx)^m\,dx$$
$$= \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^{-m}(dx)^m\right)\int \left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}x^m\,dx$$
$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^{-m}(dx)^m\right)\text{Subst}\left(\int x^{-1+3(1+m)}\left(1+\frac{bx}{a}\right)$$
$$= \frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x(dx)^m\,{}_2F_1\left(3(1+m),-2p;4+3m;-\frac{b\sqrt[3]{x}}{a}\right)}{1+m}$$

Mathematica [A] time = 0.0301352, size = 68, normalized size = 0.88

$$\frac{x(dx)^m\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}{}_2F_1\left(3(m+1),-2p;3(m+1)+1;-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

```
[Out] (((a + b*x^(1/3))^2)^p*x*(d*x)^m*Hypergeometric2F1[3*(1 + m), -2*p, 1 + 3*(
1 + m), -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*(d*x)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2 x^{\frac{2}{3}} + 2\, abx^{\frac{1}{3}} + a^2\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)
```

3.473 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx$

Optimal. Leaf size=468

$$\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+9)}-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+4)}+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^7\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+7)}$$

```
[Out] (3*a^9*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(1 +
 2*p)) - (12*a^9*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^
p)/(b^9*(1 + p)) + (84*a^9*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) + b^2
*x^(2/3))^p)/(b^9*(3 + 2*p)) - (84*a^9*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*b*x
^(1/3) + b^2*x^(2/3))^p)/(b^9*(2 + p)) + (210*a^9*(1 + (b*x^(1/3))/a)^5*(a^
2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(5 + 2*p)) - (84*a^9*(1 + (b*x^(1/
3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(3 + p)) + (84*a^9*(1
+ (b*x^(1/3))/a)^7*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(7 + 2*p)) -
 (12*a^9*(1 + (b*x^(1/3))/a)^8*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*
(4 + p)) + (3*a^9*(1 + (b*x^(1/3))/a)^9*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))
^p)/(b^9*(9 + 2*p))
```

Rubi [A] time = 0.224245, antiderivative size = 468, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1356, 266, 43}

$$\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+9)}-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+4)}+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^7\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```


```
[Out] (3*a^9*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(1 +
 2*p)) - (12*a^9*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^
p)/(b^9*(1 + p)) + (84*a^9*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) + b^2
*x^(2/3))^p)/(b^9*(3 + 2*p)) - (84*a^9*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*b*x
^(1/3) + b^2*x^(2/3))^p)/(b^9*(2 + p)) + (210*a^9*(1 + (b*x^(1/3))/a)^5*(a^
2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(5 + 2*p)) - (84*a^9*(1 + (b*x^(1/
3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(3 + p)) + (84*a^9*(1
+ (b*x^(1/3))/a)^7*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(7 + 2*p)) -
 (12*a^9*(1 + (b*x^(1/3))/a)^8*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*
(4 + p)) + (3*a^9*(1 + (b*x^(1/3))/a)^9*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))
^p)/(b^9*(9 + 2*p))
```


Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx &= \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int \left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p} x^2\,dx\\ &= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^8\left(1+\frac{bx}{a}\right)^{2p}dx,x,\sqrt[3]{x}\right)\\ &= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(\frac{a^8\left(1+\frac{bx}{a}\right)^{2p}}{b^8}-\frac{8a^8\left(1+\frac{bx}{a}\right)^{1}}{b^8}\right.\right.\\ &= \frac{3a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+2p)}-\frac{12a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+p)}+\end{aligned}$$

Mathematica [A] time = 0.224451, size = 207, normalized size = 0.44

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(-\frac{4a^7\left(a+b\sqrt[3]{x}\right)}{p+1}+\frac{28a^6\left(a+b\sqrt[3]{x}\right)^2}{2p+3}-\frac{28a^5\left(a+b\sqrt[3]{x}\right)^3}{p+2}+\frac{70a^4\left(a+b\sqrt[3]{x}\right)^4}{2p+5}-\frac{28a^3\left(a+b\sqrt[3]{x}\right)^5}{p+3}+\frac{28a^2\left(a+b\sqrt[3]{x}\right)^6}{2p+7}+\frac{a^8}{2p+1}-\frac{4a\left(a+b\sqrt[3]{x}\right)^7}{p+4}\right.}{b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

```
[Out] (3*(a^8/(1 + 2*p) - (4*a^7*(a + b*x^(1/3)))/(1 + p) + (28*a^6*(a + b*x^(1/3
))^2)/(3 + 2*p) - (28*a^5*(a + b*x^(1/3))^3)/(2 + p) + (70*a^4*(a + b*x^(1/
3))^4)/(5 + 2*p) - (28*a^3*(a + b*x^(1/3))^5)/(3 + p) + (28*a^2*(a + b*x^(1
/3))^6)/(7 + 2*p) - (4*a*(a + b*x^(1/3))^7)/(4 + p) + (a + b*x^(1/3))^8/(9
+ 2*p))*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p)/b^9
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)
```

Maxima [A] time = 1.06306, size = 489, normalized size = 1.04

$$3\left(\left(16\,p^8+288\,p^7+2184\,p^6+9072\,p^5+22449\,p^4+33642\,p^3+29531\,p^2+13698\,p+2520\right)b^9x^3+\left(16\,p^8+224\,p^7+12\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="maxima")
```

```
[Out] 3*((16*p^8 + 288*p^7 + 2184*p^6 + 9072*p^5 + 22449*p^4 + 33642*p^3 + 29531*
p^2 + 13698*p + 2520)*b^9*x^3 + (16*p^8 + 224*p^7 + 1288*p^6 + 3920*p^5 + 6
769*p^4 + 6566*p^3 + 3267*p^2 + 630*p)*a*b^8*x^(8/3) - 8*(8*p^7 + 84*p^6 +
350*p^5 + 735*p^4 + 812*p^3 + 441*p^2 + 90*p)*a^2*b^7*x^(7/3) + 28*(8*p^6 +
 60*p^5 + 170*p^4 + 225*p^3 + 137*p^2 + 30*p)*a^3*b^6*x^2 - 168*(4*p^5 + 20
*p^4 + 35*p^3 + 25*p^2 + 6*p)*a^4*b^5*x^(5/3) + 420*(4*p^4 + 12*p^3 + 11*p^
2 + 3*p)*a^5*b^4*x^(4/3) - 1680*(2*p^3 + 3*p^2 + p)*a^6*b^3*x + 2520*(2*p^2
 + p)*a^7*b^2*x^(2/3) - 5040*a^8*b*p*x^(1/3) + 2520*a^9)*(b*x^(1/3) + a)^(2
*p)/((32*p^9 + 720*p^8 + 6960*p^7 + 37800*p^6 + 126546*p^5 + 269325*p^4 + 3
61840*p^3 + 293175*p^2 + 128322*p + 22680)*b^9)
```

Fricas [A] time = 3.18854, size = 1354, normalized size = 2.89

$$3\left(2520\,a^9+\left(16\,b^9p^8+288\,b^9p^7+2184\,b^9p^6+9072\,b^9p^5+22449\,b^9p^4+33642\,b^9p^3+29531\,b^9p^2+13698\,b^9p+2520\,b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="fricas")
```

```
[Out] 3*(2520*a^9 + (16*b^9*p^8 + 288*b^9*p^7 + 2184*b^9*p^6 + 9072*b^9*p^5 + 224
49*b^9*p^4 + 33642*b^9*p^3 + 29531*b^9*p^2 + 13698*b^9*p + 2520*b^9)*x^3 +
28*(8*a^3*b^6*p^6 + 60*a^3*b^6*p^5 + 170*a^3*b^6*p^4 + 225*a^3*b^6*p^3 + 13
7*a^3*b^6*p^2 + 30*a^3*b^6*p)*x^2 - 1680*(2*a^6*b^3*p^3 + 3*a^6*b^3*p^2 + a
^6*b^3*p)*x + (5040*a^7*b^2*p^2 + 2520*a^7*b^2*p + (16*a*b^8*p^8 + 224*a*b^
8*p^7 + 1288*a*b^8*p^6 + 3920*a*b^8*p^5 + 6769*a*b^8*p^4 + 6566*a*b^8*p^3 +
 3267*a*b^8*p^2 + 630*a*b^8*p)*x^2 - 168*(4*a^4*b^5*p^5 + 20*a^4*b^5*p^4 +
35*a^4*b^5*p^3 + 25*a^4*b^5*p^2 + 6*a^4*b^5*p)*x)*x^(2/3) - 4*(1260*a^8*b*p
 + 2*(8*a^2*b^7*p^7 + 84*a^2*b^7*p^6 + 350*a^2*b^7*p^5 + 735*a^2*b^7*p^4 +
812*a^2*b^7*p^3 + 441*a^2*b^7*p^2 + 90*a^2*b^7*p)*x^2 - 105*(4*a^5*b^4*p^4
+ 12*a^5*b^4*p^3 + 11*a^5*b^4*p^2 + 3*a^5*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p^7 + 37800*b^
9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293175*b^9*p^2 +
 128322*b^9*p + 22680*b^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x**2,x)
```

[Out] Timed out

Giac [B] time = 1.18422, size = 2111, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="giac")
```

```
[Out] 3*(16*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^8*x^3 + 16*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^8*x^(8/3) + 288*(b^2*x^(2/3) + 2*a*b*x^(1/3
) + a^2)^p*b^9*p^7*x^3 + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^
7*x^(8/3) - 64*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^7*x^(7/3) +
2184*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^6*x^3 + 1288*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^6*x^(8/3) - 672*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^p*a^2*b^7*p^6*x^(7/3) + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^3*b^6*p^6*x^2 + 9072*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^5*x^3 +
 3920*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^5*x^(8/3) - 2800*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^5*x^(7/3) + 1680*(b^2*x^(2/3) + 2
*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^5*x^2 + 22449*(b^2*x^(2/3) + 2*a*b*x^(1/3)
+ a^2)^p*b^9*p^4*x^3 - 672*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p^
5*x^(5/3) + 6769*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^4*x^(8/3) -
5880*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^4*x^(7/3) + 4760*(b^2*
x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^4*x^2 + 33642*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*b^9*p^3*x^3 - 3360*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^
p*a^4*b^5*p^4*x^(5/3) + 6566*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^
3*x^(8/3) + 1680*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^4*x^(4/3)
- 6496*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^3*x^(7/3) + 6300*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^3*x^2 + 29531*(b^2*x^(2/3) + 2
*a*b*x^(1/3) + a^2)^p*b^9*p^2*x^3 - 5880*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2
)^p*a^4*b^5*p^3*x^(5/3) + 3267*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*
p^2*x^(8/3) + 5040*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^3*x^(4/3
) - 3528*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^2*x^(7/3) - 3360*(
b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^6*b^3*p^3*x + 3836*(b^2*x^(2/3) + 2*
a*b*x^(1/3) + a^2)^p*a^3*b^6*p^2*x^2 + 13698*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
 a^2)^p*b^9*p*x^3 - 4200*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p^2*
x^(5/3) + 630*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p*x^(8/3) + 4620*
(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^2*x^(4/3) - 720*(b^2*x^(2/3
) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p*x^(7/3) - 5040*(b^2*x^(2/3) + 2*a*b*x^
(1/3) + a^2)^p*a^6*b^3*p^2*x + 840*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^
3*b^6*p*x^2 + 2520*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*x^3 + 5040*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^7*b^2*p^2*x^(2/3) - 1008*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p*x^(5/3) + 1260*(b^2*x^(2/3) + 2*a*b*x^(1
/3) + a^2)^p*a^5*b^4*p*x^(4/3) - 1680*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^6*b^3*p*x + 2520*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^7*b^2*p*x^(2/3)
 - 5040*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^8*b*p*x^(1/3) + 2520*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^9)/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p
^7 + 37800*b^9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293
175*b^9*p^2 + 128322*b^9*p + 22680*b^9)
```

3.474 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx$

Optimal. Leaf size=315

$$\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+3)}-\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+5)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^2\right.}{b^6(p+2)}$$

```
[Out] (-3*a^6*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(1
+ 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))
^p)/(2*b^6*(1 + p)) - (30*a^6*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) +
b^2*x^(2/3))^p)/(b^6*(3 + 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*
b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(2 + p)) - (15*a^6*(1 + (b*x^(1/3))/a)^5*(
a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(5 + 2*p)) + (3*a^6*(1 + (b*x^(1
/3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(2*b^6*(3 + p))
```

Rubi [A] time = 0.139651, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1356, 266, 43}

$$\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+3)}-\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+5)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^2\right.}{b^6(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

```
[Out] (-3*a^6*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(1
+ 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))
^p)/(2*b^6*(1 + p)) - (30*a^6*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) +
b^2*x^(2/3))^p)/(b^6*(3 + 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*
b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(2 + p)) - (15*a^6*(1 + (b*x^(1/3))/a)^5*(
a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(5 + 2*p)) + (3*a^6*(1 + (b*x^(1
/3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(2*b^6*(3 + p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int \left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p} x\,dx$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\mathrm{Subst}\left(\int x^5\left(1+\frac{bx}{a}\right)^{2p}dx, x, \sqrt[3]{x}\right)$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\mathrm{Subst}\left(\int\left(-\frac{a^5\left(1+\frac{bx}{a}\right)^{2p}}{b^5}+\frac{5a^5\left(1+\frac{bx}{a}\right)}{b^5}\right.\right.$$

$$= -\frac{3a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(1+2p)}+\frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(1+p)}$$

Mathematica [A] time = 0.185468, size = 143, normalized size = 0.45

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(\frac{5a^4\left(a+b\sqrt[3]{x}\right)}{p+1}-\frac{20a^3\left(a+b\sqrt[3]{x}\right)^2}{2p+3}+\frac{10a^2\left(a+b\sqrt[3]{x}\right)^3}{p+2}-\frac{2a^5}{2p+1}-\frac{10a\left(a+b\sqrt[3]{x}\right)^4}{2p+5}+\frac{\left(a+b\sqrt[3]{x}\right)^5}{p+3}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

```
[Out] (3*((-2*a^5)/(1 + 2*p) + (5*a^4*(a + b*x^(1/3)))/(1 + p) - (20*a^3*(a + b*x
^(1/3))^2)/(3 + 2*p) + (10*a^2*(a + b*x^(1/3))^3)/(2 + p) - (10*a*(a + b*x^
(1/3))^4)/(5 + 2*p) + (a + b*x^(1/3))^5/(3 + p))*(a + b*x^(1/3))*((a + b*x^
(1/3))^2)^p)/(2*b^6)
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int \left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)
```

Maxima [A] time = 0.994342, size = 267, normalized size = 0.85

$$\frac{3\left(\left(8\,p^5+60\,p^4+170\,p^3+225\,p^2+137\,p+30\right)b^6x^2+2\left(4\,p^5+20\,p^4+35\,p^3+25\,p^2+6\,p\right)ab^5x^{\frac{5}{3}}-5\left(4\,p^4+12\,p^3+\right.\right.}{2\left(8\,p^6+84\,p^5+350\,p^4+735\,p^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="maxima")
```

```
[Out] 3/2*((8*p^5 + 60*p^4 + 170*p^3 + 225*p^2 + 137*p + 30)*b^6*x^2 + 2*(4*p^5 +
 20*p^4 + 35*p^3 + 25*p^2 + 6*p)*a*b^5*x^(5/3) - 5*(4*p^4 + 12*p^3 + 11*p^2
 + 3*p)*a^2*b^4*x^(4/3) + 20*(2*p^3 + 3*p^2 + p)*a^3*b^3*x - 30*(2*p^2 + p)
*a^4*b^2*x^(2/3) + 60*a^5*b*p*x^(1/3) - 30*a^6)*(b*x^(1/3) + a)^(2*p)/((8*p
^6 + 84*p^5 + 350*p^4 + 735*p^3 + 812*p^2 + 441*p + 90)*b^6)
```

Fricas [A] time = 2.5916, size = 653, normalized size = 2.07

$$-\frac{3\left(30\,a^6-\left(8\,b^6p^5+60\,b^6p^4+170\,b^6p^3+225\,b^6p^2+137\,b^6p+30\,b^6\right)x^2-20\left(2\,a^3b^3p^3+3\,a^3b^3p^2+a^3b^3p\right)x+2\left(30\,a^4b\right.\right.}{2\left(8\,b^6p^6+84\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="fricas")
```

```
[Out] -3/2*(30*a^6 - (8*b^6*p^5 + 60*b^6*p^4 + 170*b^6*p^3 + 225*b^6*p^2 + 137*b^
6*p + 30*b^6)*x^2 - 20*(2*a^3*b^3*p^3 + 3*a^3*b^3*p^2 + a^3*b^3*p)*x + 2*(3
0*a^4*b^2*p^2 + 15*a^4*b^2*p - (4*a*b^5*p^5 + 20*a*b^5*p^4 + 35*a*b^5*p^3 +
 25*a*b^5*p^2 + 6*a*b^5*p)*x)*x^(2/3) - 5*(12*a^5*b*p - (4*a^2*b^4*p^4 + 12
*a^2*b^4*p^3 + 11*a^2*b^4*p^2 + 3*a^2*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 8
12*b^6*p^2 + 441*b^6*p + 90*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14164, size = 1006, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="giac")
```

```
[Out] 3/2*(8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^5*x^2 + 8*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^5*x^(5/3) + 60*(b^2*x^(2/3) + 2*a*b*x^(1/3)
 + a^2)^p*b^6*p^4*x^2 + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^4*
x^(5/3) - 20*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^4*p^4*x^(4/3) + 17
0*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^3*x^2 + 70*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*a*b^5*p^3*x^(5/3) - 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a
^2)^p*a^2*b^4*p^3*x^(4/3) + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^
3*p^3*x + 225*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^2*x^2 + 50*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^2*x^(5/3) - 55*(b^2*x^(2/3) + 2*a*b
*x^(1/3) + a^2)^p*a^2*b^4*p^2*x^(4/3) + 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a
```

```
^2)^p*a^3*b^3*p^2*x + 137*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p*x^2 -
 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p^2*x^(2/3) + 12*(b^2*x^(
2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p*x^(5/3) - 15*(b^2*x^(2/3) + 2*a*b*x^(
1/3) + a^2)^p*a^2*b^4*p*x^(4/3) + 20*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*
a^3*b^3*p*x + 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*x^2 - 30*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p*x^(2/3) + 60*(b^2*x^(2/3) + 2*a*b*
x^(1/3) + a^2)^p*a^5*b*p*x^(1/3) - 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^6)/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 812*b^6*p^2 + 4
41*b^6*p + 90*b^6)
```

3.475 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx$

Optimal. Leaf size=142

$$\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+3)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(p+1)}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+1)}$$

```
[Out] (3*a^2*(a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1 + 2*p
)) - (3*a*(a + b*x^(1/3))^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1
+ p)) + (3*(a + b*x^(1/3))^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(3
 + 2*p))
```

Rubi [A] time = 0.0677279, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1341, 646, 43}

$$\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+3)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(p+1)}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1 + 2*p
)) - (3*a*(a + b*x^(1/3))^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1
+ p)) + (3*(a + b*x^(1/3))^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(3
 + 2*p))
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p dx = 3\,\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^p dx, x, \sqrt[3]{x}\right)$$
$$= \left(3\left(b\left(a+b\sqrt[3]{x}\right)\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^{2p} dx, x, \sqrt[3]{x}\right)$$
$$= \left(3\left(b\left(a+b\sqrt[3]{x}\right)\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^{2p}}{b^2}-\frac{2a\left(ab+b\right.}{b^3}\right.\right.$$
$$= \frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+2p)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+p)}+\frac{3(}{}$$

Mathematica [A] time = 0.0489338, size = 83, normalized size = 0.58

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(a^2-ab(2p+1)\sqrt[3]{x}+b^2\left(2p^2+3p+1\right)x^{2/3}\right)}{b^3(p+1)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

```
[Out] (3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(a^2 - a*b*(1 + 2*p)*x^(1/3) + b^2
*(1 + 3*p + 2*p^2)*x^(2/3)))/(b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [F] time = 0.005, size = 0, normalized size = 0.

$$\int\left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

Maxima [A] time = 0.972562, size = 104, normalized size = 0.73

$$\frac{3\left(\left(2\,p^2+3\,p+1\right)b^3x+\left(2\,p^2+p\right)ab^2x^{\frac{2}{3}}-2\,a^2bpx^{\frac{1}{3}}+a^3\right)\left(bx^{\frac{1}{3}}+a\right)^{2\,p}}{\left(4\,p^3+12\,p^2+11\,p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="maxima")
```

```
[Out] 3*((2*p^2 + 3*p + 1)*b^3*x + (2*p^2 + p)*a*b^2*x^(2/3) - 2*a^2*b*p*x^(1/3)
+ a^3)*(b*x^(1/3) + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Fricas [A] time = 2.12405, size = 240, normalized size = 1.69

$$-\frac{3\left(2\,a^{2}bpx^{\frac{1}{3}}-a^{3}-\left(2\,b^{3}p^{2}+3\,b^{3}p+b^{3}\right)x-\left(2\,ab^{2}p^{2}+ab^{2}p\right)x^{\frac{2}{3}}\right)\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}}{4\,b^{3}p^{3}+12\,b^{3}p^{2}+11\,b^{3}p+3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="fricas")
```

```
[Out] -3*(2*a^2*b*p*x^(1/3) - a^3 - (2*b^3*p^2 + 3*b^3*p + b^3)*x - (2*a*b^2*p^2
+ a*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(4*b^3*p^3 + 12*b
^3*p^2 + 11*b^3*p + 3*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^{2}+2ab\sqrt[3]{x}+b^{2}x^{\frac{2}{3}}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p,x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p, x)
```

Giac [A] time = 1.16262, size = 309, normalized size = 2.18

$$\frac{3\left(2\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}b^{3}p^{2}x+2\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}ab^{2}p^{2}x^{\frac{2}{3}}+3\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}b^{3}px+\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a\right.}{4\,b^{3}p^{3}+12\,b^{3}p^{2}+11\,b^{3}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="giac")
```

```
[Out] 3*(2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*p^2*x + 2*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*a*b^2*p^2*x^(2/3) + 3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^
2)^p*b^3*p*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^2*p*x^(2/3) - 2*(b
^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b*p*x^(1/3) + (b^2*x^(2/3) + 2*a*b*
x^(1/3) + a^2)^p*b^3*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3)/(4*b^3*
p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

3.476 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx$

Optimal. Leaf size=69

$$-\frac{3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{2p+1}$$

```
[Out] (-3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeometri
c2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(1 + 2*p)
```

Rubi [A] time = 0.0312997, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1356, 266, 65}

$$-\frac{3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
```

```
[Out] (-3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeometri
c2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(1 + 2*p)
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int \frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}}{x}\,dx$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(1,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right)}{1+2p}$$

Mathematica [A] time = 0.0131312, size = 58, normalized size = 0.84

$$-\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p {}_2F_1\left(1,2p+1;2p+2;\frac{\sqrt[3]{x}b}{a}+1\right)}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
```

```
[Out] (-3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 +
 2*p, 1 + (b*x^(1/3))/a])/(a*(1 + 2*p))
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x,x)
```

```
[Out] Integral(((a + b*x**(1/3))**2)**p/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

3.477 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx$

Optimal. Leaf size=75

$$\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(4,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}$$

```
[Out] (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeome
tric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 + 2*p))
```

Rubi [A] time = 0.0364101, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1356, 266, 65}

$$\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(4,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```



```
[Out] (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeome
tric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 + 2*p))
```


Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int \frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}}{x^2}\,dx$$

$$=\left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

$$=\frac{3b^3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(4,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right)}{a^3(1+2p)}$$

Mathematica [A] time = 0.0151505, size = 61, normalized size = 0.81

$$\frac{3b^3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\,{}_2F_1\left(4,2p+1;2p+2;\frac{\sqrt[3]{x}b}{a}+1\right)}{a^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```

```
[Out] (3*b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[4, 1 + 2*p,
2 + 2*p, 1 + (b*x^(1/3))/a])/(a^4*(1 + 2*p))
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

3.478 $\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx$

Optimal. Leaf size=146

$$-\frac{b^2(1-2p)(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^3\sqrt[3]{x}}+\frac{b(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^2x^{2/3}}-\frac{\left(a+b\sqrt[3]{x}\right)\left(a^2+\right.}{}$$

```
[Out] -(((a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(a*x)) + (b*(1 -
p)*(a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(a^2*x^(2/3)) - (
b^2*(1 - 2*p)*(1 - p)*(a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p
)/(a^3*x^(1/3))
```

Rubi [C] time = 0.0985437, antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 3, integrand size = 77, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {1356, 266, 65}

$$\frac{2b^3(1-2p)(1-p)p\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p{}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}+\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p*
(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

```
[Out] (2*b^3*(1 - 2*p)*(1 - p)*p*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x
^(2/3))^p*Hypergeometric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3
*(1 + 2*p)) + (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)
)^p*Hypergeometric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 +
2*p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx = -\frac{\left(2b^3(1-2p)(1-p)p\right)\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/}\right.}{x}}{3a^3}$$

$$=\left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right.$$

$$=\left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)\right.$$

$$=\frac{2b^3(1-2p)(1-p)p\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\right.}{a}$$

Mathematica [C] time = 0.0877713, size = 101, normalized size = 0.69

$$\frac{b^3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(2p\left(2p^2-3p+1\right){}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)+3\,{}_2F_1\left(4,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)\right)}{a^3(2ap+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 -
 p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

```
[Out] (b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(2*p*(1 - 3*p + 2*p^2)*Hypergeom
etric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a] + 3*Hypergeometric2F1[4,
 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a]))/(a^3*(a + 2*a*p))
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p-\frac{2\,b^3\left(1-2\,p\right)\left(1-p\right)p}{3\,a^3x}\left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a*
b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a*
b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p b^3\left(2\,p-1\right)\left(p-1\right)p}{3\,a^3x}+\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="maxima")
```

```
[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*
p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Fricas [A] time = 2.69189, size = 186, normalized size = 1.27

$$-\frac{\left(a^2bpx^{\frac{1}{3}} + a^3 + \left(2\,b^3p^2 - 3\,b^3p + b^3\right)x + 2\left(ab^2p^2 - ab^2p\right)x^{\frac{2}{3}}\right)\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="fricas")
```

```
[Out] -(a^2*b*p*x^(1/3) + a^3 + (2*b^3*p^2 - 3*b^3*p + b^3)*x + 2*(a*b^2*p^2 - a*
b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(a^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2-2/3*b**3*(1-2*p)*(1-p
)*p*(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/a**3/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p b^3\left(2\,p - 1\right)\left(p - 1\right)p}{3\,a^3x} + \frac{\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="giac")
```

```
[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*
p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

3.479 $\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\sqrt[4]{x}\left(a+b\sqrt[4]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

```
[Out] (-12*a^2)/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (2*a^3)/(b^4*(a +
 b*x^(1/4))*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (4*(a + b*x^(1/4))*x
^(1/4))/(b^3*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) - (12*a*(a + b*x^(1/4
))*Log[a + b*x^(1/4)])/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]])
```

Rubi [A] time = 0.103783, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\sqrt[4]{x}\left(a+b\sqrt[4]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

```
[Out] (-12*a^2)/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (2*a^3)/(b^4*(a +
 b*x^(1/4))*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (4*(a + b*x^(1/4))*x
^(1/4))/(b^3*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) - (12*a*(a + b*x^(1/4
))*Log[a + b*x^(1/4)])/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx = 4\,\text{Subst}\left(\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, \sqrt[4]{x}\right)$$

$$= \frac{\left(4b^3\left(a+b\sqrt[4]{x}\right)\right)\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^3}\,dx, x, \sqrt[4]{x}\right)}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

$$= \frac{\left(4b^3\left(a+b\sqrt[4]{x}\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^6}-\frac{a^3}{b^6(a+bx)^3}+\frac{3a^2}{b^6(a+bx)^2}-\frac{3a}{b^6(a+bx)}\right)dx, x, \sqrt[4]{x}\right)}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

$$= -\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\left(a+b\sqrt[4]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}}}$$

Mathematica [A] time = 0.0677754, size = 93, normalized size = 0.53

$$\frac{2\left(-4a^2b\sqrt[4]{x}-5a^3+4ab^2\sqrt{x}-6a\left(a+b\sqrt[4]{x}\right)^2\log\left(a+b\sqrt[4]{x}\right)+2b^3x^{3/4}\right)}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{\left(a+b\sqrt[4]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

```
[Out] (2*(-5*a^3 - 4*a^2*b*x^(1/4) + 4*a*b^2*Sqrt[x] + 2*b^3*x^(3/4) - 6*a*(a + b
*x^(1/4))^2*Log[a + b*x^(1/4)]))/(b^4*(a + b*x^(1/4))*Sqrt[(a + b*x^(1/4))^
2])
```

Maple [A] time = 0.016, size = 114, normalized size = 0.7

$$2\,\frac{\sqrt{a^2+2\,ab\sqrt[4]{x}+b^2\sqrt{x}}\left(2\,x^{3/4}b^3-6\,\sqrt{x}\ln\left(a+b\sqrt[4]{x}\right)ab^2+4\,\sqrt{x}ab^2-12\,\sqrt[4]{x}\ln\left(a+b\sqrt[4]{x}\right)a^2b-4\,\sqrt[4]{x}a^2b-6\,\ln\left(a+\right.\right.}{\left(a+b\sqrt[4]{x}\right)^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x)
```

```
[Out] 2*(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)*(2*x^(3/4)*b^3-6*x^(1/2)*ln(a+b*x^(
1/4))*a*b^2+4*x^(1/2)*a*b^2-12*x^(1/4)*ln(a+b*x^(1/4))*a^2*b-4*x^(1/4)*a^2*
b-6*ln(a+b*x^(1/4))*a^3-5*a^3)/(a+b*x^(1/4))^3/b^4
```

Maxima [A] time = 1.0124, size = 200, normalized size = 1.14

$$-\frac{12\,a\log\left(x^{\frac{1}{4}}+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{3}{2}}b}-\frac{18\,a^3b}{\left(b^2\right)^{\frac{7}{2}}\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^2}+\frac{4\,\sqrt{x}}{\sqrt{b^2\sqrt{x}+2\,abx^{\frac{1}{4}}+a^2b^2}}-\frac{24\,a^2x^{\frac{1}{4}}}{\left(b^2\right)^{\frac{5}{2}}\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^2}+\frac{8\,a^2}{\sqrt{b^2\sqrt{x}+2\,abx^{\frac{1}{4}}+a^2b^4}}-\frac{}{\left(b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] -12*a*log(x^(1/4) + a/b)/((b^2)^(3/2)*b) - 18*a^3*b/((b^2)^(7/2)*(x^(1/4) +
 a/b)^2) + 4*sqrt(x)/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^2) - 24*a^2
*x^(1/4)/((b^2)^(5/2)*(x^(1/4) + a/b)^2) + 8*a^2/(sqrt(b^2*sqrt(x) + 2*a*b*
x^(1/4) + a^2)*b^4) - 4*a^3/((b^2)^(3/2)*b^3*(x^(1/4) + a/b)^2)
```

Fricas [A] time = 21.1281, size = 315, normalized size = 1.79

$$\frac{2\left(9\,a^5b^4x - 5\,a^9 - 6\left(ab^8x^2 - 2\,a^5b^4x + a^9\right)\log\left(bx^{\frac{1}{4}} + a\right) - 2\left(3\,a^2b^7x - a^6b^3\right)x^{\frac{3}{4}} + \left(7\,a^3b^6x - 3\,a^7b^2\right)\sqrt{x} + 2\left(b^9x^2 - 6\,a^4\right.\right.}{b^{12}x^2 - 2\,a^4b^8x + a^8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(9*a^5*b^4*x - 5*a^9 - 6*(a*b^8*x^2 - 2*a^5*b^4*x + a^9)*log(b*x^(1/4) +
a) - 2*(3*a^2*b^7*x - a^6*b^3)*x^(3/4) + (7*a^3*b^6*x - 3*a^7*b^2)*sqrt(x)
+ 2*(b^9*x^2 - 6*a^4*b^5*x + 3*a^8*b)*x^(1/4))/(b^12*x^2 - 2*a^4*b^8*x + a^
8*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/4)+b**2*x**(1/2))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/4) + b**2*sqrt(x))**(-3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.480 $\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=268

$$\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{30a^3}{b^6\left(a+b\sqrt[6]{x}\right)\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

```
[Out] (-60*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (3*a^5)/(2*b^6*(a
 + b*x^(1/6))^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (10*a^4)/(b^6*(a
 + b*x^(1/6))^2*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (30*a^3)/(b^6*(a
 + b*x^(1/6))*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (6*(a + b*x^(1/6))
*x^(1/6))/(b^5*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (30*a*(a + b*x^(1
/6))*Log[a + b*x^(1/6)])/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)])
```

Rubi [A] time = 0.15092, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{30a^3}{b^6\left(a+b\sqrt[6]{x}\right)\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

```
[Out] (-60*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (3*a^5)/(2*b^6*(a
 + b*x^(1/6))^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (10*a^4)/(b^6*(a
 + b*x^(1/6))^2*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (30*a^3)/(b^6*(a
 + b*x^(1/6))*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (6*(a + b*x^(1/6))
*x^(1/6))/(b^5*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (30*a*(a + b*x^(1
/6))*Log[a + b*x^(1/6)])/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = 6\,\text{Subst}\left(\int \frac{x^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, \sqrt[6]{x}\right)$$
$$= \frac{\left(6b^5\left(a+b\sqrt[6]{x}\right)\right)\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^5}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$
$$= \frac{\left(6b^5\left(a+b\sqrt[6]{x}\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^{10}}-\frac{a^5}{b^{10}(a+bx)^5}+\frac{5a^4}{b^{10}(a+bx)^4}-\frac{10a^3}{b^{10}(a+bx)^3}+\frac{10a^2}{b^{10}(a+bx)^2}-\frac{5a}{b^{10}(a+bx)}\right.\right.}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$
$$= -\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{1}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a}}$$

Mathematica [A] time = 0.0997186, size = 121, normalized size = 0.45

$$\frac{-252a^3b^2\sqrt[3]{x}-48a^2b^3\sqrt{x}-248a^4b\sqrt[6]{x}-77a^5+48ab^4x^{2/3}-60a\left(a+b\sqrt[6]{x}\right)^4\log\left(a+b\sqrt[6]{x}\right)+12b^5x^{5/6}}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{\left(a+b\sqrt[6]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

```
[Out] (-77*a^5 - 248*a^4*b*x^(1/6) - 252*a^3*b^2*x^(1/3) - 48*a^2*b^3*Sqrt[x] + 4
8*a*b^4*x^(2/3) + 12*b^5*x^(5/6) - 60*a*(a + b*x^(1/6))^4*Log[a + b*x^(1/6)
])/(2*b^6*(a + b*x^(1/6))^3*Sqrt[(a + b*x^(1/6))^2])
```

Maple [A] time = 0.014, size = 174, normalized size = 0.7

$$\frac{1}{2b^6}\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}\left(12x^{5/6}b^5-60x^{2/3}\ln\left(a+b\sqrt[6]{x}\right)ab^4+48x^{2/3}ab^4-240\sqrt{x}\ln\left(a+b\sqrt[6]{x}\right)a^2b^3-48\sqrt{x}a^2b^3-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x)
```

```
[Out] 1/2*(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)*(12*x^(5/6)*b^5-60*x^(2/3)*ln(a+b
*x^(1/6))*a*b^4+48*x^(2/3)*a*b^4-240*x^(1/2)*ln(a+b*x^(1/6))*a^2*b^3-48*x^(
1/2)*a^2*b^3-360*x^(1/3)*ln(a+b*x^(1/6))*a^3*b^2-252*x^(1/3)*a^3*b^2-240*x^
(1/6)*ln(a+b*x^(1/6))*a^4*b-248*x^(1/6)*a^4*b-60*ln(a+b*x^(1/6))*a^5-77*a^5
)/(a+b*x^(1/6))^5/b^6
```

Maxima [A] time = 1.06323, size = 161, normalized size = 0.6

$$\frac{12\,b^5x^{\frac{5}{6}}+48\,ab^4x^{\frac{2}{3}}-48\,a^2b^3\sqrt{x}-252\,a^3b^2x^{\frac{1}{3}}-248\,a^4bx^{\frac{1}{6}}-77\,a^5}{2\left(b^{10}x^{\frac{2}{3}}+4\,ab^9\sqrt{x}+6\,a^2b^8x^{\frac{1}{3}}+4\,a^3b^7x^{\frac{1}{6}}+a^4b^6\right)}-\frac{30\,a\log\left(bx^{\frac{1}{6}}+a\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^5*x^(5/6) + 48*a*b^4*x^(2/3) - 48*a^2*b^3*sqrt(x) - 252*a^3*b^2*x
^(1/3) - 248*a^4*b*x^(1/6) - 77*a^5)/(b^10*x^(2/3) + 4*a*b^9*sqrt(x) + 6*a^
2*b^8*x^(1/3) + 4*a^3*b^7*x^(1/6) + a^4*b^6) - 30*a*log(b*x^(1/6) + a)/b^6
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/6)+b**2*x**(1/3))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.53807, size = 320, normalized size = 1.19

$$\frac{3\,a^4|a|\log\left(\left|x^{\frac{1}{6}}|b|\mathrm{sgn}\,(a)\,\mathrm{sgn}\,(b)+|a|\right|\right)}{4\left(a^3b^5|a||b|\mathrm{sgn}\,(a)\,\mathrm{sgn}\,(b)-a^4b^6\right)}+\frac{3\left(24\,a^5b^2|b|\mathrm{sgn}\,(a)\,\mathrm{sgn}\,(b)-25\,a^4b^3|a|\right)\log\left(\left|bx^{\frac{1}{6}}+a\right|\right)}{4\left(a^3b^8|a||b|\mathrm{sgn}\,(a)\,\mathrm{sgn}\,(b)-a^4b^9\right)}+\frac{6\,x^{\frac{1}{6}}}{b^4|b|\mathrm{sgn}\,(a)\,\mathrm{sgn}\,(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] 3/4*a^4*abs(a)*log(abs(x^(1/6)*abs(b)*sgn(a)*sgn(b) + abs(a)))/(a^3*b^5*abs
(a)*abs(b)*sgn(a)*sgn(b) - a^4*b^6) + 3/4*(24*a^5*b^2*abs(b)*sgn(a)*sgn(b)
- 25*a^4*b^3*abs(a))*log(abs(b*x^(1/6) + a))/(a^3*b^8*abs(a)*abs(b)*sgn(a)*
sgn(b) - a^4*b^9) + 6*x^(1/6)/(b^4*abs(b)*sgn(a)*sgn(b)) + 1/4*(70*a^5*abs(
b)*sgn(a)*sgn(b) - 70*a^4*b*abs(a) + 93*(a^3*b^2*abs(b)*sgn(a)*sgn(b) - a^2
*b^3*abs(a))*x^(1/3) + 159*(a^4*b*abs(b)*sgn(a)*sgn(b) - a^3*b^2*abs(a))*x^
(1/6))/((abs(a)*abs(b)*sgn(a)*sgn(b) - a*b)*(b*x^(1/6) + a)^3*b^6)
```

3.481 $\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx$

Optimal. Leaf size=179

$$\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}+\frac{6a^2b\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}-\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{\sqrt{x}\left(a+\frac{b}{\sqrt{x}}\right)}+\frac{6ab^2\log\left(\sqrt{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}$$

```
[Out] (-2*b^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]])/((a + b/Sqrt[x])*Sqrt[x]) + (6
*a^2*b*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*Sqrt[x])/(a + b/Sqrt[x]) + (a^3*
Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*x)/(a + b/Sqrt[x]) + (6*a*b^2*Sqrt[a^2
+ b^2/x + (2*a*b)/Sqrt[x]]*Log[Sqrt[x]])/(a + b/Sqrt[x])
```

Rubi [A] time = 0.0922638, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1341, 1355, 263, 43}

$$\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}+\frac{6a^2b\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}-\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{\sqrt{x}\left(a+\frac{b}{\sqrt{x}}\right)}+\frac{6ab^2\log\left(\sqrt{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

```
[Out] (-2*b^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]])/((a + b/Sqrt[x])*Sqrt[x]) + (6
*a^2*b*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*Sqrt[x])/(a + b/Sqrt[x]) + (a^3*
Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*x)/(a + b/Sqrt[x]) + (6*a*b^2*Sqrt[a^2
+ b^2/x + (2*a*b)/Sqrt[x]]*Log[Sqrt[x]])/(a + b/Sqrt[x])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = 2\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2} x\,dx, x, \sqrt{x}\right)$$

$$= \frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^3 x\,dx, x, \sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$

$$= \frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^3}{x^2} dx, x, \sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$

$$= \frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int \left(3a^2b^4+\frac{b^6}{x^2}+\frac{3ab^5}{x}+a^3b^3x\right) dx, x, \sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$

$$= -\frac{2b^4\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}}{\left(ab+\frac{b^2}{\sqrt{x}}\right)\sqrt{x}} + \frac{6a^2b^2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\sqrt{x}}{ab+\frac{b^2}{\sqrt{x}}} + \frac{a^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\,x}{a+\frac{b}{\sqrt{x}}} + \frac{3ab^3\sqrt{a^2+\frac{b^2}{x}}}{ab+}$$

Mathematica [A] time = 0.0335123, size = 66, normalized size = 0.37

$$\frac{\sqrt{\frac{\left(a\sqrt{x}+b\right)^2}{x}}\left(6a^2bx+a^3x^{3/2}+3ab^2\sqrt{x}\log(x)-2b^3\right)}{a\sqrt{x}+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Sqrt[x])^2/x]*(-2*b^3 + 6*a^2*b*x + a^3*x^(3/2) + 3*a*b^2*Sqrt
[x]*Log[x]))/(b + a*Sqrt[x])
```

Maple [A] time = 0.026, size = 68, normalized size = 0.4

$$\sqrt{\left(a^2x^{\frac{3}{2}}+b^2\sqrt{x}+2\,abx\right)x^{-\frac{3}{2}}}\left(x^{\frac{3}{2}}a^3+6\,xa^2b+3\,\sqrt{x}\ln(x)\,ab^2-2\,b^3\right)\left(a\sqrt{x}+b\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x)
```

```
[Out] ((a^2*x^(3/2)+b^2*x^(1/2)+2*a*b*x)/x^(3/2))^(1/2)*(x^(3/2)*a^3+6*x*a^2*b+3*
x^(1/2)*ln(x)*a*b^2-2*b^3)/(a*x^(1/2)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^3x + 3\,ab^2 \int \frac{1}{x}\,dx + 6\,a^2b\sqrt{x} - \frac{2\,b^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] a^3*x + 3*a*b^2*integrate(1/x, x) + 6*a^2*b*sqrt(x) - 2*b^3/sqrt(x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + \frac{2ab}{\sqrt{x}} + \frac{b^2}{x}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x+2*a*b/x**(1/2))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/sqrt(x) + b**2/x)**(3/2), x)
```

Giac [A] time = 1.18833, size = 108, normalized size = 0.6

$$a^3x\mathrm{sgn}\left(ax + b\sqrt{x}\right)\mathrm{sgn}\left(x\right) + 3\,ab^2\log\left(|x|\right)\mathrm{sgn}\left(ax + b\sqrt{x}\right)\mathrm{sgn}\left(x\right) + 6\,a^2b\sqrt{x}\mathrm{sgn}\left(ax + b\sqrt{x}\right)\mathrm{sgn}\left(x\right) - \frac{2\,b^3\mathrm{sgn}\left(ax + b\sqrt{\right.}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] a^3*x*sgn(a*x + b*sqrt(x))*sgn(x) + 3*a*b^2*log(abs(x))*sgn(a*x + b*sqrt(x)
)*sgn(x) + 6*a^2*b*sqrt(x)*sgn(a*x + b*sqrt(x))*sgn(x) - 2*b^3*sgn(a*x + b*
sqrt(x))*sgn(x)/sqrt(x)
```

3.482 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx$

Optimal. Leaf size=391

$$\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{21a^6bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{63a^5b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{105a^3b^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{63a^2b^5}{2}$$

```
[Out] (-3*b^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(4*(a + b/x^(1/3))*x^(4/
3)) - (7*a*b^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))*
x) - (63*a^2*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1/
3))*x^(2/3)) - (105*a^3*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a
+ b/x^(1/3))*x^(1/3)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)]*x^(1/3))/(a + b/x^(1/3)) + (21*a^6*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^
(1/3)]*x^(2/3))/(2*(a + b/x^(1/3))) + (a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)
/x^(1/3)]*x)/(a + b/x^(1/3)) + (105*a^4*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^(1/3))
```

Rubi [A] time = 0.187171, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{21a^6bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{63a^5b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{105a^3b^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{63a^2b^5}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (-3*b^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(4*(a + b/x^(1/3))*x^(4/
3)) - (7*a*b^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))*
x) - (63*a^2*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1/
3))*x^(2/3)) - (105*a^3*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a
+ b/x^(1/3))*x^(1/3)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)]*x^(1/3))/(a + b/x^(1/3)) + (21*a^6*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^
(1/3)]*x^(2/3))/(2*(a + b/x^(1/3))) + (a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)
/x^(1/3)]*x)/(a + b/x^(1/3)) + (105*a^4*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^(1/3))
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```

```
[p - 1/2]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = 3\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{7/2} x^2\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^7 x^2\,dx, x, \sqrt[3]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^7}{x^5}\,dx, x, \sqrt[3]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(21a^5b^9+\frac{b^{14}}{x^5}+\frac{7ab^{13}}{x^4}+\frac{21a^2b^{12}}{x^3}+\frac{35a^3b^{11}}{x^2}+\frac{35a^4b^{10}}{x}+7a^6b^8x+\right.\right.}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$= -\frac{3b^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{4/3}} - \frac{7ab^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x} - \frac{63a^2b^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}} - \frac{105a^3b^5\sqrt{a^2+}}{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right.}$$

Mathematica [A] time = 0.073284, size = 125, normalized size = 0.32

$$\frac{\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(252a^5b^2x^{5/3}-126a^2b^5x^{2/3}+140a^4b^3x^{4/3}\log(x)-420a^3b^4x+42a^6bx^2+4a^7x^{7/3}-28ab^6\sqrt[3]{x}-3b^7\right)}{4x\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(-3*b^7 - 28*a*b^6*x^(1/3) - 126*a^2*b^5*x
^(2/3) - 420*a^3*b^4*x + 252*a^5*b^2*x^(5/3) + 42*a^6*b*x^2 + 4*a^7*x^(7/3)
 + 140*a^4*b^3*x^(4/3)*Log[x]))/(4*(b + a*x^(1/3))*x)
```

Maple [A] time = 0.022, size = 115, normalized size = 0.3

$$\frac{1}{4}\left(\left(a^2x^{\frac{2}{3}}+2ab\sqrt[3]{x}+b^2\right)x^{-\frac{2}{3}}\right)^{\frac{7}{2}}\left(42a^6bx^3+140a^4b^3\ln(x)x^{7/3}+252a^5b^2x^{8/3}+4a^7x^{10/3}-28ab^6x^{4/3}-420a^3b^4x^2-126\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x)
```

```
[Out] 1/4*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(7/2)*(42*a^6*b*x^3+140*a^4*b
^3*ln(x)*x^(7/3)+252*a^5*b^2*x^(8/3)+4*a^7*x^(10/3)-28*a*b^6*x^(4/3)-420*a^
3*b^4*x^2-126*a^2*b^5*x^(5/3)-3*b^7*x)/(b+a*x^(1/3))^7
```

Maxima [A] time = 0.977742, size = 107, normalized size = 0.27

$$35\,a^4b^3\log\left(x\right) + \frac{4\,a^7x^{\frac{7}{3}} + 42\,a^6bx^2 + 252\,a^5b^2x^{\frac{5}{3}} - 420\,a^3b^4x - 126\,a^2b^5x^{\frac{2}{3}} - 28\,ab^6x^{\frac{1}{3}} - 3\,b^7}{4\,x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="maxima")
```

```
[Out] 35*a^4*b^3*log(x) + 1/4*(4*a^7*x^(7/3) + 42*a^6*b*x^2 + 252*a^5*b^2*x^(5/3)
 - 420*a^3*b^4*x - 126*a^2*b^5*x^(2/3) - 28*a*b^6*x^(1/3) - 3*b^7)/x^(4/3)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30014, size = 234, normalized size = 0.6

$$a^7x\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}\left(x\right) + 35\,a^4b^3\log\left(|x|\right)\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}\left(x\right) + \frac{21}{2}\,a^6bx^{\frac{2}{3}}\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}\left(x\right) + 63\,a^5b^2x^{\frac{1}{3}}\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="giac")
```

```
[Out] a^7*x*sgn(a*x + b*x^(2/3))*sgn(x) + 35*a^4*b^3*log(abs(x))*sgn(a*x + b*x^(2
/3))*sgn(x) + 21/2*a^6*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 63*a^5*b^2*x
^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 1/4*(420*a^3*b^4*x*sgn(a*x + b*x^(2/3)
)*sgn(x) + 126*a^2*b^5*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 28*a*b^6*x^(1/
3)*sgn(a*x + b*x^(2/3))*sgn(x) + 3*b^7*sgn(a*x + b*x^(2/3))*sgn(x))/x^(4/3)
```

3.483 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx$

Optimal. Leaf size=291

$$\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{15a^4bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^3b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{15ab^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{3b^5\sqrt{a^2}}{2x^{2/3}}$$

```
[Out] (-3*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1/3))*x^(2/
3)) - (15*a*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))
*x^(1/3)) + (30*a^3*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/
(a + b/x^(1/3)) + (15*a^4*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/
3))/(2*(a + b/x^(1/3))) + (a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)
/(a + b/x^(1/3)) + (30*a^2*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Lo
g[x^(1/3)])/(a + b/x^(1/3))
```

Rubi [A] time = 0.137337, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{15a^4bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^3b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{15ab^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{3b^5\sqrt{a^2}}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

```
[Out] (-3*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1/3))*x^(2/
3)) - (15*a*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))
*x^(1/3)) + (30*a^3*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/
(a + b/x^(1/3)) + (15*a^4*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/
3))/(2*(a + b/x^(1/3))) + (a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)
/(a + b/x^(1/3)) + (30*a^2*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Lo
g[x^(1/3)])/(a + b/x^(1/3))
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = 3\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^2\,dx, x, \sqrt[3]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^5}{x^3}\,dx, x, \sqrt[3]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(10a^3b^7+\frac{b^{10}}{x^3}+\frac{5ab^9}{x^2}+\frac{10a^2b^8}{x}+5a^4b^6x+a^5b^5x^2\right) dx, x, \sqrt[3]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= -\frac{3b^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}} - \frac{15ab^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\sqrt[3]{x}} + \frac{30a^3b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{ab+\frac{b^2}{\sqrt[3]{x}}} + \frac{15a^4b^2\sqrt{}}{2}$$

Mathematica [A] time = 0.0591039, size = 99, normalized size = 0.34

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(20a^2b^3x^{2/3}\log(x)+60a^3b^2x+15a^4bx^{4/3}+2a^5x^{5/3}-30ab^4\sqrt[3]{x}-3b^5\right)}{2x\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-3*b^5 - 30*a*b^4*x^(1/3) + 60*a^3*b^2*x + 15*a^4*b*x^(4/
3) + 2*a^5*x^(5/3) + 20*a^2*b^3*x^(2/3)*Log[x]))/(2*Sqrt[(b + a*x^(1/3))^2/
x^(2/3)]*x)
```

Maple [A] time = 0.009, size = 91, normalized size = 0.3

$$\frac{x}{2}\left(\left(a^2x^{\frac{2}{3}}+2\,ab\sqrt[3]{x}+b^2\right)x^{-\frac{2}{3}}\right)^{\frac{5}{2}}\left(15\,a^4bx^{4/3}+60\,a^3b^2x+20\,a^2b^3\ln(x)\,x^{2/3}+2\,a^5x^{5/3}-30\,ab^4\sqrt[3]{x}-3\,b^5\right)\left(b+a\sqrt[3]{x}\right)^{-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)*x*(15*a^4*b*x^(4/3)+60*
a^3*b^2*x+20*a^2*b^3*ln(x)*x^(2/3)+2*a^5*x^(5/3)-30*a*b^4*x^(1/3)-3*b^5)/(b
+a*x^(1/3))^5
```

Maxima [A] time = 1.03974, size = 77, normalized size = 0.26

$$10\,a^2b^3\log\left(x\right)+\frac{2\,a^5x^{\frac{5}{3}}+15\,a^4bx^{\frac{4}{3}}+60\,a^3b^2x-30\,ab^4x^{\frac{1}{3}}-3\,b^5}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 10*a^2*b^3*log(x) + 1/2*(2*a^5*x^(5/3) + 15*a^4*b*x^(4/3) + 60*a^3*b^2*x -
30*a*b^4*x^(1/3) - 3*b^5)/x^(2/3)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(5/2), x)
```

Giac [A] time = 1.26051, size = 173, normalized size = 0.59

$$a^5x\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)+10\,a^2b^3\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)+\frac{15}{2}\,a^4bx^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)+30\,a^3b^2x^{\frac{1}{3}}\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(2/3))*sgn(x) + 10*a^2*b^3*log(abs(x))*sgn(a*x + b*x^(2
/3))*sgn(x) + 15/2*a^4*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 30*a^3*b^2*x
^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 3/2*(10*a*b^4*x^(1/3)*sgn(a*x + b*x^(2
/3))*sgn(x) + b^5*sgn(a*x + b*x^(2/3))*sgn(x))/x^(2/3)
```

3.484 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx$

Optimal. Leaf size=189

$$\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{9a^2bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{9ab^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{3b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] (9*a*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a + b/x^(1/3))
 + (9*a^2*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^
(1/3))) + (a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))
 + (3*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^
(1/3))
```

Rubi [A] time = 0.0906669, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{9a^2bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{9ab^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{3b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```


```
[Out] (9*a*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a + b/x^(1/3))
 + (9*a^2*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^
(1/3))) + (a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))
 + (3*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^
(1/3))
```


Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = 3\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2} x^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^3 x^2\,dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^3}{x}\,dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(3ab^5+\frac{b^6}{x}+3a^2b^4x+a^3b^3x^2\right) dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{9ab^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{ab+\frac{b^2}{\sqrt[3]{x}}} + \frac{9a^2b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)} + \frac{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}} + \frac{b^4\sqrt{a^2+}}{a}$$

Mathematica [A] time = 0.0291729, size = 77, normalized size = 0.41

$$\frac{\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(9a^2bx+2a^3x^{4/3}+18ab^2x^{2/3}+2b^3\sqrt[3]{x}\log(x)\right)}{2\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(18*a*b^2*x^(2/3) + 9*a^2*b*x + 2*a^3*x^(4
/3) + 2*b^3*x^(1/3)*Log[x]))/(2*(b + a*x^(1/3)))
```

Maple [A] time = 0.005, size = 69, normalized size = 0.4

$$\frac{x}{2}\left(\left(a^2x^{\frac{2}{3}}+2\,ab\sqrt[3]{x}+b^2\right)x^{-\frac{2}{3}}\right)^{\frac{3}{2}}\left(9\,x^{2/3}a^2b+18\,ab^2\sqrt[3]{x}+2\,b^3\ln(x)+2\,a^3x\right)\left(b+a\sqrt[3]{x}\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)*x*(9*x^(2/3)*a^2*b+18*a
*b^2*x^(1/3)+2*b^3*ln(x)+2*a^3*x)/(b+a*x^(1/3))^3
```

Maxima [A] time = 1.03323, size = 41, normalized size = 0.22

$$a^3x + b^3\log(x) + \frac{9}{2}a^2bx^{\frac{2}{3}} + 9ab^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
```

```
[Out] a^3*x + b^3*log(x) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(3/2), x)
```

Giac [A] time = 1.17087, size = 107, normalized size = 0.57

$$a^3x\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) + b^3\log(|x|)\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) + \frac{9}{2}a^2bx^{\frac{2}{3}}\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) + 9ab^2x^{\frac{1}{3}}\operatorname{sgn}\left(ax + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

```
[Out] a^3*x*sgn(a*x + b*x^(2/3))*sgn(x) + b^3*log(abs(x))*sgn(a*x + b*x^(2/3))*sg
n(x) + 9/2*a^2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 9*a*b^2*x^(1/3)*sgn(
a*x + b*x^(2/3))*sgn(x)
```

3.485 $\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=88

$$\frac{3bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{ax\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] (3*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^(1/3)))
 + (a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))
```

Rubi [A] time = 0.0525367, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 1355, 14}

$$\frac{3bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{ax\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (3*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^(1/3)))
 + (a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx = 3\,\text{Subst}\left(\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}x^2\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)x^2\,dx, x, \sqrt[3]{x}\right)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(b^2x+abx^2\right)dx, x, \sqrt[3]{x}\right)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$
$$= \frac{3b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}+\frac{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}}$$

Mathematica [A] time = 0.0147499, size = 49, normalized size = 0.56

$$\frac{\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(2ax^{4/3}+3bx\right)}{2\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(3*b*x + 2*a*x^(4/3)))/(2*(b + a*x^(1/3)))
```

Maple [A] time = 0.004, size = 50, normalized size = 0.6

$$\frac{1}{2}\sqrt{\left(a^2x^{\frac{2}{3}}+2\,ab\sqrt[3]{x}+b^2\right)x^{-\frac{2}{3}}}\sqrt[3]{x}\left(3\,x^{2/3}b+2\,ax\right)\left(b+a\sqrt[3]{x}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)*x^(1/3)*(3*x^(2/3)*b+2*
a*x)/(b+a*x^(1/3))
```

Maxima [A] time = 0.968008, size = 14, normalized size = 0.16

$$ax+\frac{3}{2}bx^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
```

```
[Out] a*x + 3/2*b*x^(2/3)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

Giac [A] time = 1.15607, size = 46, normalized size = 0.52

$$ax\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}\left(x\right) + \frac{3}{2}\,bx^{\frac{2}{3}}\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
```

```
[Out] a*x*sgn(a*x + b*x^(2/3))*sgn(x) + 3/2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x)
```

3.486 $\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx$

Optimal. Leaf size=190

$$\frac{3b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^2\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

```
[Out] (3*b^2*(a + b/x^(1/3))*x^(1/3))/(a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)]) - (3*b*(a + b/x^(1/3))*x^(2/3))/(2*a^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)]) - (3*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^4*Sqrt[a^2 + b^2/x^(2/
3) + (2*a*b)/x^(1/3)])
```

Rubi [A] time = 0.118063, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{3b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^2\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (3*b^2*(a + b/x^(1/3))*x^(1/3))/(a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)]) - (3*b*(a + b/x^(1/3))*x^(2/3))/(2*a^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)]) - (3*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^4*Sqrt[a^2 + b^2/x^(2/
3) + (2*a*b)/x^(1/3)])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2}{x}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\\
&= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^3}{b^2+abx}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\\
&= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{b}{a^3}-\frac{x}{a^2}+\frac{x^2}{ab}-\frac{b^2}{a^3(b+ax)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\\
&= \frac{3\left(ab^2+\frac{b^3}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}}{2a^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}+\frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}
\end{aligned}$$

Mathematica [A] time = 0.0410827, size = 86, normalized size = 0.45

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(-3a^2bx^{2/3}+2a^3x+6ab^2\sqrt[3]{x}-6b^3\log\left(a\sqrt[3]{x}+b\right)\right)}{2a^4\sqrt[3]{x}\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] ((b + a*x^(1/3))*(6*a*b^2*x^(1/3) - 3*a^2*b*x^(2/3) + 2*a^3*x - 6*b^3*Log[b
 + a*x^(1/3)]))/(2*a^4*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3))
```

Maple [A] time = 0.006, size = 78, normalized size = 0.4

$$-\frac{1}{2\,a^4}\left(b+a\sqrt[3]{x}\right)\left(3\,x^{2/3}a^2b-6\,ab^2\sqrt[3]{x}+6\,b^3\ln\left(b+a\sqrt[3]{x}\right)-2\,a^3x\right)\frac{1}{\sqrt{\left(a^2x^{\frac{2}{3}}+2\,ab\sqrt[3]{x}+b^2\right)x^{-\frac{2}{3}}}}\frac{1}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x)
```

```
[Out] -1/2/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)/x^(1/3)*(b+a*x^(1/3))*
(3*x^(2/3)*a^2*b-6*a*b^2*x^(1/3)+6*b^3*ln(b+a*x^(1/3))-2*a^3*x)/a^4
```

Maxima [A] time = 1.01861, size = 59, normalized size = 0.31

$$-\frac{3\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^4}+\frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
```

```
[Out] -3*b^3*log(a*x^(1/3) + b)/a^4 + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3
))/a^3
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

Giac [A] time = 1.20538, size = 104, normalized size = 0.55

$$-\frac{3\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^4\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}+\frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
```

```
[Out] -3*b^3*log(abs(a*x^(1/3) + b))/(a^4*sgn(a*x + b*x^(2/3))*sgn(x)) + 1/2*(2*a
^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3))/(a^3*sgn(a*x + b*x^(2/3))*sgn(x))
```

3.487 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx$

Optimal. Leaf size=300

$$\frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)}+\frac{18b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{9bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x(}{a^3\sqrt{a^2}}$$

```
[Out] (3*b^5*(a + b/x^(1/3)))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b
 + a*x^(1/3))^2) - (15*b^4*(a + b/x^(1/3)))/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (
2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (18*b^2*(a + b/x^(1/3))*x^(1/3))/(a^5*Sq
rt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (9*b*(a + b/x^(1/3))*x^(2/3))/(2
*a^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^3*
Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (30*b^3*(a + b/x^(1/3))*Log[b
+ a*x^(1/3)])/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])
```

Rubi [A] time = 0.187466, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)}+\frac{18b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{9bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x(}{a^3\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

```
[Out] (3*b^5*(a + b/x^(1/3)))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b
 + a*x^(1/3))^2) - (15*b^4*(a + b/x^(1/3)))/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (
2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (18*b^2*(a + b/x^(1/3))*x^(1/3))/(a^5*Sq
rt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (9*b*(a + b/x^(1/3))*x^(2/3))/(2
*a^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^3*
Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (30*b^3*(a + b/x^(1/3))*Log[b
+ a*x^(1/3)])/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+\frac{b^2}{x}\right)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^5}{\left(b^2+abx\right)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{6}{a^5b}-\frac{3x}{a^4b^2}+\frac{x^2}{a^3b^3}-\frac{b^2}{a^5(b+ax)^3}+\frac{5b}{a^5(b+ax)^2}-\frac{10}{a^5(b+ax)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^2}-\frac{15\left(ab^4+\frac{b^5}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)}+\frac{18\left(ab^2+\frac{b^3}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-$$

Mathematica [A] time = 0.0897873, size = 126, normalized size = 0.42

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(63a^2b^3x^{2/3}+20a^3b^2x-5a^4bx^{4/3}+2a^5x^{5/3}+6ab^4\sqrt[3]{x}-60b^3\left(a\sqrt[3]{x}+b\right)^2\log\left(a\sqrt[3]{x}+b\right)-27b^5\right)}{2a^6x\left(\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-27*b^5 + 6*a*b^4*x^(1/3) + 63*a^2*b^3*x^(2/3) + 20*a^3*b
^2*x - 5*a^4*b*x^(4/3) + 2*a^5*x^(5/3) - 60*b^3*(b + a*x^(1/3))^2*Log[b + a
*x^(1/3)]))/(2*a^6*((b + a*x^(1/3))^2/x^(2/3))^(3/2)*x)
```

Maple [A] time = 0.01, size = 141, normalized size = 0.5

$$\frac{1}{2\,xa^6}\left(2\,a^5x^{5/3}-5\,a^4bx^{4/3}-60\,x^{2/3}\ln\left(b+a\sqrt[3]{x}\right)a^2b^3+63\,x^{2/3}a^2b^3-120\,\sqrt[3]{x}\ln\left(b+a\sqrt[3]{x}\right)ab^4+6\,ab^4\sqrt[3]{x}-60\,\ln\left(b+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x)
```

```
[Out] 1/2/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)/x*(2*a^5*x^(5/3)-5*a^4*
b*x^(4/3)-60*x^(2/3)*ln(b+a*x^(1/3))*a^2*b^3+63*x^(2/3)*a^2*b^3-120*x^(1/3)
*ln(b+a*x^(1/3))*a*b^4+6*a*b^4*x^(1/3)-60*ln(b+a*x^(1/3))*b^5+20*a^3*b^2*x-
27*b^5)*(b+a*x^(1/3))/a^6
```

Maxima [A] time = 1.02366, size = 131, normalized size = 0.44

$$\frac{2\,a^5x^{\frac{5}{3}} - 5\,a^4bx^{\frac{4}{3}} + 20\,a^3b^2x + 63\,a^2b^3x^{\frac{2}{3}} + 6\,ab^4x^{\frac{1}{3}} - 27\,b^5}{2\left(a^8x^{\frac{2}{3}} + 2\,a^7bx^{\frac{1}{3}} + a^6b^2\right)} - \frac{30\,b^3\log\left(ax^{\frac{1}{3}} + b\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^5*x^(5/3) - 5*a^4*b*x^(4/3) + 20*a^3*b^2*x + 63*a^2*b^3*x^(2/3) +
6*a*b^4*x^(1/3) - 27*b^5)/(a^8*x^(2/3) + 2*a^7*b*x^(1/3) + a^6*b^2) - 30*b^
3*log(a*x^(1/3) + b)/a^6
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2} + \frac{2ab}{\sqrt[3]{x}} + \frac{b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-3/2), x)
```

Giac [A] time = 1.22877, size = 163, normalized size = 0.54

$$-\frac{30\,b^3\log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{a^6\mathrm{sgn}\left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}}\right)} - \frac{3\left(10\,ab^4x^{\frac{1}{3}} + 9\,b^5\right)}{2\left(ax^{\frac{1}{3}} + b\right)^2a^6\mathrm{sgn}\left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}}\right)} + \frac{2\,a^6x - 9\,a^5bx^{\frac{2}{3}} + 36\,a^4b^2x^{\frac{1}{3}}}{2\,a^9\mathrm{sgn}\left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

```
[Out] -30*b^3*log(abs(a*x^(1/3) + b))/(a^6*sgn(a*x^(2/3) + b*x^(1/3))) - 3/2*(10*
a*b^4*x^(1/3) + 9*b^5)/((a*x^(1/3) + b)^2*a^6*sgn(a*x^(2/3) + b*x^(1/3))) +
 1/2*(2*a^6*x - 9*a^5*b*x^(2/3) + 36*a^4*b^2*x^(1/3))/(a^9*sgn(a*x^(2/3) +
b*x^(1/3)))
```

3.488 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx$

Optimal. Leaf size=410

$$\frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^4}-\frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^3}+\frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{105b^4\left(a\right.\cdots}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\cdots}}$$

```
[Out] (3*b^7*(a + b/x^(1/3)))/(4*a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b
 + a*x^(1/3))^4) - (7*b^6*(a + b/x^(1/3)))/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2
*a*b)/x^(1/3)]*(b + a*x^(1/3))^3) + (63*b^5*(a + b/x^(1/3)))/(2*a^8*Sqrt[a^
2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))^2) - (105*b^4*(a + b/x^(
1/3)))/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (4
5*b^2*(a + b/x^(1/3))*x^(1/3))/(a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)]) - (15*b*(a + b/x^(1/3))*x^(2/3))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(
1/3)]) - (105*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^8*Sqrt[a^2 + b^2/x
^(2/3) + (2*a*b)/x^(1/3)])
```

Rubi [A] time = 0.267721, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^4}-\frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^3}+\frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{105b^4\left(a\right.\cdots}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] (3*b^7*(a + b/x^(1/3)))/(4*a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b
 + a*x^(1/3))^4) - (7*b^6*(a + b/x^(1/3)))/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2
*a*b)/x^(1/3)]*(b + a*x^(1/3))^3) + (63*b^5*(a + b/x^(1/3)))/(2*a^8*Sqrt[a^
2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))^2) - (105*b^4*(a + b/x^(
1/3)))/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (4
5*b^2*(a + b/x^(1/3))*x^(1/3))/(a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)]) - (15*b*(a + b/x^(1/3))*x^(2/3))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(
1/3)]) - (105*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^8*Sqrt[a^2 + b^2/x
^(2/3) + (2*a*b)/x^(1/3)])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+\frac{b^2}{x}\right)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^7}{\left(b^2+abx\right)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{15}{a^7b^3}-\frac{5x}{a^6b^4}+\frac{x^2}{a^5b^5}-\frac{b^2}{a^7(b+ax)^5}+\frac{7b}{a^7(b+ax)^4}-\frac{21}{a^7(b+ax)^3}+\frac{35}{a^7b(b+ax)^2}\right.\right.}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^7+\frac{b^8}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^4} - \frac{7\left(ab^6+\frac{b^7}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^3} + \frac{63\left(ab^5+\right.}{2a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

Mathematica [A] time = 0.138987, size = 152, normalized size = 0.37

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(84a^5b^2x^{5/3}+556a^4b^3x^{4/3}-444a^2b^5x^{2/3}+544a^3b^4x-14a^6bx^2+4a^7x^{7/3}-856ab^6\sqrt[3]{x}-420b^3\left(a\sqrt[3]{x}+b\right)^4\right.}{4a^8x^{5/3}\left(\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-319*b^7 - 856*a*b^6*x^(1/3) - 444*a^2*b^5*x^(2/3) + 544*
a^3*b^4*x + 556*a^4*b^3*x^(4/3) + 84*a^5*b^2*x^(5/3) - 14*a^6*b*x^2 + 4*a^7
*x^(7/3) - 420*b^3*(b + a*x^(1/3))^4*Log[b + a*x^(1/3)]))/(4*a^8*((b + a*x^
```

```
(1/3))^2/x^(2/3))^(5/2)*x^(5/3))
```

Maple [A] time = 0.011, size = 199, normalized size = 0.5

$$-\frac{1}{4a^8}\left(14a^6bx^2+420\ln\left(b+a\sqrt[3]{x}\right)x^{4/3}a^4b^3-84a^5b^2x^{5/3}-4x^{7/3}a^7+1680\ln\left(b+a\sqrt[3]{x}\right)xa^3b^4-556x^{4/3}a^4b^3+2520\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x)
```

```
[Out] -1/4/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)/x^(5/3)*(14*a^6*b*x^2+
420*ln(b+a*x^(1/3))*x^(4/3)*a^4*b^3-84*a^5*b^2*x^(5/3)-4*x^(7/3)*a^7+1680*l
n(b+a*x^(1/3))*x*a^3*b^4-556*x^(4/3)*a^4*b^3+2520*ln(b+a*x^(1/3))*x^(2/3)*a
^2*b^5-544*x*a^3*b^4+1680*ln(b+a*x^(1/3))*x^(1/3)*a*b^6+444*x^(2/3)*a^2*b^5
+420*ln(b+a*x^(1/3))*b^7+856*x^(1/3)*a*b^6+319*b^7)*(b+a*x^(1/3))/a^8
```

Maxima [A] time = 1.00687, size = 188, normalized size = 0.46

$$\frac{4a^7x^{\frac{7}{3}}-14a^6bx^2+84a^5b^2x^{\frac{5}{3}}+556a^4b^3x^{\frac{4}{3}}+544a^3b^4x-444a^2b^5x^{\frac{2}{3}}-856ab^6x^{\frac{1}{3}}-319b^7}{4\left(a^{12}x^{\frac{4}{3}}+4a^{11}bx+6a^{10}b^2x^{\frac{2}{3}}+4a^9b^3x^{\frac{1}{3}}+a^8b^4\right)}-\frac{105b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^7*x^(7/3) - 14*a^6*b*x^2 + 84*a^5*b^2*x^(5/3) + 556*a^4*b^3*x^(4/3
) + 544*a^3*b^4*x - 444*a^2*b^5*x^(2/3) - 856*a*b^6*x^(1/3) - 319*b^7)/(a^1
2*x^(4/3) + 4*a^11*b*x + 6*a^10*b^2*x^(2/3) + 4*a^9*b^3*x^(1/3) + a^8*b^4)
- 105*b^3*log(a*x^(1/3) + b)/a^8
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-5/2), x)
```

Giac [A] time = 1.21239, size = 190, normalized size = 0.46

$$-\frac{105\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^8\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}-\frac{420\,a^3b^4x+1134\,a^2b^5x^{\frac{2}{3}}+1036\,ab^6x^{\frac{1}{3}}+319\,b^7}{4\left(ax^{\frac{1}{3}}+b\right)^4a^8\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}+\frac{2\,a^{10}x-15\,a^9bx^{\frac{2}{3}}+90\,a^8b^2x^{\frac{1}{3}}}{2\,a^{15}\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] -105*b^3*log(abs(a*x^(1/3) + b))/(a^8*sgn(a*x^(2/3) + b*x^(1/3))) - 1/4*(42
0*a^3*b^4*x + 1134*a^2*b^5*x^(2/3) + 1036*a*b^6*x^(1/3) + 319*b^7)/((a*x^(1
/3) + b)^4*a^8*sgn(a*x^(2/3) + b*x^(1/3))) + 1/2*(2*a^10*x - 15*a^9*b*x^(2/
3) + 90*a^8*b^2*x^(1/3))/(a^15*sgn(a*x^(2/3) + b*x^(1/3)))
```

3.489 $\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx$

Optimal. Leaf size=289

$$\frac{20a^4bx^{3/4}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{3\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^3b^2\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{40a^2b^3\sqrt[4]{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}-\frac{4b^5\sqrt{a^2+}}{\sqrt[4]{x}\left(a\right.}$$

```
[Out] (-4*b^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)])/((a + b/x^(1/4))*x^(1/4)
) + (40*a^2*b^3*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(1/4))/(a + b/x
^(1/4)) + (20*a^3*b^2*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Sqrt[x])/(a
 + b/x^(1/4)) + (20*a^4*b*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(3/4)
)/(3*(a + b/x^(1/4))) + (a^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x)/(
a + b/x^(1/4)) + (20*a*b^4*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Log[x^
(1/4)])/(a + b/x^(1/4))
```

Rubi [A] time = 0.138081, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{20a^4bx^{3/4}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{3\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^3b^2\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{40a^2b^3\sqrt[4]{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}-\frac{4b^5\sqrt{a^2+}}{\sqrt[4]{x}\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```


```
[Out] (-4*b^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)])/((a + b/x^(1/4))*x^(1/4)
) + (40*a^2*b^3*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(1/4))/(a + b/x
^(1/4)) + (20*a^3*b^2*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Sqrt[x])/(a
 + b/x^(1/4)) + (20*a^4*b*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(3/4)
)/(3*(a + b/x^(1/4))) + (a^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x)/(
a + b/x^(1/4)) + (20*a*b^4*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Log[x^
(1/4)])/(a + b/x^(1/4))
```


Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = 4\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^3\,dx, x, \sqrt[4]{x}\right)$$
$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^3\,dx, x, \sqrt[4]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$
$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^5}{x^2}\,dx, x, \sqrt[4]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$
$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \left(10a^2b^8+\frac{b^{10}}{x^2}+\frac{5ab^9}{x}+10a^3b^7x+5a^4b^6x^2+a^5b^5x^3\right) dx, x\right.}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$
$$= -\frac{4b^6\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}}{\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)\sqrt[4]{x}} + \frac{40a^2b^4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt[4]{x}}{ab+\frac{b^2}{\sqrt[4]{x}}} + \frac{20a^3b^3\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt{x}}{ab+\frac{b^2}{\sqrt[4]{x}}} + \frac{20a^4}{}$$

Mathematica [A] time = 0.052928, size = 98, normalized size = 0.34

$$\frac{\sqrt{\frac{\left(a\sqrt[4]{x}+b\right)^2}{\sqrt{x}}}\left(60a^3b^2x^{3/4}+120a^2b^3\sqrt{x}+20a^4bx+3a^5x^{5/4}+15ab^4\sqrt[4]{x}\log(x)-12b^5\right)}{3\left(a\sqrt[4]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/4))^2/Sqrt[x]]*(-12*b^5 + 120*a^2*b^3*Sqrt[x] + 60*a^3*b^
2*x^(3/4) + 20*a^4*b*x + 3*a^5*x^(5/4) + 15*a*b^4*x^(1/4)*Log[x]))/(3*(b +
a*x^(1/4)))
```

Maple [A] time = 0.032, size = 94, normalized size = 0.3

$$\frac{1}{3}\sqrt{\left(a^2x^{\frac{3}{4}}+2\,ab\sqrt{x}+b^2\sqrt[4]{x}\right)x^{-\frac{3}{4}}}\left(20\,xa^4b+15\,\ln(x)\sqrt[4]{x}ab^4+120\,\sqrt{x}a^2b^3+60\,x^{3/4}a^3b^2+3\,x^{5/4}a^5-12\,b^5\right)\left(a\sqrt[4]{x}+b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x)
```

```
[Out] 1/3*((a^2*x^(3/4)+2*a*b*x^(1/2)+b^2*x^(1/4))/x^(3/4))^(1/2)*(20*x*a^4*b+15*
ln(x)*x^(1/4)*a*b^4+120*x^(1/2)*a^2*b^3+60*x^(3/4)*a^3*b^2+3*x^(5/4)*a^5-12
*b^5)/(a*x^(1/4)+b)
```

Maxima [A] time = 0.97714, size = 77, normalized size = 0.27

$$5\,ab^4\log\left(x\right)+\frac{3\,a^5x^{\frac{5}{4}}+20\,a^4bx+60\,a^3b^2x^{\frac{3}{4}}+120\,a^2b^3\sqrt{x}-12\,b^5}{3\,x^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] 5*a*b^4*log(x) + 1/3*(3*a^5*x^(5/4) + 20*a^4*b*x + 60*a^3*b^2*x^(3/4) + 120
*a^2*b^3*sqrt(x) - 12*b^5)/x^(1/4)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b/x**(1/4)+b**2/x**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23674, size = 170, normalized size = 0.59

$$a^5x\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}\left(x\right)+5\,ab^4\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}\left(x\right)+\frac{20}{3}\,a^4bx^{\frac{3}{4}}\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}\left(x\right)+20\,a^3b^2\sqrt{x}\mathrm{sgn}\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(3/4))*sgn(x) + 5*a*b^4*log(abs(x))*sgn(a*x + b*x^(3/4)
)*sgn(x) + 20/3*a^4*b*x^(3/4)*sgn(a*x + b*x^(3/4))*sgn(x) + 20*a^3*b^2*sqrt
(x)*sgn(a*x + b*x^(3/4))*sgn(x) + 40*a^2*b^3*x^(1/4)*sgn(a*x + b*x^(3/4))*s
gn(x) - 4*b^5*sgn(a*x + b*x^(3/4))*sgn(x)/x^(1/4)
```

3.490 $\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2}\,dx$

Optimal. Leaf size=291

$$\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^4bx^{4/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{4\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{50a^3b^2x^{3/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{3\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{25a^2b^3x^{2/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25ab^4}{}$$

```
[Out] (25*a*b^4*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(1/5))/(a + b/x^(1/5)
) + (25*a^2*b^3*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(2/5))/(a + b/x
^(1/5)) + (50*a^3*b^2*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(3/5))/(3
*(a + b/x^(1/5))) + (25*a^4*b*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(
4/5))/(4*(a + b/x^(1/5))) + (a^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*
x)/(a + b/x^(1/5)) + (5*b^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*Log[x
^(1/5)])/(a + b/x^(1/5))
```

Rubi [A] time = 0.136913, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^4bx^{4/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{4\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{50a^3b^2x^{3/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{3\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{25a^2b^3x^{2/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25ab^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```


```
[Out] (25*a*b^4*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(1/5))/(a + b/x^(1/5)
) + (25*a^2*b^3*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(2/5))/(a + b/x
^(1/5)) + (50*a^3*b^2*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(3/5))/(3
*(a + b/x^(1/5))) + (25*a^4*b*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(
4/5))/(4*(a + b/x^(1/5))) + (a^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*
x)/(a + b/x^(1/5)) + (5*b^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*Log[x
^(1/5)])/(a + b/x^(1/5))
```


Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = 5\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^4\,dx, x, \sqrt[5]{x}\right)$$

$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^4\,dx, x, \sqrt[5]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^5}{x}\,dx, x, \sqrt[5]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \left(5ab^9+\frac{b^{10}}{x}+10a^2b^8x+10a^3b^7x^2+5a^4b^6x^3+a^5b^5x^4\right) \right.}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

$$= \frac{25ab^5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\sqrt[5]{x}}{ab+\frac{b^2}{\sqrt[5]{x}}} + \frac{25a^2b^4\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{2/5}}{ab+\frac{b^2}{\sqrt[5]{x}}} + \frac{50a^3b^3\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{3/5}}{3\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

Mathematica [A] time = 0.0508853, size = 103, normalized size = 0.35

$$\frac{\sqrt{\frac{\left(a\sqrt[5]{x}+b\right)^2}{x^{2/5}}}\left(200a^3b^2x^{4/5}+300a^2b^3x^{3/5}+75a^4bx+12a^5x^{6/5}+300ab^4x^{2/5}+12b^5\sqrt[5]{x}\log(x)\right)}{12\left(a\sqrt[5]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/5))^2/x^(2/5)]*(300*a*b^4*x^(2/5) + 300*a^2*b^3*x^(3/5) +
 200*a^3*b^2*x^(4/5) + 75*a^4*b*x + 12*a^5*x^(6/5) + 12*b^5*x^(1/5)*Log[x])
)/(12*(b + a*x^(1/5)))
```

Maple [A] time = 0.02, size = 91, normalized size = 0.3

$$\frac{x}{12}\left(\left(a^2x^{\frac{2}{5}}+2\,ab\sqrt[5]{x}+b^2\right)x^{-\frac{2}{5}}\right)^{\frac{5}{2}}\left(75\,a^4bx^{4/5}+200\,a^3b^2x^{3/5}+300\,a^2b^3x^{2/5}+300\,ab^4\sqrt[5]{x}+12\,b^5\ln(x)+12\,a^5x\right)\left(a\sqrt[5]{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x)
```

```
[Out] 1/12*((a^2*x^(2/5)+2*a*b*x^(1/5)+b^2)/x^(2/5))^(5/2)*x*(75*a^4*b*x^(4/5)+20
0*a^3*b^2*x^(3/5)+300*a^2*b^3*x^(2/5)+300*a*b^4*x^(1/5)+12*b^5*ln(x)+12*a^5
*x)/(a*x^(1/5)+b)^5
```

Maxima [A] time = 0.989778, size = 70, normalized size = 0.24

$$a^5x + b^5 \log(x) + \frac{25}{4} a^4 b x^{\frac{4}{5}} + \frac{50}{3} a^3 b^2 x^{\frac{3}{5}} + 25 a^2 b^3 x^{\frac{2}{5}} + 25 a b^4 x^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="maxima")
```

```
[Out] a^5*x + b^5*log(x) + 25/4*a^4*b*x^(4/5) + 50/3*a^3*b^2*x^(3/5) + 25*a^2*b^3
*x^(2/5) + 25*a*b^4*x^(1/5)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/5)+2*a*b/x**(1/5))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18118, size = 169, normalized size = 0.58

$$a^5 x \operatorname{sgn}\left(ax + bx^{\frac{4}{5}}\right) \operatorname{sgn}(x) + b^5 \log(|x|) \operatorname{sgn}\left(ax + bx^{\frac{4}{5}}\right) \operatorname{sgn}(x) + \frac{25}{4} a^4 b x^{\frac{4}{5}} \operatorname{sgn}\left(ax + bx^{\frac{4}{5}}\right) \operatorname{sgn}(x) + \frac{50}{3} a^3 b^2 x^{\frac{3}{5}} \operatorname{sgn}\left(ax + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(4/5))*sgn(x) + b^5*log(abs(x))*sgn(a*x + b*x^(4/5))*sg
n(x) + 25/4*a^4*b*x^(4/5)*sgn(a*x + b*x^(4/5))*sgn(x) + 50/3*a^3*b^2*x^(3/5
)*sgn(a*x + b*x^(4/5))*sgn(x) + 25*a^2*b^3*x^(2/5)*sgn(a*x + b*x^(4/5))*sgn
(x) + 25*a*b^4*x^(1/5)*sgn(a*x + b*x^(4/5))*sgn(x)
```

3.491 $\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x}}$$

```
[Out] (20*a)/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (5*a^4)/(4*b^5*(a +
b*x^(1/5))^3*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (20*a^3)/(3*b^5*(a
+ b*x^(1/5))^2*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (15*a^2)/(b^5*(a
+ b*x^(1/5))*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (5*(a + b*x^(1/5))*
Log[a + b*x^(1/5)])/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)])
```

Rubi [A] time = 0.125948, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] (20*a)/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (5*a^4)/(4*b^5*(a +
b*x^(1/5))^3*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (20*a^3)/(3*b^5*(a
+ b*x^(1/5))^2*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (15*a^2)/(b^5*(a
+ b*x^(1/5))*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (5*(a + b*x^(1/5))*
Log[a + b*x^(1/5)])/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)])
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = 5\,\text{Subst}\left(\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, \sqrt[5]{x}\right)$$
$$= \frac{\left(5b^5\left(a+b\sqrt[5]{x}\right)\right)\text{Subst}\left(\int \frac{x^4}{\left(ab+b^2x\right)^5}\,dx, x, \sqrt[5]{x}\right)}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$
$$= \frac{\left(5b^5\left(a+b\sqrt[5]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^4}{b^9(a+bx)^5}-\frac{4a^3}{b^9(a+bx)^4}+\frac{6a^2}{b^9(a+bx)^3}-\frac{4a}{b^9(a+bx)^2}+\frac{1}{b^9(a+bx)}\right)dx, x,}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$
$$= \frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{}{3b^5\left(a+b\sqrt[5]{x}\right)^2}$$

Mathematica [A] time = 0.0966441, size = 98, normalized size = 0.44

$$\frac{5a\left(88a^2b\sqrt[5]{x}+25a^3+108ab^2x^{2/5}+48b^3x^{3/5}\right)+60\left(a+b\sqrt[5]{x}\right)^4\log\left(a+b\sqrt[5]{x}\right)}{12b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{\left(a+b\sqrt[5]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] (5*a*(25*a^3 + 88*a^2*b*x^(1/5) + 108*a*b^2*x^(2/5) + 48*b^3*x^(3/5)) + 60*
(a + b*x^(1/5))^4*Log[a + b*x^(1/5)])/(12*b^5*(a + b*x^(1/5))^3*Sqrt[(a + b
*x^(1/5))^2])
```

Maple [A] time = 0.011, size = 152, normalized size = 0.7

$$\frac{5}{12\,b^5}\sqrt{a^2+2\,ab\sqrt[5]{x}+b^2x^{\frac{2}{5}}}\left(12\,x^{4/5}\ln\left(a+b\sqrt[5]{x}\right)b^4+48\,x^{3/5}\ln\left(a+b\sqrt[5]{x}\right)ab^3+48\,x^{3/5}ab^3+72\,x^{2/5}\ln\left(a+b\sqrt[5]{x}\right)a^2b^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x)
```

```
[Out] 5/12*(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)*(12*x^(4/5)*ln(a+b*x^(1/5))*b^4+
48*x^(3/5)*ln(a+b*x^(1/5))*a*b^3+48*x^(3/5)*a*b^3+72*x^(2/5)*ln(a+b*x^(1/5)
)*a^2*b^2+108*x^(2/5)*a^2*b^2+48*x^(1/5)*ln(a+b*x^(1/5))*a^3*b+88*x^(1/5)*a
^3*b+12*ln(a+b*x^(1/5))*a^4+25*a^4)/(a+b*x^(1/5))^5/b^5
```

Maxima [A] time = 1.01338, size = 134, normalized size = 0.6

$$\frac{5\left(48\,ab^3x^{\frac{3}{5}}+108\,a^2b^2x^{\frac{2}{5}}+88\,a^3bx^{\frac{1}{5}}+25\,a^4\right)}{12\left(b^9x^{\frac{4}{5}}+4\,ab^8x^{\frac{3}{5}}+6\,a^2b^7x^{\frac{2}{5}}+4\,a^3b^6x^{\frac{1}{5}}+a^4b^5\right)}+\frac{5\,\log\left(bx^{\frac{1}{5}}+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="maxima")
```

```
[Out] 5/12*(48*a*b^3*x^(3/5) + 108*a^2*b^2*x^(2/5) + 88*a^3*b*x^(1/5) + 25*a^4)/(
b^9*x^(4/5) + 4*a*b^8*x^(3/5) + 6*a^2*b^7*x^(2/5) + 4*a^3*b^6*x^(1/5) + a^4
*b^5) + 5*log(b*x^(1/5) + a)/b^5
```

Fricas [A] time = 2.19283, size = 703, normalized size = 3.17

$$\frac{5\left(300\,a^5b^{15}x^3 + 100\,a^{15}b^5x + 25\,a^{20} + 12\left(b^{20}x^4 + 4\,a^5b^{15}x^3 + 6\,a^{10}b^{10}x^2 + 4\,a^{15}b^5x + a^{20}\right)\log\left(bx^{\frac{1}{5}} + a\right) + \left(48\,ab^{19}x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="fricas")
```

```
[Out] 5/12*(300*a^5*b^15*x^3 + 100*a^15*b^5*x + 25*a^20 + 12*(b^20*x^4 + 4*a^5*b^
15*x^3 + 6*a^10*b^10*x^2 + 4*a^15*b^5*x + a^20)*log(b*x^(1/5) + a) + (48*a*
b^19*x^3 - 226*a^6*b^14*x^2 + 104*a^11*b^9*x + 3*a^16*b^4)*x^(4/5) - (84*a^
2*b^18*x^3 - 228*a^7*b^13*x^2 + 67*a^12*b^8*x + 4*a^17*b^3)*x^(3/5) + (136*
a^3*b^17*x^3 - 197*a^8*b^12*x^2 + 48*a^13*b^7*x + 6*a^18*b^2)*x^(2/5) - (20
7*a^4*b^16*x^3 - 124*a^9*b^11*x^2 + 56*a^14*b^6*x + 12*a^19*b)*x^(1/5))/(b^
25*x^4 + 4*a^5*b^20*x^3 + 6*a^10*b^15*x^2 + 4*a^15*b^10*x + a^20*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[5]{x} + b^2x^{\frac{2}{5}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/5)+b**2*x**(2/5))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/5) + b**2*x**(2/5))**(-5/2), x)
```

Giac [A] time = 1.13323, size = 113, normalized size = 0.51

$$\frac{5\,\log\left(\left|bx^{\frac{1}{5}} + a\right|\right)}{b^5\mathrm{sgn}\left(bx^{\frac{1}{5}} + a\right)} + \frac{5\left(48\,ab^2x^{\frac{3}{5}} + 108\,a^2bx^{\frac{2}{5}} + 88\,a^3x^{\frac{1}{5}} + \frac{25\,a^4}{b}\right)}{12\left(bx^{\frac{1}{5}} + a\right)^4 b^4\mathrm{sgn}\left(bx^{\frac{1}{5}} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="giac")
```

```
[Out] 5*log(abs(b*x^(1/5) + a))/(b^5*sgn(b*x^(1/5) + a)) + 5/12*(48*a*b^2*x^(3/5)
 + 108*a^2*b*x^(2/5) + 88*a^3*x^(1/5) + 25*a^4/b)/((b*x^(1/5) + a)^4*b^4*sg
n(b*x^(1/5) + a))
```

3.492 $\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2}\,dx$

Optimal. Leaf size=391

$$\frac{42a^6bx^{5/6}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{5\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{63a^5b^2x^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{70a^4b^3\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{105a^3b^4}{}$$

```
[Out] (-6*b^7*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)])/((a + b/x^(1/6))*x^(1/6)
) + (126*a^2*b^5*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/6))/(a + b/
x^(1/6)) + (105*a^3*b^4*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/3))/
(a + b/x^(1/6)) + (70*a^4*b^3*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*Sqr
t[x])/(a + b/x^(1/6)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6
)]*x^(2/3))/(2*(a + b/x^(1/6))) + (42*a^6*b*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b
)/x^(1/6)]*x^(5/6))/(5*(a + b/x^(1/6))) + (a^7*Sqrt[a^2 + b^2/x^(1/3) + (2*
a*b)/x^(1/6)]*x)/(a + b/x^(1/6)) + (42*a*b^6*Sqrt[a^2 + b^2/x^(1/3) + (2*a*
b)/x^(1/6)]*Log[x^(1/6)])/(a + b/x^(1/6))
```

Rubi [A] time = 0.179674, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{42a^6bx^{5/6}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{5\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{63a^5b^2x^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{70a^4b^3\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{105a^3b^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```


```
[Out] (-6*b^7*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)])/((a + b/x^(1/6))*x^(1/6)
) + (126*a^2*b^5*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/6))/(a + b/
x^(1/6)) + (105*a^3*b^4*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/3))/
(a + b/x^(1/6)) + (70*a^4*b^3*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*Sqr
t[x])/(a + b/x^(1/6)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6
)]*x^(2/3))/(2*(a + b/x^(1/6))) + (42*a^6*b*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b
)/x^(1/6)]*x^(5/6))/(5*(a + b/x^(1/6))) + (a^7*Sqrt[a^2 + b^2/x^(1/3) + (2*
a*b)/x^(1/6)]*x)/(a + b/x^(1/6)) + (42*a*b^6*Sqrt[a^2 + b^2/x^(1/3) + (2*a*
b)/x^(1/6)]*Log[x^(1/6)])/(a + b/x^(1/6))
```


Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```

```
[p - 1/2]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = 6\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{7/2} x^5\,dx, x, \sqrt[6]{x}\right)$$

$$= \frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^7 x^5\,dx, x, \sqrt[6]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$= \frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^7}{x^2}\,dx, x, \sqrt[6]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$= \frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int \left(21a^2b^{12}+\frac{b^{14}}{x^2}+\frac{7ab^{13}}{x}+35a^3b^{11}x+35a^4b^{10}x^2+21a^5b^9x\right.\right.}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$= -\frac{6b^8\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}}{\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)\sqrt[6]{x}} + \frac{126a^2b^6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[6]{x}}{ab+\frac{b^2}{\sqrt[6]{x}}} + \frac{105a^3b^5\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[3]{x}}{ab+\frac{b^2}{\sqrt[6]{x}}} + 7$$

Mathematica [A] time = 0.0701364, size = 124, normalized size = 0.32

$$\frac{\sqrt{\frac{\left(a\sqrt[6]{x}+b\right)^2}{\sqrt[3]{x}}}\left(315a^5b^2x^{5/6}+700a^4b^3x^{2/3}+1050a^3b^4\sqrt{x}+1260a^2b^5\sqrt[3]{x}+84a^6bx+10a^7x^{7/6}+70ab^6\sqrt[6]{x}\log(x)-60b^7\right)}{10\left(a\sqrt[6]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/6))^2/x^(1/3)]*(-60*b^7 + 1260*a^2*b^5*x^(1/3) + 1050*a^3
*b^4*Sqrt[x] + 700*a^4*b^3*x^(2/3) + 315*a^5*b^2*x^(5/6) + 84*a^6*b*x + 10*
a^7*x^(7/6) + 70*a*b^6*x^(1/6)*Log[x]))/(10*(b + a*x^(1/6)))
```

Maple [A] time = 0.024, size = 116, normalized size = 0.3

$$\frac{1}{10}\sqrt{\left(\sqrt{x}a^2+2\,ab\sqrt[3]{x}+b^2\sqrt[6]{x}\right)\frac{1}{\sqrt{x}}}\left(84\,a^6bx+315\,a^5b^2x^{5/6}+70\,ab^6\ln(x)\sqrt[6]{x}+1050\,a^3b^4\sqrt{x}+1260\,a^2b^5\sqrt[3]{x}+700\,a^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x)
```

```
[Out] 1/10*((x^(1/2)*a^2+2*a*b*x^(1/3)+b^2*x^(1/6))/x^(1/2))^(1/2)*(84*a^6*b*x+31
5*a^5*b^2*x^(5/6)+70*a*b^6*ln(x)*x^(1/6)+1050*a^3*b^4*x^(1/2)+1260*a^2*b^5*
x^(1/3)+700*a^4*b^3*x^(2/3)+10*a^7*x^(7/6)-60*b^7)/(a*x^(1/6)+b)
```

Maxima [A] time = 0.993768, size = 107, normalized size = 0.27

$$7\,ab^6\log\left(x\right)+\frac{10\,a^7x^{\frac{7}{6}}+84\,a^6bx+315\,a^5b^2x^{\frac{5}{6}}+700\,a^4b^3x^{\frac{2}{3}}+1050\,a^3b^4\sqrt{x}+1260\,a^2b^5x^{\frac{1}{3}}-60\,b^7}{10\,x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="maxima")
```

```
[Out] 7*a*b^6*log(x) + 1/10*(10*a^7*x^(7/6) + 84*a^6*b*x + 315*a^5*b^2*x^(5/6) +
700*a^4*b^3*x^(2/3) + 1050*a^3*b^4*sqrt(x) + 1260*a^2*b^5*x^(1/3) - 60*b^7)
/x^(1/6)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(1/3)+2*a*b/x**(1/6))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36387, size = 232, normalized size = 0.59

$$a^7x\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}\left(x\right)+7\,ab^6\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}\left(x\right)+\frac{42}{5}\,a^6bx^{\frac{5}{6}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}\left(x\right)+\frac{63}{2}\,a^5b^2x^{\frac{2}{3}}\mathrm{sgn}\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="giac")
```

```
[Out] a^7*x*sgn(a*x + b*x^(5/6))*sgn(x) + 7*a*b^6*log(abs(x))*sgn(a*x + b*x^(5/6)
)*sgn(x) + 42/5*a^6*b*x^(5/6)*sgn(a*x + b*x^(5/6))*sgn(x) + 63/2*a^5*b^2*x^
(2/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 70*a^4*b^3*sqrt(x)*sgn(a*x + b*x^(5/6))
*sgn(x) + 105*a^3*b^4*x^(1/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 126*a^2*b^5*x^(
1/6)*sgn(a*x + b*x^(5/6))*sgn(x) - 6*b^7*sgn(a*x + b*x^(5/6))*sgn(x)/x^(1/6
)
```

3.493 $\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=46

$$\frac{b^2\log(b+cx^n)}{c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b^2*Log[b + c*x^n])/(c^3*n)
```

Rubi [A] time = 0.036353, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1584, 266, 43}

$$\frac{b^2\log(b+cx^n)}{c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b^2*Log[b + c*x^n])/(c^3*n)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1+3n}}{b+cx^n}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{b+cx}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{b^2}{c^2(b+cx)}\right)dx,x,x^n\right)}{n}\\
&= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b^2\log(b+cx^n)}{c^3n}
\end{aligned}$$

Mathematica [A] time = 0.0288737, size = 38, normalized size = 0.83

$$\frac{2b^2\log(b+cx^n)+cx^n(cx^n-2b)}{2c^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (c*x^n*(-2*b + c*x^n) + 2*b^2*Log[b + c*x^n])/(2*c^3*n)
```

Maple [A] time = 0.023, size = 62, normalized size = 1.4

$$\frac{1}{e^{n\ln(x)}}\left(\frac{\left(e^{n\ln(x)}\right)^3}{2\,cn}-\frac{b\left(e^{n\ln(x)}\right)^2}{c^2n}\right)+\frac{b^2\ln\left(ce^{n\ln(x)}+b\right)}{c^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (1/2/c/n*exp(n*ln(x))^3-b/c^2/n*exp(n*ln(x))^2)/exp(n*ln(x))+b^2/c^3/n*ln(c
*exp(n*ln(x))+b)
```

Maxima [A] time = 0.975811, size = 61, normalized size = 1.33

$$\frac{b^2\log\left(\frac{cx^n+b}{c}\right)}{c^3n}+\frac{cx^{2\,n}-2\,bx^n}{2\,c^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] b^2*log((c*x^n + b)/c)/(c^3*n) + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n)
```

Fricas [A] time = 1.93044, size = 84, normalized size = 1.83

$$\frac{c^2x^{2\,n}-2\,bcx^n+2\,b^2\log(cx^n+b)}{2\,c^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*x^(2*n) - 2*b*c*x^n + 2*b^2*log(c*x^n + b))/(c^3*n)
```

Sympy [A] time = 31.2943, size = 42, normalized size = 0.91

$$\frac{b^2\left(\begin{cases}\frac{x^n}{b} & \text{for } c=0\\ \frac{\log(b+cx^n)}{c} & \text{otherwise}\end{cases}\right)}{c^2n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] b**2*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(c**2*n) - b*
x**n/(c**2*n) + x**(2*n)/(2*c*n)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4n-1}}{cx^{2n} + bx^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.494 $\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=28

$$\frac{x^n}{cn}-\frac{b\log(b+cx^n)}{c^2n}$$

```
[Out] x^n/(c*n) - (b*Log[b + c*x^n])/(c^2*n)
```

Rubi [A] time = 0.0258639, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1584, 266, 43}

$$\frac{x^n}{cn}-\frac{b\log(b+cx^n)}{c^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]

[Out] x^n/(c*n) - (b*Log[b + c*x^n])/(c^2*n)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx&=\int \frac{x^{-1+2n}}{b+cx^n}\,dx\\&=\frac{\text{Subst}\left(\int \frac{x}{b+cx}\,dx,x,x^n\right)}{n}\\&=\frac{\text{Subst}\left(\int \left(\frac{1}{c}-\frac{b}{c(b+cx)}\right)dx,x,x^n\right)}{n}\\&=\frac{x^n}{cn}-\frac{b\log(b+cx^n)}{c^2n}\end{aligned}$$

Mathematica [A] time = 0.0141799, size = 26, normalized size = 0.93

$$\frac{\frac{x^n}{c}-\frac{b\log(b+cx^n)}{c^2}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (x^n/c - (b*Log[b + c*x^n])/c^2)/n
```

Maple [A] time = 0.02, size = 33, normalized size = 1.2

$$\frac{e^{n\ln(x)}}{cn}-\frac{b\ln\left(c e^{n\ln(x)}+b\right)}{c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/c/n*exp(n*ln(x))-b/c^2/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 0.980079, size = 43, normalized size = 1.54

$$\frac{x^n}{cn}-\frac{b\log\left(\frac{cx^n+b}{c}\right)}{c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] x^n/(c*n) - b*log((c*x^n + b)/c)/(c^2*n)
```

Fricas [A] time = 1.9112, size = 49, normalized size = 1.75

$$\frac{cx^n-b\log\left(cx^n+b\right)}{c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] (c*x^n - b*log(c*x^n + b))/(c^2*n)
```

Sympy [A] time = 81.3623, size = 26, normalized size = 0.93

$$-\frac{b\left(\begin{cases}\frac{x^n}{b} & \text{for } c=0\\ \frac{\log(b+cx^n)}{c} & \text{otherwise}\end{cases}\right)}{cn}+\frac{x^n}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -b*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(c*n) + x**n/(c
*n)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3n-1}}{cx^{2n} + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.495 $\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{cn}$$

```
[Out] Log[b + c*x^n]/(c*n)
```

Rubi [A] time = 0.0124989, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1584, 260}

$$\frac{\log(b+cx^n)}{cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[b + c*x^n]/(c*n)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1+n}}{b+cx^n}\,dx\\ &= \frac{\log(b+cx^n)}{cn}\end{aligned}$$

Mathematica [A] time = 0.0037878, size = 15, normalized size = 1.

$$\frac{\log(b+cx^n)}{cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[b + c*x^n]/(c*n)
```

Maple [A] time = 0.017, size = 18, normalized size = 1.2

$$\frac{\ln\left(c\mathrm{e}^{n\ln(x)}+b\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/c/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 0.987509, size = 26, normalized size = 1.73

$$\frac{\log\left(\frac{cx^n+b}{c}\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log((c*x^n + b)/c)/(c*n)
```

Fricas [A] time = 1.90315, size = 30, normalized size = 2.

$$\frac{\log\left(cx^n+b\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] log(c*x^n + b)/(c*n)
```

Sympy [A] time = 14.0103, size = 37, normalized size = 2.47

$$\begin{cases} \frac{\log(x)}{b} & \text{for}\: c = 0 \wedge n = 0 \\ \frac{\log(x)}{b+c} & \text{for}\: n = 0 \\ \frac{x^n}{bn} & \text{for}\: c = 0 \\ -\frac{\log(x)}{c} + \frac{\log\left(\frac{bx^n}{c}+x^{2n}\right)}{cn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (log(x)/(b + c), Eq(n, 0)), (x**
n/(b*n), Eq(c, 0)), (-log(x)/c + log(b*x**n/c + x**(2*n))/(c*n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{cx^{2n} + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.496 $\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{b}-\frac{\log(b+cx^n)}{bn}$$

```
[Out] Log[x]/b - Log[b + c*x^n]/(b*n)
```

Rubi [A] time = 0.0185387, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1584, 266, 36, 29, 31}

$$\frac{\log(x)}{b}-\frac{\log(b+cx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]

[Out] Log[x]/b - Log[b + c*x^n]/(b*n)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x(b + cx^n)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(b+cx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{bn} - \frac{c\,\text{Subst}\left(\int \frac{1}{b+cx}\,dx, x, x^n\right)}{bn}$$
$$= \frac{\log(x)}{b} - \frac{\log(b + cx^n)}{bn}$$

Mathematica [A] time = 0.006655, size = 22, normalized size = 0.96

$$\frac{n\log(x) - \log(b + cx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (n*Log[x] - Log[b + c*x^n])/(b*n)
```

Maple [A] time = 0.02, size = 26, normalized size = 1.1

$$\frac{\ln(x)}{b} - \frac{\ln\left(ce^{n\ln(x)} + b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] ln(x)/b-1/b/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 0.989826, size = 36, normalized size = 1.57

$$\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log(x)/b - log((c*x^n + b)/c)/(b*n)
```

Fricas [A] time = 1.85103, size = 47, normalized size = 2.04

$$\frac{n\log(x) - \log(cx^n + b)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] (n*log(x) - log(c*x^n + b))/(b*n)
```

Sympy [A] time = 14.6952, size = 42, normalized size = 1.83

$$\begin{cases} \frac{\log(x)}{c} & \text{for}\ b = 0 \wedge n = 0 \\ \frac{\log(x)}{b+c} & \text{for}\ n = 0 \\ -\frac{x^{-n}}{cn} & \text{for}\ b = 0 \\ \frac{2\log(x)}{b} - \frac{\log\left(x^n + \frac{cx^{2n}}{b}\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((log(x)/c, Eq(b, 0) & Eq(n, 0)), (log(x)/(b + c), Eq(n, 0)), (-x*
*(-n)/(c*n), Eq(b, 0)), (2*log(x)/b - log(x**n + c*x**(2*n)/b)/(b*n), True)
)
```

Giac [A] time = 1.10034, size = 34, normalized size = 1.48

$$\frac{\log(|x|)}{b} - \frac{\log(|cx^n + b|)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] log(abs(x))/b - log(abs(c*x^n + b))/(b*n)
```

3.497 $\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=57

$$-\frac{c^2\log(b+cx^n)}{b^3n}+\frac{c^2\log(x)}{b^3}+\frac{cx^{-n}}{b^2n}-\frac{x^{-2n}}{2bn}$$

```
[Out] -1/(2*b*n*x^(2*n)) + c/(b^2*n*x^n) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^n]
)/(b^3*n)
```

Rubi [A] time = 0.0421494, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1584, 266, 44}

$$-\frac{c^2\log(b+cx^n)}{b^3n}+\frac{c^2\log(x)}{b^3}+\frac{cx^{-n}}{b^2n}-\frac{x^{-2n}}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(2*b*n*x^(2*n)) + c/(b^2*n*x^n) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^n]
)/(b^3*n)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-2n}}{b+cx^n}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^3(b+cx)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^3}-\frac{c}{b^2x^2}+\frac{c^2}{b^3x}-\frac{c^3}{b^3(b+cx)}\right)dx,x,x^n\right)}{n}\\
&= -\frac{x^{-2n}}{2bn}+\frac{cx^{-n}}{b^2n}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log(b+cx^n)}{b^3n}
\end{aligned}$$

Mathematica [A] time = 0.0652911, size = 49, normalized size = 0.86

$$\frac{-2c^2\log(b+cx^n)+bx^{-2n}(2cx^n-b)+2c^2n\log(x)}{2b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] ((b*(-b + 2*c*x^n))/x^(2*n) + 2*c^2*n*Log[x] - 2*c^2*Log[b + c*x^n])/(2*b^3
*n)
```

Maple [A] time = 0.022, size = 69, normalized size = 1.2

$$\frac{1}{\left(e^{n\ln(x)}\right)^2}\left(\frac{ce^{n\ln(x)}}{b^2n}-\frac{1}{2\,bn}+\frac{c^2\ln(x)\left(e^{n\ln(x)}\right)^2}{b^3}\right)-\frac{c^2\ln\left(ce^{n\ln(x)}+b\right)}{b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (c/b^2/n*exp(n*ln(x))-1/2/b/n+c^2/b^3*ln(x)*exp(n*ln(x))^2)/exp(n*ln(x))^2-
c^2/b^3/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 0.962629, size = 78, normalized size = 1.37

$$\frac{c^2\log(x)}{b^3}-\frac{c^2\log\left(\frac{cx^n+b}{c}\right)}{b^3n}+\frac{2\,cx^n-b}{2\,b^2nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*log(x)/b^3 - c^2*log((c*x^n + b)/c)/(b^3*n) + 1/2*(2*c*x^n - b)/(b^2*n*
x^(2*n))
```

Fricas [A] time = 1.76319, size = 128, normalized size = 2.25

$$\frac{2\,c^2nx^{2\,n}\log(x)-2\,c^2x^{2\,n}\log(cx^n+b)+2\,bcx^n-b^2}{2\,b^3nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*c^2*n*x^(2*n)*log(x) - 2*c^2*x^(2*n)*log(c*x^n + b) + 2*b*c*x^n - b^
2)/(b^3*n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-n-1}}{cx^{2n} + bx^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.498 $\int \frac{x^{-1-2n}}{bx^n+cx^{2n}} \, dx$

Optimal. Leaf size=76

$$-\frac{c^2x^{-n}}{b^3n}+\frac{c^3\log\left(b+cx^n\right)}{b^4n}-\frac{c^3\log(x)}{b^4}+\frac{cx^{-2n}}{2b^2n}-\frac{x^{-3n}}{3bn}$$

```
[Out] -1/(3*b*n*x^(3*n)) + c/(2*b^2*n*x^(2*n)) - c^2/(b^3*n*x^n) - (c^3*Log[x])/b
^4 + (c^3*Log[b + c*x^n])/(b^4*n)
```

Rubi [A] time = 0.0471033, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1584, 266, 44}

$$-\frac{c^2x^{-n}}{b^3n}+\frac{c^3\log\left(b+cx^n\right)}{b^4n}-\frac{c^3\log(x)}{b^4}+\frac{cx^{-2n}}{2b^2n}-\frac{x^{-3n}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(3*b*n*x^(3*n)) + c/(2*b^2*n*x^(2*n)) - c^2/(b^3*n*x^n) - (c^3*Log[x])/b
^4 + (c^3*Log[b + c*x^n])/(b^4*n)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-2n}}{bx^n+cx^{2n}} \, dx &= \int \frac{x^{-1-3n}}{b+cx^n} \, dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^4(b+cx)} \, dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^4}-\frac{c}{b^2x^3}+\frac{c^2}{b^3x^2}-\frac{c^3}{b^4x}+\frac{c^4}{b^4(b+cx)}\right) \, dx, x, x^n\right)}{n}\\
&= -\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(b+cx^n\right)}{b^4n}
\end{aligned}$$

Mathematica [A] time = 0.0792356, size = 62, normalized size = 0.82

$$-\frac{bx^{-3n}\left(2b^2-3bcx^n+6c^2x^{2n}\right)-6c^3\log\left(b+cx^n\right)+6c^3n\log(x)}{6b^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*(2*b^2 - 3*b*c*x^n + 6*c^2*x^(2*n)))/x^(3*n) + 6*c^3*n*Log[x] - 6*c^3*
Log[b + c*x^n])/(6*b^4*n)
```

Maple [A] time = 0.025, size = 88, normalized size = 1.2

$$\frac{1}{\left(e^{n\ln(x)}\right)^3}\left(-\frac{1}{3\,bn}+\frac{ce^{n\ln(x)}}{2\,b^2n}-\frac{c^2\left(e^{n\ln(x)}\right)^2}{b^3n}-\frac{c^3\ln(x)\left(e^{n\ln(x)}\right)^3}{b^4}\right)+\frac{c^3\ln\left(ce^{n\ln(x)}+b\right)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (-1/3/b/n+1/2*c/b^2/n*exp(n*ln(x))-c^2/b^3/n*exp(n*ln(x))^2-c^3/b^4*ln(x)*e
xp(n*ln(x))^3)/exp(n*ln(x))^3+c^3/b^4/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 1.0671, size = 96, normalized size = 1.26

$$-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(\frac{cx^n+b}{c}\right)}{b^4n}-\frac{6\,c^2x^{2\,n}-3\,bcx^n+2\,b^2}{6\,b^3nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c^3*log(x)/b^4 + c^3*log((c*x^n + b)/c)/(b^4*n) - 1/6*(6*c^2*x^(2*n) - 3*b
*c*x^n + 2*b^2)/(b^3*n*x^(3*n))
```

Fricas [A] time = 1.57293, size = 159, normalized size = 2.09

$$-\frac{6\,c^3nx^{3\,n}\log(x)-6\,c^3x^{3\,n}\log\left(cx^n+b\right)+6\,bc^2x^{2\,n}-3\,b^2cx^n+2\,b^3}{6\,b^4nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/6*(6*c^3*n*x^(3*n)*log(x) - 6*c^3*x^(3*n)*log(c*x^n + b) + 6*b*c^2*x^(2*
n) - 3*b^2*c*x^n + 2*b^3)/(b^4*n*x^(3*n))
```

Sympy [A] time = 144.718, size = 73, normalized size = 0.96

$$-\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}+\frac{c^4\left(\begin{cases}\frac{x^n}{b} & \text{for } c=0\\ \frac{\log(b+cx^n)}{c} & \text{otherwise}\end{cases}\right)}{b^4n}-\frac{c^3\log(x^n)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -x**(-3*n)/(3*b*n) + c*x**(-2*n)/(2*b**2*n) - c**2*x**(-n)/(b**3*n) + c**4*
Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(b**4*n) - c**3*lo
g(x**n)/(b**4*n)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-2n-1}}{cx^{2n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.499 $\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=93

$$-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}-\frac{c^4\log(b+cx^n)}{b^5n}+\frac{c^4\log(x)}{b^5}+\frac{cx^{-3n}}{3b^2n}-\frac{x^{-4n}}{4bn}$$

```
[Out] -1/(4*b*n*x^(4*n)) + c/(3*b^2*n*x^(3*n)) - c^2/(2*b^3*n*x^(2*n)) + c^3/(b^4
*n*x^n) + (c^4*Log[x])/b^5 - (c^4*Log[b + c*x^n])/(b^5*n)
```

Rubi [A] time = 0.0538467, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1584, 266, 44}

$$-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}-\frac{c^4\log(b+cx^n)}{b^5n}+\frac{c^4\log(x)}{b^5}+\frac{cx^{-3n}}{3b^2n}-\frac{x^{-4n}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(4*b*n*x^(4*n)) + c/(3*b^2*n*x^(3*n)) - c^2/(2*b^3*n*x^(2*n)) + c^3/(b^4
*n*x^n) + (c^4*Log[x])/b^5 - (c^4*Log[b + c*x^n])/(b^5*n)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-4n}}{b+cx^n}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^5(b+cx)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^5}-\frac{c}{b^2x^4}+\frac{c^2}{b^3x^3}-\frac{c^3}{b^4x^2}+\frac{c^4}{b^5x}-\frac{c^5}{b^5(b+cx)}\right)dx,x,x^n\right)}{n}\\
&= -\frac{x^{-4n}}{4bn}+\frac{cx^{-3n}}{3b^2n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}+\frac{c^4\log(x)}{b^5}-\frac{c^4\log(b+cx^n)}{b^5n}
\end{aligned}$$

Mathematica [A] time = 0.106363, size = 75, normalized size = 0.81

$$-\frac{bx^{-4n}\left(-4b^2cx^n+3b^3+6bc^2x^{2n}-12c^3x^{3n}\right)+12c^4\log\left(b+cx^n\right)-12c^4n\log(x)}{12b^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*(3*b^3 - 4*b^2*c*x^n + 6*b*c^2*x^(2*n) - 12*c^3*x^(3*n)))/x^(4*n) - 12
*c^4*n*Log[x] + 12*c^4*Log[b + c*x^n])/(12*b^5*n)
```

Maple [A] time = 0.026, size = 105, normalized size = 1.1

$$\frac{1}{\left(e^{n\ln(x)}\right)^4}\left(\frac{c^3\left(e^{n\ln(x)}\right)^3}{b^4n}-\frac{1}{4\,bn}+\frac{ce^{n\ln(x)}}{3\,b^2n}-\frac{c^2\left(e^{n\ln(x)}\right)^2}{2\,b^3n}+\frac{c^4\ln(x)\left(e^{n\ln(x)}\right)^4}{b^5}\right)-\frac{c^4\ln\left(ce^{n\ln(x)}+b\right)}{b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (c^3/b^4/n*exp(n*ln(x))^3-1/4/b/n+1/3*c/b^2/n*exp(n*ln(x))-1/2*c^2/b^3/n*ex
p(n*ln(x))^2+c^4/b^5*ln(x)*exp(n*ln(x))^4)/exp(n*ln(x))^4-c^4/b^5/n*ln(c*ex
p(n*ln(x))+b)
```

Maxima [A] time = 1.00449, size = 113, normalized size = 1.22

$$\frac{c^4\log(x)}{b^5}-\frac{c^4\log\left(\frac{cx^n+b}{c}\right)}{b^5n}+\frac{12\,c^3x^{3\,n}-6\,bc^2x^{2\,n}+4\,b^2cx^n-3\,b^3}{12\,b^4nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^4*log(x)/b^5 - c^4*log((c*x^n + b)/c)/(b^5*n) + 1/12*(12*c^3*x^(3*n) - 6*
b*c^2*x^(2*n) + 4*b^2*c*x^n - 3*b^3)/(b^4*n*x^(4*n))
```

Fricas [A] time = 1.56316, size = 190, normalized size = 2.04

$$\frac{12\,c^4nx^{4\,n}\log(x)-12\,c^4x^{4\,n}\log\left(cx^n+b\right)+12\,bc^3x^{3\,n}-6\,b^2c^2x^{2\,n}+4\,b^3cx^n-3\,b^4}{12\,b^5nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*c^4*n*x^(4*n)*log(x) - 12*c^4*x^(4*n)*log(c*x^n + b) + 12*b*c^3*x^
(3*n) - 6*b^2*c^2*x^(2*n) + 4*b^3*c*x^n - 3*b^4)/(b^5*n*x^(4*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-3n-1}}{cx^{2n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.500 $\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=236

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}+\frac{\sqrt{2}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n}-\frac{\sqrt{2}$$

```
[Out] -4/(3*b*n*x^((3*n)/4)) + (Sqrt[2]*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x^(n/
4))/b^(1/4)])/(b^(7/4)*n) - (Sqrt[2]*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x^
(n/4))/b^(1/4)])/(b^(7/4)*n) + (c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n) - (c^(3/4)*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n)
```

Rubi [A] time = 0.206301, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {1584, 362, 345, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}+\frac{\sqrt{2}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n}-\frac{\sqrt{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```


```
[Out] -4/(3*b*n*x^((3*n)/4)) + (Sqrt[2]*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x^(n/
4))/b^(1/4)])/(b^(7/4)*n) - (Sqrt[2]*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x^
(n/4))/b^(1/4)])/(b^(7/4)*n) + (c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n) - (c^(3/4)*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n)
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-\frac{3n}{4}}}{b+cx^n}\,dx$$

$$= -\frac{4x^{-3n/4}}{3bn} - \frac{c\int \frac{x^{\frac{1}{4}(-4+n)}}{b+cx^n}\,dx}{b}$$

$$= -\frac{4x^{-3n/4}}{3bn} - \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{bn}$$

$$= -\frac{4x^{-3n/4}}{3bn} - \frac{(2c)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n} - \frac{(2c)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n}$$

$$= -\frac{4x^{-3n/4}}{3bn} - \frac{\sqrt{c}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n} - \frac{\sqrt{c}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, x^{1+\frac{1}{4}(-4}\right.}{b^{3/2}n}$$

$$= -\frac{4x^{-3n/4}}{3bn} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n} - \frac{c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n} - \frac{(\sqrt{2}}{}$$

$$= -\frac{4x^{-3n/4}}{3bn} + \frac{\sqrt{2}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} - \frac{\sqrt{2}c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x\right.}{\sqrt{2}b^{7/4}n}$$

Mathematica [C] time = 0.0093805, size = 34, normalized size = 0.14

$$-\frac{4x^{-3n/4}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{cx^n}{b}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-4*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^n)/b)])/(3*b*n*x^((3*n)/4))
```

Maple [C] time = 0.087, size = 54, normalized size = 0.2

$$-\frac{4}{3\,bn}\left(x^{\frac{n}{4}}\right)^{-3} + \sum_{\_R=\mathrm{RootOf}\left(b^7n^4\_Z^4+c^3\right)} \_R\ln\left(x^{\frac{n}{4}} - \frac{b^2n\_R}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] -4/3/b/n/(x^(1/4*n))^3+sum(_R*ln(x^(1/4*n)-b^2*n/c*_R),_R=RootOf(_Z^4*b^7*n
^4+c^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-c\int \frac{x^{\frac{1}{4}n}}{bcxx^n+b^2x}\,dx - \frac{4}{3\,bnx^{\frac{3}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(1/4*n)/(b*c*x*x^n + b^2*x), x) - 4/3/(b*n*x^(3/4*n))
```

Fricas [A] time = 1.70372, size = 625, normalized size = 2.65

$$-\frac{12\,bnx^{3}x^{\frac{3}{4}\,n-3}\left(-\frac{c^{3}}{b^{7}n^{4}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{5}cn^{3}xx^{\frac{1}{4}\,n-1}\left(-\frac{c^{3}}{b^{7}n^{4}}\right)^{\frac{3}{4}}-b^{5}n^{3}x\sqrt{\frac{b^{4}n^{2}\sqrt{-\frac{c^{3}}{b^{7}n^{4}}}+c^{2}x^{2}x^{\frac{1}{2}\,n-2}}{x^{2}}}\left(-\frac{c^{3}}{b^{7}n^{4}}\right)^{\frac{3}{4}}}{c^{3}}\right)+3\,bnx^{3}x^{\frac{3}{4}\,n-3}\left(-\frac{c^{3}}{b^{7}n^{4}}\right)^{\frac{1}{4}}\log\left(\frac{b^{2}n\cdots}{\cdots}\right.}{3\,bnx^{3}x^{\frac{3}{4}\,n-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/3*(12*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*arctan(-(b^5*c*n^3*x*
x^(1/4*n - 1)*(-c^3/(b^7*n^4))^(3/4) - b^5*n^3*x*sqrt((b^4*n^2*sqrt(-c^3/(b
^7*n^4)) + c^2*x^2*x^(1/2*n - 2))/x^2)*(-c^3/(b^7*n^4))^(3/4))/c^3) + 3*b*n
*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*log((b^2*n*(-c^3/(b^7*n^4))^(1/4)
 + c*x*x^(1/4*n - 1))/x) - 3*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*l
og(-(b^2*n*(-c^3/(b^7*n^4))^(1/4) - c*x*x^(1/4*n - 1))/x) + 4)/(b*n*x^3*x^(
3/4*n - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-n}x^{\frac{n}{4}-1}}{b+cx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x**(-n)*x**(n/4 - 1)/(b + c*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.501 $\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=160

$$-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}+\frac{\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{3x^{-2n/3}}{2bn}$$

```
[Out] -3/(2*b*n*x^((2*n)/3)) + (Sqrt[3]*c^(2/3)*ArcTan[(b^(1/3) - 2*c^(1/3)*x^(n/
3))/(Sqrt[3]*b^(1/3))])/(b^(5/3)*n) - (c^(2/3)*Log[b^(1/3) + c^(1/3)*x^(n/3
)])/(b^(5/3)*n) + (c^(2/3)*Log[b^(2/3) - b^(1/3)*c^(1/3)*x^(n/3) + c^(2/3)*
x^((2*n)/3)])/(2*b^(5/3)*n)
```

Rubi [A] time = 0.132147, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {1584, 362, 345, 200, 31, 634, 617, 204, 628}

$$-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}+\frac{\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{3x^{-2n/3}}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(2*b*n*x^((2*n)/3)) + (Sqrt[3]*c^(2/3)*ArcTan[(b^(1/3) - 2*c^(1/3)*x^(n/
3))/(Sqrt[3]*b^(1/3))])/(b^(5/3)*n) - (c^(2/3)*Log[b^(1/3) + c^(1/3)*x^(n/3
)])/(b^(5/3)*n) + (c^(2/3)*Log[b^(2/3) - b^(1/3)*c^(1/3)*x^(n/3) + c^(2/3)*
x^((2*n)/3)])/(2*b^(5/3)*n)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-\frac{2n}{3}}}{b+cx^n}\,dx\\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{c\int \frac{x^{\frac{1}{3}(-3+n)}}{b+cx^n}\,dx}{b}\\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{(3c)\,\text{Subst}\left(\int \frac{1}{b+cx^3}\,dx,x,x^{1+\frac{1}{3}(-3+n)}\right)}{bn}\\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{c}x}\,dx,x,x^{1+\frac{1}{3}(-3+n)}\right)}{b^{5/3}n}-\frac{c\,\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{c}x}{b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+c^{2/3}x^2}\,dx,x,x^{1+\frac{1}{3}(-3+n)}\right)}{b^{5/3}n}\\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{b}\sqrt[3]{c}+2c^{2/3}x}{b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+c^{2/3}x^2}\,dx,x,x^{1+\frac{1}{3}(-3+n)}\right)}{2b^{5/3}n}-\frac{(3c)\,\text{Subst}\left(\right.}{}\\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}-\frac{\left(3c^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\right.}{b^{5/3}}\\
&= -\frac{3x^{-2n/3}}{2bn}+\frac{\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n}\right.}{2b^{5/3}n}
\end{aligned}$$

Mathematica [C] time = 0.0096091, size = 34, normalized size = 0.21

$$-\frac{3x^{-2n/3}\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{cx^n}{b}\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-3*Hypergeometric2F1[-2/3, 1, 1/3, -((c*x^n)/b)])/(2*b*n*x^((2*n)/3))
```

Maple [C] time = 0.058, size = 54, normalized size = 0.3

$$-\frac{3}{2\,bn}\left(x^{\frac{n}{3}}\right)^{-2}+\sum_{\_R=\mathrm{RootOf}\left(b^5n^3\_Z^3+c^2\right)}\_R\ln\left(x^{\frac{n}{3}}-\frac{b^2n\_R}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] -3/2/b/n/(x^(1/3*n))^2+sum(_R*ln(x^(1/3*n)-b^2*n/c*_R),_R=RootOf(_Z^3*b^5*n
^3+c^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-c\int\frac{x^{\frac{1}{3}\,n}}{bcxx^n+b^2x}\,dx-\frac{3}{2\,bnx^{\frac{2}{3}\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(1/3*n)/(b*c*x*x^n + b^2*x), x) - 3/2/(b*n*x^(2/3*n))
```

Fricas [A] time = 1.70976, size = 502, normalized size = 3.14

$$\frac{2\,\sqrt{3}x^2x^{\frac{2}{3}\,n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bxx^{\frac{1}{3}\,n-1}\left(-\frac{c^2}{b^2}\right)^{\frac{2}{3}}-\sqrt{3}c}{3\,c}\right)+2\,x^2x^{\frac{2}{3}\,n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{cxx^{\frac{1}{3}\,n-1}-b\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}}{x}\right)-x^2x^{\frac{2}{3}\,n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log\left(\right.}{2\,bnx^2x^{\frac{2}{3}\,n-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x^2*x^(2/3*n - 2)*(-c^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x
*x^(1/3*n - 1)*(-c^2/b^2)^(2/3) - sqrt(3)*c)/c) + 2*x^2*x^(2/3*n - 2)*(-c^2
/b^2)^(1/3)*log((c*x*x^(1/3*n - 1) - b*(-c^2/b^2)^(1/3))/x) - x^2*x^(2/3*n
- 2)*(-c^2/b^2)^(1/3)*log((c^2*x^2*x^(2/3*n - 2) + b*c*x*x^(1/3*n - 1)*(-c^
2/b^2)^(1/3) + b^2*(-c^2/b^2)^(2/3))/x^2) - 3)/(b*n*x^2*x^(2/3*n - 2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{3}n-1}}{cx^{2n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.502 $\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}-\frac{2x^{-n/2}}{bn}$$

```
[Out] -2/(b*n*x^(n/2)) + (2*Sqrt[c]*ArcTan[Sqrt[b]/(Sqrt[c]*x^(n/2))])/(b^(3/2)*n
)
```

Rubi [A] time = 0.0347027, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1584, 345, 193, 321, 205}

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}-\frac{2x^{-n/2}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(b*n*x^(n/2)) + (2*Sqrt[c]*ArcTan[Sqrt[b]/(Sqrt[c]*x^(n/2))])/(b^(3/2)*n
)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-\frac{n}{2}}}{b+cx^n}\,dx$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^2}}\,dx, x, x^{-n/2}\right)}{n}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{c+bx^2}\,dx, x, x^{-n/2}\right)}{n}$$
$$= -\frac{2x^{-n/2}}{bn} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{c+bx^2}\,dx, x, x^{-n/2}\right)}{bn}$$
$$= -\frac{2x^{-n/2}}{bn} + \frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}$$

Mathematica [C] time = 0.0071318, size = 32, normalized size = 0.64

$$-\frac{2x^{-n/2}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{cx^n}{b}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, -((c*x^n)/b)])/(b*n*x^(n/2))
```

Maple [A] time = 0.062, size = 79, normalized size = 1.6

$$-2\frac{1}{bnx^{n/2}} + \frac{1}{b^2n}\sqrt{-bc}\ln\left(x^{\frac{n}{2}} - \frac{1}{c}\sqrt{-bc}\right) - \frac{1}{b^2n}\sqrt{-bc}\ln\left(x^{\frac{n}{2}} + \frac{1}{c}\sqrt{-bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] -2/b/n/(x^(1/2*n))+1/b^2*(-b*c)^(1/2)/n*ln(x^(1/2*n)-(-b*c)^(1/2)/c)-1/b^2*
(-b*c)^(1/2)/n*ln(x^(1/2*n)+(-b*c)^(1/2)/c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-c\int \frac{x^{\frac{1}{2}n}}{bcxx^n+b^2x}\,dx - \frac{2}{bnx^{\frac{1}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(1/2*n)/(b*c*x*x^n + b^2*x), x) - 2/(b*n*x^(1/2*n))
```

Fricas [A] time = 1.66699, size = 323, normalized size = 6.46

$$\left[\frac{xx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2x^{n-2}-2\,bxx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}-b}{cx^2x^{n-2}+b}\right)-2}{bnxx^{\frac{1}{2}n-1}}, \frac{2\left(xx^{\frac{1}{2}n-1}\sqrt{\frac{c}{b}}\arctan\left(\frac{b\sqrt{\frac{c}{b}}}{cxx^{\frac{1}{2}n-1}}\right)-1\right)}{bnxx^{\frac{1}{2}n-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [(x*x^(1/2*n - 1)*sqrt(-c/b)*log((c*x^2*x^(n - 2) - 2*b*x*x^(1/2*n - 1)*sqr
t(-c/b) - b)/(c*x^2*x^(n - 2) + b)) - 2)/(b*n*x*x^(1/2*n - 1)), 2*(x*x^(1/2
*n - 1)*sqrt(c/b)*arctan(b*sqrt(c/b)/(c*x*x^(1/2*n - 1))) - 1)/(b*n*x*x^(1/
2*n - 1))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{2}n-1}}{cx^{2n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.503 $\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=68

$$-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}+\frac{2cx^{-n/2}}{b^2n}-\frac{2x^{-3n/2}}{3bn}$$

```
[Out] -2/(3*b*n*x^((3*n)/2)) + (2*c)/(b^2*n*x^(n/2)) - (2*c^(3/2)*ArcTan[Sqrt[b]/
(Sqrt[c]*x^(n/2))])/(b^(5/2)*n)
```

Rubi [A] time = 0.0408412, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1584, 362, 345, 193, 321, 205}

$$-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}+\frac{2cx^{-n/2}}{b^2n}-\frac{2x^{-3n/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(3*b*n*x^((3*n)/2)) + (2*c)/(b^2*n*x^(n/2)) - (2*c^(3/2)*ArcTan[Sqrt[b]/
(Sqrt[c]*x^(n/2))])/(b^(5/2)*n)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-\frac{3n}{2}}}{b+cx^n}\,dx \\
&= -\frac{2x^{-3n/2}}{3bn} - \frac{c\int \frac{x^{-1-\frac{n}{2}}}{b+cx^n}\,dx}{b} \\
&= -\frac{2x^{-3n/2}}{3bn} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^2}}\,dx, x, x^{-n/2}\right)}{bn} \\
&= -\frac{2x^{-3n/2}}{3bn} + \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{c+bx^2}\,dx, x, x^{-n/2}\right)}{bn} \\
&= -\frac{2x^{-3n/2}}{3bn} + \frac{2cx^{-n/2}}{b^2n} - \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{c+bx^2}\,dx, x, x^{-n/2}\right)}{b^2n} \\
&= -\frac{2x^{-3n/2}}{3bn} + \frac{2cx^{-n/2}}{b^2n} - \frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}
\end{aligned}$$

Mathematica [C] time = 0.0081641, size = 34, normalized size = 0.5

$$-\frac{2x^{-3n/2}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{cx^n}{b}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, -((c*x^n)/b)])/(3*b*n*x^((3*n)/2))
```

Maple [A] time = 0.075, size = 97, normalized size = 1.4

$$2\frac{c}{nb^2x^{n/2}} - \frac{2}{3\,bn}\left(x^{\frac{n}{2}}\right)^{-3} + \frac{c}{b^3n}\sqrt{-bc}\ln\left(x^{\frac{n}{2}} + \frac{1}{c}\sqrt{-bc}\right) - \frac{c}{b^3n}\sqrt{-bc}\ln\left(x^{\frac{n}{2}} - \frac{1}{c}\sqrt{-bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 2*c/b^2/n/(x^(1/2*n))-2/3/b/n/(x^(1/2*n))^3+1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/
2*n)+(-b*c)^(1/2)/c)-1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/2*n)-(-b*c)^(1/2)/c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c^2\int \frac{x^{\frac{1}{2}n}}{b^2cxx^n + b^3x}\,dx + \frac{2\,(3\,cx^n - b)}{3\,b^2nx^{\frac{3}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(1/2*n)/(b^2*c*x*x^n + b^3*x), x) + 2/3*(3*c*x^n - b)/(b^2*
n*x^(3/2*n))
```

Fricas [A] time = 1.70623, size = 379, normalized size = 5.57

$$\left[-\frac{2\,bx^{3}x^{-\frac{3}{2}\,n-3}-6\,cxx^{-\frac{1}{2}\,n-1}-3\,c\sqrt{-\frac{c}{b}}\log\left(\frac{bx^{2}x^{-n-2}-2\,bxx^{-\frac{1}{2}\,n-1}\sqrt{-\frac{c}{b}}-c}{bx^{2}x^{-n-2}+c}\right)}{3\,b^{2}n},-\frac{2\left(bx^{3}x^{-\frac{3}{2}\,n-3}-3\,cxx^{-\frac{1}{2}\,n-1}-3\,c\sqrt{\frac{c}{b}}\arctan\left(\frac{\sqrt{\frac{c}{b}}}{xx^{-\frac{1}{2}\,n-1}}\right)\right)}{3\,b^{2}n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*b*x^3*x^(-3/2*n - 3) - 6*c*x*x^(-1/2*n - 1) - 3*c*sqrt(-c/b)*log((
b*x^2*x^(-n - 2) - 2*b*x*x^(-1/2*n - 1)*sqrt(-c/b) - c)/(b*x^2*x^(-n - 2) +
 c)))/(b^2*n), -2/3*(b*x^3*x^(-3/2*n - 3) - 3*c*x*x^(-1/2*n - 1) - 3*c*sqrt
(c/b)*arctan(sqrt(c/b)/(x*x^(-1/2*n - 1))))/(b^2*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.504 $\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=176

$$-\frac{c^{4/3}\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{b^{7/3}n}+\frac{c^{4/3}\log\left(b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}\right)}{2b^{7/3}n}+\frac{\sqrt{3}c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{\sqrt{3}\sqrt[3]{c}}\right)}{b^{7/3}n}+\frac{3cx^{-n/3}}{b^2n}-\frac{3x^{-4n/3}}{4bn}$$

```
[Out] -3/(4*b*n*x^((4*n)/3)) + (3*c)/(b^2*n*x^(n/3)) + (Sqrt[3]*c^(4/3)*ArcTan[(c
^(1/3) - (2*b^(1/3))/x^(n/3))/(Sqrt[3]*c^(1/3))])/(b^(7/3)*n) - (c^(4/3)*Lo
g[c^(1/3) + b^(1/3)/x^(n/3)])/(b^(7/3)*n) + (c^(4/3)*Log[c^(2/3) + b^(2/3)/
x^((2*n)/3) - (b^(1/3)*c^(1/3))/x^(n/3)])/(2*b^(7/3)*n)
```

Rubi [A] time = 0.142492, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {1584, 362, 345, 193, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{c^{4/3}\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{b^{7/3}n}+\frac{c^{4/3}\log\left(b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}\right)}{2b^{7/3}n}+\frac{\sqrt{3}c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{\sqrt{3}\sqrt[3]{c}}\right)}{b^{7/3}n}+\frac{3cx^{-n/3}}{b^2n}-\frac{3x^{-4n/3}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(4*b*n*x^((4*n)/3)) + (3*c)/(b^2*n*x^(n/3)) + (Sqrt[3]*c^(4/3)*ArcTan[(c
^(1/3) - (2*b^(1/3))/x^(n/3))/(Sqrt[3]*c^(1/3))])/(b^(7/3)*n) - (c^(4/3)*Lo
g[c^(1/3) + b^(1/3)/x^(n/3)])/(b^(7/3)*n) + (c^(4/3)*Log[c^(2/3) + b^(2/3)/
x^((2*n)/3) - (b^(1/3)*c^(1/3))/x^(n/3)])/(2*b^(7/3)*n)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-\frac{4n}{3}}}{b+cx^n}\,dx$$
$$= -\frac{3x^{-4n/3}}{4bn} - \frac{c\int \frac{x^{-1-\frac{n}{3}}}{b+cx^n}\,dx}{b}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{(3c)\,\mathrm{Subst}\left(\int \frac{1}{b+\frac{c}{x^3}}\,dx, x, x^{-n/3}\right)}{bn}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{(3c)\,\mathrm{Subst}\left(\int \frac{x^3}{c+bx^3}\,dx, x, x^{-n/3}\right)}{bn}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{(3c^2)\,\mathrm{Subst}\left(\int \frac{1}{c+bx^3}\,dx, x, x^{-n/3}\right)}{b^2n}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{b}x}\,dx, x, x^{-n/3}\right)}{b^2n} - \frac{c^{4/3}\,\mathrm{Subst}\left(\int \frac{2\sqrt[3]{c}-\sqrt[3]{b}x}{c^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+b^{2/3}x^2}\,dx, x, x^-\right.}{b^2n}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\,\mathrm{Subst}\left(\int \frac{-\sqrt[3]{b}\sqrt[3]{c}+2b^{2/3}x}{c^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+b^{2/3}x^2}\,dx, x, x^{-n/3}\right)}{2b^{7/3}n} - \frac{(3c}{}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\log\left(c^{2/3}+b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}\right)}{2b^{7/3}n} - \frac{(3c^{4/3})}{}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} + \frac{\sqrt{3}c^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x^{-n/3}}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{b^{7/3}n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\log\left(c^{2/3}+b^{2/3}x^{-}\right.}{2b^{7/}}$$

Mathematica [C] time = 0.0078412, size = 34, normalized size = 0.19

$$-\frac{3x^{-4n/3}\,{}_2F_1\left(-\frac{4}{3},1;-\frac{1}{3};-\frac{cx^n}{b}\right)}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-3*Hypergeometric2F1[-4/3, 1, -1/3, -((c*x^n)/b)])/(4*b*n*x^((4*n)/3))
```

Maple [C] time = 0.065, size = 73, normalized size = 0.4

$$3\,\frac{c}{b^2nx^{n/3}} - \frac{3}{4\,bn}\left(x^{\frac{n}{3}}\right)^{-4} + \sum_{\_R=\mathrm{RootOf}\left(b^7n^3\_Z^3+c^4\right)} \_R\,\ln\left(x^{\frac{n}{3}} + \frac{b^5n^2\_R^2}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 3*c/b^2/n/(x^(1/3*n))-3/4/b/n/(x^(1/3*n))^4+sum(_R*ln(x^(1/3*n)+b^5*n^2/c^3
*_R^2),_R=RootOf(_Z^3*b^7*n^3+c^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c^2 \int \frac{x^{\frac{2}{3}n}}{b^2cxx^n + b^3x}\,dx + \frac{3\,(4\,cx^n - b)}{4\,b^2nx^{\frac{4}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(2/3*n)/(b^2*c*x*x^n + b^3*x), x) + 3/4*(4*c*x^n - b)/(b^2*
n*x^(4/3*n))
```

Fricas [A] time = 1.67486, size = 427, normalized size = 2.43

$$-\frac{3\,bx^4x^{-\frac{4}{3}n-4} - 12\,cxx^{-\frac{1}{3}n-1} - 4\,\sqrt{3}c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bxx^{-\frac{1}{3}n-1}\left(-\frac{c}{b}\right)^{\frac{2}{3}}-\sqrt{3}c}{3\,c}\right) - 4\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\log\left(\frac{xx^{-\frac{1}{3}n-1}-\left(-\frac{c}{b}\right)^{\frac{1}{3}}}{x}\right) + 2\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\mathrm{lo}}{4\,b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/4*(3*b*x^4*x^(-4/3*n - 4) - 12*c*x*x^(-1/3*n - 1) - 4*sqrt(3)*c*(-c/b)^(
1/3)*arctan(1/3*(2*sqrt(3)*b*x*x^(-1/3*n - 1)*(-c/b)^(2/3) - sqrt(3)*c)/c)
- 4*c*(-c/b)^(1/3)*log((x*x^(-1/3*n - 1) - (-c/b)^(1/3))/x) + 2*c*(-c/b)^(1
/3)*log((x^2*x^(-2/3*n - 2) + x*x^(-1/3*n - 1)*(-c/b)^(1/3) + (-c/b)^(2/3))
/x^2))/(b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n} + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.505 $\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=252

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}b^{9/4}n}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}b^{9/4}n}+\frac{\sqrt{2}c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}$$

```
[Out] -4/(5*b*n*x^((5*n)/4)) + (4*c)/(b^2*n*x^(n/4)) + (Sqrt[2]*c^(5/4)*ArcTan[1
- (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) - (Sqrt[2]*c^(5/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) + (c^(5/4)*Log[Sqr
t[c] + Sqrt[b]/x^(n/2) - (Sqrt[2]*b^(1/4)*c^(1/4))/x^(n/4)])/(Sqrt[2]*b^(9/
4)*n) - (c^(5/4)*Log[Sqrt[c] + Sqrt[b]/x^(n/2) + (Sqrt[2]*b^(1/4)*c^(1/4))/
x^(n/4)])/(Sqrt[2]*b^(9/4)*n)
```

Rubi [A] time = 0.224505, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {1584, 362, 345, 193, 321, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}b^{9/4}n}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}b^{9/4}n}+\frac{\sqrt{2}c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```


```
[Out] -4/(5*b*n*x^((5*n)/4)) + (4*c)/(b^2*n*x^(n/4)) + (Sqrt[2]*c^(5/4)*ArcTan[1
- (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) - (Sqrt[2]*c^(5/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) + (c^(5/4)*Log[Sqr
t[c] + Sqrt[b]/x^(n/2) - (Sqrt[2]*b^(1/4)*c^(1/4))/x^(n/4)])/(Sqrt[2]*b^(9/
4)*n) - (c^(5/4)*Log[Sqrt[c] + Sqrt[b]/x^(n/2) + (Sqrt[2]*b^(1/4)*c^(1/4))/
x^(n/4)])/(Sqrt[2]*b^(9/4)*n)
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-\frac{5n}{4}}}{b+cx^n}\,dx$$
$$= -\frac{4x^{-5n/4}}{5bn} - \frac{c\int \frac{x^{-1-\frac{n}{4}}}{b+cx^n}\,dx}{b}$$
$$= -\frac{4x^{-5n/4}}{5bn} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{b+\frac{c}{x^4}}\,dx, x, x^{-n/4}\right)}{bn}$$
$$= -\frac{4x^{-5n/4}}{5bn} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{x^4}{c+bx^4}\,dx, x, x^{-n/4}\right)}{bn}$$
$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} - \frac{(4c^2)\,\mathrm{Subst}\left(\int \frac{1}{c+bx^4}\,dx, x, x^{-n/4}\right)}{b^2n}$$
$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} - \frac{(2c^{3/2})\,\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{b}x^2}{c+bx^4}\,dx, x, x^{-n/4}\right)}{b^2n} - \frac{(2c^{3/2})\,\mathrm{Subst}\left(\int \frac{\sqrt{c}+\sqrt{b}x^2}{c+bx^4}\,dx, x, x^{-n/}\right.}{b^2n}$$
$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{c^{5/4}\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{c}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{b}}-x^2}\,dx, x, x^{-n/4}\right)}{\sqrt{2}b^{9/4}n} + \frac{c^{5/4}\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{c}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{b}}-x^2}\,dx, x\right.}{\sqrt{2}b^{9/4}n}$$
$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}x^{-n/2}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}\right)}{\sqrt{2}b^{9/4}n} - \frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}x^{-n/2}+\sqrt{2}\sqrt[4]{b}\cdot\right.}{\sqrt{2}b^{9/4}n}$$
$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{\sqrt{2}c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n} - \frac{\sqrt{2}c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n} + \frac{c^{5/4}\log\left(\sqrt{c}\right.}{}$$

Mathematica [C] time = 0.0081725, size = 34, normalized size = 0.13

$$-\frac{4x^{-5n/4}\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{cx^n}{b}\right)}{5bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-4*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^n)/b)])/(5*b*n*x^((5*n)/4))
```

Maple [C] time = 0.075, size = 73, normalized size = 0.3

$$4\frac{c}{b^2nx^{n/4}} - \frac{4}{5\,bn}\left(x^{\frac{n}{4}}\right)^{-5} + \sum_{\_R=\mathrm{RootOf}\left(b^9n^4\_Z^4+c^5\right)} \_R\ \ln\left(x^{\frac{n}{4}} + \frac{b^7n^3\_R^3}{c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 4*c/b^2/n/(x^(1/4*n))-4/5/b/n/(x^(1/4*n))^5+sum(_R*ln(x^(1/4*n)+b^7*n^3/c^4
*_R^3),_R=RootOf(_Z^4*b^9*n^4+c^5))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c^2 \int \frac{x^{\frac{3}{4}n}}{b^2 c x x^n + b^3 x}\, dx + \frac{4\,(5\,c x^n - b)}{5\,b^2 n x^{\frac{5}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(3/4*n)/(b^2*c*x*x^n + b^3*x), x) + 4/5*(5*c*x^n - b)/(b^2*
n*x^(5/4*n))
```

Fricas [A] time = 1.72154, size = 605, normalized size = 2.4

$$-\frac{4\,bx^5x^{-\frac{5}{4}n-5} + 20\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^7cn^3xx^{-\frac{1}{4}n-1}\left(-\frac{c^5}{b^9n^4}\right)^{\frac{3}{4}} - b^7n^3x\sqrt{\frac{b^4n^2\sqrt{-\frac{c^5}{b^9n^4}} + c^2x^2x^{-\frac{1}{2}n-2}}{x^2}}\left(-\frac{c^5}{b^9n^4}\right)^{\frac{3}{4}}}{c^5}\right) + 5\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\log\left(\right.}{5\,b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/5*(4*b*x^5*x^(-5/4*n - 5) + 20*b^2*n*(-c^5/(b^9*n^4))^(1/4)*arctan(-(b^7
*c*n^3*x*x^(-1/4*n - 1)*(-c^5/(b^9*n^4))^(3/4) - b^7*n^3*x*sqrt((b^4*n^2*sq
rt(-c^5/(b^9*n^4)) + c^2*x^2*x^(-1/2*n - 2))/x^2)*(-c^5/(b^9*n^4))^(3/4))/c
^5) + 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log((b^2*n*(-c^5/(b^9*n^4))^(1/4) + c*
x*x^(-1/4*n - 1))/x) - 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log(-(b^2*n*(-c^5/(b^
9*n^4))^(1/4) - c*x*x^(-1/4*n - 1))/x) - 20*c*x*x^(-1/4*n - 1))/(b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n} + bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.506 $\int x^{-1-n(-1+p)} \left(bx^n + cx^{2n}\right)^p \, dx$

Optimal. Leaf size=37

$$\frac{x^{-n(p+1)} \left(bx^n + cx^{2n}\right)^{p+1}}{cn(p+1)}$$

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(c*n*(1 + p)*x^(n*(1 + p)))
```

Rubi [A] time = 0.0395401, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2014}

$$\frac{x^{-n(p+1)} \left(bx^n + cx^{2n}\right)^{p+1}}{cn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(c*n*(1 + p)*x^(n*(1 + p)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1-n(-1+p)} \left(bx^n + cx^{2n}\right)^p \, dx = \frac{x^{-n(1+p)} \left(bx^n + cx^{2n}\right)^{1+p}}{cn(1+p)}$$

Mathematica [A] time = 0.0170824, size = 38, normalized size = 1.03

$$\frac{x^{-np} (b + cx^n) (x^n (b + cx^n))^p}{cn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((b + c*x^n)*(x^n*(b + c*x^n))^p)/(c*n*(1 + p)*x^(n*p))
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int x^{-1-n(-1+p)} \left(bx^n + cx^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
```

Maxima [A] time = 1.08022, size = 58, normalized size = 1.57

$$\frac{(cx^n + b)e^{\left(-np\log(x)+p\log(cx^n+b)+p\log(x^n)\right)}}{cn\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] (c*x^n + b)*e^(-n*p*log(x) + p*log(c*x^n + b) + p*log(x^n))/(c*n*(p + 1))
```

Fricas [A] time = 1.58892, size = 126, normalized size = 3.41

$$\frac{\left(cxx^{-np+n-1}x^n + bxx^{-np+n-1}\right)\left(cx^{2\,n} + bx^n\right)^p}{\left(cnp + cn\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] (c*x*x^(-n*p + n - 1)*x^n + b*x*x^(-n*p + n - 1))*(c*x^(2*n) + b*x^n)^p/((c
*n*p + c*n)*x^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*(-1+p))*(b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2\,n} + bx^n\right)^p x^{-n\left(p-1\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(p - 1) - 1), x)
```

3.507 $\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=38

$$-\frac{x^{-2n(p+1)}\left(bx^n+cx^{2n}\right)^{p+1}}{bn(p+1)}$$

```
[Out] -((b*x^n + c*x^(2*n))^(1 + p)/(b*n*(1 + p)*x^(2*n*(1 + p))))
```

Rubi [A] time = 0.0389941, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2014}

$$-\frac{x^{-2n(p+1)}\left(bx^n+cx^{2n}\right)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] -((b*x^n + c*x^(2*n))^(1 + p)/(b*n*(1 + p)*x^(2*n*(1 + p))))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=-\frac{x^{-2n(1+p)}\left(bx^n+cx^{2n}\right)^{1+p}}{bn(1+p)}$$

Mathematica [A] time = 0.0205104, size = 43, normalized size = 1.13

$$-\frac{x^{-n(2p+1)}\left(b+cx^n\right)\left(x^n\left(b+cx^n\right)\right)^p}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] -(((b + c*x^n)*(x^n*(b + c*x^n))^p)/(b*n*(1 + p)*x^(n*(1 + 2*p))))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n\right)^p x^{-n(2p+1)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

Fricas [A] time = 1.56053, size = 124, normalized size = 3.26

$$-\frac{\left(cxx^{-2\,np-n-1}x^n + bxx^{-2\,np-n-1}\right)\left(cx^{2\,n} + bx^n\right)^p}{bnp + bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] -(c*x*x^(-2*n*p - n - 1)*x^n + b*x*x^(-2*n*p - n - 1))*(c*x^(2*n) + b*x^n)^
p/(b*n*p + b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*(1+2*p))*(b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n\right)^p x^{-n(2p+1)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

3.508 $\int x^{-1+2n} \left(a^2 + 2abx^n + b^2x^{2n}\right)^{5/2} dx$

Optimal. Leaf size=112

$$\frac{(a+bx^n)^7 \sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\left(ab^2+b^3x^n\right)} - \frac{a(a+bx^n)^6 \sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\left(ab^2+b^3x^n\right)}$$

```
[Out] -(a*(a + b*x^n)^6*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(6*n*(a*b^2 + b^3*x^
n)) + ((a + b*x^n)^7*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(7*n*(a*b^2 + b^3
*x^n))
```

Rubi [A] time = 0.0416955, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{(a+bx^n)^7 \sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\left(ab^2+b^3x^n\right)} - \frac{a(a+bx^n)^6 \sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\left(ab^2+b^3x^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -(a*(a + b*x^n)^6*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(6*n*(a*b^2 + b^3*x^
n)) + ((a + b*x^n)^7*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(7*n*(a*b^2 + b^3
*x^n))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}dx=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}\left(ab+b^2x^n\right)^5dx}{b^4\left(ab+b^2x^n\right)}$$
$$=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\operatorname{Subst}\left(\int x\left(ab+b^2x\right)^5dx,x,x^n\right)}{b^4n\left(ab+b^2x^n\right)}$$
$$=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\operatorname{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^5}{b}+\frac{\left(ab+b^2x\right)^6}{b^2}\right)dx,x,x^n\right)}{b^4n\left(ab+b^2x^n\right)}$$
$$=-\frac{a\left(a+bx^n\right)^6\sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\left(ab^2+b^3x^n\right)}+\frac{\left(a+bx^n\right)^7\sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\left(ab^2+b^3x^n\right)}$$

Mathematica [A] time = 0.0573805, size = 40, normalized size = 0.36

$$-\frac{\left(a-6bx^n\right)\left(a+bx^n\right)^5\sqrt{\left(a+bx^n\right)^2}}{42b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -((a - 6*b*x^n)*(a + b*x^n)^5*Sqrt[(a + b*x^n)^2])/(42*b^2*n)
```

Maple [A] time = 0.049, size = 208, normalized size = 1.9

$$\frac{b^5\left(x^n\right)^7}{\left(7a+7bx^n\right)n}\sqrt{\left(a+bx^n\right)^2}+\frac{5ab^4\left(x^n\right)^6}{\left(6a+6bx^n\right)n}\sqrt{\left(a+bx^n\right)^2}+2\frac{\sqrt{\left(a+bx^n\right)^2}a^2b^3\left(x^n\right)^5}{\left(a+bx^n\right)n}+\frac{5a^3b^2\left(x^n\right)^4}{\left(2a+2bx^n\right)n}\sqrt{\left(a+bx^n\right)^2}+\frac{5}{(3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

```
[Out] 1/7*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^5/n*(x^n)^7+5/6*((a+b*x^n)^2)^(1/2)/(a+
b*x^n)*a*b^4/n*(x^n)^6+2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b^3/n*(x^n)^5+5/
2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*b^2/n*(x^n)^4+5/3*((a+b*x^n)^2)^(1/2)/(
a+b*x^n)*a^4*b/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^5/n*(x^n)^2
```

Maxima [A] time = 1.00644, size = 100, normalized size = 0.89

$$\frac{6b^5x^{7n}+35ab^4x^{6n}+84a^2b^3x^{5n}+105a^3b^2x^{4n}+70a^4bx^{3n}+21a^5x^{2n}}{42n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*x
^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

Fricas [A] time = 1.59756, size = 165, normalized size = 1.47

$$\frac{6 b^5 x^{7n} + 35 ab^4 x^{6n} + 84 a^2 b^3 x^{5n} + 105 a^3 b^2 x^{4n} + 70 a^4 b x^{3n} + 21 a^5 x^{2n}}{42 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas
")
```

```
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*x
^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2 x^{2n} + 2\, abx^n + a^2\right)^{\frac{5}{2}} x^{2n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2)*x^(2*n - 1), x)
```

3.509 $\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=112

$$\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\left(ab^2+b^3x^n\right)}-\frac{a(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\left(ab^2+b^3x^n\right)}$$

```
[Out] -(a*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(4*n*(a*b^2 + b^3*x^
n)) + ((a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(5*n*(a*b^2 + b^3
*x^n))
```

Rubi [A] time = 0.0396355, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\left(ab^2+b^3x^n\right)}-\frac{a(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\left(ab^2+b^3x^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] -(a*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(4*n*(a*b^2 + b^3*x^
n)) + ((a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(5*n*(a*b^2 + b^3
*x^n))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}\left(ab+b^2x^n\right)^3dx}{b^2\left(ab+b^2x^n\right)}$$
$$=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\mathrm{Subst}\left(\int x\left(ab+b^2x\right)^3dx,x,x^n\right)}{b^2n\left(ab+b^2x^n\right)}$$
$$=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\mathrm{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^3}{b}+\frac{\left(ab+b^2x\right)^4}{b^2}\right)dx,x,x^n\right)}{b^2n\left(ab+b^2x^n\right)}$$
$$=-\frac{a\left(a+bx^n\right)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\left(ab^2+b^3x^n\right)}+\frac{\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\left(ab^2+b^3x^n\right)}$$

Mathematica [A] time = 0.0379362, size = 40, normalized size = 0.36

$$-\frac{\left(a-4bx^n\right)\left(a+bx^n\right)^3\sqrt{\left(a+bx^n\right)^2}}{20b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] -((a - 4*b*x^n)*(a + b*x^n)^3*Sqrt[(a + b*x^n)^2])/(20*b^2*n)
```

Maple [A] time = 0.023, size = 135, normalized size = 1.2

$$\frac{b^3\left(x^n\right)^5}{\left(5a+5bx^n\right)n}\sqrt{\left(a+bx^n\right)^2}+\frac{3ab^2\left(x^n\right)^4}{\left(4a+4bx^n\right)n}\sqrt{\left(a+bx^n\right)^2}+\frac{a^2b\left(x^n\right)^3}{\left(a+bx^n\right)n}\sqrt{\left(a+bx^n\right)^2}+\frac{a^3\left(x^n\right)^2}{\left(2a+2bx^n\right)n}\sqrt{\left(a+bx^n\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] 1/5*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3/n*(x^n)^5+3/4*((a+b*x^n)^2)^(1/2)/(a+
b*x^n)*a*b^2/n*(x^n)^4+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/n*(x^n)^3+1/2*((
a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/n*(x^n)^2
```

Maxima [A] time = 0.986123, size = 65, normalized size = 0.58

$$\frac{4\,b^3x^{5\,n}+15\,ab^2x^{4\,n}+20\,a^2bx^{3\,n}+10\,a^3x^{2\,n}}{20\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n))
/n
```

Fricas [A] time = 1.59965, size = 107, normalized size = 0.96

$$\frac{4\,b^3x^{5\,n} + 15\,ab^2x^{4\,n} + 20\,a^2bx^{3\,n} + 10\,a^3x^{2\,n}}{20\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n))
/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{3}{2}} x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^(2*n - 1), x)
```

3.510 $\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=99

$$\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(a+bx^n\right)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}$$

```
[Out] (a*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a + b*x^n)) + (b^2*x^
(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n))
```

Rubi [A] time = 0.0310233, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1355, 14}

$$\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(a+bx^n\right)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a + b*x^n)) + (b^2*x^
(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}\left(ab+b^2x^n\right)dx}{ab+b^2x^n}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(abx^{-1+2n}+b^2x^{-1+3n}\right)dx}{ab+b^2x^n}\\
&= \frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(a+bx^n\right)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}
\end{aligned}$$

Mathematica [A] time = 0.0230789, size = 44, normalized size = 0.44

$$\frac{x^{2n}\sqrt{\left(a+bx^n\right)^2}\left(3a+2bx^n\right)}{6n\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(3*a + 2*b*x^n))/(6*n*(a + b*x^n))
```

Maple [A] time = 0.02, size = 64, normalized size = 0.7

$$\frac{b\,(x^n)^3}{(3\,a+3\,bx^n)\,n}\sqrt{(a+bx^n)^2}+\frac{a\,(x^n)^2}{(2\,a+2\,bx^n)\,n}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] 1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*
x^n)*a/n*(x^n)^2
```

Maxima [A] time = 1.01323, size = 30, normalized size = 0.3

$$\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

Fricas [A] time = 1.575, size = 47, normalized size = 0.47

$$\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2n-1}\sqrt{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)*sqrt((a + b*x**n)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b^2x^{2n} + 2abx^n + a^2}x^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^(2*n - 1), x)
```

3.511 $\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=90

$$\frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (x^n*(a + b*x^n))/(b*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - (a*(a + b*x^n
)*Log[a + b*x^n])/(b^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0438885, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^n*(a + b*x^n))/(b*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - (a*(a + b*x^n
)*Log[a + b*x^n])/(b^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{(ab+b^2x^n)\int \frac{x^{-1+2n}}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{(ab+b^2x^n)\,\text{Subst}\left(\int \frac{x}{ab+b^2x}\,dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{(ab+b^2x^n)\,\text{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a}{b^2(a+bx)}\right)dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] time = 0.0321583, size = 46, normalized size = 0.51

$$\frac{(a+bx^n)\left(\frac{x^n}{b}-\frac{a\log(a+bx^n)}{b^2}\right)}{n\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] ((a + b*x^n)*(x^n/b - (a*Log[a + b*x^n])/b^2))/(n*Sqrt[(a + b*x^n)^2])
```

Maple [A] time = 0.03, size = 71, normalized size = 0.8

$$\frac{x^n}{(a+bx^n)\,bn}\sqrt{(a+bx^n)^2} - \frac{a}{(a+bx^n)\,b^2n}\sqrt{(a+bx^n)^2}\ln\left(x^n+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)/b/n*x^n-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/b^2/n
*ln(x^n+a/b)
```

Maxima [A] time = 1.01878, size = 43, normalized size = 0.48

$$\frac{x^n}{bn} - \frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima
")
```

```
[Out] x^n/(b*n) - a*log((b*x^n + a)/b)/(b^2*n)
```

Fricas [A] time = 1.58961, size = 49, normalized size = 0.54

$$\frac{bx^n - a\log(bx^n + a)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas
")
```

```
[Out] (b*x^n - a*log(b*x^n + a))/(b^2*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{\sqrt{(a + bx^n)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)/sqrt((a + b*x**n)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.512 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{x^{2n}}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] x^(2*n)/(2*a*n*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0268692, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1355, 264}

$$\frac{x^{2n}}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] x^(2*n)/(2*a*n*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{x^{-1+2n}}{\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^{2n}}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.0165093, size = 35, normalized size = 0.73

$$\frac{x^{2n}\left(a+bx^n\right)}{2an\left(\left(a+bx^n\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^(2*n)*(a + b*x^n))/(2*a*n*((a + b*x^n)^2)^(3/2))
```

Maple [A] time = 0.023, size = 37, normalized size = 0.8

$$-\frac{2\,bx^{n}+a}{2\,(a+bx^{n})^{3}\,b^{2}n}\sqrt{(a+bx^{n})^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] -1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^3*(2*b*x^n+a)/b^2/n
```

Maxima [A] time = 0.990497, size = 55, normalized size = 1.15

$$-\frac{2\,bx^{n}+a}{2\left(b^{4}nx^{2\,n}+2\,ab^{3}nx^{n}+a^{2}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

Fricas [A] time = 1.55798, size = 86, normalized size = 1.79

$$-\frac{2\,bx^{n}+a}{2\left(b^{4}nx^{2\,n}+2\,ab^{3}nx^{n}+a^{2}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

3.513 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{a}{4b^2n\,(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{3b^2n\,(a+bx^n)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] a/(4*b^2*n*(a + b*x^n)^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(3*b^2*n*
(a + b*x^n)^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0515943, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{a}{4b^2n\,(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{3b^2n\,(a+bx^n)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] a/(4*b^2*n*(a + b*x^n)^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(3*b^2*n*
(a + b*x^n)^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^n\right)\right)\int \frac{x^{-1+2n}}{\left(ab+b^2x^n\right)^5}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(b^4\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^5}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(b^4\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \left(-\frac{a}{b^6(a+bx)^5}+\frac{1}{b^6(a+bx)^4}\right)dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{a}{4b^2n\left(a+bx^n\right)^3\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{3b^2n\left(a+bx^n\right)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] time = 0.0327514, size = 40, normalized size = 0.45

$$-\frac{a+4bx^n}{12b^2n\left(a+bx^n\right)^3\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -(a + 4*b*x^n)/(12*b^2*n*(a + b*x^n)^3*Sqrt[(a + b*x^n)^2])
```

Maple [A] time = 0.024, size = 37, normalized size = 0.4

$$-\frac{4\,bx^n+a}{12\,\left(a+bx^n\right)^5b^2n}\sqrt{\left(a+bx^n\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

```
[Out] -1/12*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^5*(4*b*x^n+a)/b^2/n
```

Maxima [A] time = 1.02756, size = 93, normalized size = 1.06

$$-\frac{4\,bx^n+a}{12\,\left(b^6nx^{4\,n}+4\,ab^5nx^{3\,n}+6\,a^2b^4nx^{2\,n}+4\,a^3b^3nx^n+a^4b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*n
) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

Fricas [A] time = 1.56607, size = 147, normalized size = 1.67

$$-\frac{4 \, b x^{n} + a}{12 \, {\left(b^{6} n x^{4 \, n} + 4 \, a b^{5} n x^{3 \, n} + 6 \, a^{2} b^{4} n x^{2 \, n} + 4 \, a^{3} b^{3} n x^{n} + a^{4} b^{2} n\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*n
) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2 \, n - 1}}{{\left(b^{2} x^{2 \, n} + 2 \, a b x^{n} + a^{2}\right)}^{\frac{5}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2), x)
```

3.514 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx$

Optimal. Leaf size=88

$$\frac{a}{6b^2n\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\left(a+bx^n\right)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] a/(6*b^2*n*(a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(5*b^2*n*
(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0513487, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{a}{6b^2n\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\left(a+bx^n\right)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

```
[Out] a/(6*b^2*n*(a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(5*b^2*n*
(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx = \frac{\left(b^6\left(ab+b^2x^n\right)\right)\int \frac{x^{-1+2n}}{\left(ab+b^2x^n\right)^7}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(b^6\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^7}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(b^6\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \left(-\frac{a}{b^8(a+bx)^7}+\frac{1}{b^8(a+bx)^6}\right)dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{a}{6b^2n\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\left(a+bx^n\right)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] time = 0.0341907, size = 40, normalized size = 0.45

$$-\frac{a+6bx^n}{30b^2n\left(a+bx^n\right)^5\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

```
[Out] -(a + 6*b*x^n)/(30*b^2*n*(a + b*x^n)^5*Sqrt[(a + b*x^n)^2])
```

Maple [A] time = 0.032, size = 37, normalized size = 0.4

$$-\frac{6\,bx^n+a}{30\,\left(a+bx^n\right)^7b^2n}\sqrt{\left(a+bx^n\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x)
```

```
[Out] -1/30*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^7*(6*b*x^n+a)/b^2/n
```

Maxima [A] time = 1.02007, size = 131, normalized size = 1.49

$$-\frac{6\,bx^n+a}{30\left(b^8nx^{6\,n}+6\,ab^7nx^{5\,n}+15\,a^2b^6nx^{4\,n}+20\,a^3b^5nx^{3\,n}+15\,a^4b^4nx^{2\,n}+6\,a^5b^3nx^n+a^6b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="maxima
")
```

```
[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4*
n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*b^
2*n)
```

Fricas [A] time = 1.55046, size = 211, normalized size = 2.4

$$-\frac{6\,bx^{n}+a}{30\,\left(b^{8}nx^{6\,n}+6\,ab^{7}nx^{5\,n}+15\,a^{2}b^{6}nx^{4\,n}+20\,a^{3}b^{5}nx^{3\,n}+15\,a^{4}b^{4}nx^{2\,n}+6\,a^{5}b^{3}nx^{n}+a^{6}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="fricas
")
```

```
[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4*
n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*b^
2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2\,n-1}}{\left(b^{2}x^{2\,n}+2\,abx^{n}+a^{2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(7/2), x)
```

3.515 $\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=108

$$\frac{b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{a(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\left(a+bx^n\right)}$$

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x^n
)) + (b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m +
n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.0415609, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1355, 14, 20, 30}

$$\frac{b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{a(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x^n
)) + (b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m +
n)*(a*b + b^2*x^n))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int (dx)^m\left(ab+b^2x^n\right)dx}{ab+b^2x^n}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(ab(dx)^m+b^2x^n(dx)^m\right)dx}{ab+b^2x^n}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\,(a+bx^n)} + \frac{\left(b^2\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^n(dx)^m\,dx}{ab+b^2x^n}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\,(a+bx^n)} + \frac{\left(b^2x^{-m}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^{m+n}\,dx}{ab+b^2x^n}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\,(a+bx^n)} + \frac{b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\left(ab+b^2x^n\right)}$$

Mathematica [A] time = 0.0318535, size = 55, normalized size = 0.51

$$\frac{x(dx)^m\sqrt{(a+bx^n)^2}\,(a(m+n+1)+b(m+1)x^n)}{(m+1)(m+n+1)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^n)^2]*(a*(1 + m + n) + b*(1 + m)*x^n))/((1 + m)*(1
 + m + n)*(a + b*x^n))
```

Maple [C] time = 0.041, size = 132, normalized size = 1.2

$$\frac{x\,(mbx^n+am+an+bx^n+a)}{(a+bx^n)\,(1+m)\,(1+m+n)}\sqrt{(a+bx^n)^2}\,e^{\frac{m\left(-i(\mathrm{csgn}(idx))^3\pi+i(\mathrm{csgn}(idx))^2\mathrm{csgn}(id)\pi+i(\mathrm{csgn}(idx))^2\mathrm{csgn}(ix)\pi-i\,\mathrm{csgn}(idx)\mathrm{csgn}(id)\mathrm{csgn}(ix)\pi+2\ln(x)+2\,\mathrm{l}}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x*(m*b*x^n+a*m+a*n+b*x^n+a)/(1+m)/(1+m+n)*exp
(1/2*m*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*csgn(I*d)*Pi+I*csgn(I*d*x)^2*cs
gn(I*x)*Pi-I*csgn(I*d*x)*csgn(I*d)*csgn(I*x)*Pi+2*ln(x)+2*ln(d)))
```

Maxima [A] time = 1.05472, size = 63, normalized size = 0.58

$$\frac{ad^m(m+n+1)xx^m + bd^m(m+1)xe^{(m\log(x)+n\log(x))}}{m^2+m(n+2)+n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] (a*d^m*(m + n + 1)*x*x^m + b*d^m*(m + 1)*x*e^(m*log(x) + n*log(x)))/(m^2 +
m*(n + 2) + n + 1)
```

Fricas [A] time = 1.63823, size = 155, normalized size = 1.44

$$\frac{(bm+b)xx^n e^{(m\log(d)+m\log(x))} + (am+an+a)xe^{(m\log(d)+m\log(x))}}{m^2+(m+1)n+2m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m + a*n + a)*x*e^(m*log(d) +
m*log(x)))/(m^2 + (m + 1)*n + 2*m + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \sqrt{(a+bx^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**n)**2), x)
```

Giac [A] time = 1.11743, size = 234, normalized size = 2.17

$$\frac{bmxx^n e^{(m\log(d)+m\log(x))}\mathrm{sgn}\left(bx^n+a\right) + amxe^{(m\log(d)+m\log(x))}\mathrm{sgn}\left(bx^n+a\right) + bmxe^{(m\log(d)+m\log(x))}\mathrm{sgn}\left(bx^n+a\right) + anxe^{(m\log}}{m^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] (b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*m*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*n*
x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*x^n*e^(m*log(d) + m*log(x))*
sgn(b*x^n + a) + a*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a))/(m^2 + m*n + 2*m + n + 1)
```

3.516 $\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=93

$$\frac{b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}$$

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^2*x^(3 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.0283172, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$\frac{b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^2*x^(3 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b^2*x^n))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2\left(ab+b^2x^n\right)\,dx}{ab+b^2x^n}\\&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(abx^2+b^2x^{2+n}\right)\,dx}{ab+b^2x^n}\\&=\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}+\frac{b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A] time = 0.023155, size = 46, normalized size = 0.49

$$\frac{x^3\sqrt{\left(a+bx^n\right)^2}\left(a(n+3)+3bx^n\right)}{3(n+3)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a*(3 + n) + 3*b*x^n))/(3*(3 + n)*(a + b*x^n))
```

Maple [A] time = 0.015, size = 61, normalized size = 0.7

$$\frac{ax^3}{3\,a+3\,bx^n}\sqrt{(a+bx^n)^2}+\frac{bx^3x^n}{(a+bx^n)\,(3+n)}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] 1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x^3+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(3+
n)*x^3*x^n
```

Maxima [A] time = 1.0194, size = 34, normalized size = 0.37

$$\frac{3\,bx^3x^n+a(n+3)x^3}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*x^3*x^n + a*(n + 3)*x^3)/(n + 3)
```

Fricas [A] time = 1.53308, size = 61, normalized size = 0.66

$$\frac{3\,bx^3x^n+(an+3\,a)x^3}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x^3*x^n + (a*n + 3*a)*x^3)/(n + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a + b*x**n)**2), x)
```

Giac [A] time = 1.12569, size = 72, normalized size = 0.77

$$\frac{3\,bx^3x^n\mathrm{sgn}\left(bx^n+a\right)+anx^3\mathrm{sgn}\left(bx^n+a\right)+3\,ax^3\mathrm{sgn}\left(bx^n+a\right)}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b*x^3*x^n*sgn(b*x^n + a) + a*n*x^3*sgn(b*x^n + a) + 3*a*x^3*sgn(b*x^
n + a))/(n + 3)
```

3.517 $\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=93

$$\frac{b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}$$

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (b^2*x^(2 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.0247172, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (b^2*x^(2 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b + b^2*x^n))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x\left(ab+b^2x^n\right)dx}{ab+b^2x^n}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(abx+b^2x^{1+n}\right)dx}{ab+b^2x^n}\\
&= \frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}+\frac{b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)\left(ab+b^2x^n\right)}
\end{aligned}$$

Mathematica [A] time = 0.0222121, size = 46, normalized size = 0.49

$$\frac{x^2\sqrt{(a+bx^n)^2}\,(a(n+2)+2bx^n)}{2(n+2)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a*(2 + n) + 2*b*x^n))/(2*(2 + n)*(a + b*x^n))
```

Maple [A] time = 0.013, size = 61, normalized size = 0.7

$$\frac{ax^2}{2a+2bx^n}\sqrt{(a+bx^n)^2}+\frac{bx^2x^n}{(a+bx^n)(2+n)}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] 1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(2+
n)*x^2*x^n
```

Maxima [A] time = 0.997503, size = 34, normalized size = 0.37

$$\frac{2bx^2x^n+a(n+2)x^2}{2(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^2*x^n + a*(n + 2)*x^2)/(n + 2)
```

Fricas [A] time = 1.57154, size = 61, normalized size = 0.66

$$\frac{2bx^2x^n+(an+2a)x^2}{2(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*x^n + (a*n + 2*a)*x^2)/(n + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a + b*x**n)**2), x)
```

Giac [A] time = 1.11286, size = 72, normalized size = 0.77

$$\frac{2\,bx^2x^n\mathrm{sgn}\left(bx^n+a\right)+anx^2\mathrm{sgn}\left(bx^n+a\right)+2\,ax^2\mathrm{sgn}\left(bx^n+a\right)}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*x^2*x^n*sgn(b*x^n + a) + a*n*x^2*sgn(b*x^n + a) + 2*a*x^2*sgn(b*x^
n + a))/(n + 2)
```

3.518 $\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=88

$$\frac{b^2x^{n+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)\left(ab+b^2x^n\right)}+\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(a + b*x^n) + (b^2*x^(1 + n)*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.0185866, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1343}

$$\frac{b^2x^{n+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)\left(ab+b^2x^n\right)}+\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(a + b*x^n) + (b^2*x^(1 + n)*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + n)*(a*b + b^2*x^n))
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(2ab+2b^2x^n\right)\,dx}{2ab+2b^2x^n}\\ &= \frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}+\frac{b^2x^{1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+n)\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A] time = 0.01403, size = 39, normalized size = 0.44

$$\frac{x\sqrt{(a+bx^n)^2}\,(an+a+bx^n)}{(n+1)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*Sqrt[(a + b*x^n)^2]*(a + a*n + b*x^n))/((1 + n)*(a + b*x^n))
```

Maple [A] time = 0.013, size = 56, normalized size = 0.6

$$\frac{ax}{a+b x^{n}}\sqrt{\left(a+b x^{n}\right)^{2}}+\frac{b x x^{n}}{\left(a+b x^{n}\right)\left(1+n\right)}\sqrt{\left(a+b x^{n}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(1+n)*x*x
^n
```

Maxima [A] time = 0.960536, size = 26, normalized size = 0.3

$$\frac{a(n+1)x+b x x^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] (a*(n + 1)*x + b*x*x^n)/(n + 1)
```

Fricas [A] time = 1.59111, size = 45, normalized size = 0.51

$$\frac{b x x^{n}+(a n+a)x}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] (b*x*x^n + (a*n + a)*x)/(n + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^{2}+2abx^{n}+b^{2}x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

Giac [A] time = 1.1015, size = 34, normalized size = 0.39

$$\left(ax+\frac{b x^{n+1}}{n+1}\right)\operatorname{sgn}\left(b x^{n}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")

[Out] (a*x + b*x^(n + 1)/(n + 1))*sgn(b*x^n + a)
```

3.519 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx$

Optimal. Leaf size=85

$$\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

```
[Out] (b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (a*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rubi [A] time = 0.0261973, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

```
[Out] (b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (a*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x}\,dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x}+b^2x^{-1+n}\right)dx}{ab+b^2x^n}\\ &= \frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}\end{aligned}$$

Mathematica [A] time = 0.015956, size = 37, normalized size = 0.44

$$\frac{\sqrt{(a+bx^n)^2}\,(an\log(x)+bx^n)}{n\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(b*x^n + a*n*Log[x]))/(n*(a + b*x^n))
```

Maple [A] time = 0.019, size = 54, normalized size = 0.6

$$\frac{a\ln(x)}{a+bx^n}\sqrt{(a+bx^n)^2}+\frac{bx^n}{(a+bx^n)\,n}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*ln(x)+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n*x^n
```

Maxima [A] time = 1.00288, size = 18, normalized size = 0.21

$$a\log(x)+\frac{bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*log(x) + b*x^n/n
```

Fricas [A] time = 1.62657, size = 32, normalized size = 0.38

$$\frac{an\log(x)+bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] (a*n*log(x) + b*x^n)/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(a+bx^n)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x, x)
```

3.520 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx$

Optimal. Leaf size=94

$$-\frac{b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}$$

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (b^2*x^(-1 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.0330818, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$-\frac{b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (b^2*x^(-1 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x^2}\,dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x^2}+b^2x^{-2+n}\right)dx}{ab+b^2x^n}\\ &= -\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}-\frac{b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A] time = 0.0251198, size = 42, normalized size = 0.45

$$\frac{\sqrt{\left(a+bx^n\right)^2}\left(-an+a+bx^n\right)}{(n-1)x\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a - a*n + b*x^n))/((-1 + n)*x*(a + b*x^n))
```

Maple [A] time = 0.018, size = 61, normalized size = 0.7

$$-\frac{a}{(a+bx^n)\,x}\sqrt{(a+bx^n)^2}+\frac{bx^n}{(a+bx^n)\,(-1+n)\,x}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x)
```

```
[Out] -((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*b/x
*x^n
```

Maxima [A] time = 1.00533, size = 30, normalized size = 0.32

$$-\frac{a(n-1)-bx^n}{(n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -(a*(n - 1) - b*x^n)/((n - 1)*x)
```

Fricas [A] time = 1.60517, size = 43, normalized size = 0.46

$$-\frac{an-bx^n-a}{(n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*n - b*x^n - a)/((n - 1)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(a+bx^n)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b^2 x^{2n} + 2\,abx^n + a^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^2, x)
```

3.521 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3} dx$

Optimal. Leaf size=96

$$-\frac{b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\,(a+bx^n)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) - (b^2*x^(-2 +
 n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.0293044, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$-\frac{b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) - (b^2*x^(-2 +
 n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a*b + b^2*x^n))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x^3}\,dx}{ab+b^2x^n}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x^3}+b^2x^{-3+n}\right)dx}{ab+b^2x^n}\\
&= -\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\,(a+bx^n)}-\frac{b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}
\end{aligned}$$

Mathematica [A] time = 0.0260375, size = 47, normalized size = 0.49

$$\frac{\sqrt{(a+bx^n)^2}\,(2bx^n-a(n-2))}{2(n-2)x^2\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(-(a*(-2 + n)) + 2*b*x^n))/(2*(-2 + n)*x^2*(a + b*x^n)
)
```

Maple [A] time = 0.019, size = 61, normalized size = 0.6

$$-\frac{a}{(2a+2bx^n)x^2}\sqrt{(a+bx^n)^2}+\frac{bx^n}{(a+bx^n)(-2+n)x^2}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x)
```

```
[Out] -1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-2+
n)*b/x^2*x^n
```

Maxima [A] time = 0.98262, size = 30, normalized size = 0.31

$$-\frac{a(n-2)-2bx^n}{2(n-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*(n - 2) - 2*b*x^n)/((n - 2)*x^2)
```

Fricas [A] time = 1.54761, size = 57, normalized size = 0.59

$$-\frac{an-2bx^n-2a}{2(n-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*n - 2*b*x^n - 2*a)/((n - 2)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(a+bx^n)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^3, x)
```

3.522 $\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$

Optimal. Leaf size=238

$$\frac{3a^2b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+2n+1)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+3n+1)\left(ab+b^2x^n\right)}+$$

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x
^n)) + (3*a^2*b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((
1 + m + n)*(a*b + b^2*x^n)) + (3*a*b^3*x^(1 + 2*n)*(d*x)^m*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)])/((1 + m + 2*n)*(a*b + b^2*x^n)) + (b^4*x^(1 + 3*n)*(d*
x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m + 3*n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.0980303, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1355, 270, 20, 30}

$$\frac{3a^2b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+2n+1)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+3n+1)\left(ab+b^2x^n\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x
^n)) + (3*a^2*b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((
1 + m + n)*(a*b + b^2*x^n)) + (3*a*b^3*x^(1 + 2*n)*(d*x)^m*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)])/((1 + m + 2*n)*(a*b + b^2*x^n)) + (b^4*x^(1 + 3*n)*(d*
x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m + 3*n)*(a*b + b^2*x^n))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int (dx)^m \left(ab+b^2x^n\right)^3 dx}{b^2\left(ab+b^2x^n\right)}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int \left(a^3b^3(dx)^m+3a^2b^4x^n(dx)^m+3ab^5x^{2n}(dx)^m+b^6x^{3n}(dx)^m\right)}{b^2\left(ab+b^2x^n\right)}$$
$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)(a+bx^n)} + \frac{\left(3a^2b^2\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^n(dx)^m\,dx}{ab+b^2x^n} + \frac{(3}{}$$
$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)(a+bx^n)} + \frac{\left(3a^2b^2x^{-m}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^{m+n}\,d}{ab+b^2x^n}$$
$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)(a+bx^n)} + \frac{3a^2b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\left(ab+b^2x^n\right)} + \frac{3ab^3x^{1+}}{(1}$$

Mathematica [A] time = 0.109328, size = 90, normalized size = 0.38

$$\frac{x(dx)^m\left((a+bx^n)^2\right)^{3/2}\left(\frac{3a^2bx^n}{m+n+1}+\frac{a^3}{m+1}+\frac{3ab^2x^{2n}}{m+2n+1}+\frac{b^3x^{3n}}{m+3n+1}\right)}{(a+bx^n)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^n)^2)^(3/2)*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) +
 (3*a*b^2*x^(2*n))/(1 + m + 2*n) + (b^3*x^(3*n))/(1 + m + 3*n)))/(a + b*x^n
)^3
```

Maple [C] time = 0.055, size = 532, normalized size = 2.2

$$\frac{x\left(a^3+3b^3m^2n(x^n)^3+2b^3mn^2(x^n)^3+3ab^2m^3(x^n)^2+6b^3mn(x^n)^3+3a^2bm^3x^n+9ab^2m^2(x^n)^2+9ab^2n^2(x^n)^2+9a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x*(a^3+3*b^3*m^2*n*(x^n)^3+2*b^3*m*n^2*(x^n)^
3+3*a*b^2*m^3*(x^n)^2+6*b^3*m*n*(x^n)^3+3*a^2*b*m^3*x^n+9*a*b^2*m^2*(x^n)^2
+9*a*b^2*n^2*(x^n)^2+9*a^2*b*m^2*x^n+18*a^2*b*n^2*x^n+9*m*a*b^2*(x^n)^2+12*
a*b^2*(x^n)^2*n+9*m*a^2*b*x^n+15*a^2*b*n*x^n+b^3*(x^n)^3+a^3*m^3+3*a^3*m^2+
11*a^3*n^2+6*a^3*n+6*a^3*n^3+3*m*a^3+6*a^3*m^2*n+11*a^3*m*n^2+12*a^3*m*n+b^
3*m^3*(x^n)^3+3*b^3*m^2*(x^n)^3+2*b^3*n^2*(x^n)^3+3*m*b^3*(x^n)^3+3*b^3*(x^
n)^3*n+3*a^2*b*x^n+3*a*b^2*(x^n)^2+12*a*b^2*m^2*n*(x^n)^2+9*a*b^2*m*n^2*(x^
n)^2+15*a^2*b*m^2*n*x^n+18*a^2*b*m*n^2*x^n+24*a*b^2*m*n*(x^n)^2+30*a^2*b*m*
n*x^n)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)*exp(1/2*m*(-I*csgn(I*d*x)^3*Pi+I*c
sgn(I*d*x)^2*csgn(I*d)*Pi+I*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I
*d)*csgn(I*x)*Pi+2*ln(x)+2*ln(d)))
```

Maxima [A] time = 1.01157, size = 373, normalized size = 1.57

$$\frac{\left(m^3+3\,m^2(2\,n+1)+6\,n^3+\left(11\,n^2+12\,n+3\right)m+11\,n^2+6\,n+1\right)a^3d^mxx^m+\left(m^3+3\,m^2(n+1)+\left(2\,n^2+6\,n+3\right)m\right.}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1)
*a^3*d^m*x*x^m + (m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n +
 1)*b^3*d^m*x*e^(m*log(x) + 3*n*log(x)) + 3*(m^3 + m^2*(4*n + 3) + (3*n^2 +
 8*n + 3)*m + 3*n^2 + 4*n + 1)*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x)) + 3*(m
^3 + m^2*(5*n + 3) + (6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*a^2*b*d^m*x*e^
(m*log(x) + n*log(x)))/(m^4 + 2*m^3*(3*n + 2) + (11*n^2 + 18*n + 6)*m^2 + 6
*n^3 + 2*(3*n^3 + 11*n^2 + 9*n + 2)*m + 11*n^2 + 6*n + 1)
```

Fricas [A] time = 1.63259, size = 887, normalized size = 3.73

$$\frac{\left(b^3m^3+3\,b^3m^2+3\,b^3m+b^3+2\left(b^3m+b^3\right)n^2+3\left(b^3m^2+2\,b^3m+b^3\right)n\right)xx^{3\,n}e^{(m\log(d)+m\log(x))}+3\left(ab^2m^3+3\,ab^2m^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3 + 2*(b^3*m + b^3)*n^2 + 3*(b^3*m^2 +
2*b^3*m + b^3)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*(a*b^2*m^3 + 3*a*b^
2*m^2 + 3*a*b^2*m + a*b^2 + 3*(a*b^2*m + a*b^2)*n^2 + 4*(a*b^2*m^2 + 2*a*b^
2*m + a*b^2)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^3 + 3*a^2*b*
m^2 + 3*a^2*b*m + a^2*b + 6*(a^2*b*m + a^2*b)*n^2 + 5*(a^2*b*m^2 + 2*a^2*b*
m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^3*m^3 + 6*a^3*n^3 + 3*a^3*
m^2 + 3*a^3*m + a^3 + 11*(a^3*m + a^3)*n^2 + 6*(a^3*m^2 + 2*a^3*m + a^3)*n)
*x*e^(m*log(d) + m*log(x)))/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 11*(m^2 + 2*m +
1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39822, size = 3671, normalized size = 15.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] (b^3*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x
^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(3*n)*e^(m*
log(d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^(2*n)*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b^3*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + 12*a*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3
*b^3*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m*n^2
*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(2*n)*e
^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*x^n*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + b^3*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*m^2
*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*m^2*n*x*x^n*e^(m
*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x^n*e^(m*log(d) + m*log(
x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + 9*a*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*
m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*m^3*x*e^(m*log(d)
+ m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^
n + a) + 3*a*b^2*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*m^3*x*e
^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*m^2*n*x*e^(m*log(d) + m*log(x
))*sgn(b*x^n + a) + 15*a^2*b*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
 + 12*a*b^2*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*
e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 11*a^3*m*n^2*x*e^(m*log(d) + m*log
(x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n +
a) + 9*a*b^2*m*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x
*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*n^3*x*e^(m*log(d) + m*log(x
))*sgn(b*x^n + a) + 3*b^3*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n +
 a) + 6*b^3*m*n*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^
2*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m^2*x*x^(2*n)*
e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*x*x^(2*n)*e^(m*log(d) +
m*log(x))*sgn(b*x^n + a) + 24*a*b^2*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*s
gn(b*x^n + a) + 6*b^3*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
+ 9*a*b^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^2*
x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m^2*x*x^n*e^(m*l
og(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m^2*x*x^n*e^(m*log(d) + m*log(x)
)*sgn(b*x^n + a) + 3*b^3*m^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) +
 30*a^2*b*m*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 24*a*b^2*m*n*x
*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*b^3*m*n*x*x^n*e^(m*log(d) +
 m*log(x))*sgn(b*x^n + a) + 18*a^2*b*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(
b*x^n + a) + 9*a*b^2*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b
^3*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^3*m^2*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m^2*x*e^(m*log(d) + m*log(x))*sgn(b
*x^n + a) + 9*a*b^2*m^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^
2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a^3*m*n*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + 30*a^2*b*m*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n +
a) + 24*a*b^2*m*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*b^3*m*n*x*e^
(m*log(d) + m*log(x))*sgn(b*x^n + a) + 11*a^3*n^2*x*e^(m*log(d) + m*log(x))
*sgn(b*x^n + a) + 18*a^2*b*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9
*a*b^2*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^2*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^(3*n)*e^(m*log(d) + m*log(x))*s
gn(b*x^n + a) + 3*b^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) +
9*a*b^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^(2
*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*n*x*x^(2*n)*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*s
gn(b*x^n + a) + 9*a^2*b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*
a*b^2*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^n*e^(m*l
og(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*n*x*x^n*e^(m*log(d) + m*log(x))
*sgn(b*x^n + a) + 12*a*b^2*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) +
 3*b^3*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^3*m*x*e^(m*log(
```

```
d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m*x*e^(m*log(d) + m*log(x))*sgn(b*x
^n + a) + 9*a*b^2*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*e^
(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*n*x*e^(m*log(d) + m*log(x))*sg
n(b*x^n + a) + 15*a^2*b*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b
^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*n*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a
) + 3*a*b^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*x*x^(2*n
)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*x*x^n*e^(m*log(d) + m*lo
g(x))*sgn(b*x^n + a) + 3*a*b^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
 + b^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*x*e^(m*log(d) + m
*log(x))*sgn(b*x^n + a) + 3*a^2*b*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
+ 3*a*b^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*x*e^(m*log(d) + m*
log(x))*sgn(b*x^n + a))/(m^4 + 6*m^3*n + 11*m^2*n^2 + 6*m*n^3 + 4*m^3 + 18*
m^2*n + 22*m*n^2 + 6*n^3 + 6*m^2 + 18*m*n + 11*n^2 + 4*m + 6*n + 1)
```

3.523 $\int x^2 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=212

$$\frac{b^4x^{3(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(n+1)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)\left(ab+b^2x^n\right)}+\frac{a^3x^3\sqrt{a^2+2abx^n+}}{3\left(a+bx^n\right)}$$

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^4*x^(3*(
1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(1 + n)*(a*b + b^2*x^n)) +
(3*a^2*b^2*x^(3 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b
^2*x^n)) + (3*a*b^3*x^(3 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 +
2*n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.0624851, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 270}

$$\frac{b^4x^{3(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(n+1)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)\left(ab+b^2x^n\right)}+\frac{a^3x^3\sqrt{a^2+2abx^n+}}{3\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^4*x^(3*(
1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(1 + n)*(a*b + b^2*x^n)) +
(3*a^2*b^2*x^(3 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b
^2*x^n)) + (3*a*b^3*x^(3 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 +
2*n)*(a*b + b^2*x^n))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2\left(ab+b^2x^n\right)^3\,dx}{b^2\left(ab+b^2x^n\right)}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(a^3b^3x^2+3ab^5x^{2(1+n)}+3a^2b^4x^{2+n}+b^6x^{2+3n}\right)dx}{b^2\left(ab+b^2x^n\right)}\\ &= \frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}+\frac{b^4x^{3(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(1+n)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{3+n}\sqrt{a^2+2abx^n}}{(3+n)\left(ab+b^2x\right.}\end{aligned}$$

Mathematica [A] time = 0.0804647, size = 123, normalized size = 0.58

$$\frac{x^3\sqrt{(a+bx^n)^2}\left(9a^2b\left(2n^2+5n+3\right)x^n+a^3\left(2n^3+11n^2+18n+9\right)+9ab^2\left(n^2+4n+3\right)x^{2n}+b^3\left(2n^2+9n+9\right)x^{3n}\right)}{3(n+1)(n+3)(2n+3)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a^3*(9 + 18*n + 11*n^2 + 2*n^3) + 9*a^2*b*(3 + 5*
n + 2*n^2)*x^n + 9*a*b^2*(3 + 4*n + n^2)*x^(2*n) + b^3*(9 + 9*n + 2*n^2)*x^
(3*n)))/(3*(1 + n)*(3 + n)*(3 + 2*n)*(a + b*x^n))
```

Maple [A] time = 0.016, size = 146, normalized size = 0.7

$$\frac{x^3a^3}{3\,a+3\,bx^n}\sqrt{(a+bx^n)^2}+\frac{b^3x^3\,(x^n)^3}{(3\,a+3\,bx^n)\,(1+n)}\sqrt{(a+bx^n)^2}+3\,\frac{\sqrt{(a+bx^n)^2}ab^2x^3\,(x^n)^2}{(a+bx^n)\,(3+2\,n)}+3\,\frac{\sqrt{(a+bx^n)^2}a^2bx^3x^n}{(a+bx^n)\,(3+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] 1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x^3*a^3+1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)
*b^3*x^3/(1+n)*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^2/(3+2*n)*x^3*(x
^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(3+n)*x^3*x^n
```

Maxima [A] time = 1.01564, size = 146, normalized size = 0.69

$$\frac{\left(2\,n^2+9\,n+9\right)b^3x^3x^{3\,n}+9\left(n^2+4\,n+3\right)ab^2x^3x^{2\,n}+9\left(2\,n^2+5\,n+3\right)a^2bx^3x^n+\left(2\,n^3+11\,n^2+18\,n+9\right)a^3x^3}{3\left(2\,n^3+11\,n^2+18\,n+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*((2*n^2 + 9*n + 9)*b^3*x^3*x^(3*n) + 9*(n^2 + 4*n + 3)*a*b^2*x^3*x^(2*n
) + 9*(2*n^2 + 5*n + 3)*a^2*b*x^3*x^n + (2*n^3 + 11*n^2 + 18*n + 9)*a^3*x^3
)/(2*n^3 + 11*n^2 + 18*n + 9)
```

Fricas [A] time = 1.55789, size = 304, normalized size = 1.43

$$\frac{\left(2\,b^3n^2+9\,b^3n+9\,b^3\right)x^3x^{3\,n}+9\left(ab^2n^2+4\,ab^2n+3\,ab^2\right)x^3x^{2\,n}+9\left(2\,a^2bn^2+5\,a^2bn+3\,a^2b\right)x^3x^n+\left(2\,a^3n^3+11\,a^3n\cdots\right.}{3\left(2\,n^3+11\,n^2+18\,n+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*b^3*n^2 + 9*b^3*n + 9*b^3)*x^3*x^(3*n) + 9*(a*b^2*n^2 + 4*a*b^2*n +
 3*a*b^2)*x^3*x^(2*n) + 9*(2*a^2*b*n^2 + 5*a^2*b*n + 3*a^2*b)*x^3*x^n + (2*
```

```
a^3*n^3 + 11*a^3*n^2 + 18*a^3*n + 9*a^3)*x^3)/(2*n^3 + 11*n^2 + 18*n + 9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)

[Out] Timed out
```

Giac [A] time = 1.14995, size = 394, normalized size = 1.86

$$\frac{2 b^3 n^2 x^3 x^{3 n} \mathrm{sgn}(bx^n + a) + 9 ab^2 n^2 x^3 x^{2 n} \mathrm{sgn}(bx^n + a) + 18 a^2 bn^2 x^3 x^n \mathrm{sgn}(bx^n + a) + 2 a^3 n^3 x^3 \mathrm{sgn}(bx^n + a) + 9 b^3 nx^3 x^{3 n} \ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")

[Out] 1/3*(2*b^3*n^2*x^3*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^3*x^(2*n)*sgn(b*x
^n + a) + 18*a^2*b*n^2*x^3*x^n*sgn(b*x^n + a) + 2*a^3*n^3*x^3*sgn(b*x^n + a
) + 9*b^3*n*x^3*x^(3*n)*sgn(b*x^n + a) + 36*a*b^2*n*x^3*x^(2*n)*sgn(b*x^n +
 a) + 45*a^2*b*n*x^3*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^3*sgn(b*x^n + a) + 9
*b^3*x^3*x^(3*n)*sgn(b*x^n + a) + 27*a*b^2*x^3*x^(2*n)*sgn(b*x^n + a) + 27*
a^2*b*x^3*x^n*sgn(b*x^n + a) + 18*a^3*n*x^3*sgn(b*x^n + a) + 9*a^3*x^3*sgn(
b*x^n + a))/(2*n^3 + 11*n^2 + 18*n + 9)
```

3.524 $\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=211

$$\frac{3ab^3x^{2(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(n+1)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)\left(ab+b^2x^n\right)}+\frac{a^3x^2\sqrt{a^2+2ab\cdots}}{2\left(a+b\cdots\right.}$$

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (3*a*b^3*x^
(2*(1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 + n)*(a*b + b^2*x^n)
) + (3*a^2*b^2*x^(2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b
 + b^2*x^n)) + (b^4*x^(2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 +
3*n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.0581511, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{3ab^3x^{2(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(n+1)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)\left(ab+b^2x^n\right)}+\frac{a^3x^2\sqrt{a^2+2ab\cdots}}{2\left(a+b\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```



```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (3*a*b^3*x^
(2*(1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 + n)*(a*b + b^2*x^n)
) + (3*a^2*b^2*x^(2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b
 + b^2*x^n)) + (b^4*x^(2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 +
3*n)*(a*b + b^2*x^n))
```


Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x\left(ab+b^2x^n\right)^3\,dx}{b^2\left(ab+b^2x^n\right)}\\&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(a^3b^3x+3a^2b^4x^{1+n}+3ab^5x^{1+2n}+b^6x^{1+3n}\right)dx}{b^2\left(ab+b^2x^n\right)}\\&=\frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}+\frac{3ab^3x^{2(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1+n)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{2+n}\sqrt{a^2+2\cdots}}{(2+n)\left(ab+\cdots\right.}\end{aligned}$$

Mathematica [A] time = 0.0726074, size = 124, normalized size = 0.59

$$\frac{x^2\sqrt{(a+bx^n)^2}\left(6a^2b\left(3n^2+5n+2\right)x^n+a^3\left(3n^3+11n^2+12n+4\right)+3ab^2\left(3n^2+8n+4\right)x^{2n}+2b^3\left(n^2+3n+2\right)x^{3n}\right)}{2(n+1)(n+2)(3n+2)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a^3*(4 + 12*n + 11*n^2 + 3*n^3) + 6*a^2*b*(2 + 5*
n + 3*n^2)*x^n + 3*a*b^2*(4 + 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 + 3*n + n^2)*
x^(3*n)))/(2*(1 + n)*(2 + n)*(2 + 3*n)*(a + b*x^n))
```

Maple [A] time = 0.016, size = 145, normalized size = 0.7

$$\frac{x^2a^3}{2a+2bx^n}\sqrt{(a+bx^n)^2}+\frac{b^3x^2(x^n)^3}{(a+bx^n)(2+3n)}\sqrt{(a+bx^n)^2}+\frac{3ab^2x^2(x^n)^2}{(2a+2bx^n)(1+n)}\sqrt{(a+bx^n)^2}+3\frac{\sqrt{(a+bx^n)^2}a^2bx^2x^n}{(a+bx^n)(2+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] 1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x^2*a^3+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3
/(2+3*n)*x^2*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^2*x^2/(1+n)*(x^n
)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(2+n)*x^2*x^n
```

Maxima [A] time = 1.00437, size = 147, normalized size = 0.7

$$\frac{2\left(n^2+3n+2\right)b^3x^2x^{3n}+3\left(3n^2+8n+4\right)ab^2x^2x^{2n}+6\left(3n^2+5n+2\right)a^2bx^2x^n+\left(3n^3+11n^2+12n+4\right)a^3x^2}{2\left(3n^3+11n^2+12n+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(n^2 + 3*n + 2)*b^3*x^2*x^(3*n) + 3*(3*n^2 + 8*n + 4)*a*b^2*x^2*x^(2
*n) + 6*(3*n^2 + 5*n + 2)*a^2*b*x^2*x^n + (3*n^3 + 11*n^2 + 12*n + 4)*a^3*x
^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

Fricas [A] time = 1.57025, size = 306, normalized size = 1.45

$$\frac{2\left(b^3n^2+3b^3n+2b^3\right)x^2x^{3n}+3\left(3ab^2n^2+8ab^2n+4ab^2\right)x^2x^{2n}+6\left(3a^2bn^2+5a^2bn+2a^2b\right)x^2x^n+\left(3a^3n^3+11a^3n^2\right.}{2\left(3n^3+11n^2+12n+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^2*x^(3*n) + 3*(3*a*b^2*n^2 + 8*a*b^2*n
 + 4*a*b^2)*x^2*x^(2*n) + 6*(3*a^2*b*n^2 + 5*a^2*b*n + 2*a^2*b)*x^2*x^n + (
```

```
3*a^3*n^3 + 11*a^3*n^2 + 12*a^3*n + 4*a^3)*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14155, size = 394, normalized size = 1.87

$$\frac{2\,b^3n^2x^2x^{3\,n}\operatorname{sgn}\left(bx^n+a\right)+9\,ab^2n^2x^2x^{2\,n}\operatorname{sgn}\left(bx^n+a\right)+18\,a^2bn^2x^2x^n\operatorname{sgn}\left(bx^n+a\right)+3\,a^3n^3x^2\operatorname{sgn}\left(bx^n+a\right)+6\,b^3nx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*n^2*x^2*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^2*x^(2*n)*sgn(b*x
^n + a) + 18*a^2*b*n^2*x^2*x^n*sgn(b*x^n + a) + 3*a^3*n^3*x^2*sgn(b*x^n + a
) + 6*b^3*n*x^2*x^(3*n)*sgn(b*x^n + a) + 24*a*b^2*n*x^2*x^(2*n)*sgn(b*x^n +
 a) + 30*a^2*b*n*x^2*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^2*sgn(b*x^n + a) + 4
*b^3*x^2*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*x^2*x^(2*n)*sgn(b*x^n + a) + 12*
a^2*b*x^2*x^n*sgn(b*x^n + a) + 12*a^3*n*x^2*sgn(b*x^n + a) + 4*a^3*x^2*sgn(
b*x^n + a))/(3*n^3 + 11*n^2 + 12*n + 4)
```

3.525 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$

Optimal. Leaf size=206

$$\frac{3a^2b^4x^{n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(n+1)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{2n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(2n+1)\left(ab+b^2x^n\right)^3}+\frac{b^6x^{3n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(3n+1)\left(ab+b^2x^n\right)^3}+\frac{a^3x\left(a^2+2\right.}{(a}$$

```
[Out] (a^3*x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/(a + b*x^n)^3 + (3*a^2*b^4*x^
(1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + n)*(a*b + b^2*x^n)^3)
+ (3*a*b^5*x^(1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + 2*n)*(a
*b + b^2*x^n)^3) + (b^6*x^(1 + 3*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/
((1 + 3*n)*(a*b + b^2*x^n)^3)
```

Rubi [A] time = 0.0518595, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1343, 244}

$$\frac{3a^2b^4x^{n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(n+1)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{2n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(2n+1)\left(ab+b^2x^n\right)^3}+\frac{b^6x^{3n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(3n+1)\left(ab+b^2x^n\right)^3}+\frac{a^3x\left(a^2+2\right.}{(a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/(a + b*x^n)^3 + (3*a^2*b^4*x^
(1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + n)*(a*b + b^2*x^n)^3)
+ (3*a*b^5*x^(1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + 2*n)*(a
*b + b^2*x^n)^3) + (b^6*x^(1 + 3*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/
((1 + 3*n)*(a*b + b^2*x^n)^3)
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx &= \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\int \left(2ab+2b^2x^n\right)^3 dx}{\left(2ab+2b^2x^n\right)^3}\\ &= \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\int \left(8a^3b^3+24a^2b^4x^n+24ab^5x^{2n}+8b^6x^{3n}\right) dx}{\left(2ab+2b^2x^n\right)^3}\\ &= \frac{a^3x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(a+bx^n)^3}+\frac{3a^2b^4x^{1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(1+n)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{1+2n}\left(a^2+2ab\right.}{(1+2n)\left(ab-\right.}\end{aligned}$$

Mathematica [A] time = 0.0755736, size = 122, normalized size = 0.59

$$\frac{x\sqrt{(a+bx^n)^2}\left(3a^2b\left(6n^2+5n+1\right)x^n+a^3\left(6n^3+11n^2+6n+1\right)+3ab^2\left(3n^2+4n+1\right)x^{2n}+b^3\left(2n^2+3n+1\right)x^{3n}\right)}{(n+1)(2n+1)(3n+1)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^n)^2]*(a^3*(1 + 6*n + 11*n^2 + 6*n^3) + 3*a^2*b*(1 + 5*n +
 6*n^2)*x^n + 3*a*b^2*(1 + 4*n + 3*n^2)*x^(2*n) + b^3*(1 + 3*n + 2*n^2)*x^(
3*n)))/((1 + n)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n))
```

Maple [A] time = 0.014, size = 138, normalized size = 0.7

$$\frac{a^3x}{a+bx^n}\sqrt{(a+bx^n)^2}+\frac{b^3x\,(x^n)^3}{(a+bx^n)\,(1+3\,n)}\sqrt{(a+bx^n)^2}+3\,\frac{\sqrt{(a+bx^n)^2}ab^2x\,(x^n)^2}{(a+bx^n)\,(1+2\,n)}+3\,\frac{\sqrt{(a+bx^n)^2}a^2bxx^n}{(a+bx^n)\,(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3/(1+3*
n)*x*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^2/(1+2*n)*x*(x^n)^2+3*((a+
b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(1+n)*x*x^n
```

Maxima [A] time = 0.988257, size = 136, normalized size = 0.66

$$\frac{\left(2\,n^2+3\,n+1\right)b^3xx^{3\,n}+3\left(3\,n^2+4\,n+1\right)ab^2xx^{2\,n}+3\left(6\,n^2+5\,n+1\right)a^2bxx^n+\left(6\,n^3+11\,n^2+6\,n+1\right)a^3x}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] ((2*n^2 + 3*n + 1)*b^3*x*x^(3*n) + 3*(3*n^2 + 4*n + 1)*a*b^2*x*x^(2*n) + 3*
(6*n^2 + 5*n + 1)*a^2*b*x*x^n + (6*n^3 + 11*n^2 + 6*n + 1)*a^3*x)/(6*n^3 +
11*n^2 + 6*n + 1)
```

Fricas [A] time = 1.56252, size = 277, normalized size = 1.34

$$\frac{\left(2\,b^3n^2+3\,b^3n+b^3\right)xx^{3\,n}+3\left(3\,ab^2n^2+4\,ab^2n+ab^2\right)xx^{2\,n}+3\left(6\,a^2bn^2+5\,a^2bn+a^2b\right)xx^n+\left(6\,a^3n^3+11\,a^3n^2+6\right.}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] ((2*b^3*n^2 + 3*b^3*n + b^3)*x*x^(3*n) + 3*(3*a*b^2*n^2 + 4*a*b^2*n + a*b^2
)*x*x^(2*n) + 3*(6*a^2*b*n^2 + 5*a^2*b*n + a^2*b)*x*x^n + (6*a^3*n^3 + 11*a
```

```
^3*n^2 + 6*a^3*n + a^3)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + 2abx^n + b^2x^{2n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(3/2), x)
```

Giac [A] time = 1.15787, size = 355, normalized size = 1.72

$$\frac{6\,a^3n^3x\mathrm{sgn}\left(bx^n+a\right)+2\,b^3n^2xx^{3\,n}\mathrm{sgn}\left(bx^n+a\right)+9\,ab^2n^2xx^{2\,n}\mathrm{sgn}\left(bx^n+a\right)+18\,a^2bn^2xx^n\mathrm{sgn}\left(bx^n+a\right)+11\,a^3n^2x\mathrm{sgn}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] (6*a^3*n^3*x*sgn(b*x^n + a) + 2*b^3*n^2*x*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*
n^2*x*x^(2*n)*sgn(b*x^n + a) + 18*a^2*b*n^2*x*x^n*sgn(b*x^n + a) + 11*a^3*n
^2*x*sgn(b*x^n + a) + 3*b^3*n*x*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*n*x*x^(2*
n)*sgn(b*x^n + a) + 15*a^2*b*n*x*x^n*sgn(b*x^n + a) + 6*a^3*n*x*sgn(b*x^n +
 a) + b^3*x*x^(3*n)*sgn(b*x^n + a) + 3*a*b^2*x*x^(2*n)*sgn(b*x^n + a) + 3*a
^2*b*x*x^n*sgn(b*x^n + a) + a^3*x*sgn(b*x^n + a))/(6*n^3 + 11*n^2 + 6*n + 1
)
```

3.526 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=196

$$\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(ab+b^2x^n\right)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}+\frac{a^3\log(x)\sqrt{a^2+2abx^n+}}{a+bx^n}$$

```
[Out] (3*a^2*b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (
3*a*b^3*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a*b + b^2*x^n))
+ (b^4*x^(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n)) +
 (a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rubi [A] time = 0.0516225, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1355, 266, 43}

$$\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(ab+b^2x^n\right)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}+\frac{a^3\log(x)\sqrt{a^2+2abx^n+}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

```
[Out] (3*a^2*b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (
3*a*b^3*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a*b + b^2*x^n))
+ (b^4*x^(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n)) +
 (a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(ab+b^2x^n\right)^3}{x}\,dx}{b^2\left(ab+b^2x^n\right)}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x}\,dx, x, x^n\right)}{b^2n\left(ab+b^2x^n\right)}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)dx, x, x^n\right)}{b^2n\left(ab+b^2x^n\right)}$$
$$= \frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)} + \frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(ab+b^2x^n\right)} + \frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2}}{3n\left(ab+b^2x^n\right)}$$

Mathematica [A] time = 0.0385719, size = 68, normalized size = 0.35

$$\frac{\left((a+bx^n)^2\right)^{3/2}\left(3a^2bx^n+a^3n\log(x)+\frac{3}{2}ab^2x^{2n}+\frac{1}{3}b^3x^{3n}\right)}{n\left(a+bx^n\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

```
[Out] (((a + b*x^n)^2)^(3/2)*(3*a^2*b*x^n + (3*a*b^2*x^(2*n))/2 + (b^3*x^(3*n))/3
 + a^3*n*Log[x]))/(n*(a + b*x^n)^3)
```

Maple [A] time = 0.016, size = 127, normalized size = 0.7

$$\frac{a^3\ln(x)}{a+bx^n}\sqrt{(a+bx^n)^2}+\frac{b^3(x^n)^3}{(3a+3bx^n)n}\sqrt{(a+bx^n)^2}+\frac{3ab^2(x^n)^2}{(2a+2bx^n)n}\sqrt{(a+bx^n)^2}+3\frac{\sqrt{(a+bx^n)^2}a^2bx^n}{(a+bx^n)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*ln(x)+1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b
^3/n*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^2/n*(x^n)^2+3*((a+b*x^n)
^2)^(1/2)/(a+b*x^n)*a^2*b/n*x^n
```

Maxima [A] time = 0.983491, size = 58, normalized size = 0.3

$$a^3\log(x)+\frac{2b^3x^{3n}+9ab^2x^{2n}+18a^2bx^n}{6n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

Fricas [A] time = 1.56861, size = 99, normalized size = 0.51

$$\frac{6\,a^3n\log\left(x\right)+2\,b^3x^{3\,n}+9\,ab^2x^{2\,n}+18\,a^2bx^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x,x)
```

```
[Out] Integral(((a + b*x**n)**2)**(3/2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x, x)
```

3.527 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=212

$$-\frac{3a^2b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}$$

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (3*a^2*b^2*x
^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n)) -
(3*a*b^3*x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - 2*n)*(a*b
+ b^2*x^n)) - (b^4*x^(-1 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 -
3*n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.0698802, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 270}

$$-\frac{3a^2b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (3*a^2*b^2*x
^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n)) -
(3*a*b^3*x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - 2*n)*(a*b
+ b^2*x^n)) - (b^4*x^(-1 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 -
3*n)*(a*b + b^2*x^n))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(ab+b^2x^n\right)^3}{x^2}\,dx}{b^2\left(ab+b^2x^n\right)}$$

$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{a^3b^3}{x^2}+3a^2b^4x^{-2+n}+3ab^5x^{2(-1+n)}+b^6x^{-2+3n}\right)dx}{b^2\left(ab+b^2x^n\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)} - \frac{3a^2b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)} - \frac{3ab^3x^{-1+2n}\sqrt{a^2+2ab}}{(1-2n)\left(ab+b\right.}$$

Mathematica [A] time = 0.0941215, size = 124, normalized size = 0.58

$$\frac{\sqrt{\left(a+bx^n\right)^2}\left(3a^2b\left(6n^2-5n+1\right)x^n+a^3\left(-6n^3+11n^2-6n+1\right)+3ab^2\left(3n^2-4n+1\right)x^{2n}+b^3\left(2n^2-3n+1\right)x^{3n}\right)}{(n-1)(2n-1)(3n-1)x\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(1 - 6*n + 11*n^2 - 6*n^3) + 3*a^2*b*(1 - 5*n + 6
*n^2)*x^n + 3*a*b^2*(1 - 4*n + 3*n^2)*x^(2*n) + b^3*(1 - 3*n + 2*n^2)*x^(3*
n)))/((-1 + n)*(-1 + 2*n)*(-1 + 3*n)*x*(a + b*x^n))
```

Maple [A] time = 0.024, size = 147, normalized size = 0.7

$$-\frac{a^3}{\left(a+bx^n\right)x}\sqrt{\left(a+bx^n\right)^2}+\frac{b^3\left(x^n\right)^3}{\left(a+bx^n\right)\left(-1+3n\right)x}\sqrt{\left(a+bx^n\right)^2}+3\frac{\sqrt{\left(a+bx^n\right)^2}ab^2\left(x^n\right)^2}{\left(a+bx^n\right)\left(-1+2n\right)x}+3\frac{\sqrt{\left(a+bx^n\right)^2}a^2bx^n}{\left(a+bx^n\right)\left(-1+n\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x)
```

```
[Out] -((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+3*n)
*b^3/x*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+2*n)*a*b^2/x*(x^n)^2+3*(
(a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*a^2*b/x*x^n
```

Maxima [A] time = 1.03693, size = 136, normalized size = 0.64

$$\frac{\left(2n^2-3n+1\right)b^3x^{3n}+3\left(3n^2-4n+1\right)ab^2x^{2n}+3\left(6n^2-5n+1\right)a^2bx^n-\left(6n^3-11n^2+6n-1\right)a^3}{\left(6n^3-11n^2+6n-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*b^3*x^(3*n) + 3*(3*n^2 - 4*n + 1)*a*b^2*x^(2*n) + 3*(6*n
^2 - 5*n + 1)*a^2*b*x^n - (6*n^3 - 11*n^2 + 6*n - 1)*a^3)/((6*n^3 - 11*n^2
+ 6*n - 1)*x)
```

Fricas [A] time = 1.58915, size = 270, normalized size = 1.27

$$-\frac{6\,a^3n^3-11\,a^3n^2+6\,a^3n-a^3-\left(2\,b^3n^2-3\,b^3n+b^3\right)x^{3\,n}-3\left(3\,ab^2n^2-4\,ab^2n+ab^2\right)x^{2\,n}-3\left(6\,a^2bn^2-5\,a^2bn+a^2b\right)x^n}{\left(6\,n^3-11\,n^2+6\,n-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(6*a^3*n^3 - 11*a^3*n^2 + 6*a^3*n - a^3 - (2*b^3*n^2 - 3*b^3*n + b^3)*x^(3
*n) - 3*(3*a*b^2*n^2 - 4*a*b^2*n + a*b^2)*x^(2*n) - 3*(6*a^2*b*n^2 - 5*a^2*
b*n + a^2*b)*x^n)/((6*n^3 - 11*n^2 + 6*n - 1)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx^n)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**n)**2)**(3/2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^2, x)
```

3.528 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=218

$$-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\left(ab+b^2x^n\right)}-\frac{3a^2b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+2abx}}{2x^2\,(a+}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) - (3*a*b^3*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 - n)*x^(2*(1 - n))*(a*b + b^2*x^n
)) - (3*a^2*b^2*x^(-2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a
*b + b^2*x^n)) - (b^4*x^(-2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2
 - 3*n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.0698201, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 270}

$$-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\left(ab+b^2x^n\right)}-\frac{3a^2b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+2abx}}{2x^2\,(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) - (3*a*b^3*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 - n)*x^(2*(1 - n))*(a*b + b^2*x^n
)) - (3*a^2*b^2*x^(-2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a
*b + b^2*x^n)) - (b^4*x^(-2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2
 - 3*n)*(a*b + b^2*x^n))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(ab+b^2x^n\right)^3}{x^3}\,dx}{b^2\left(ab+b^2x^n\right)}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{a^3b^3}{x^3}+3a^2b^4x^{-3+n}+b^6x^{3(-1+n)}+3ab^5x^{-3+2n}\right)dx}{b^2\left(ab+b^2x^n\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)} - \frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\left(ab+b^2x^n\right)} - \frac{3a^2b^2x^{-2+n}\sqrt{a^2+2abx^n}}{(2-n)\left(ab+b^2x\right.}$$

Mathematica [A] time = 0.102713, size = 124, normalized size = 0.57

$$\frac{\sqrt{\left(a+bx^n\right)^2}\left(6a^2b\left(3n^2-5n+2\right)x^n+a^3\left(-3n^3+11n^2-12n+4\right)+3ab^2\left(3n^2-8n+4\right)x^{2n}+2b^3\left(n^2-3n+2\right)x^{3n}\right)}{2(n-2)(n-1)(3n-2)x^2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(4 - 12*n + 11*n^2 - 3*n^3) + 6*a^2*b*(2 - 5*n +
3*n^2)*x^n + 3*a*b^2*(4 - 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 - 3*n + n^2)*x^(3
*n)))/(2*(-2 + n)*(-1 + n)*(-2 + 3*n)*x^2*(a + b*x^n))
```

Maple [A] time = 0.023, size = 145, normalized size = 0.7

$$-\frac{a^3}{(2a+2bx^n)x^2}\sqrt{(a+bx^n)^2}+\frac{b^3(x^n)^3}{(a+bx^n)(-2+3n)x^2}\sqrt{(a+bx^n)^2}+\frac{3ab^2(x^n)^2}{(2a+2bx^n)(-1+n)x^2}\sqrt{(a+bx^n)^2}+3\frac{\sqrt{(a+b}}{(a+bx^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x)
```

```
[Out] -1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-
2+3*n)*b^3/x^2*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*a*b^2/x^2*(
x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-2+n)*a^2*b/x^2*x^n
```

Maxima [A] time = 1.01281, size = 136, normalized size = 0.62

$$\frac{2\left(n^2-3n+2\right)b^3x^{3n}+3\left(3n^2-8n+4\right)ab^2x^{2n}+6\left(3n^2-5n+2\right)a^2bx^n-\left(3n^3-11n^2+12n-4\right)a^3}{2\left(3n^3-11n^2+12n-4\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(n^2 - 3*n + 2)*b^3*x^(3*n) + 3*(3*n^2 - 8*n + 4)*a*b^2*x^(2*n) + 6*
(3*n^2 - 5*n + 2)*a^2*b*x^n - (3*n^3 - 11*n^2 + 12*n - 4)*a^3)/((3*n^3 - 11
*n^2 + 12*n - 4)*x^2)
```

Fricas [A] time = 1.60612, size = 292, normalized size = 1.34

$$-\frac{3\,a^3n^3 - 11\,a^3n^2 + 12\,a^3n - 4\,a^3 - 2\,\left(b^3n^2 - 3\,b^3n + 2\,b^3\right)x^{3\,n} - 3\,\left(3\,ab^2n^2 - 8\,ab^2n + 4\,ab^2\right)x^{2\,n} - 6\,\left(3\,a^2bn^2 - 5\,a^2bn\right.}{2\,\left(3\,n^3 - 11\,n^2 + 12\,n - 4\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^3*n^3 - 11*a^3*n^2 + 12*a^3*n - 4*a^3 - 2*(b^3*n^2 - 3*b^3*n + 2*
b^3)*x^(3*n) - 3*(3*a*b^2*n^2 - 8*a*b^2*n + 4*a*b^2)*x^(2*n) - 6*(3*a^2*b*n
^2 - 5*a^2*b*n + 2*a^2*b)*x^n)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^n\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**n)**2)**(3/2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^3, x)
```

3.529 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=76

$$\frac{(dx)^{m+1}(a+bx^n)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -
((b*x^n)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0389876, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1355, 364}

$$\frac{(dx)^{m+1}(a+bx^n)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -
((b*x^n)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(ab+b^2x^n\right)\int \frac{(dx)^m}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{(dx)^{1+m}(a+bx^n)\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.0224633, size = 62, normalized size = 0.82

$$\frac{x(dx)^m(a+bx^n)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+1}{n}+1;-\frac{bx^n}{a}\right)}{a(m+1)\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, 1 + (1 + m)/n, -((b*
x^n)/a)])/(a*(1 + m)*Sqrt[(a + b*x^n)^2])
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{a^2 + 2\,abx^n + b^2x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{(a + bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt((a + b*x**n)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.530 $\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3(a+bx^n)\,_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0267265, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$\frac{x^3(a+bx^n)\,_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(ab+b^2x^n\right)\int \frac{x^2}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^3(a+bx^n)\,_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.0147394, size = 53, normalized size = 0.83

$$\frac{x^3(a+bx^n)\,_2F_1\left(1,\frac{3}{n};1+\frac{3}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, 1 + 3/n, -((b*x^n)/a)])/(3*a*Sqr
t[(a + b*x^n)^2])
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{a^2 + 2\,ab x^n + b^2 x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{b^2 x^{2\,n} + 2\,ab x^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\sqrt{b^2 x^{2\,n} + 2\,ab x^n + a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{(a + bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt((a + b*x**n)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{b^2 x^{2n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.531 $\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2 (a + bx^n)\ {}_2F_1\left(1, \frac{2}{n}; \frac{n+2}{n}; -\frac{bx^n}{a}\right)}{2a\sqrt{a^2 + 2abx^n + b^2x^{2n}}}$$

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0218741, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 364}

$$\frac{x^2 (a + bx^n)\ {}_2F_1\left(1, \frac{2}{n}; \frac{n+2}{n}; -\frac{bx^n}{a}\right)}{2a\sqrt{a^2 + 2abx^n + b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx &= \frac{\left(ab + b^2x^n\right)\int \frac{x}{ab+b^2x^n}\,dx}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\\ &= \frac{x^2 (a + bx^n)\ {}_2F_1\left(1, \frac{2}{n}; \frac{2+n}{n}; -\frac{bx^n}{a}\right)}{2a\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.0136308, size = 53, normalized size = 0.83

$$\frac{x^2 (a + bx^n)\ {}_2F_1\left(1, \frac{2}{n}; 1 + \frac{2}{n}; -\frac{bx^n}{a}\right)}{2a\sqrt{(a + bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a*Sqr
t[(a + b*x^n)^2])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x \frac{1}{\sqrt{a^2 + 2\,abx^n + b^2x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(a + bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x/sqrt((a + b*x**n)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{b^2 x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.532 $\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=55

$$\frac{x(a+bx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0173405, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1343, 245}

$$\frac{x(a+bx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(2ab+2b^2x^n\right)\int \frac{1}{2ab+2b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x(a+bx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.0103412, size = 44, normalized size = 0.8

$$\frac{x(a+bx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*S
qrt[(a + b*x^n)^2])
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2 + 2\,abx^n + b^2x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.533 $\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=85

$$\frac{\log(x)\left(a+bx^n\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{\left(a+bx^n\right)\log\left(a+bx^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] ((a + b*x^n)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - ((a + b*x^n)
*Log[a + b*x^n])/(a*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0361088, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1355, 266, 36, 29, 31}

$$\frac{\log(x)\left(a+bx^n\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{\left(a+bx^n\right)\log\left(a+bx^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] ((a + b*x^n)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - ((a + b*x^n)
*Log[a + b*x^n])/(a*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{\left(ab+b^2x^n\right)\int \frac{1}{x\left(ab+b^2x^n\right)}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(ab+b^2x^n\right)\operatorname{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)}\,dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(ab+b^2x^n\right)\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{abn\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{\left(b\left(ab+b^2x^n\right)\right)\operatorname{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, x^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{(a+bx^n)\log(x)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{(a+bx^n)\log\left(a+bx^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] time = 0.0148707, size = 42, normalized size = 0.49

$$\frac{(a+bx^n)\left(n\log(x)-\log\left(a+bx^n\right)\right)}{an\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] ((a + b*x^n)*(n*Log[x] - Log[a + b*x^n]))/(a*n*Sqrt[(a + b*x^n)^2])
```

Maple [A] time = 0.018, size = 66, normalized size = 0.8

$$\frac{\ln(x)}{(a+bx^n)\,a}\sqrt{(a+bx^n)^2} - \frac{1}{(a+bx^n)\,an}\sqrt{(a+bx^n)^2}\ln\left(x^n+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*ln(x)/a-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a/n*ln(
x^n+a/b)
```

Maxima [A] time = 0.993845, size = 36, normalized size = 0.42

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{bx^n+a}{b}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] log(x)/a - log((b*x^n + a)/b)/(a*n)
```

Fricas [A] time = 1.59713, size = 47, normalized size = 0.55

$$\frac{n\log\left(x\right) - \log\left(bx^{n} + a\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] (n*log(x) - log(b*x^n + a))/(a*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\left(a + bx^{n}\right)^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a + b*x**n)**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^{2}x^{2\,n} + 2\,abx^{n} + a^{2}}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x), x)
```

3.534 $\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=65

$$-\frac{(a+bx^n)\,_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(
a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rubi [A] time = 0.0258666, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$-\frac{(a+bx^n)\,_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(
a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(ab+b^2x^n\right)\int \frac{1}{x^2\left(ab+b^2x^n\right)}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= -\frac{(a+bx^n)\,_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.0140003, size = 51, normalized size = 0.78

$$-\frac{(a+bx^n)\,_2F_1\left(1,-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), 1 - n^(-1), -((b*x^n)/a)])/(a*
x*Sqrt[(a + b*x^n)^2]))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^{2n}+2abx^n+a^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^{2n}+2abx^n+a^2}}{b^2x^2x^{2n}+2abx^2x^n+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^2*x^2*x^(2*n) + 2*a*b*x^2*x
^n + a^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((a + b*x**n)**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

3.535 $\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=67

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -((a + b*x^n)*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a*
x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0261127, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -((a + b*x^n)*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a*
x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(ab+b^2x^n\right)\int \frac{1}{x^3\left(ab+b^2x^n\right)}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= -\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.0134131, size = 53, normalized size = 0.79

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -((a + b*x^n)*Hypergeometric2F1[1, -2/n, 1 - 2/n, -((b*x^n)/a)])/(2*a*x^2*S
qrt[(a + b*x^n)^2])
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}} \frac{1}{\sqrt{a^{2}+2\,ab x^{n}+b^{2} x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^{2} x^{2\,n}+2\,ab x^{n}+a^{2}}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b^{2} x^{2\,n}+2\,ab x^{n}+a^{2}}}{b^{2} x^{3} x^{2\,n}+2\,ab x^{3} x^{n}+a^{2} x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^2*x^3*x^(2*n) + 2*a*b*x^3*x
^n + a^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\sqrt{\left(a+bx^{n}\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((a + b*x**n)**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

3.536 $\int \frac{(dx)^m}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{(dx)^{m+1}(a+bx^n)\,{}_2F_1\left(3,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -
((b*x^n)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0372914, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1355, 364}

$$\frac{(dx)^{m+1}(a+bx^n)\,{}_2F_1\left(3,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -
((b*x^n)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{(dx)^m}{\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{(dx)^{1+m}(a+bx^n)\,{}_2F_1\left(3,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.0211479, size = 61, normalized size = 0.8

$$\frac{x(dx)^m(a+bx^n)\,{}_2F_1\left(3,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^3(m+1)\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((b*
x^n)/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^n)^2])
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (dx)^m \left(a^2 + 2\,abx^n + b^2x^{2\,n}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(m^2 - m(3\,n-2) + 2\,n^2 - 3\,n + 1\right)d^m \int \frac{x^m}{2\left(a^2bn^2x^n + a^3n^2\right)}\,dx - \frac{ad^m(m-3\,n+1)xx^m + bd^m(m-2\,n+1)xe^{(m\log(x)+n\log}}{2\left(a^2b^2n^2x^{2\,n} + 2\,a^3bn^2x^n + a^4n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*d^m*integrate(1/2*x^m/(a^2*b*n^2*x^n
+ a^3*n^2), x) - 1/2*(a*d^m*(m - 3*n + 1)*x*x^m + b*d^m*(m - 2*n + 1)*x*e^(
m*log(x) + n*log(x)))/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}\,(dx)^m}{b^4x^{4\,n} + 4\,a^2b^2x^{2\,n} + 4\,a^3bx^n + a^4 + 2\left(2\,ab^3x^n + a^2b^2\right)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x)^m/(b^4*x^(4*n) + 4*a^2*b
^2*x^(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**n)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

3.537 $\int \frac{x^2}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}} \, dx$

Optimal. Leaf size=64

$$\frac{x^3 \left(a+bx^n\right) {}_2F_1\left(3,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[3, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^3
*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0266486, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$\frac{x^3 \left(a+bx^n\right) {}_2F_1\left(3,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[3, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^3
*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}} \, dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{x^2}{\left(ab+b^2x^n\right)^3} \, dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^3 \left(a+bx^n\right) {}_2F_1\left(3,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.0178028, size = 55, normalized size = 0.86

$$\frac{x^3 \left(a+bx^n\right)^3 {}_2F_1\left(3,\frac{3}{n};1+\frac{3}{n};-\frac{bx^n}{a}\right)}{3a^3\left(\left(a+bx^n\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(a + b*x^n)^3*Hypergeometric2F1[3, 3/n, 1 + 3/n, -((b*x^n)/a)])/(3*a^3
*((a + b*x^n)^2)^(3/2))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int x^{2} \left(a^{2}+2 abx^{n}+b^{2} x^{2 n}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(2 n^{2}-9 n+9\right) \int \frac{x^{2}}{2 \left(a^{2} b n^{2} x^{n}+a^{3} n^{2}\right)} dx + \frac{b(2 n-3) x^{3} x^{n}+3 a(n-1) x^{3}}{2 \left(a^{2} b^{2} n^{2} x^{2 n}+2 a^{3} b n^{2} x^{n}+a^{4} n^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (2*n^2 - 9*n + 9)*integrate(1/2*x^2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b*
(2*n - 3)*x^3*x^n + 3*a*(n - 1)*x^3)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n
 + a^4*n^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^{2} x^{2 n}+2 abx^{n}+a^{2}} x^{2}}{b^{4} x^{4 n}+4 a^{2} b^{2} x^{2 n}+4 a^{3} bx^{n}+a^{4}+2 \left(2 ab^{3} x^{n}+a^{2} b^{2}\right) x^{2 n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2/(b^4*x^(4*n) + 4*a^2*b^2*x
^(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(\left(a+bx^{n}\right)^{2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**n)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

3.538 $\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\left(a+bx^n\right)\,_2F_1\left(3,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[3, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^3
*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0223194, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 364}

$$\frac{x^2\left(a+bx^n\right)\,_2F_1\left(3,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[3, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^3
*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{x}{\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^2\left(a+bx^n\right)\,_2F_1\left(3,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.0167804, size = 55, normalized size = 0.86

$$\frac{x^2\left(a+bx^n\right)^3\,_2F_1\left(3,\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2a^3\left(\left(a+bx^n\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(a + b*x^n)^3*Hypergeometric2F1[3, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a^3
*((a + b*x^n)^2)^(3/2))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int x \left(a^{2}+2 a b x^{n}+b^{2} x^{2 n}\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(n^{2}-3 n+2\right) \int \frac{x}{a^{2} b n^{2} x^{n}+a^{3} n^{2}}\, d x+\frac{2 b(n-1) x^{2} x^{n}+a(3 n-2) x^{2}}{2\left(a^{2} b^{2} n^{2} x^{2 n}+2 a^{3} b n^{2} x^{n}+a^{4} n^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (n^2 - 3*n + 2)*integrate(x/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(2*b*(n - 1
)*x^2*x^n + a*(3*n - 2)*x^2)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n
^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^{2} x^{2 n}+2 a b x^{n}+a^{2}}\, x}{b^{4} x^{4 n}+4 a^{2} b^{2} x^{2 n}+4 a^{3} b x^{n}+a^{4}+2\left(2 a b^{3} x^{n}+a^{2} b^{2}\right) x^{2 n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x/(b^4*x^(4*n) + 4*a^2*b^2*x^(
2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\left(a+b x^{n}\right)^{2}\right)^{\frac{3}{2}}}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**n)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

3.539 $\int \frac{1}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{x\,(a+bx^n)^3\;{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}$$

```
[Out] (x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a
^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))
```

Rubi [A] time = 0.0158974, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1343, 245}

$$\frac{x\,(a+bx^n)^3\;{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a
^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(2ab+2b^2x^n\right)^3\int \frac{1}{\left(2ab+2b^2x^n\right)^3}\,dx}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\\ &= \frac{x\,(a+bx^n)^3\;{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0117737, size = 46, normalized size = 0.81

$$\frac{x\,(a+bx^n)^3\;{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a
^3*((a + b*x^n)^2)^(3/2))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \left(a^2 + 2\,abx^n + b^2x^{2\,n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(2\,n^2 - 3\,n + 1\right)\int \frac{1}{2\left(a^2bn^2x^n + a^3n^2\right)}\,dx + \frac{b(2\,n-1)xx^n + a(3\,n-1)x}{2\left(a^2b^2n^2x^{2\,n} + 2\,a^3bn^2x^n + a^4n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (2*n^2 - 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b*(2*n
 - 1)*x*x^n + a*(3*n - 1)*x)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n
^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}{b^4x^{4\,n} + 4\,a^2b^2x^{2\,n} + 4\,a^3bx^n + a^4 + 2\left(2\,ab^3x^n + a^2b^2\right)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^(4*n) + 4*a^2*b^2*x^(2*
n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2abx^n + b^2x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2 x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-3/2), x)
```

3.540 $\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=159

$$\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\log(x)\left(a+bx^n\right)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{\left(a+bx^n\right)\log\left(a+bx^n\right)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/(a^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) + 1/(2*a*n*(a + b*x^n)*Sqrt[a
^2 + 2*a*b*x^n + b^2*x^(2*n)]) + ((a + b*x^n)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)]) - ((a + b*x^n)*Log[a + b*x^n])/(a^3*n*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0830614, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1355, 266, 44}

$$\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\log(x)\left(a+bx^n\right)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{\left(a+bx^n\right)\log\left(a+bx^n\right)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] 1/(a^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) + 1/(2*a*n*(a + b*x^n)*Sqrt[a
^2 + 2*a*b*x^n + b^2*x^(2*n)]) + ((a + b*x^n)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)]) - ((a + b*x^n)*Log[a + b*x^n])/(a^3*n*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{1}{x\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^3}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\left(a+bx^n\right)\log(x)}{a^3\sqrt{a^2+2abx^n+b^2}}$$

Mathematica [A] time = 0.0673037, size = 76, normalized size = 0.48

$$\frac{\left(a+bx^n\right)^3\left(\frac{1}{a^2(a+bx^n)}-\frac{\log(a+bx^n)}{a^3}+\frac{n\log(x)}{a^3}+\frac{1}{2a(a+bx^n)^2}\right)}{n\left(\left(a+bx^n\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] ((a + b*x^n)^3*(1/(2*a*(a + b*x^n)^2) + 1/(a^2*(a + b*x^n)) + (n*Log[x])/a^
3 - Log[a + b*x^n]/a^3))/(n*((a + b*x^n)^2)^(3/2))
```

Maple [A] time = 0.02, size = 104, normalized size = 0.7

$$\frac{\ln(x)}{(a+bx^n)a^3}\sqrt{(a+bx^n)^2}+\frac{2bx^n+3a}{2(a+bx^n)^3a^2n}\sqrt{(a+bx^n)^2}-\frac{1}{(a+bx^n)a^3n}\sqrt{(a+bx^n)^2}\ln\left(x^n+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*ln(x)/a^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^3
*(2*b*x^n+3*a)/a^2/n-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a^3/n*ln(x^n+a/b)
```

Maxima [A] time = 0.962489, size = 95, normalized size = 0.6

$$\frac{2bx^n+3a}{2\left(a^2b^2nx^{2n}+2a^3bnx^n+a^4n\right)}+\frac{\log(x)}{a^3}-\frac{\log\left(\frac{bx^n+a}{b}\right)}{a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) + log(x)/a^
3 - log((b*x^n + a)/b)/(a^3*n)
```

Fricas [A] time = 1.86974, size = 244, normalized size = 1.53

$$\frac{2\,b^{2}nx^{2\,n}\log\left(x\right)+2\,a^{2}n\log\left(x\right)+3\,a^{2}+2\,(2\,abn\log\left(x\right)+ab)x^{n}-2\,\left(b^{2}x^{2\,n}+2\,abx^{n}+a^{2}\right)\log\left(bx^{n}+a\right)}{2\,\left(a^{3}b^{2}nx^{2\,n}+2\,a^{4}bnx^{n}+a^{5}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n*log(x) +
a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/(a^3*b^2*n*x^(
2*n) + 2*a^4*b*n*x^n + a^5*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\left(a+bx^{n}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**n)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^{2}x^{2\,n}+2\,abx^{n}+a^{2}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x), x)
```

3.541 $\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[3, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(
a^3*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rubi [A] time = 0.0259398, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$-\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[3, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(
a^3*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{1}{x^2\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= -\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.0153982, size = 53, normalized size = 0.82

$$-\frac{(a+bx^n)^3\,{}_2F_1\left(3,-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3x\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -(((a + b*x^n)^3*Hypergeometric2F1[3, -n^(-1), 1 - n^(-1), -((b*x^n)/a)])/(
a^3*x*((a + b*x^n)^2)^(3/2)))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a^2+2\,ab x^n+b^2 x^{2\,n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(2\,n^2+3\,n+1\right)\int \frac{1}{2\left(a^2bn^2x^2x^n+a^3n^2x^2\right)}\,dx+\frac{b(2\,n+1)x^n+a(3\,n+1)}{2\left(a^2b^2n^2xx^{2\,n}+2\,a^3bn^2xx^n+a^4n^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (2*n^2 + 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^2*x^n + a^3*n^2*x^2), x) + 1/2
*(b*(2*n + 1)*x^n + a*(3*n + 1))/(a^2*b^2*n^2*x*x^(2*n) + 2*a^3*b*n^2*x*x^n
 + a^4*n^2*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}{b^4x^2x^{4\,n}+4\,a^2b^2x^2x^{2\,n}+4\,a^3bx^2x^n+a^4x^2+2\left(2\,ab^3x^2x^n+a^2b^2x^2\right)x^{2\,n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^2*x^(4*n) + 4*a^2*b^2*x
^2*x^(2*n) + 4*a^3*b*x^2*x^n + a^4*x^2 + 2*(2*a*b^3*x^2*x^n + a^2*b^2*x^2)*
x^(2*n)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**n)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2), x)
```

3.542 $\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{(a+bx^n)\ {}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -((a + b*x^n)*Hypergeometric2F1[3, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a^
3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0275665, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$-\frac{(a+bx^n)\ {}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -((a + b*x^n)*Hypergeometric2F1[3, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a^
3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{1}{x^3\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= -\frac{(a+bx^n)\ {}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.0159934, size = 55, normalized size = 0.82

$$-\frac{(a+bx^n)^3\ {}_2F_1\left(3,-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -((a + b*x^n)^3*Hypergeometric2F1[3, -2/n, 1 - 2/n, -((b*x^n)/a)])/(2*a^3*x
^2*((a + b*x^n)^2)^(3/2))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a^2+2\,abx^n+b^2x^{2\,n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(n^2+3\,n+2\right)\int \frac{1}{a^2bn^2x^3x^n+a^3n^2x^3}\,dx+\frac{2\,b(n+1)x^n+a(3\,n+2)}{2\left(a^2b^2n^2x^2x^{2\,n}+2\,a^3bn^2x^2x^n+a^4n^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (n^2 + 3*n + 2)*integrate(1/(a^2*b*n^2*x^3*x^n + a^3*n^2*x^3), x) + 1/2*(2*
b*(n + 1)*x^n + a*(3*n + 2))/(a^2*b^2*n^2*x^2*x^(2*n) + 2*a^3*b*n^2*x^2*x^n
 + a^4*n^2*x^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}{b^4x^3x^{4\,n}+4\,a^2b^2x^3x^{2\,n}+4\,a^3bx^3x^n+a^4x^3+2\left(2\,ab^3x^3x^n+a^2b^2x^3\right)x^{2\,n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^3*x^(4*n) + 4*a^2*b^2*x
^3*x^(2*n) + 4*a^3*b*x^3*x^n + a^4*x^3 + 2*(2*a*b^3*x^3*x^n + a^2*b^2*x^3)*
x^(2*n)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**n)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^3), x)
```

3.543 $\int \left(a^2+b^2 x^{-\frac{2}{1+2p}}+2 a b x^{-\frac{1}{1+2p}}\right)^p \, dx$

Optimal. Leaf size=52

$$\frac{x\left(a+bx^{\frac{1}{-2p-1}}\right)\left(a^2+2abx^{\frac{1}{-2p-1}}+b^2x^{-\frac{2}{2p+1}}\right)^p}{a}$$

```
[Out] (x*(a + b*x^(-1 - 2*p)^(-1))*(a^2 + 2*a*b*x^(-1 - 2*p)^(-1) + b^2/x^(2/(1 +
 2*p)))^p)/a
```

Rubi [A] time = 0.0195689, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1343, 191}

$$\frac{x\left(a+bx^{\frac{1}{-2p-1}}\right)\left(a^2+2abx^{\frac{1}{-2p-1}}+b^2x^{-\frac{2}{2p+1}}\right)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

```
[Out] (x*(a + b*x^(-1 - 2*p)^(-1))*(a^2 + 2*a*b*x^(-1 - 2*p)^(-1) + b^2/x^(2/(1 +
 2*p)))^p)/a
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\,dx &= \left(\left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\left(2ab+2b^2x^{-\frac{1}{1+2p}}\right)^{-2p}\right)\int \left(2ab+2b^2x^{-\frac{1}{1+2p}}\right)^{2p}\,dx\\ &= \frac{x\left(a+bx^{\frac{1}{-1-2p}}\right)\left(a^2+2abx^{\frac{1}{-1-2p}}+b^2x^{-\frac{2}{1+2p}}\right)^p}{a}\end{aligned}$$

Mathematica [A] time = 0.0240551, size = 58, normalized size = 1.12

$$\frac{x^{\frac{2p}{2p+1}}\left(ax^{\frac{1}{2p+1}}+b\right)\left(x^{-\frac{2}{2p+1}}\left(ax^{\frac{1}{2p+1}}+b\right)^2\right)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

```
[Out] (x^((2*p)/(1 + 2*p))*(b + a*x^(1 + 2*p)^(-1))*((b + a*x^(1 + 2*p)^(-1))^2/x
^(2/(1 + 2*p)))^p)/a
```

Maple [F] time = 0.243, size = 0, normalized size = 0.

$$\int \left(a^2 + b^2 \left(x^{2\,(1+2\,p)^{-1}}\right)^{-1} + 2\,\frac{ab}{x^{(1+2\,p)^{-1}}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x)
```

```
[Out] int((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + \frac{b^2}{x^{\frac{2}{2\,p+1}}} + \frac{2\,ab}{x^{\left(\frac{1}{2\,p+1}\right)}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="m
axima")
```

```
[Out] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p, x)
```

Fricas [A] time = 1.65135, size = 163, normalized size = 3.13

$$\frac{\left(axx^{\left(\frac{1}{2\,p+1}\right)} + bx\right)\left(\frac{a^2x^{\frac{2}{2\,p+1}}+2\,abx^{\left(\frac{1}{2\,p+1}\right)}+b^2}{x^{\frac{2}{2\,p+1}}}\right)^p}{ax^{\left(\frac{1}{2\,p+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="f
ricas")
```

```
[Out] (a*x*x^(1/(2*p + 1)) + b*x)*((a^2*x^(2/(2*p + 1)) + 2*a*b*x^(1/(2*p + 1)) +
 b^2)/x^(2/(2*p + 1)))^p/(a*x^(1/(2*p + 1)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/(x**(2/(1+2*p)))+2*a*b/(x**(1/(1+2*p))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + \frac{b^2}{x^{\frac{2}{2p+1}}} + \frac{2ab}{x^{\left(\frac{1}{2p+1}\right)}}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="g
iac")
```

```
[Out] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p, x)
```

3.544 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}\,dx$

Optimal. Leaf size=43

$$\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}}{a}$$

```
[Out] (x*(a + b*x^n))/(a*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((1 + n)/(2*n)))
```

Rubi [A] time = 0.0134285, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1343, 191}

$$\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n))/(a*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((1 + n)/(2*n)))
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}\,dx &= \left(\left(2ab+2b^2x^n\right)^{\frac{1+n}{n}}\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}\right)\int \left(2ab+2b^2x^n\right)^{-\frac{1+n}{n}}\,dx\\ &= \frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}}{a}\end{aligned}$$

Mathematica [A] time = 0.0609589, size = 32, normalized size = 0.74

$$\frac{x\left(a+bx^n\right)\left(\left(a+bx^n\right)^2\right)^{-\frac{n+1}{2n}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n))/(a*((a + b*x^n)^2)^((1 + n)/(2*n)))
```

Maple [A] time = 0.035, size = 51, normalized size = 1.2

$$\left(x+\frac{bx e^{n\ln(x)}}{a}\right)\left(e^{\frac{(1+n)\ln\left(a^2+2abe^{n\ln(x)}+b^2\left(e^{n\ln(x)}\right)^2\right)}{2n}}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x)
```

```
[Out] (x+b/a*x*exp(n*ln(x)))/exp(1/2*(1+n)/n*ln(a^2+2*a*b*exp(n*ln(x))+b^2*exp(n*
ln(x))^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^{2n}+2\,abx^n+a^2\right)^{\frac{n+1}{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

Fricas [A] time = 1.64416, size = 93, normalized size = 2.16

$$\frac{bxx^n+ax}{\left(b^2x^{2n}+2\,abx^n+a^2\right)^{\frac{n+1}{2n}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="frica
s")
```

```
[Out] (b*x*x^n + a*x)/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)*a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+n)/n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{n+1}{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="giac"
)
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

3.545 $\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p dx$

Optimal. Leaf size=130

$$\frac{2(p+1)x\left(a+bx^{-\frac{1}{2(p+1)}}\right)\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a(2p+1)}-\frac{x\left(a+bx^{-\frac{1}{2(p+1)}}\right)^2\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a^2(2p+1)}$$

```
[Out] (2*(1 + p)*x*(a + b/x^(1/(2*(1 + p))))*(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/
x^(1/(2*(1 + p))))^p)/(a*(1 + 2*p)) - (x*(a + b/x^(1/(2*(1 + p))))^2*(a^2 +
 b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p)/(a^2*(1 + 2*p))
```

Rubi [A] time = 0.0552074, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {1343, 192, 191}

$$\frac{2(p+1)x\left(a+bx^{-\frac{1}{2(p+1)}}\right)\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a(2p+1)}-\frac{x\left(a+bx^{-\frac{1}{2(p+1)}}\right)^2\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

```
[Out] (2*(1 + p)*x*(a + b/x^(1/(2*(1 + p))))*(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/
x^(1/(2*(1 + p))))^p)/(a*(1 + 2*p)) - (x*(a + b/x^(1/(2*(1 + p))))^2*(a^2 +
 b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p)/(a^2*(1 + 2*p))
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p dx=\left(\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\left(2ab+2b^2x^{-\frac{1}{2(1+p)}}\right)^{-2p}\right)\int\left(2ab+2b^2x^{-\frac{1}{2(1+p)}}\right)^{2p}$$

$$=\frac{2(1+p)x\left(a+bx^{-\frac{1}{2(1+p)}}\right)\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a(1+2p)}-\frac{\left(\left(a^2+b^2x^{-\frac{1}{1+p}}+2ab\right.\right.}{}$$

$$=\frac{2(1+p)x\left(a+bx^{-\frac{1}{2(1+p)}}\right)\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a(1+2p)}-\frac{x\left(a+bx^{-\frac{1}{2(1+p)}}\right)^2\left(a^2\right.}{a^2(}$$

Mathematica [A] time = 0.0647322, size = 80, normalized size = 0.62

$$\frac{x^{\frac{p}{p+1}}\left(ax^{\frac{1}{2p+2}}+b\right)\left(x^{-\frac{1}{p+1}}\left(ax^{\frac{1}{2p+2}}+b\right)^2\right)^p\left(a(2p+1)x^{\frac{1}{2p+2}}-b\right)}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

```
[Out] (x^(p/(1 + p))*(b + a*x^(2 + 2*p)^(-1))*((b + a*x^(2 + 2*p)^(-1))^2/x^(1 +
p)^(-1))^p*(-b + a*(1 + 2*p)*x^(2 + 2*p)^(-1)))/(a^2*(1 + 2*p))
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int\left(a^2+\frac{b^2}{x^{(1+p)^{-1}}}+2\,ab\left(x^{1/2\,(1+p)^{-1}}\right)^{-1}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x)
```

```
[Out] int((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2+\frac{2\,ab}{x^{\frac{1}{2\,(p+1)}}}+\frac{b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="max
ima")
```

```
[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)
```

Fricas [A] time = 1.69303, size = 231, normalized size = 1.78

$$\frac{\left(2\,abpxx^{\frac{1}{2\,(p+1)}}-b^{2}x+\left(2\,a^{2}p+a^{2}\right)xx^{\left(\frac{1}{p+1}\right)}\right)\left(\frac{2\,abx^{\frac{1}{2\,(p+1)}}+a^{2}x^{\left(\frac{1}{p+1}\right)}+b^{2}}{x^{\left(\frac{1}{p+1}\right)}}\right)^{p}}{\left(2\,a^{2}p+a^{2}\right)x^{\left(\frac{1}{p+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="fri
cas")
```

```
[Out] (2*a*b*p*x*x^(1/2/(p + 1)) - b^2*x + (2*a^2*p + a^2)*x*x^(1/(p + 1)))*((2*a
*b*x^(1/2/(p + 1)) + a^2*x^(1/(p + 1)) + b^2)/x^(1/(p + 1)))^p/((2*a^2*p +
a^2)*x^(1/(p + 1)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/(x**(1/(1+p)))+2*a*b/(x**(1/2/(1+p))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{2}+\frac{2\,ab}{x^{\frac{1}{2\,(p+1)}}}+\frac{b^{2}}{x^{\left(\frac{1}{p+1}\right)}}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="gia
c")
```

```
[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)
```

3.546 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+2n}{2n}}\,dx$

Optimal. Leaf size=102

$$\frac{nx\,(a+bx^n)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a^2(n+1)}+\frac{x\,(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a(n+1)}$$

```
[Out] (x*(a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/(a*(1 + n
)) + (n*x*(a + b*x^n)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/
(a^2*(1 + n))
```

Rubi [A] time = 0.0442048, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1343, 192, 191}

$$\frac{nx\,(a+bx^n)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a^2(n+1)}+\frac{x\,(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + 2*n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/(a*(1 + n
)) + (n*x*(a + b*x^n)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/
(a^2*(1 + n))
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+2n}{2n}} dx = \left(\left(2ab+2b^2x^n\right)^{\frac{1+2n}{n}} \left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+2n}{2n}}\right) \int \left(2ab+2b^2x^n\right)^{-\frac{1+2n}{n}} dx$$

$$= \frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)} + \frac{\left(n\left(2ab+2b^2x^n\right)^{\frac{1+2n}{n}}\left(a^2+2abx^n+b^2x^{2n}\right)\right.}{2ab(1+n)}$$

$$= \frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)} + \frac{nx\left(a+bx^n\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a^2(1+n)}$$

Mathematica [C] time = 0.0441615, size = 59, normalized size = 0.58

$$\frac{x\left(\left(a+bx^n\right)^2\right)^{-\frac{1}{2}/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}} {}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + 2*n)/(2*n)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a^2*((a + b*x^n)^2)^(1/(2*n)))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \left(\left(a^2+2\,abx^n+b^2x^{2\,n}\right)^{\frac{1+2\,n}{2\,n}}\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x)
```

```
[Out] int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{2\,n+1}{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)
```

Fricas [A] time = 1.67306, size = 171, normalized size = 1.68

$$\frac{b^2nxx^{2\,n}+(2\,abn+ab)xx^n+\left(a^2n+a^2\right)x}{\left(a^2n+a^2\right)\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{2\,n+1}{2\,n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="fri
cas")
```

```
[Out] (b^2*n*x*x^(2*n) + (2*a*b*n + a*b)*x*x^n + (a^2*n + a^2)*x)/((a^2*n + a^2)*
(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+2*n)/n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{2\,n+1}{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)
```

3.547 $\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx$

Optimal. Leaf size=117

$$\frac{(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^{p+1}}{2a^2dn(p+1)(2p+1)}-\frac{(a+bx^n)(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{adn(2p+1)}$$

```
[Out] -(((a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(a*d*n*(1 + 2*p)*(d*x)^(2
*n*(1 + p)))) + (a^2 + 2*a*b*x^n + b^2*x^(2*n))^(1 + p)/(2*a^2*d*n*(1 + p)*
(1 + 2*p)*(d*x)^(2*n*(1 + p)))
```

Rubi [A] time = 0.0642789, antiderivative size = 124, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1356, 273, 264}

$$\frac{\left(\frac{bx^n}{a}+1\right)^2(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2dn\left(2p^2+3p+1\right)}-\frac{\left(\frac{bx^n}{a}+1\right)(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -(((1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(d*n*(1 + 2*p)*(d*x)^
(2*n*(1 + p)))) + ((1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*
d*n*(1 + 3*p + 2*p^2)*(d*x)^(2*n*(1 + p)))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (dx)^{-1-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^p dx=\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\int (dx)^{-1-2n(1+p)}\left(1+\frac{bx^n}{a}\right)^{2p} dx$$

$$=-\frac{(dx)^{-2n(1+p)}\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(1+2p)}+\frac{\left((-2n(1+p)+n(1+2p\right.}{}$$

$$=-\frac{(dx)^{-2n(1+p)}\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(1+2p)}+\frac{(dx)^{-2n(1+p)}\left(1+\frac{bx^n}{a}\right)^2\left(\right.}{2dn\left(1+3p\right.}$$

Mathematica [C] time = 0.0327773, size = 75, normalized size = 0.64

$$-\frac{x(dx)^{-2n(p+1)-1}\left((a+bx^n)^2\right)^p\left(\frac{bx^n}{a}+1\right)^{-2p}{}_2F_1\left(-2p,-2(p+1);1-2(p+1);-\frac{bx^n}{a}\right)}{2n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -(x*(d*x)^(-1 - 2*n*(1 + p))*((a + b*x^n)^2)^p*Hypergeometric2F1[-2*p, -2*(
1 + p), 1 - 2*(1 + p), -((b*x^n)/a)])/(2*n*(1 + p)*(1 + (b*x^n)/a)^(2*p))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int (dx)^{-1-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

```
[Out] int((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^{2n}+2abx^n+a^2\right)^p (dx)^{-2n(p+1)-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
maxima")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

Fricas [A] time = 1.62185, size = 401, normalized size = 3.43

$$-\frac{\left(2abpxx^ne^{\left(-(2np+2n+1)\log(d)-(2np+2n+1)\log(x)\right)}-b^2xx^{2n}e^{\left(-(2np+2n+1)\log(d)-(2np+2n+1)\log(x)\right)}+\left(2a^2p+a^2\right)xe^{\left(-(2np+2n\right.}}{2\left(2a^2np^2+3a^2np+a^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
fricas")
```

```
[Out] -1/2*(2*a*b*p*x*x^n*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x)
) - b^2*x*x^(2*n)*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x))
+ (2*a^2*p + a^2)*x*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x)
))*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^p/(2*a^2*n*p^2 + 3*a^2*n*p + a^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(-1-2*n*(1+p))*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2 x^{2n} + 2\,abx^n + a^2\right)^p (dx)^{-2n(p+1)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

3.548 $\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{a^2\left(\frac{bx^n}{a}+1\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2b^2n(p+1)}-\frac{a^2\left(\frac{bx^n}{a}+1\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(2p+1)}$$

```
[Out] -((a^2*(1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(b^2*n*(1 + 2*p))
) + (a^2*(1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*b^2*n*(1 +
 p))
```

Rubi [A] time = 0.0647758, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1356, 266, 43}

$$\frac{a^2\left(\frac{bx^n}{a}+1\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2b^2n(p+1)}-\frac{a^2\left(\frac{bx^n}{a}+1\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -((a^2*(1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(b^2*n*(1 + 2*p))
) + (a^2*(1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*b^2*n*(1 +
 p))
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\int x^{-1+2n}\left(1+\frac{bx^n}{a}\right)^{2p}\,dx$$

$$=\frac{\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\text{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x^n\right)}{n}$$

$$=\frac{\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b}+\frac{a\left(1+\frac{bx}{a}\right)^{1+2p}}{b}\right)dx,x,x^n\right)}{n}$$

$$=-\frac{a^2\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(1+2p)}+\frac{a^2\left(1+\frac{bx^n}{a}\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2b^2n(1+p)}$$

Mathematica [A] time = 0.0288713, size = 54, normalized size = 0.52

$$\frac{(a+bx^n)\left((a+bx^n)^2\right)^p\left(b(2p+1)x^n-a\right)}{2b^2n(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] ((a + b*x^n)*((a + b*x^n)^2)^p*(-a + b*(1 + 2*p)*x^n))/(2*b^2*n*(1 + p)*(1
+ 2*p))
```

Maple [C] time = 0.071, size = 148, normalized size = 1.4

$$-\frac{-2\,b^2p\,(x^n)^2-2\,apx^nb-b^2\,(x^n)^2+a^2}{(2+4\,p)\,(1+p)\,nb^2}\mathrm{e}^{-\frac{p\left(i\pi\left(\mathrm{csgn}\left(i(a+bx^n)^2\right)\right)^3-2\,i\pi\left(\mathrm{csgn}\left(i(a+bx^n)^2\right)\right)^2\mathrm{csgn}\left(i(a+bx^n)\right)+i\pi\,\mathrm{csgn}\left(i(a+bx^n)^2\right)\left(\mathrm{csgn}\left(i(a+bx^n)\right)\right)^2-4\,\ln(a+bx^n)\right)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

```
[Out] -1/2*(-2*b^2*p*(x^n)^2-2*a*p*x^n*b-b^2*(x^n)^2+a^2)/(1+2*p)/(1+p)/n/b^2*exp
(-1/2*p*(I*Pi*csgn(I*(a+b*x^n)^2)^3-2*I*Pi*csgn(I*(a+b*x^n)^2)^2*csgn(I*(a+
b*x^n))+I*Pi*csgn(I*(a+b*x^n)^2)*csgn(I*(a+b*x^n))^2-4*ln(a+b*x^n)))
```

Maxima [A] time = 1.00132, size = 80, normalized size = 0.78

$$\frac{\left(b^2(2\,p+1)x^{2\,n}+2\,abpx^n-a^2\right)(bx^n+a)^{2\,p}}{2\left(2\,p^2+3\,p+1\right)b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(2*p + 1)*x^(2*n) + 2*a*b*p*x^n - a^2)*(b*x^n + a)^(2*p)/((2*p^2 +
 3*p + 1)*b^2*n)
```

Fricas [A] time = 1.6522, size = 161, normalized size = 1.56

$$\frac{\left(2\,abpx^{n}-a^{2}+\left(2\,b^{2}p+b^{2}\right)x^{2\,n}\right)\left(b^{2}x^{2\,n}+2\,abx^{n}+a^{2}\right)^{p}}{2\left(2\,b^{2}np^{2}+3\,b^{2}np+b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*p*x^n - a^2 + (2*b^2*p + b^2)*x^(2*n))*(b^2*x^(2*n) + 2*a*b*x^n
+ a^2)^p/(2*b^2*n*p^2 + 3*b^2*n*p + b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^{2}x^{2\,n}+2\,abx^{n}+a^{2}\right)^{p}x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*x^(2*n - 1), x)
```

3.549 $\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=111

$$\frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2c^3n}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3n\sqrt{b^2-4ac}}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^
n)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[a + b*x
^n + c*x^(2*n)])/(2*c^3*n)
```

Rubi [A] time = 0.121727, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 701, 634, 618, 206, 628}

$$\frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2c^3n}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3n\sqrt{b^2-4ac}}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^
n)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[a + b*x
^n + c*x^(2*n)])/(2*c^3*n)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx, x, x^n\right)}{n} \\
&= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{\text{Subst}\left(\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx, x, x^n\right)}{c^2n} \\
&= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}-\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2c^3n}+\frac{\left(b^2-ac\right)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x,\right.}{2c^3n} \\
&= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c\right.}{c^3n} \\
&= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,n}+\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}
\end{aligned}
$$

Mathematica [A] time = 0.202551, size = 93, normalized size = 0.84

$$
\frac{\left(b^2-ac\right)\log\left(a+x^n\left(b+cx^n\right)\right)+\frac{2b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}+cx^n\left(cx^n-2b\right)}{2c^3n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (c*x^n*(-2*b + c*x^n) + (2*b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 -
 4*a*c]])/Sqrt[b^2 - 4*a*c] + (b^2 - a*c)*Log[a + x^n*(b + c*x^n)])/(2*c^3*
n)
```

Maple [B] time = 0.138, size = 973, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -1/c^2*ln(x)*a+1/c^3*ln(x)*b^2+1/2/c/n*(x^n)^2-b*x^n/c^2/n+4/(4*a*c^4*n^2-b
^2*c^3*n^2)*n^2*ln(x)*a^2*c^2-5/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*a*b^2*c
```

```
+1/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*b^4-2/c/(4*a*c-b^2)/n*ln(x^n+1/2*(3*
a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a
*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c
^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*a*b^2-1/2/c^3/(4*
a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b
^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4+1/2/c^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a
*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*
c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)-2/c/(4*a*c-b^
2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+
b^8)^(1/2))/c/b/(3*a*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2
*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^
2))*a*b^2-1/2/c^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3
+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4-1/2/c^3/(4*a*c-
b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*
c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b
^8)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^2-ac\right)\log\left(x\right)}{c^3}+\frac{cx^{2n}-2bx^n}{2c^2n}+\int -\frac{ab^2-a^2c+\left(b^3-2abc\right)x^n}{c^4xx^{2n}+bc^3xx^n+ac^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] (b^2 - a*c)*log(x)/c^3 + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n) + integrate(-(a*
b^2 - a^2*c + (b^3 - 2*a*b*c)*x^n)/(c^4*x*x^(2*n) + b*c^3*x*x^n + a*c^3*x),
 x)
```

Fricas [A] time = 1.66456, size = 761, normalized size = 6.86

$$\left[-\frac{\left(b^3-3abc\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^{2n}+b^2-2ac+2\left(bc-\sqrt{b^2-4ac}c\right)x^n-\sqrt{b^2-4ac}b}{cx^{2n}+bx^n+a}\right)-\left(b^2c^2-4ac^3\right)x^{2n}+2\left(b^3c-4abc^2\right)x^n-\left(b^4-5a\right.}{2\left(b^2c^3-4ac^4\right)n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c +
 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^
n + a)) - (b^2*c^2 - 4*a*c^3)*x^(2*n) + 2*(b^3*c - 4*a*b*c^2)*x^n - (b^4 -
5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^3 - 4*a*c^4)*n),
 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*
x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^2*c^2 - 4*a*c^3)*x^(2*n) -
2*(b^3*c - 4*a*b*c^2)*x^n + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b
*x^n + a))/((b^2*c^3 - 4*a*c^4)*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.550 $\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=87

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2n\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}+\frac{x^n}{cn}$$

```
[Out] x^n/(c*n) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c^2*S
qrt[b^2 - 4*a*c]*n) - (b*Log[a + b*x^n + c*x^(2*n)])/(2*c^2*n)
```

Rubi [A] time = 0.074099, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1357, 703, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2n\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}+\frac{x^n}{cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x^n/(c*n) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c^2*S
qrt[b^2 - 4*a*c]*n) - (b*Log[a + b*x^n + c*x^(2*n)])/(2*c^2*n)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^n\right)}{n}\\
&= \frac{x^n}{cn} + \frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^n\right)}{cn}\\
&= \frac{x^n}{cn} - \frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{2c^2n} + \frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2c^2n}\\
&= \frac{x^n}{cn} - \frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n} - \frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^n\right)}{c^2n}\\
&= \frac{x^n}{cn} - \frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,n} - \frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}
\end{aligned}$$

Mathematica [A] time = 0.128354, size = 80, normalized size = 0.92

$$\frac{-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} - \frac{b\log(a+x^n(b+cx^n))}{2c} + x^n}{cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x^n - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2
 - 4*a*c]) - (b*Log[a + x^n*(b + c*x^n)])/(2*c))/(c*n)
```

Maple [B] time = 0.091, size = 664, normalized size = 7.6

$$-\frac{b\ln(x)}{c^2} + \frac{x^n}{cn} + 4\frac{n^2\ln(x)\,abc}{4\,ac^3n^2-b^2c^2n^2} - \frac{n^2\ln(x)\,b^3}{4\,ac^3n^2-b^2c^2n^2} - 2\frac{ab}{c\left(4\,ac-b^2\right)n}\ln\left(x^n - 1/2\frac{-2\,abc+b^3+\sqrt{-16\,a^3c^3+20}}{c\left(2\,ac-b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -b/c^2*ln(x)+x^n/c/n+4/(4*a*c^3*n^2-b^2*c^2*n^2)*n^2*ln(x)*a*b*c-1/(4*a*c^3
*n^2-b^2*c^2*n^2)*n^2*ln(x)*b^3-2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+
(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*a*b+1/2/c^
2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^
```

```
4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3+1/2/c^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b
*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-1
6*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)-2/c/(4*a*c-b^2)/n*ln(x^n+1/2*
(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2
))*a*b+1/2/c^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^
2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1/2/c^2/(4*a*c-b^2)/n*ln(x^n
+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*
c-b^2))*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b\log(x)}{c^2}+\frac{x^n}{cn}-\int -\frac{ab+\left(b^2-ac\right)x^n}{c^3xx^{2n}+bc^2xx^n+ac^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -b*log(x)/c^2 + x^n/(c*n) - integrate(-(a*b + (b^2 - a*c)*x^n)/(c^3*x*x^(2*
n) + b*c^2*x*x^n + a*c^2*x), x)
```

Fricas [A] time = 1.69275, size = 633, normalized size = 7.28

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc+\sqrt{b^2-4\,ac}c\right)x^n+\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)-2\left(b^2c-4\,ac^2\right)x^n+\left(b^3-4\,abc\right)\log\left(cx^{2\,n}+bx^n+\right.}{2\left(b^2c^2-4\,ac^3\right)n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2
*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n
+ a)) - 2*(b^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a
))/((b^2*c^2 - 4*a*c^3)*n), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 2*(b
^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^2
 - 4*a*c^3)*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3n-1}}{cx^{2n} + bx^{n} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.551 $\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=68

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{cn\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*n) + Log[
a + b*x^n + c*x^(2*n)]/(2*c*n)
```

Rubi [A] time = 0.0505273, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1357, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{cn\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*n) + Log[
a + b*x^n + c*x^(2*n)]/(2*c*n)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{2cn} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{2cn}\\
&= \frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^n\right)}{cn}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,n} + \frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}
\end{aligned}
$$

Mathematica [A] time = 0.0673643, size = 62, normalized size = 0.91

$$
\frac{\frac{2b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} + \log\left(a+x^n\left(b+cx^n\right)\right)}{2cn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((2*b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c] + Log[a +
 x^n*(b + c*x^n)])/(2*c*n)
```

Maple [B] time = 0.076, size = 402, normalized size = 5.9

$$
\frac{\ln(x)}{c} - 4\,\frac{n^2\ln(x)\,ac}{4\,ac^2n^2-b^2cn^2} + \frac{n^2\ln(x)\,b^2}{4\,ac^2n^2-b^2cn^2} + 2\,\frac{a}{\left(4\,ac-b^2\right)n}\ln\left(x^n - 1/2\,\frac{-b^2+\sqrt{-4\,ab^2c+b^4}}{bc}\right) - \frac{b^2}{2\,c\left(4\,ac-b^2\right)n}\ln
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/c*ln(x)-4/(4*a*c^2*n^2-b^2*c*n^2)*n^2*ln(x)*a*c+1/(4*a*c^2*n^2-b^2*c*n^2)
*n^2*ln(x)*b^2+2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c
)*a-1/2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2+1
/2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*
c+b^4)^(1/2)+2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*a
-1/2/c/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/c
/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\frac{\log(x)}{c} - \int \frac{bx^n + a}{c^2xx^{2n} + bcxx^n + acx}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log(x)/c - integrate((b*x^n + a)/(c^2*x*x^(2*n) + b*c*x*x^n + a*c*x), x)
```

Fricas [A] time = 1.62732, size = 517, normalized size = 7.6

$$\left[\frac{\sqrt{b^2-4\,acb}\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc+\sqrt{b^2-4\,ac}c\right)x^n+\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^{2\,n}+bx^n+a\right)}{2\left(b^2c-4\,ac^2\right)n}, \frac{2\sqrt{-b^2+4\,ac}b\arctan\left(-\frac{2}{\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(
b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + (b^2
- 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a*c^2)*n), 1/2*(2*sqrt(-b^
2 + 4*a*c)*b*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b
^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a*c^2)
*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.552 $\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=39

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

```
[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)
```

Rubi [A] time = 0.0329309, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1352, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{n}\\ &= -\frac{2\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^n\right)}{n}\\ &= -\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4acn}}\end{aligned}$$

Mathematica [A] time = 0.0620931, size = 39, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)
```

Maple [B] time = 0.046, size = 113, normalized size = 2.9

$$-\frac{1}{n}\ln\left(x^n+\frac{1}{2c}\left(b^2-4ac+b\sqrt{-4ac+b^2}\right)\frac{1}{\sqrt{-4ac+b^2}}\right)\frac{1}{\sqrt{-4ac+b^2}}+\frac{1}{n}\ln\left(x^n+\frac{1}{2c}\left(b\sqrt{-4ac+b^2}+4ac-b^2\right)\frac{1}{\sqrt{-4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))/c/(-4*a
*c+b^2)^(1/2))+1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*(b*(-4*a*c+b^2)^(1/2)+4*a*
c-b^2)/c/(-4*a*c+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n-1}}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [B] time = 1.57392, size = 351, normalized size = 9.

$$\left[\frac{\log\left(\frac{2c^2x^{2n}+b^2-2ac+2\left(bc-\sqrt{b^2-4ac}c\right)x^n-\sqrt{b^2-4ac}b}{cx^{2n}+bx^n+a}\right)}{\sqrt{b^2-4ac}n}, -\frac{2\sqrt{-b^2+4ac}\arctan\left(-\frac{2\sqrt{-b^2+4ac}cx^n+\sqrt{-b^2+4ac}b}{b^2-4ac}\right)}{(b^2-4ac)n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqr
t(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a))/(sqrt(b^2 - 4*a*c)*n), -2*sqrt(-
b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b
```

```
^2 - 4*a*c))/((b^2 - 4*a*c)*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09947, size = 53, normalized size = 1.36

$$\frac{2\arctan\left(\frac{2cx^n+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x^n + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*n)
```

3.553 $\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=98

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2n\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{b\log(x)}{a^2}-\frac{x^{-n}}{an}$$

```
[Out] -(1/(a*n*x^n)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(
a^2*Sqrt[b^2 - 4*a*c]*n) - (b*Log[x])/a^2 + (b*Log[a + b*x^n + c*x^(2*n)])/
(2*a^2*n)
```

Rubi [A] time = 0.126038, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2n\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{b\log(x)}{a^2}-\frac{x^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(1/(a*n*x^n)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(
a^2*Sqrt[b^2 - 4*a*c]*n) - (b*Log[x])/a^2 + (b*Log[a + b*x^n + c*x^(2*n)])/
(2*a^2*n)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^n\right)}{n}\\
&= -\frac{x^{-n}}{an}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx,x,x^n\right)}{an}\\
&= -\frac{x^{-n}}{an}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^n\right)}{an}\\
&= -\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,x^n\right)}{a^2n}\\
&= -\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{2a^2n}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{2a^2n}\\
&= -\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^n\right)}{a^2n}\\
&= -\frac{x^{-n}}{an}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}n}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}
\end{aligned}
$$

Mathematica [A] time = 0.614541, size = 135, normalized size = 1.38

$$
-\frac{-\frac{4c^2\log\left(x^{-n}\left(b-\sqrt{b^2-4ac}\right)+2c\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^2}+\frac{4c^2\log\left(x^{-n}\left(\sqrt{b^2-4ac}+b\right)+2c\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^2}+\frac{x^{-n}}{a}}{n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((1/(a*x^n) - (4*c^2*Log[2*c + (b - Sqrt[b^2 - 4*a*c])/x^n])/(Sqrt[b^2 - 4
*a*c]*(b - Sqrt[b^2 - 4*a*c])^2) + (4*c^2*Log[2*c + (b + Sqrt[b^2 - 4*a*c])
```

```
/x^n])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^2))/n)
```

Maple [B] time = 0.114, size = 658, normalized size = 6.7

$$-\frac{1}{anx^n} - 4\frac{n^2\ln(x)abc}{4a^3cn^2 - a^2b^2n^2} + \frac{n^2\ln(x)b^3}{4a^3cn^2 - a^2b^2n^2} + 2\frac{bc}{a(4ac-b^2)n}\ln\left(x^n - 1/2\frac{-2abc + b^3 + \sqrt{-16a^3c^3 + 20a^2b^2c^2 - 8}}{c(2ac-b^2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -1/a/n/(x^n)-4/(4*a^3*c*n^2-a^2*b^2*n^2)*n^2*ln(x)*a*b*c+1/(4*a^3*c*n^2-a^2
*b^2*n^2)*n^2*ln(x)*b^3+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3
*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b*c-1/2/a^2/(4*a*c
-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)
^(1/2))/c/(2*a*c-b^2))*b^3+1/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(
-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-16*a^3*c^
3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c
-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b*c-1
/2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*
a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1/2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*
a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*
(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{anx^n} - \int \frac{cx^n + b}{acxx^{2n} + abxx^n + a^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -1/(a*n*x^n) - integrate((c*x^n + b)/(a*c*x*x^(2*n) + a*b*x*x^n + a^2*x), x
)
```

Fricas [A] time = 1.70989, size = 738, normalized size = 7.53

$$\left[-\frac{2(b^3 - 4abc)nx^n\log(x) + (b^2 - 2ac)\sqrt{b^2 - 4ac}x^n\log\left(\frac{2c^2x^{2n} + b^2 - 2ac + 2(bc + \sqrt{b^2-4ac}c)x^n + \sqrt{b^2-4ac}b}{cx^{2n} + bx^n + a}\right) + 2ab^2 - 8a^2c - (b^3 -}{2(a^2b^2 - 4a^3c)nx^n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^3 - 4*a*b*c)*n*x^n*log(x) + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^n
*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqr
t(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a
*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a^2*b^2 - 4*a^3*c)*n*x^n), -1/2*(2*
```

```
(b^3 - 4*a*b*c)*n*x^n*log(x) + 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^n*arcta
n(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + 2*a
*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a^2*b^2
- 4*a^3*c)*n*x^n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.554 $\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=126

$$-\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3n\sqrt{b^2-4ac}}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

```
[Out] -1/(2*a*n*x^(2*n)) + b/(a^2*n*x^n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)
/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[x])/a^3 -
 ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^3*n)
```

Rubi [A] time = 0.170971, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3n\sqrt{b^2-4ac}}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(2*a*n*x^(2*n)) + b/(a^2*n*x^n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)
/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[x])/a^3 -
 ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^3*n)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx+cx^2)}\,dx, x, x^n\right)}{n}\\
&= -\frac{x^{-2n}}{2an}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x^2(a+bx+cx^2)}\,dx, x, x^n\right)}{an}\\
&= -\frac{x^{-2n}}{2an}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax^2}+\frac{b^2-ac}{a^2x}+\frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)}\right)dx, x, x^n\right)}{an}\\
&= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{(b^2-ac)\log(x)}{a^3}+\frac{\text{Subst}\left(\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2}\,dx, x, x^n\right)}{a^3n}\\
&= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2a^3n}-\frac{(b^2-ac)\,\text{Su}}{}\\
&= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2a^3n}+\frac{(b(b^2-3ac))\,\text{Subst}\left(\int\right.}{a^3}\\
&= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\,n}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log(a+bx^n+c}{2a^3n}
\end{aligned}$$

Mathematica [A] time = 0.339076, size = 112, normalized size = 0.89

$$\frac{-a^2x^{-2n}-(b^2-ac)\log(a+x^n(b+cx^n))+\frac{2b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}+2n\log(x)(b^2-ac)+2abx^{-n}}{2a^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-(a^2/x^(2*n)) + (2*a*b)/x^n + (2*b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)/Sq
rt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c] + 2*(b^2 - a*c)*n*Log[x] - (b^2 - a*c)*
```

```
Log[a + x^n*(b + c*x^n)])/(2*a^3*n)
```

Maple [B] time = 0.135, size = 958, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] b/a^2/n/(x^n)-1/2/a/n/(x^n)^2-4/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a^2*c^2
+5/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a*b^2*c-1/(4*a^4*c*n^2-a^3*b^2*n^2)*
n^2*ln(x)*b^4+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+
33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*c^2-5/2/a^2/(4*a*c-b
^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+
b^8)^(1/2))/c/b/(3*a*c-b^2))*b^2*c+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^
2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b
^2))*b^4+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+3
3*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a
^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b
^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*
c^2-5/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^
2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^2*c+1/2/a^3/(4*a*c-b^2)
/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^
8)^(1/2))/c/b/(3*a*c-b^2))*b^4-1/2/a^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c
+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2)
)*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,bx^n - a}{2\,a^2nx^{2\,n}} + \int \frac{bcx^n + b^2 - ac}{a^2cxx^{2\,n} + a^2bxx^n + a^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n - a)/(a^2*n*x^(2*n)) + integrate((b*c*x^n + b^2 - a*c)/(a^2*c*
x*x^(2*n) + a^2*b*x*x^n + a^3*x), x)
```

Fricas [A] time = 1.67542, size = 938, normalized size = 7.44

$$\left[-\frac{a^2b^2 - 4\,a^3c - 2\left(b^4 - 5\,ab^2c + 4\,a^2c^2\right)nx^{2\,n}\log\left(x\right) + \left(b^3 - 3\,abc\right)\sqrt{b^2 - 4\,ac}x^{2\,n}\log\left(\frac{2\,c^2x^{2\,n} + b^2 - 2\,ac + 2\left(bc - \sqrt{b^2 - 4\,ac}c\right)x^n - \sqrt{b^2 - 4}}{cx^{2\,n} + bx^n + a}\right.}{2\left(a^3b^2 - 4\,a^4c\right)nx^{2\,n}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x)
 + (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^(2*n)*log((2*c^2*x^(2*n) + b^2 - 2*a
*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) +
b*x^n + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*n)*log(c*x^(2*n) + b*x^n +
 a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b^2 - 4*a^4*c)*n*x^(2*n)), -1/2*(a^2
*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x) - 2*(b^3
- 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^(2*n)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n +
 sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*
n)*log(c*x^(2*n) + b*x^n + a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b^2 - 4*a^
4*c)*n*x^(2*n))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-2n-1}}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.555 $\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=164

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4n\sqrt{b^2-4ac}}-\frac{x^{-n}\left(b^2-ac\right)}{a^3n}+\frac{b\left(b^2-2ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}+\frac{bx^{-2}}{2a^2}$$

```
[Out] -1/(3*a*n*x^(3*n)) + b/(2*a^2*n*x^(2*n)) - (b^2 - a*c)/(a^3*n*x^n) - ((b^4
 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqr
t[b^2 - 4*a*c]*n) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a +
 b*x^n + c*x^(2*n)])/(2*a^4*n)
```

Rubi [A] time = 0.231263, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4n\sqrt{b^2-4ac}}-\frac{x^{-n}\left(b^2-ac\right)}{a^3n}+\frac{b\left(b^2-2ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}+\frac{bx^{-2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(3*a*n*x^(3*n)) + b/(2*a^2*n*x^(2*n)) - (b^2 - a*c)/(a^3*n*x^n) - ((b^4
 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqr
t[b^2 - 4*a*c]*n) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a +
 b*x^n + c*x^(2*n)])/(2*a^4*n)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx+cx^2)}\,dx,x,x^n\right)}{n}$$

$$= -\frac{x^{-3n}}{3an}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x^3(a+bx+cx^2)}\,dx,x,x^n\right)}{an}$$

$$= -\frac{x^{-3n}}{3an}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax^3}+\frac{b^2-ac}{a^2x^2}+\frac{-b^3+2abc}{a^3x}+\frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a^3(a+bx+cx^2)}\right)dx,x,x^n\right)}{an}$$

$$= -\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{(b^2-ac)x^{-n}}{a^3n}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{\text{Subst}\left(\int \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a+bx+cx^2}\,dx,x\right.}{a^4n}$$

$$= -\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{(b^2-ac)x^{-n}}{a^3n}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{(b(b^2-2ac))\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x\right.}{2a^4n}$$

$$= -\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{(b^2-ac)x^{-n}}{a^3n}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log(a+bx^n+cx^{2n})}{2a^4n}-\frac{(b}{}$$

$$= -\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{(b^2-ac)x^{-n}}{a^3n}-\frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}\,n}-\frac{b(b^2-2ac)\log(x)}{a^4}$$

Mathematica [A] time = 0.457669, size = 143, normalized size = 0.87

$$\frac{-\frac{6(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}+3a^2bx^{-2n}-2a^3x^{-3n}+6ax^{-n}(ac-b^2)+3b(b^2-2ac)\log(a+x^n(b+cx^n))-6bn\log}{6a^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((-2*a^3)/x^(3*n) + (3*a^2*b)/x^(2*n) + (6*a*(-b^2 + a*c))/x^n - (6*(b^4 -
4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 -
```

```
 4*a*c] - 6*b*(b^2 - 2*a*c)*n*Log[x] + 3*b*(b^2 - 2*a*c)*Log[a + x^n*(b + c
*x^n)])/(6*a^4*n)
```

Maple [B] time = 0.16, size = 1300, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/a^2/n/(x^n)*c-1/a^3/n/(x^n)*b^2+1/2*b/a^2/n/(x^n)^2-1/3/a/n/(x^n)^3+8/(4*
a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*a^2*b*c^2-6/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*
ln(x)*a*b^3*c+1/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*b^5-4/a^2/(4*a*c-b^2)/n
*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b
^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*
b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^
5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2
*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^
2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-1
2*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^5+1/2/a^4/(4*a*c-b^2)
/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3
*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4)
)*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10
)^(1/2)-4/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5
*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c
/(2*a^2*c^2-4*a*b^2*c+b^4))*b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*
c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2
-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-
b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-9
6*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c
+b^4))*b^5-1/2/a^4/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-1
6*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/
2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3
+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,abx^{n}-2\,a^{2}-6\left(b^{2}-ac\right)x^{2\,n}}{6\,a^{3}nx^{3\,n}}+\int -\frac{b^{3}-2\,abc+\left(b^{2}c-ac^{2}\right)x^{n}}{a^{3}cxx^{2\,n}+a^{3}bxx^{n}+a^{4}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b*x^n - 2*a^2 - 6*(b^2 - a*c)*x^(2*n))/(a^3*n*x^(3*n)) + integrate
(-(b^3 - 2*a*b*c + (b^2*c - a*c^2)*x^n)/(a^3*c*x*x^(2*n) + a^3*b*x*x^n + a^
4*x), x)
```

Fricas [A] time = 1.70359, size = 1130, normalized size = 6.89

$$\left[-\frac{2\,a^3b^2 - 8\,a^4c + 6\left(b^5 - 6\,ab^3c + 8\,a^2bc^2\right)nx^{3\,n}\log\left(x\right) - 3\left(b^4 - 4\,ab^2c + 2\,a^2c^2\right)\sqrt{b^2 - 4\,ac}x^{3\,n}\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc\right.}{cx^{2}}\right.}{6\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a^3*b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*lo
g(x) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^(3*n)*log((2*c^2
*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a
*c)*b)/(c*x^(2*n) + b*x^n + a)) - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^(3*n)
*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^(2*n) -
 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4*a^5*c)*n*x^(3*n)), -1/6*(2*a^3*
b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*log(x) + 6*(b^4
 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^(3*n)*arctan(-(2*sqrt(-b^2 +
 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 3*(b^5 - 6*a*b^3*c +
 8*a^2*b*c^2)*x^(3*n)*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c +
 4*a^3*c^2)*x^(2*n) - 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4*a^5*c)*n*x
^(3*n))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-3\,n-1}}{cx^{2\,n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.556 $\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=353

$$\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}}{n\sqrt{b^2-4ac}\left(\sqrt{b^2\right.}}$$

```
[Out] (2*2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)
*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) + (2*2^(3/4)*c^(3/4)*A
rcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2
 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)*c^(3/4)*ArcTanh[(2
^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rubi [A] time = 0.62696, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1381, 1347, 212, 208, 205}

$$\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}}{n\sqrt{b^2-4ac}\left(\sqrt{b^2\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)
*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) + (2*2^(3/4)*c^(3/4)*A
rcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2
 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)*c^(3/4)*ArcTanh[(2
^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \frac{4\,\text{Subst}\left(\int \frac{1}{a+bx^4+cx^8}\,dx, x, x^{n/4}\right)}{n}$$

$$= \frac{(4c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\,n} - \frac{(4c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\,n}$$

$$= \frac{(4c)\,\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}\,n} + \frac{(4c)\,\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}\,n}$$

$$= \frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n} + \frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\cdots\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-\cdots}\right.}$$

Mathematica [A] time = 0.878652, size = 340, normalized size = 0.96

$$\frac{2\,2^{3/4}c^{3/4}\left(-\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\cdots}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*2^(3/4)*c^(3/4)*(-(((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/
4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])) - ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(
Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((-b - Sqrt[b^2 - 4*a*c
])^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])
/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) - ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-
b + Sqrt[b^2 - 4*a*c])^(1/4)]/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(
3/4))))/n
```

Maple [C] time = 0.256, size = 280, normalized size = 0.8

$$\sum_{\_R=\mathrm{RootOf}\left(\left(256\,a^7c^4n^8-256\,a^6b^2c^3n^8+96\,a^5b^4c^2n^8-16\,a^4b^6cn^8+a^3b^8n^8\right)\_Z^8+\left(-48\,a^3bc^3n^4+40\,a^2b^3c^2n^4-11\,ab^5cn^4+b^7n^4\right)\_Z^4+c^3\right)} \_R \ln\left(x^{\frac{n}{4}}+\left(16\,\frac{n}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] sum(_R*ln(x^(1/4*n)+(16/(a*c^2-b^2*c)*n^5*b*a^5*c^2-8/(a*c^2-b^2*c)*n^5*b^3
*a^4*c+1/(a*c^2-b^2*c)*n^5*b^5*a^3)*_R^5+(2/(a*c^2-b^2*c)*n*a^2*c^2-4/(a*c^
2-b^2*c)*n*b^2*a*c+1/(a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((256*a^7*c^4*n^8-25
6*a^6*b^2*c^3*n^8+96*a^5*b^4*c^2*n^8-16*a^4*b^6*c*n^8+a^3*b^8*n^8)*_Z^8+(-4
8*a^3*b*c^3*n^4+40*a^2*b^3*c^2*n^4-11*a*b^5*c*n^4+b^7*n^4)*_Z^4+c^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [B] time = 4.15006, size = 9276, normalized size = 26.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 6
4*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4
)))*arctan(1/16*sqrt(2)*(2*sqrt(2)*((a^3*b^10*c - 15*a^4*b^8*c^2 + 86*a^5*b
^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5 - 128*a^8*c^6)*n^7*x*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)*n^8)) - (b^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4
*b*c^5)*n^3*x)*x^(1/4*n - 1)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^
4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2
)*n^4)) + sqrt(2)*((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c
^3 + 128*a^7*c^4)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^7 - 9*a*b^5*c + 24*a^2*b^3
*c^2 - 16*a^3*b*c^3)*n^3*x)*sqrt((4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2*x
^(1/2*n - 2) - sqrt(2)*((a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*
b^3*c^3 + 64*a^7*b*c^4)*n^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12
*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^8 - 8*a*b^6*c + 21*a^2
*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4)*n^2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c +
```

16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2)*n^4)))/x^2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^
4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2
)*n^4)))*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt(
(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))
)/(b^4*c^3 - 2*a*b^2*c^4 + a^2*c^5)) + 2*sqrt(2)*sqrt(sqrt(2)*sqrt(((a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*arctan(1/8*(2*((a^3*b^10*c - 15*a^
4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5 - 128*a^8*c^
6)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8
*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*
a^3*b^3*c^4 + 16*a^4*b*c^5)*n^3*x)*x^(1/4*n - 1)*sqrt(sqrt(2)*sqrt(((a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*
a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48
*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2)*n^4)) + ((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^
2*c^3 + 128*a^7*c^4)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^7 - 9*a*b^5*c + 24*a^2*
b^3*c^2 - 16*a^3*b*c^3)*n^3*x)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)*n^4)))*sqrt((4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2*x^(1/2
n - 2) + sqrt(2)((a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c
^3 + 64*a^7*b*c^4)*n^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^8 - 8*a*b^6*c + 21*a^2*b^4*
c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4)*n^2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2)*n^4)))/x^2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt(
(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))
)/(b^4*c^3 - 2*a*b^2*c^4 + a^2*c^5)) + 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3
*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/(
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/
4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8
*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) - 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((
a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((
a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c
)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^
(1/4*n - 1) - sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*
c^3)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6
- 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*
b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) - 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(
((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b
*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*

```
x^(1/4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)*n^8)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(((a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3
*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) + 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt
(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*
b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x
*x^(1/4*n - 1) - sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a
^9*c^3)*n^8)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(((a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{n}{4}-1}}{a + bx^n + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x**(n/4 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{4}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.557 $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=610

$$\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{b-\right.}{\sqrt[3]{2}n\sqrt{}}$$

```
[Out] -((2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b - Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*n)) + (2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3
))/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^
2 - 4*a*c])^(2/3)*n) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) +
 2^(1/3)*c^(1/3)*x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)
*n) - (2^(2/3)*c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*
x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) - (c^(2/3)*Lo
g[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(
1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b
- Sqrt[b^2 - 4*a*c])^(2/3)*n) + (c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3)
- 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x
^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n)
```

Rubi [A] time = 1.15229, antiderivative size = 610, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1381, 1347, 200, 31, 634, 617, 204, 628}

$$\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{b-\right.}{\sqrt[3]{2}n\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b - Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*n)) + (2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3
))/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^
2 - 4*a*c])^(2/3)*n) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) +
 2^(1/3)*c^(1/3)*x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)
*n) - (2^(2/3)*c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*
x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) - (c^(2/3)*Lo
g[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(
1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b
- Sqrt[b^2 - 4*a*c])^(2/3)*n) + (c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3)
- 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x
^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n)
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^3+cx^6}\,dx, x, x^{n/3}\right)}{n}$$

$$= \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}n} - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}n}$$

$$= \frac{\left(2^{2/3}c\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(2^{2/3}c\right)\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\right.}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{c^{2/}}{}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{c^{2/}}{}$$

$$= -\frac{2^{2/3}\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2}}\right.}{\sqrt{b^2-4ac}\left(b-\sqrt{}\right.}$$

Mathematica [A] time = 0.77329, size = 526, normalized size = 0.86

$$\frac{c^{2/3}\left(-\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)+\left(b-\sqrt{b^2-4ac}\right)^{2/3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (c^(2/3)*(-2*Sqrt[3]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x^(n/3))/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]] + 2*Sqrt[3]*(b - Sq
rt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b + Sqrt[b^
2 - 4*a*c])^(1/3))/Sqrt[3]] + 2*(b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt
[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] - 2*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] - (b +
Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3
)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)] + (b
 - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(
1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)]))
/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*(b + Sqrt[b^2 - 4
*a*c])^(2/3)*n)
```

Maple [C] time = 0.215, size = 260, normalized size = 0.4

$$\sum_{\_R=\mathrm{RootOf}\left(\left(64\,a^5c^3n^6-48\,a^4b^2c^2n^6+12\,a^3b^4cn^6-a^2b^6n^6\right)\_Z^6+\left(16\,a^2bc^2n^3-8\,ab^3cn^3+b^5n^3\right)\_Z^3+c^2\right)} \_R\ln\left(x^{\frac{n}{3}}+\left(-16\,\frac{n^4ba^4c^2}{2\,c^2a-b^2c}+8\,\frac{n^4b^3a^3c}{2\,c^2a-b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] sum(_R*ln(x^(1/3*n)+(-16/(2*a*c^2-b^2*c)*n^4*b*a^4*c^2+8/(2*a*c^2-b^2*c)*n^
4*b^3*a^3*c-1/(2*a*c^2-b^2*c)*n^4*b^5*a^2)*_R^4+(4/(2*a*c^2-b^2*c)*n*a^2*c^
2-5/(2*a*c^2-b^2*c)*n*b^2*a*c+1/(2*a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((64*a^
5*c^3*n^6-48*a^4*b^2*c^2*n^6+12*a^3*b^4*c*n^6-a^2*b^6*n^6)*_Z^6+(16*a^2*b*c
^2*n^3-8*a*b^3*c*n^3+b^5*n^3)*_Z^3+c^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{3}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [B] time = 4.07959, size = 9999, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a
^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/
((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b
^8*c - 14*a^3*b^6*c^2 + 72*a^4*b^4*c^3 - 160*a^5*b^2*c^4 + 128*a^6*c^5)*n^5
*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)*n^6)) - sqrt(3)*(b^7*c - 8*a*b^5*c^2 + 20*a^2*b^3*c^3 -
16*a^3*b*c^4)*n^2*x)*x^(1/3*n - 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*
a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3
)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3) + sqrt(2)*(1/2)^(2/3)*(sqrt(3
)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*n^5*x*sqrt((b^4 -
4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c
^3)*n^6)) - sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n^2*x)*sqrt((2*(b^4*c^2
 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) - (1/2)^(1/3)*((a^2*b^7*c - 1
0*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c
+ 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6))
- (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*((
(a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a
^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3
```

))^(1/3) - (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3))/x^2)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3) + 2*sqrt(3)*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4))/(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)) - 2*sqrt(3)*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b^8*c - 14*a^3*b^6*c^2 + 72*a^4*b^4*c^3 - 160*a^5*b^2*c^4 + 128*a^6*c^5)*n^5*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + sqrt(3)*(b^7*c - 8*a*b^5*c^2 + 20*a^2*b^3*c^3 - 16*a^3*b*c^4)*n^2*x)*x^(1/3*n - 1)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3) + sqrt(2)*(1/2)^(2/3)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*n^5*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n^2*x)*sqrt((2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) + (1/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3) + (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3))/x^2)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3) - 2*sqrt(3)*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4))/(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)) + (1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(2*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) + (1/2)^(1/3)*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3))/x) + (1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(2*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) - (1/2)^(1/3)*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3))/x) - 1/2*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(8*(2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) - (1/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*

```
a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)
/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3) - (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c +
72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c
 + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6))
 - (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(
((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*
a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^
3))^(2/3))/x^2) - 1/2*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*
a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3
)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(8*(2*(b^4*c^2 - 4*a*b^2*c
^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) + (1/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^
2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)
/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^6*c - 8
*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*(-((a^2*b^2 -
4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c +
48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3) +
(1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 1
28*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4
*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^
2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4
- 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7
*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3))/x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{3}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.558 $\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=169

$$\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (2*Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*n) - (2*Sqrt[2]*Sqrt[
c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*n)
```

Rubi [A] time = 0.191817, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1381, 1093, 205}

$$\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*n) - (2*Sqrt[2]*Sqrt[
c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*n)
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{a+bx^2+cx^4}\,dx, x, x^{n/2}\right)}{n}$$
$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^{n/2}\right)}{\sqrt{b^2-4ac}\,n} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^{n/2}\right)}{\sqrt{b^2-4ac}\,n}$$
$$= \frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,n} - \frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,n}$$

Mathematica [A] time = 0.279386, size = 145, normalized size = 0.86

$$\frac{2\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4*
a*c]]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[
b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[b^2 - 4*a*c]*n)
```

Maple [C] time = 0.113, size = 114, normalized size = 0.7

$$\sum_{\_R=\text{RootOf}\left(\left(16\,a^3c^2n^4-8\,a^2b^2cn^4+ab^4n^4\right)\_Z^4+\left(-4\,abcn^2+b^3n^2\right)\_Z^2+c\right)} \_R\ln\left(x^{\frac{n}{2}}+\left(4\,n^3ba^2-\frac{n^3b^3a}{c}\right)\_R^3+\left(2\,an-\frac{b^2n}{c}\right)\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] sum(_R*ln(x^(1/2*n)+(4*n^3*b*a^2-1/c*n^3*b^3*a)*_R^3+(2*a*n-1/c*n*b^2)*_R),
_R=RootOf((16*a^3*c^2*n^4-8*a^2*b^2*c*n^4+a*b^4*n^4)*_Z^4+(-4*a*b*c*n^2+b^3
*n^2)*_Z^2+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [B] time = 1.79865, size = 1705, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4))
+ b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) + sqrt(2)*((a*b^3 -
4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 - 4*a*c)*n)*sqrt(-(
(a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^
2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2
- 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) - s
qrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 -
4*a*c)*n)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) +
b)/((a*b^2 - 4*a^2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(((a*b^2 - 4*a^2*c)*n^2*s
qrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x
^(1/2*n - 1) + sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)
*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*
a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x) + 1/2*sqrt(2)*sqrt(((a*b^2 -
 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2
))*log((4*c*x*x^(1/2*n - 1) - sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2
*b^2 - 4*a^3*c)*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1
/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{2}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.559 $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=205

$$\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}n\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a^{3/2}n\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2x^{-n/2}}{an}$$

```
[Out] -2/(a*n*x^(n/2)) + (Sqrt[2]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[a])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]*n) + (Sqrt[2]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[a])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*n)
```

Rubi [A] time = 0.395062, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1381, 1340, 1122, 1166, 205}

$$\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}n\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a^{3/2}n\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2x^{-n/2}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(a*n*x^(n/2)) + (Sqrt[2]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[a])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]*n) + (Sqrt[2]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[a])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*n)
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
```

```
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{a+\frac{c}{x^4}+\frac{b}{x^2}}\,dx, x, x^{-n/2}\right)}{n}\\
&= -\frac{2\,\text{Subst}\left(\int \frac{x^4}{c+bx^2+ax^4}\,dx, x, x^{-n/2}\right)}{n}\\
&= -\frac{2x^{-n/2}}{an}+\frac{2\,\text{Subst}\left(\int \frac{c+bx^2}{c+bx^2+ax^4}\,dx, x, x^{-n/2}\right)}{an}\\
&= -\frac{2x^{-n/2}}{an}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^2}\,dx, x, x^{-n/2}\right)}{an}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{}}\right.}{an}\\
&= -\frac{2x^{-n/2}}{an}+\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b-\sqrt{b^2-4ac}}n}+\frac{\sqrt{2}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b+\sqrt{b^2-4ac}}n}
\end{aligned}
$$

Mathematica [C] time = 0.203583, size = 127, normalized size = 0.62

$$\frac{4cx^{-n/2}\left(\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (4*c*(Hypergeometric2F1[-1/2, 1, 1/2, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/2, 1, 1/2, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n
/2))
```

Maple [C] time = 0.201, size = 268, normalized size = 1.3

$$-2\,\frac{1}{anx^{n/2}}+\sum_{\_R=\text{RootOf}\left(\left(16\,a^5c^2n^4-8\,a^4b^2cn^4+a^3b^4n^4\right)\_Z^4+\left(12\,a^2bc^2n^2-7\,ab^3cn^2+b^5n^2\right)\_Z^2+c^3\right)}\_R\ln\left(x^{\frac{n}{2}}+\left(-8\,\frac{a^5n^3c^2}{ac^3-b^2c^2}+6\,\frac{n^3b^2}{ac^3-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -2/a/n/(x^(1/2*n))+sum(_R*ln(x^(1/2*n)+(-8/(a*c^3-b^2*c^2)*n^3*a^5*c^2+6/(a
*c^3-b^2*c^2)*n^3*b^2*a^4*c-1/(a*c^3-b^2*c^2)*n^3*b^4*a^3)*_R^3+(-5/(a*c^3-
b^2*c^2)*n*b*a^2*c^2+5/(a*c^3-b^2*c^2)*n*b^3*a*c-1/(a*c^3-b^2*c^2)*n*b^5)*_
R),_R=RootOf((16*a^5*c^2*n^4-8*a^4*b^2*c*n^4+a^3*b^4*n^4)*_Z^4+(12*a^2*b*c^
2*n^2-7*a*b^3*c*n^2+b^5*n^2)*_Z^2+c^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{anx^{\frac{1}{2}n}}-\int\frac{cx^{\frac{3}{2}n}+bx^{\frac{1}{2}n}}{acxx^{2n}+abxx^{n}+a^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -2/(a*n*x^(1/2*n)) - integrate((c*x^(3/2*n) + b*x^(1/2*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

Fricas [B] time = 1.99945, size = 2527, normalized size = 12.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)
)*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*((a^3*b^3 - 4*a^4*b*c)
*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - (b^4 - 5
*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4
*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*
a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) - sqrt(2)*((a^3*b^3 -
 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4))
 - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*
b^2 - 4*a^4*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n^2*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*c)/((a^
3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*(
(a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^
7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(((a^3*b^2 - 4*a^4*c)*n^2
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*
c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) + sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n
^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*
b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) -
sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b
^2 - 4*a^7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(((a^3*b^2 - 4*a
^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3
```

```
+ 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) - 4*x*x^(-1/2*n - 1))/(a*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-\frac{1}{2}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.560 $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=699

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\cdots}{\cdots}$$

```
[Out] -3/(a*n*x^(n/3)) - (Sqrt[3]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1
 - (2*2^(1/3)*a^(1/3))/((b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3)))/Sqrt[3]])/(
2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n) - (Sqrt[3]*(b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*a^(1/3))/((b + Sqrt[b^2 - 4*a
*c])^(1/3)*x^(n/3)))/Sqrt[3]])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/
3)*n) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^
(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c]
)^(2/3)*n) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a
*c])^(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4
*a*c])^(2/3)*n) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2
- 4*a*c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b - Sqr
t[b^2 - 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*n) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*
c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)
*n)
```

Rubi [A] time = 1.49293, antiderivative size = 699, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1381, 1340, 1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(a*n*x^(n/3)) - (Sqrt[3]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1
 - (2*2^(1/3)*a^(1/3))/((b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3)))/Sqrt[3]])/(
2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n) - (Sqrt[3]*(b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*a^(1/3))/((b + Sqrt[b^2 - 4*a
*c])^(1/3)*x^(n/3)))/Sqrt[3]])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/
3)*n) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^
(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c]
)^(2/3)*n) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a
*c])^(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4
*a*c])^(2/3)*n) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2
- 4*a*c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b - Sqr
t[b^2 - 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*n) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*
```

```
c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)
*n)
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
```

```
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = -\frac{3\,\text{Subst}\left(\int \frac{1}{a+\frac{c}{x^6}+\frac{b}{x^3}}\,dx, x, x^{-n/3}\right)}{n}$$

$$= -\frac{3\,\text{Subst}\left(\int \frac{x^6}{c+bx^3+ax^6}\,dx, x, x^{-n/3}\right)}{n}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{3\,\text{Subst}\left(\int \frac{c+bx^3}{c+bx^3+ax^6}\,dx, x, x^{-n/3}\right)}{an}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(3\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^3}\,dx, x, x^{-n/3}\right)}{2an} + \frac{\left(3\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\right.}{2a}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{a}x}\,dx, x, x^{-n/3}\right)}{\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{2^{2/}}{\frac{\left(b-\sqrt{b^2-4ac}\right.}{2^{2/3}}}\right.}{\sqrt[3]{2}a\left(b-\sqrt{\right.}}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4\right.}}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4a\right.}}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4\right.}}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4a\right.}}$$

$$= -\frac{3x^{-n/3}}{an} - \frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{\sqrt{3}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b-\right.}{}$$

Mathematica [C] time = 0.149777, size = 127, normalized size = 0.18

$$\frac{6cx^{-n/3}\left(\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (6*c*(Hypergeometric2F1[-1/3, 1, 2/3, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/3, 1, 2/3, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n
/3))
```

Maple [C] time = 0.398, size = 534, normalized size = 0.8

$$-3\frac{1}{anx^{n/3}}+\sum_{\_R=\mathrm{RootOf}\left(\left(64a^7c^3n^6-48a^6b^2c^2n^6+12a^5b^4cn^6-a^4b^6n^6\right)\_Z^6+\left(-32a^3bc^3n^3+32a^2b^3c^2n^3-10ab^5cn^3+b^7n^3\right)\_Z^3+c^4\right)}\_R\ln\left(x^{\frac{n}{3}}+\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -3/a/n/(x^(1/3*n))+sum(_R*ln(x^(1/3*n)+(-64/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)
*n^5*a^8*c^4+112/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^2*a^7*c^3-60/(2*a^2*
c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^4*a^6*c^2+13/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)
*n^5*b^6*a^5*c-1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^8*a^4)*_R^5+(28/(2*a
^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b*a^4*c^4-63/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3
)*n^2*b^3*a^3*c^3+42/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^5*a^2*c^2-11/(2*
a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^7*a*c+1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*
n^2*b^9)*_R^2),_R=RootOf((64*a^7*c^3*n^6-48*a^6*b^2*c^2*n^6+12*a^5*b^4*c*n^
6-a^4*b^6*n^6)*_Z^6+(-32*a^3*b*c^3*n^3+32*a^2*b^3*c^2*n^3-10*a*b^5*c*n^3+b^
7*n^3)*_Z^3+c^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3}{anx^{\frac{1}{3}n}}-\int\frac{cx^{\frac{5}{3}n}+bx^{\frac{2}{3}n}}{acxx^{2n}+abxx^n+a^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -3/(a*n*x^(1/3*n)) - integrate((c*x^(5/3*n) + b*x^(2/3*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

Fricas [B] time = 14.3671, size = 13524, normalized size = 19.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(3)*(1/2)^(1/3)*a*n*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^
6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c
 + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*
c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^4*b^12*c - 17*a^5*b^1
0*c^2 + 114*a^6*b^8*c^3 - 378*a^7*b^6*c^4 + 632*a^8*b^4*c^5 - 480*a^9*b^2*c
^6 + 128*a^10*c^7)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^
2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3
)*n^6)) - sqrt(3)*(b^13*c - 15*a*b^11*c^2 + 88*a^2*b^9*c^3 - 252*a^3*b^7*c^
4 + 356*a^4*b^5*c^5 - 220*a^5*b^3*c^6 + 48*a^6*b*c^7)*n^2*x)*x^(-1/3*n - 1)
*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c
^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3) - sqrt(2)*(1/2)
^(2/3)*(sqrt(3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3
+ 64*a^8*c^4)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6
)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b
*c^4)*n^2*x)*sqrt((2*(b^8*c^2 - 8*a*b^6*c^3 + 20*a^2*b^4*c^4 - 16*a^3*b^2*c
^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) - (1/2)^(1/3)*((a^4*b^9*c - 12*a^5*b^7*c
^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c^5)*n^4*x*sqrt((b^8 - 8*a*
b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4
*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^10*c - 12*a*b^8*c^2 + 52*a^2
*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a^5*c^6)*n*x)*x^(-1/3*n - 1
)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3
*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*
c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3) - (1/2)^(2/3)*
((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*a^7*b^5*c^3 + 352*a^8*b^3*
c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^
2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3
)*n^6)) - (b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*
b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt
((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6
- 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^
4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*
a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b
^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*
a^5*c)*n^3))^(2/3) + 2*sqrt(3)*(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - 16
*a^3*b^2*c^7 + 4*a^4*c^8))/(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - 16*a^3
*b^2*c^7 + 4*a^4*c^8)) - 4*sqrt(3)*(1/2)^(1/3)*a*n*(-((a^4*b^2 - 4*a^5*c)*n
^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a
^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*
c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^
4*b^12*c - 17*a^5*b^10*c^2 + 114*a^6*b^8*c^3 - 378*a^7*b^6*c^4 + 632*a^8*b^
4*c^5 - 480*a^9*b^2*c^6 + 128*a^10*c^7)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*
b^2*c^2 - 64*a^11*c^3)*n^6)) + sqrt(3)*(b^13*c - 15*a*b^11*c^2 + 88*a^2*b^9
*c^3 - 252*a^3*b^7*c^4 + 356*a^4*b^5*c^5 - 220*a^5*b^3*c^6 + 48*a^6*b*c^7)*
n^2*x)*x^(-1/3*n - 1)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^
10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))
^(2/3) - sqrt(2)*(1/2)^(2/3)*(sqrt(3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*
c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*
c^2 - 64*a^11*c^3)*n^6)) + sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*
a^3*b^3*c^3 + 24*a^4*b*c^4)*n^2*x)*sqrt((2*(b^8*c^2 - 8*a*b^6*c^3 + 20*a^2*
b^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) + (1/2)^(1/3)*((a^
```

```
4*b^9*c - 12*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c^5)*
n^4*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/
((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^10*c -
 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a^5*c
^6)*n*x)*x^(-1/3*n - 1)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*
a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3
))^(1/3) + (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*a^7
*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 - 200
*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(-((a^4
*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6
)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2)*(-((a^4*b^2 - 4*
a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*
c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3
+ 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3) - 2*sqrt(3)*(b^8*c^4 - 8*a*b^6*
c^5 + 20*a^2*b^4*c^6 - 16*a^3*b^2*c^7 + 4*a^4*c^8))/(b^8*c^4 - 8*a*b^6*c^5
+ 20*a^2*b^4*c^6 - 16*a^3*b^2*c^7 + 4*a^4*c^8)) + 2*(1/2)^(1/3)*a*n*(((a^4*
b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
+ 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)
) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log((2*(b^4*c - 4*a*b^2
*c^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) + (1/2)^(1/3)*((a^4*b^5 - 8*a^5*b^3*c +
16*a^6*b*c^2)*n^4*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 +
 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6))
 - (b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n)*(((a^4*b^2 - 4*a^5*c)*
n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((
a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b
*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3))/x) + 2*(1/2)^(1/3)*a*n*(-((a^4*b^2 -
4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^
4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^
3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log((2*(b^4*c - 4*a*b^2*c^2 +
 2*a^2*c^3)*x*x^(-1/3*n - 1) - (1/2)^(1/3)*((a^4*b^5 - 8*a^5*b^3*c + 16*a^6
*b*c^2)*n^4*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4
*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^
6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n)*(-((a^4*b^2 - 4*a^5*c)*n^3*s
qrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b
^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/(
(a^4*b^2 - 4*a^5*c)*n^3))^(1/3))/x) - (1/2)^(1/3)*a*n*(((a^4*b^2 - 4*a^5*c)
*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(
(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*
b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log(8*(2*(b^8*c^2 - 8*a*b^6*c^3 + 20*
a^2*b^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) - (1/2)^(1/3)*
((a^4*b^9*c - 12*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c
^5)*n^4*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c
^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^10
*c - 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a
^5*c^6)*n*x)*x^(-1/3*n - 1)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c
 + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c +
48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*
n^3))^(1/3) - (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*
a^7*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*
a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 -
200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(((a
^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c
^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n
^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2) - (1/2)^(1/3)*
```

```
a*n*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*
a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^
11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log(8*(2*(b
^8*c^2 - 8*a*b^6*c^3 + 20*a^2*b^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)*x^2*x^(
-2/3*n - 2) + (1/2)^(1/3)*((a^4*b^9*c - 12*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 8
0*a^7*b^3*c^4 + 32*a^8*b*c^5)*n^4*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
- 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
64*a^11*c^3)*n^6)) + (b^10*c - 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c
^4 + 68*a^4*b^2*c^5 - 16*a^5*c^6)*n*x)*x^(-1/3*n - 1)*(-((a^4*b^2 - 4*a^5*c
)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/
((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a
*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3) + (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^
9*c + 98*a^6*b^7*c^2 - 280*a^7*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n
^5*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a
^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^12 - 14*a
*b^10*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*
c^5 + 32*a^6*c^6)*n^2)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a
^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3)
)^(2/3))/x^2) - 6*x*x^(-1/3*n - 1))/(a*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.561 $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=414

$$-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{ax}}{\sqrt[4]{-\sqrt{b^2-}}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] -4/(a*n*x^(n/4)) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2
^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/4)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/
4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4
))])/(a^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])^(
1/4)*x^(n/4))])/(a^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rubi [A] time = 0.786885, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1381, 1340, 1367, 1422, 212, 208, 205}

$$-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{ax}}{\sqrt[4]{-\sqrt{b^2-}}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(a*n*x^(n/4)) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2
^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/4)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/
4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4
))])/(a^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])^(
1/4)*x^(n/4))])/(a^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
```

```
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = -\frac{4\,\text{Subst}\left(\int \frac{1}{a+\frac{c}{x^8}+\frac{b}{x^4}}\,dx, x, x^{-n/4}\right)}{n}$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{x^8}{c+bx^4+ax^8}\,dx, x, x^{-n/4}\right)}{n}$$
$$= -\frac{4x^{-n/4}}{an} + \frac{4\,\text{Subst}\left(\int \frac{c+bx^4}{c+bx^4+ax^8}\,dx, x, x^{-n/4}\right)}{an}$$
$$= -\frac{4x^{-n/4}}{an} + \frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^4}\,dx, x, x^{-n/4}\right)}{an} + \frac{\left(2\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\right.}{}$$
$$= -\frac{4x^{-n/4}}{an} - \frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{a}x^2}\,dx, x, x^{-n/4}\right)}{a\sqrt{-b+\sqrt{b^2-4ac}n}} - \frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Sul}}{a}$$
$$= -\frac{4x^{-n/4}}{an} - \frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n} -$$

Mathematica [C] time = 0.141982, size = 127, normalized size = 0.31

$$\frac{8cx^{-n/4}\left(\frac{{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (8*c*(Hypergeometric2F1[-1/4, 1, 3/4, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/4, 1, 3/4, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n
/4))
```

Maple [C] time = 0.549, size = 630, normalized size = 1.5

$$-4\frac{1}{anx^{n/4}} + \sum_{\_R=\text{RootOf}\left(\left(256\,a^9c^4n^8-256\,a^8b^2c^3n^8+96\,a^7b^4c^2n^8-16\,a^6b^6cn^8+a^5b^8n^8\right)\_Z^8+\left(80\,a^4bc^4n^4-120\,a^3b^3c^3n^4+61\,a^2b^5c^2n^4-13\,ab^7cn^4+b^9n^4\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -4/a/n/(x^(1/4*n))+sum(_R*ln(x^(1/4*n)+(-128/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*
n^7*a^10*c^5+352/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^2*a^9*c^4-280/(a^2*c^6
-3*a*b^2*c^5+b^4*c^4)*n^7*b^4*a^8*c^3+98/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*
b^6*a^7*c^2-16/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^8*a^6*c+1/(a^2*c^6-3*a*b
^2*c^5+b^4*c^4)*n^7*b^10*a^5)*_R^7+(-36/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b
*a^5*c^5+129/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^3*a^4*c^4-138/(a^2*c^6-3*a
```

```
*b^2*c^5+b^4*c^4)*n^3*b^5*a^3*c^3+63/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^7*
a^2*c^2-13/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^9*a*c+1/(a^2*c^6-3*a*b^2*c^5
+b^4*c^4)*n^3*b^11)*_R^3),_R=RootOf((256*a^9*c^4*n^8-256*a^8*b^2*c^3*n^8+96
*a^7*b^4*c^2*n^8-16*a^6*b^6*c*n^8+a^5*b^8*n^8)*_Z^8+(80*a^4*b*c^4*n^4-120*a
^3*b^3*c^3*n^4+61*a^2*b^5*c^2*n^4-13*a*b^7*c*n^4+b^9*n^4)*_Z^4+c^5))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4}{anx^{\frac{1}{4}n}} - \int \frac{cx^{\frac{7}{4}n} + bx^{\frac{3}{4}n}}{acxx^{2n} + abxx^n + a^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -4/(a*n*x^(1/4*n)) - integrate((c*x^(7/4*n) + b*x^(3/4*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

Fricas [B] time = 8.34506, size = 12146, normalized size = 29.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^1
0*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^
3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*arctan(1/16
*sqrt(2)*(2*sqrt(2)*((a^5*b^14*c - 19*a^6*b^12*c^2 + 147*a^7*b^10*c^3 - 590
*a^8*b^8*c^4 + 1290*a^9*b^6*c^5 - 1464*a^10*b^4*c^6 + 736*a^11*b^2*c^7 - 12
8*a^12*c^8)*n^7*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8))
+ (b^15*c - 16*a*b^13*c^2 + 103*a^2*b^11*c^3 - 340*a^3*b^9*c^4 + 605*a^4*b^
7*c^5 - 554*a^5*b^5*c^6 + 224*a^6*b^3*c^7 - 32*a^7*b*c^8)*n^3*x)*x^(-1/4*n
- 1)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)) - sqrt(2)*((a^5*b^10 - 16*a^6*b^8*c + 9
8*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*n^7*x*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^11 - 13*a*b^9*c
+ 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5)*n^3*x)
*sqrt((4*(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)
*x^2*x^(-1/2*n - 2) + sqrt(2)*((a^5*b^11 - 15*a^6*b^9*c + 85*a^7*b^7*c^2 -
220*a^8*b^5*c^3 + 240*a^9*b^3*c^4 - 64*a^10*b*c^5)*n^6*sqrt((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c +
48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2
 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6)*n^2)*sqr
t(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2
*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^
6*b^2*c + 16*a^7*c^2)*n^4)))/x^2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
```

)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 - 6*a^3*b^2*c^8 + a^4*c^9)) - 4*sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*arctan(1/8*(2*((a^5*b^14*c - 19*a^6*b^12*c^2 + 147*a^7*b^10*c^3 - 590*a^8*b^8*c^4 + 1290*a^9*b^6*c^5 - 1464*a^10*b^4*c^6 + 736*a^11*b^2*c^7 - 128*a^12*c^8)*n^7*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^15*c - 16*a*b^13*c^2 + 103*a^2*b^11*c^3 - 340*a^3*b^9*c^4 + 605*a^4*b^7*c^5 - 554*a^5*b^5*c^6 + 224*a^6*b^3*c^7 - 32*a^7*b*c^8)*n^3*x)*x^(-1/4*n - 1)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)) - ((a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*n^7*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5)*n^3*x)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*sqrt((4*(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*x^2*x^(-1/2*n - 2) - sqrt(2)*((a^5*b^11 - 15*a^6*b^9*c + 85*a^7*b^7*c^2 - 220*a^8*b^5*c^3 + 240*a^9*b^3*c^4 - 64*a^10*b*c^5)*n^6*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6)*n^2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))/x^2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 - 6*a^3*b^2*c^8 + a^4*c^9)) + sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -

```
 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64
*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c +
 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1)
 - sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c +
48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 -
4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c
+ 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - sqrt(2)*a
*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*
b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^
2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2
 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c
^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^6 - 7
*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a
^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*
c^2)*n^4))))/x) + sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b
^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*
log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) - sqrt(2)*((a^5*b^5
 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64
*a^13*c^3)*n^8)) + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(s
qrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - 8*x*x^(-1/4*n - 1))/(a*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.562 $\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=140

$$-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] (-2*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (2*c*x^3*Hypergeometric2
F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*(b^2 - 4*a*c
+ b*Sqrt[b^2 - 4*a*c]))
```

Rubi [A] time = 0.118454, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1383, 364}

$$-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (2*c*x^3*Hypergeometric2
F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*(b^2 - 4*a*c
+ b*Sqrt[b^2 - 4*a*c]))
```

Rule 1383

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[(2*c)/q, Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx &= \frac{(2c)\int \frac{x^2}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{x^2}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\ &= -\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)}-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)}\end{aligned}$$

Mathematica [A] time = 0.127976, size = 129, normalized size = 0.92

$$\frac{x^3\left(\left(\sqrt{b^2-4ac}+b\right)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+\left(\sqrt{b^2-4ac}-b\right)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)\right)}{6a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x^3*((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 3/n, (3 + n)/n, (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])] + (-b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1,
 3/n, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]))/(6*a*Sqrt[b^2 - 4*a*
c])
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{cx^{2n}+bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x**2/(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^{2n} + bx^n + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

3.563 $\int \frac{x}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=136

$$-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] -((c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (c*x^2*Hypergeometric2F1[1,
2/n, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[
b^2 - 4*a*c])
```

Rubi [A] time = 0.0401216, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1383, 364}

$$-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (c*x^2*Hypergeometric2F1[1,
2/n, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[
b^2 - 4*a*c])
```

Rule 1383

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[(2*c)/q, Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx^n+cx^{2n}}\,dx &= \frac{(2c)\int \frac{x}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{x}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\ &= -\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.106266, size = 129, normalized size = 0.95

$$\frac{x^2\left(\left(\sqrt{b^2-4ac}+b\right){}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+\left(\sqrt{b^2-4ac}-b\right){}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)\right)}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x^2*((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 2/n, (2 + n)/n, (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])] + (-b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1,
 2/n, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]))/(4*a*Sqrt[b^2 - 4*a*
c])
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(x/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{cx^{2n}+bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(x/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x/(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cx^{2n} + bx^n + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

3.564 $\int \frac{1}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=124

$$-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] (-2*c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (2*c*x*Hypergeometric2F1[1
, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b
*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.0629608, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1347, 245}

$$-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(-1),x]
```

```
[Out] (-2*c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (2*c*x*Hypergeometric2F1[1
, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b
*Sqrt[b^2 - 4*a*c])
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+bx^n+cx^{2n}}\,dx &= \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{\sqrt{b^2-4ac}}\\ &= -\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.0916139, size = 119, normalized size = 0.96

$$\frac{x\left(\left(\sqrt{b^2-4ac}+b\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+\left(\sqrt{b^2-4ac}-b\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)\right)}{2a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(-1),x]
```

```
[Out] (x*((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2*c*x
^n)/(-b + Sqrt[b^2 - 4*a*c])] + (-b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[
1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]))/(2*a*Sqrt[b^2
- 4*a*c])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \left(a+bx^n+cx^{2n}\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{cx^{2n}+bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(1/(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

3.565 $\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}\,dx$

Optimal. Leaf size=74

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{an\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]*n) + Log[
x]/a - Log[a + b*x^n + c*x^(2*n)]/(2*a*n)
```

Rubi [A] time = 0.0658304, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1357, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{an\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]*n) + Log[
x]/a - Log[a + b*x^n + c*x^(2*n)]/(2*a*n)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^n\right)}{an} \\
&= \frac{\log(x)}{a} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{2an} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2an} \\
&= \frac{\log(x)}{a} - \frac{\log\left(a+bx^n+cx^{2n}\right)}{2an} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^n\right)}{an} \\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,n} + \frac{\log(x)}{a} - \frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}
\end{aligned}$$

Mathematica [A] time = 0.115239, size = 74, normalized size = 1.

$$-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{4ac-b^2}}\right)}{n\sqrt{4ac-b^2}} + \frac{\log(a+x^n(b+cx^n))}{n} - 2\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*n) - 2
*Log[x] + Log[a + x^n*(b + c*x^n)]/n)/(2*a)
```

Maple [B] time = 0.066, size = 397, normalized size = 5.4

$$4\,\frac{n^2\ln(x)\,ac}{4\,a^2cn^2-ab^2n^2} - \frac{n^2\ln(x)\,b^2}{4\,a^2cn^2-ab^2n^2} - 2\,\frac{c}{\left(4\,ac-b^2\right)n}\ln\left(x^n - 1/2\,\frac{-b^2+\sqrt{-4\,ab^2c+b^4}}{bc}\right) + \frac{b^2}{2\,a\left(4\,ac-b^2\right)n}\ln\left(x^n - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] 4/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)*a*c-1/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)
*b^2-2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/
(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2+1/2/a/(4*a*
c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/
2)-2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/(4*
a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/a/(4*a*c-b^
2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n} + bx^n + a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

Fricas [A] time = 1.60941, size = 590, normalized size = 7.97

$$\left[\frac{2\left(b^2-4\,ac\right)n\log\left(x\right)+\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc+\sqrt{b^2-4\,ac}c\right)x^n+\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^{2\,n}+bx^n+a\right)}{2\left(ab^2-4\,a^2c\right)n}, \frac{2\left(b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2 - 4*a*c)*n*log(x) + sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b
^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^
(2*n) + b*x^n + a)) - (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4
*a^2*c)*n), 1/2*(2*(b^2 - 4*a*c)*n*log(x) + 2*sqrt(-b^2 + 4*a*c)*b*arctan(-
(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - (b^2 -
 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4*a^2*c)*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n} + bx^n + a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

3.566 $\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx$

Optimal. Leaf size=142

$$\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] (2*c*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x) + (2*c*Hypergeometric2F1
[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a
*c + b*Sqrt[b^2 - 4*a*c])*x)
```

Rubi [A] time = 0.0462026, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1383, 364}

$$\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2*c*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x) + (2*c*Hypergeometric2F1
[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a
*c + b*Sqrt[b^2 - 4*a*c])*x)
```

Rule 1383

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[(2*c)/q, Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx&=\frac{(2c)\int \frac{1}{x^2\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{1}{x^2\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}\\&=\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x}+\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x}\end{aligned}$$

Mathematica [A] time = 0.122172, size = 129, normalized size = 0.91

$$-\frac{\left(\sqrt{b^2-4ac}+b\right)\,{}_2F_1\left(1,-\frac{1}{n};\frac{n-1}{n};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+\left(\sqrt{b^2-4ac}-b\right)\,{}_2F_1\left(1,-\frac{1}{n};\frac{n-1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, (2*c*x^
n)/(-b + Sqrt[b^2 - 4*a*c])] + (-b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1
, -n^(-1), (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[b^2 -
 4*a*c]*x)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{cx^2x^{2n}+bx^2x^n+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2\,n} + bx^{n} + a\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

3.567 $\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx$

Optimal. Leaf size=140

$$\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] (c*Hypergeometric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*
c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x^2) + (c*Hypergeometric2F1[1, -
2/n, -((2 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*S
qrt[b^2 - 4*a*c])*x^2)
```

Rubi [A] time = 0.0481798, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1383, 364}

$$\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*Hypergeometric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*
c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x^2) + (c*Hypergeometric2F1[1, -
2/n, -((2 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*S
qrt[b^2 - 4*a*c])*x^2)
```

Rule 1383

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[(2*c)/q, Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx &= \frac{(2c)\int \frac{1}{x^3\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{1}{x^3\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}\\ &= \frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2}+\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x^2}\end{aligned}$$

Mathematica [A] time = 0.123382, size = 129, normalized size = 0.92

$$-\frac{\left(\sqrt{b^2-4ac}+b\right)\,{}_2F_1\left(1,-\frac{2}{n};\frac{n-2}{n};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+\left(\sqrt{b^2-4ac}-b\right)\,{}_2F_1\left(1,-\frac{2}{n};\frac{n-2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4ax^2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, -2/n, (-2 + n)/n, (2*c*x^n)/
(-b + Sqrt[b^2 - 4*a*c])] + (-b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, -
2/n, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[b^2 - 4*a*c
]*x^2)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/x^3/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{cx^3x^{2n}+bx^3x^n+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

3.568 $\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=148

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{4}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -1/2, -1/2, (4 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rubi [A] time = 0.177984, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{4}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -1/2, -1/2, (4 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx=\frac{\sqrt{a+bx^n+cx^{2n}}\int x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$=\frac{x^4\sqrt{a+bx^n+cx^{2n}}\operatorname{F}_1\left(\frac{4}{n};-\frac{1}{2},-\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.788825, size = 365, normalized size = 2.47

$$\frac{x^4\left(2bnx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n+4}{n};\frac{1}{2},\frac{1}{2};2+\frac{4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+an(n+4)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\cdots}\right.}{4(n+4)^2\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*(4*(4 + n)*(a + x^n*(b + c*x^n)) + a*n*(4 + n)*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*b*n*x^n*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(4 + n)/n, 1/2
, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^
2 - 4*a*c])]))/(4*(4 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^{2n}+bx^n+a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n}+bx^n+a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

3.569 $\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=148

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{3}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -1/2, -1/2, (3 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rubi [A] time = 0.14724, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{3}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -1/2, -1/2, (3 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}\int x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{x^3\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{3}{n};-\frac{1}{2},-\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.737382, size = 366, normalized size = 2.47

$$\frac{x^3\left(3bnx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n+3}{n};\frac{1}{2},\frac{1}{2};2+\frac{3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+2an(n+3)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\cdots}\right.}{6(n+3)^2\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*(6*(3 + n)*(a + x^n*(b + c*x^n)) + 2*a*n*(3 + n)*Sqrt[(b - Sqrt[b^2 -
4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*n*x^n
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 + n)/n, 1
/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])]))/(6*(3 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n}+bx^n+a}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n}+bx^n+a}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

3.570 $\int x\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=148

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{2}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -1/2, -1/2, (2 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rubi [A] time = 0.11389, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{2}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -1/2, -1/2, (2 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}\int x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{x^2\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{2}{n};-\frac{1}{2},-\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.681144, size = 364, normalized size = 2.46

$$\frac{x^2\left(bnx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n+2}{n};\frac{1}{2},\frac{1}{2};2+\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+an(n+2)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\right.}{2(n+2)^2\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*(2*(2 + n)*(a + x^n*(b + c*x^n)) + a*n*(2 + n)*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n*Sqr
t[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(2 + n)/n, 1/2,
1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])]))/(2*(2 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n}+bx^n+a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a + bx^n + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2\,n} + bx^n + a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

3.571 $\int \sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=139

$$\frac{x\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -1/2, -1/2, 1 + n^(-1), (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rubi [A] time = 0.0840802, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1348, 429}

$$\frac{x\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -1/2, -1/2, 1 + n^(-1), (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}\int \sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{x\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.618538, size = 351, normalized size = 2.53

$$\frac{x\left(bnx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+2(n+1)\left(an\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\right.\right.}{2(n+1)^2\sqrt{a+x^n(b+cx^n)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1
 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 + n)*(a + x^n*(b + c*x^n) + a*n*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1
 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4
*a*c])])))/(2*(1 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx^n + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.572 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}$$

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]/n - (Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*S
qrt[a + b*x^n + c*x^(2*n)])])/n + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[
a + b*x^n + c*x^(2*n)])])/(2*Sqrt[c]*n)
```

Rubi [A] time = 0.0952366, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1357, 734, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]/n - (Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*S
qrt[a + b*x^n + c*x^(2*n)])])/n + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[
a + b*x^n + c*x^(2*n)])])/(2*Sqrt[c]*n)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^n\right)}{2n}\\
&= \frac{\sqrt{a+bx^n+cx^{2n}}}{n}+\frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^n\right)}{n}+\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^n\right)}{2n}\\
&= \frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{(2a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n}\\
&= \frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}
\end{aligned}$$

Mathematica [A] time = 0.149285, size = 110, normalized size = 0.92

$$\frac{\sqrt{a+x^n\left(b+cx^n\right)}-\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+x^n\left(b+cx^n\right)}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+x^n\left(b+cx^n\right)}}\right)}{2\sqrt{c}}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] (Sqrt[a + x^n*(b + c*x^n)] - Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[
a + x^n*(b + c*x^n)])] + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[a + x^n*(
b + c*x^n)])])/(2*Sqrt[c]))/n
```

Maple [A] time = 0.108, size = 125, normalized size = 1.1

$$\frac{1}{n}\sqrt{a+b\mathrm{e}^{n\ln(x)}+c\left(\mathrm{e}^{n\ln(x)}\right)^2}+\frac{b}{2n}\ln\left(\left(\frac{b}{2}+c\mathrm{e}^{n\ln(x)}\right)\frac{1}{\sqrt{c}}+\sqrt{a+b\mathrm{e}^{n\ln(x)}+c\left(\mathrm{e}^{n\ln(x)}\right)^2}\right)\frac{1}{\sqrt{c}}-\frac{1}{n}\sqrt{a}\ln\left(\frac{1}{\mathrm{e}^{n\ln(x)}}\left(2a+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(1/2)/x,x)
```

```
[Out] 1/n*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)+1/2/n*b*ln((1/2*b+c*exp(n*ln(
x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/c^(1/2)-1/n*a^(1/2)
*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)
)/exp(n*ln(x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)
```

Fricas [A] time = 2.00975, size = 1554, normalized size = 13.06

$$\left[\frac{b\sqrt{c}\log\left(-8\,c^2x^{2n} - 8\,bcx^n - b^2 - 4\,ac - 4\left(2\,c^{\frac{3}{2}}x^n + b\sqrt{c}\right)\sqrt{cx^{2n} + bx^n + a}\right) + 2\,\sqrt{a}c\log\left(-\frac{8\,abx^n+8\,a^2+\left(b^2+4\,ac\right)x^{2n}-4\left(\sqrt{a}bx^n \cdots\right.}{x^{2n}}\right.}{4\,cn}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)
*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 2*sqrt(a)*c*log(-(8*a*b*x^
n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^
(2*n) + b*x^n + a))/x^(2*n)) + 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), -1/2
*(b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*
x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) - sqrt(a)*c*log(-(8*a*b*x^n + 8*a^2
 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b
*x^n + a))/x^(2*n)) - 2*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/4*(4*sqrt(-
a)*c*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)
/(a*c*x^(2*n) + a*b*x^n + a^2)) + b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n
- b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a))
+ 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/2*(2*sqrt(-a)*c*arctan(1/2*(sqr
t(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a*b*
x^n + a^2)) - b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*
x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*sqrt(c*x^(2*n) + b*
x^n + a)*c)/(c*n)]
```

<u>Sympy</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{2\,n} + bx^n + a}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)
```

3.573 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{1}{n};-\frac{1}{2},-\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -((Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -1/2, -1/2, -((1 - n)/n),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x
*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]))
```

Rubi [A] time = 0.149699, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$-\frac{\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{1}{n};-\frac{1}{2},-\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -1/2, -1/2, -((1 - n)/n),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x
*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]))
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}\int \frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}{x^2}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{1}{2},-\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.662031, size = 365, normalized size = 2.45

$$\frac{bnx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n-1}{n};\frac{1}{2},\frac{1}{2};2-\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-2a(n-1)n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2 \cdots}}{\cdots}}}{2(n-1)^2x\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

```
[Out] (2*(-1 + n)*(a + x^n*(b + c*x^n)) - 2*a*(-1 + n)*n*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(-1 + n)/n,
1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + S
qrt[b^2 - 4*a*c])])/(2*(-1 + n)^2*x*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{a+bx^n+cx^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{2\,n}+bx^n+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

3.574 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3} dx$

Optimal. Leaf size=151

$$-\frac{\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{2}{n};-\frac{1}{2},-\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -(Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -1/2, -1/2, -((2 - n)/n), (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2
*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])
```

Rubi [A] time = 0.148533, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$-\frac{\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{2}{n};-\frac{1}{2},-\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

```
[Out] -(Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -1/2, -1/2, -((2 - n)/n), (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2
*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}\int \frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}{x^3}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{2}{n};-\frac{1}{2},-\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.643472, size = 365, normalized size = 2.42

$$\frac{bnx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n-2}{n};\frac{1}{2},\frac{1}{2};2-\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-a(n-2)n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac+}}{\sqrt{b^2-4}}}}{2(n-2)^2x^2\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

```
[Out] (2*(-2 + n)*(a + x^n*(b + c*x^n)) - a*(-2 + n)*n*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n*Sqrt
[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(-2 + n)/n, 1/2,
1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])])/(2*(-2 + n)^2*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{a+bx^n+cx^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{2\,n}+bx^n+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

3.575 $\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=149

$$\frac{ax^4\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{4}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -3/2, -3/2, (4 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rubi [A] time = 0.153812, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{ax^4\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{4}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -3/2, -3/2, (4 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x^3\left(a+bx^n+cx^{2n}\right)^{3/2}dx=\frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int x^3\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$=\frac{ax^4\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{4}{n};-\frac{3}{2},-\frac{3}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 1.55728, size = 518, normalized size = 3.48

$$\frac{x^4\left(2(n+4)\left(32a^2c\left(n^2+3n+2\right)+a\left(3b^2n^2+2bc\left(23n^2+84n+64\right)x^n+8c^2\left(5n^2+18n+16\right)x^{2n}\right)+x^n(b+cx^n)\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*(2*(4 + n)*(32*a^2*c*(2 + 3*n + n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*
b*c*(32 + 36*n + 7*n^2)*x^n + 8*c^2*(8 + 6*n + n^2)*x^(2*n)) + a*(3*b^2*n^2
 + 2*b*c*(64 + 84*n + 23*n^2)*x^n + 8*c^2*(16 + 18*n + 5*n^2)*x^(2*n))) - 6
*a*n^2*(4 + n)*(b^2 - 2*a*c*(2 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(b^2*(8 + n) - 4*a
*c*(8 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appe
llF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(16*c*(2 + n)*(4 + n)^2*(4 + 3*n)*Sqrt[a
+ x^n*(b + c*x^n)])
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int x^3\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^{n} + a\right)^{\frac{3}{2}} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

3.576 $\int x^2 \left(a+bx^n+cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=149

$$\frac{ax^3\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{3}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -3/2, -3/2, (3 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rubi [A] time = 0.150804, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{ax^3\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{3}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -3/2, -3/2, (3 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x^2\left(a+bx^n+cx^{2n}\right)^{3/2}dx=\frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int x^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$=\frac{ax^3\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{3}{n};-\frac{3}{2},-\frac{3}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 1.52096, size = 524, normalized size = 3.52

$$\frac{x^3\left(2(n+3)\left(4a^2c\left(8n^2+18n+9\right)+a\left(3b^2n^2+2bc\left(23n^2+63n+36\right)x^n+4c^2\left(10n^2+27n+18\right)x^{2n}\right)+x^n\left(b+cx^n\right)\left(3\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(2*(3 + n)*(4*a^2*c*(9 + 18*n + 8*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 +
2*b*c*(18 + 27*n + 7*n^2)*x^n + 4*c^2*(9 + 9*n + 2*n^2)*x^(2*n)) + a*(3*b^2
*n^2 + 2*b*c*(36 + 63*n + 23*n^2)*x^n + 4*c^2*(18 + 27*n + 10*n^2)*x^(2*n))
) + 2*a*n^2*(3 + n)*(-3*b^2 + 4*a*c*(3 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(-12*a*c
*(2 + n) + b^2*(6 + n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(24*c*(1 + n)*(3 + n)^2*(3 + 2*
n)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int x^2\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

3.577 $\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=149

$$\frac{ax^2\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{2}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -3/2, -3/2, (2 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rubi [A] time = 0.113792, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$\frac{ax^2\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{2}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -3/2, -3/2, (2 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}dx=\frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int x\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$=\frac{ax^2\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{2}{n};-\frac{3}{2},-\frac{3}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 1.52875, size = 520, normalized size = 3.49

$$x^2\left(2(n+2)\left(16a^2c\left(2n^2+3n+1\right)+a\left(3b^2n^2+2bc\left(23n^2+42n+16\right)x^n+8c^2\left(5n^2+9n+4\right)x^{2n}\right)+x^n\left(b+cx^n\right)\left(3b\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(2*(2 + n)*(16*a^2*c*(1 + 3*n + 2*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 +
2*b*c*(8 + 18*n + 7*n^2)*x^n + 8*c^2*(2 + 3*n + n^2)*x^(2*n)) + a*(3*b^2*n^
2 + 2*b*c*(16 + 42*n + 23*n^2)*x^n + 8*c^2*(4 + 9*n + 5*n^2)*x^(2*n))) - 6*
a*n^2*(2 + n)*(b^2 - 4*a*c*(1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(b^2*(4 + n) - 4*a*
c*(4 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(16*c*(1 + n)*(2 + n)^2*(2 + 3*n)*Sqrt[a +
 x^n*(b + c*x^n)])
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int x\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

3.578 $\int \left(a + bx^n + cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=140

$$\frac{ax\sqrt{a + bx^n + cx^{2n}}F_1\left(\frac{1}{n}; -\frac{3}{2}, -\frac{3}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -3/2, -3/2, 1 + n^(-1), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rubi [A] time = 0.0827008, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1348, 429}

$$\frac{ax\sqrt{a + bx^n + cx^{2n}}F_1\left(\frac{1}{n}; -\frac{3}{2}, -\frac{3}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -3/2, -3/2, 1 + n^(-1), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{ax\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 1.53744, size = 513, normalized size = 3.66

$$\frac{x\left(2(n+1)\left(4a^2c\left(8n^2+6n+1\right)-3an^2\left(b^2-4ac(2n+1)\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(-3*b*n^2*(b^2*(2 + n) - 4*a*c*(2 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 +
 n)*(4*a^2*c*(1 + 6*n + 8*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*b*c*(2 + 9*
n + 7*n^2)*x^n + 4*c^2*(1 + 3*n + 2*n^2)*x^(2*n)) + a*(3*b^2*n^2 + 2*b*c*(4
 + 21*n + 23*n^2)*x^n + 4*c^2*(2 + 9*n + 10*n^2)*x^(2*n)) - 3*a*n^2*(b^2 -
4*a*c*(1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(8*c*(1 + n)^2*(1 + 2*n)*(1 + 3*n)*Sqrt[a
 + x^n*(b + c*x^n)])
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2\,n} + bx^{n} + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.579 $\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=173

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}+\frac{\left(8ac+b^2+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\left(a+bx\right.\cdots$$

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^n)*Sqrt[a + b*x^n + c*x^(2*n)])/(8*c*n) + (a + b*x^
n + c*x^(2*n))^(3/2)/(3*n) - (a^(3/2)*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt
[a + b*x^n + c*x^(2*n)])])/n - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2*S
qrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])])/(16*c^(3/2)*n)
```

Rubi [A] time = 0.158621, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1357, 734, 814, 843, 621, 206, 724}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}+\frac{\left(8ac+b^2+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\left(a+bx\right.\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^n)*Sqrt[a + b*x^n + c*x^(2*n)])/(8*c*n) + (a + b*x^
n + c*x^(2*n))^(3/2)/(3*n) - (a^(3/2)*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt
[a + b*x^n + c*x^(2*n)])])/n - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2*S
qrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])])/(16*c^(3/2)*n)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
```

```
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n}-\frac{\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx,x,x^n\right)}{2n}\\
&= \frac{\left(b^2+8ac+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n}+\frac{\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b\left(b^2-12ac\right)x}{x\sqrt{a+bx+cx^2}}\,d\ldots\right.}{8cn}\\
&= \frac{\left(b^2+8ac+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x\ldots\right.}{n}\\
&= \frac{\left(b^2+8ac+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n}-\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\ldots\right.}{n}\\
&= \frac{\left(b^2+8ac+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n}-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}
\end{aligned}
$$

Mathematica [A] time = 0.292796, size = 158, normalized size = 0.91

$$\frac{-48a^{3/2}c^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+x^n(b+cx^n)}}\right)+2\sqrt{c}\sqrt{a+x^n\left(b+cx^n\right)}\left(8c\left(4a+cx^{2n}\right)+3b^2+14bcx^n\right)-3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{\cdots}{2\cdots}\right.}{48c^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x^n*(b + c*x^n)]*(3*b^2 + 14*b*c*x^n + 8*c*(4*a + c*x^(
2*n))) - 48*a^(3/2)*c^(3/2)*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + x^n*(
b + c*x^n)])] - 3*b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[a
+ x^n*(b + c*x^n)])])/(48*c^(3/2)*n)
```

Maple [A] time = 0.045, size = 209, normalized size = 1.2

$$\frac{8c^2\left(e^{n\ln(x)}\right)^2+14be^{n\ln(x)}c+32ac+3b^2}{24cn}\sqrt{a+be^{n\ln(x)}+c\left(e^{n\ln(x)}\right)^2}+\frac{3ab}{4n}\ln\left(\left(\frac{b}{2}+ce^{n\ln(x)}\right)\frac{1}{\sqrt{c}}+\sqrt{a+be^{n\ln(x)}+c\left(\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(3/2)/x,x)
```

```
[Out] 1/24*(8*c^2*exp(n*ln(x))^2+14*b*exp(n*ln(x))*c+32*a*c+3*b^2)*(a+b*exp(n*ln(
x))+c*exp(n*ln(x))^2)^(1/2)/c/n+3/4/c^(1/2)/n*a*b*ln((1/2*b+c*exp(n*ln(x)))
/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))-1/16/c^(3/2)/n*b^3*ln((
1/2*b+c*exp(n*ln(x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))-1/
n*a^(3/2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))
^2)^(1/2))/exp(n*ln(x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

Fricas [A] time = 3.16363, size = 1955, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(
sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) - 3*(b^3 -
 12*a*b*c)*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3
/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b
*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(
24*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)
*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 3*(b^3 - 12*a*b
*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*
x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*(8*c^3*x^(2*n) + 14*b*c^2*x^n +
 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/96*(96*sqrt(-a
)*a*c^2*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n +
 a)/(a*c*x^(2*n) + a*b*x^n + a^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*
x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^
(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)
*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(48*sqrt(-a)*a*c^2*arctan(1/2*(
sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a
*b*x^n + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n +
 b*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2
*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n
 + a))/(c^2*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

3.580 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx$

Optimal. Leaf size=150

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{1}{n};-\frac{3}{2},-\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -((a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -3/2, -3/2, -((1 - n)/n)
, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]))
```

Rubi [A] time = 0.150677, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{1}{n};-\frac{3}{2},-\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```


```
[Out] -((a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -3/2, -3/2, -((1 - n)/n)
, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]))
```


Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int \frac{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^2}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{3}{2},-\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 1.3956, size = 526, normalized size = 3.51

$$\frac{2(n-1)\left(4a^2c\left(8n^2-6n+1\right)+a\left(3b^2n^2+2bc\left(23n^2-21n+4\right)x^n+4c^2\left(10n^2-9n+2\right)x^{2n}\right)+x^n\left(b+cx^n\right)\left(3b^2n^2+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] (2*(-1 + n)*(4*a^2*c*(1 - 6*n + 8*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*b*c
*(2 - 9*n + 7*n^2)*x^n + 4*c^2*(1 - 3*n + 2*n^2)*x^(2*n)) + a*(3*b^2*n^2 +
2*b*c*(4 - 21*n + 23*n^2)*x^n + 4*c^2*(2 - 9*n + 10*n^2)*x^(2*n))) - 6*a*(-
1 + n)*n^2*(b^2 + 4*a*c*(-1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*(4*a*c*(2 - 3*n) +
 b^2*(-2 + n))*n^2*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(8*c*(-1 + n)^2*(-1 + 2*n)*(-1 +
3*n)*x*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**2,x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + bx^{n} + a\right)^{\frac{3}{2}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

3.581 $\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=152

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{2}{n};-\frac{3}{2},-\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -(a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -3/2, -3/2, -((2 - n)/n), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*x
^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])
```

Rubi [A] time = 0.152111, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{2}{n};-\frac{3}{2},-\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] -(a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -3/2, -3/2, -((2 - n)/n), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*x
^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int \frac{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^3}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{a\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{2}{n};-\frac{3}{2},-\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 1.39259, size = 520, normalized size = 3.42

$$\frac{2(n-2)\left(16a^2c\left(2n^2-3n+1\right)+a\left(3b^2n^2+2bc\left(23n^2-42n+16\right)x^n+8c^2\left(5n^2-9n+4\right)x^{2n}\right)+x^n\left(b+cx^n\right)\left(3b^2n^2+2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] (2*(-2 + n)*(16*a^2*c*(1 - 3*n + 2*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*b*
c*(8 - 18*n + 7*n^2)*x^n + 8*c^2*(2 - 3*n + n^2)*x^(2*n)) + a*(3*b^2*n^2 +
2*b*c*(16 - 42*n + 23*n^2)*x^n + 8*c^2*(4 - 9*n + 5*n^2)*x^(2*n))) - 6*a*(b
^2 + 4*a*c*(-1 + n))*(-2 + n)*n^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*(4*a*c*(4 - 3*n) + b^2*
(-4 + n))*n^2*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appe
llF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(16*c*(-2 + n)^2*(-1 + n)*(-2 + 3*n)*x^2*
Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}{x^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2\,n} + bx^{n} + a\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

3.582 $\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=148

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rubi [A] time = 0.152993, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}$$
$$= \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A] time = 0.170156, size = 175, normalized size = 1.18

$$\frac{x^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{4\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2
, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/(4*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int x^3\frac{1}{\sqrt{a+bx^n+cx^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.583 $\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=148

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rubi [A] time = 0.147951, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^2}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}$$
$$= \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A] time = 0.15302, size = 175, normalized size = 1.18

$$\frac{x^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{3\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2
, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/(3*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int x^2\frac{1}{\sqrt{a+bx^n+cx^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.584 $\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}} \, dx$

Optimal. Leaf size=148

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rubi [A] time = 0.110621, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int\frac{x}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}$$
$$= \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A] time = 0.140502, size = 175, normalized size = 1.18

$$\frac{x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{2\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2
, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/(2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + bx^n + cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^{2n} + bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.585 $\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=139

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^n + c*x^(
2*n)]
```

Rubi [A] time = 0.083524, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1348, 429}

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^n + c*x^(
2*n)]
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$
$$= \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A] time = 0.0866659, size = 166, normalized size = 1.19

$$\frac{x\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{a+x^n(b+cx^n)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/
2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])])/Sqrt[a + x^n*(b + c*x^n)]
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx^n + cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2n} + bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.586 $\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

```
[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*n
))
```

Rubi [A] time = 0.0334797, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1357, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*n
))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{n}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n}$$
$$= -\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

Mathematica [A] time = 0.0656383, size = 47, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*n
))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{a+bx^n+cx^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2\,n}+bx^n+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

Fricas [A] time = 1.86126, size = 347, normalized size = 7.38

$$\left[\frac{\log\left(-\frac{8\,abx^{n}+8\,a^{2}+\left(b^{2}+4\,ac\right)x^{2\,n}-4\left(\sqrt{a}bx^{n}+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+bx^{n}+a}}{x^{2\,n}}\right)}{2\,\sqrt{a}n}, \frac{\sqrt{-a}\arctan\left(\frac{\left(\sqrt{-a}bx^{n}+2\,\sqrt{-a}a\right)\sqrt{cx^{2\,n}+bx^{n}+a}}{2\left(acx^{2\,n}+abx^{n}+a^{2}\right)}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2
*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n))/(sqrt(a)*n), sqrt(-a)*arcta
n(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2
*n) + a*b*x^n + a^2))/(a*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+bx^{n}+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**n + c*x**(2*n))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2\,n}+bx^{n}+a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

3.587 $\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^n +
 c*x^(2*n)]))
```

Rubi [A] time = 0.147036, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^n +
 c*x^(2*n)]))
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}$$
$$= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A] time = 0.151772, size = 173, normalized size = 1.16

$$-\frac{\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};\frac{n-1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{x\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -((Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1
/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(x*Sqrt[a + x^n*(b + c*x^n)]))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{a+bx^n+cx^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2\,n}+bx^n+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a + bx^n + cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**n + c*x**(2*n))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2\,n} + bx^n + a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

3.588 $\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=151

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*Sqrt[a + b*x^n +
 c*x^(2*n)])
```

Rubi [A] time = 0.146022, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*Sqrt[a + b*x^n +
 c*x^(2*n)])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A] time = 0.160041, size = 175, normalized size = 1.16

$$-\frac{\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n-2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{2x^2\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + 
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 
1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])])/(2*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2n}+bx^n+a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**n + c*x**(2*n))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

3.589 $\int \frac{x^3}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{n};\frac{3}{2},\frac{3}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/n, 3/2, 3/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rubi [A] time = 0.151205, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{n};\frac{3}{2},\frac{3}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/n, 3/2, 3/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{n};\frac{3}{2},\frac{3}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] time = 0.885126, size = 398, normalized size = 2.64

$$\frac{x^4\left(32bcx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n+4}{n};\frac{1}{2},\frac{1}{2};2+\frac{4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-(n+4)\left(b^2(n-8)-4ac(n-4)\right)\right.}{4an(n+4)\left(4ac-b^2\right)\sqrt{a+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*(-8*(4 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-8 + n) - 4*a*c*(-4 + n))*
(4 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n,
1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + 32*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])]*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(4*a*(-b^2 + 4*a*c)*n*(4 + n
)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.012, size = 0, normalized size = 0.

$$\int x^3\left(a+bx^n+cx^{2\,n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**3/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.590 $\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{n};\frac{3}{2},\frac{3}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[3/n, 3/2, 3/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rubi [A] time = 0.149151, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{n};\frac{3}{2},\frac{3}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[3/n, 3/2, 3/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^2}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{n};\frac{3}{2},\frac{3}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] time = 0.859429, size = 398, normalized size = 2.64

$$\frac{x^3\left(18bcx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n+3}{n};\frac{1}{2},\frac{1}{2};2+\frac{3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-(n+3)\left(b^2(n-6)-4ac(n-\right.}{3an(n+3)\left(4ac-b^2\right)\sqrt{a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(-6*(3 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-6 + n) - 4*a*c*(-3 + n))*
(3 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n,
1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + 18*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])]*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(3*a*(-b^2 + 4*a*c)*n*(3 + n
)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int x^2\left(a+bx^n+cx^{2n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.591 $\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{n};\frac{3}{2},\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/n, 3/2, 3/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rubi [A] time = 0.112107, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1385, 510}

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{n};\frac{3}{2},\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/n, 3/2, 3/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$
$$= \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{n};\frac{3}{2},\frac{3}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] time = 0.847491, size = 398, normalized size = 2.64

$$\frac{x^2\left(8bcx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n+2}{n};\frac{1}{2},\frac{1}{2};2+\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-(n+2)\left(b^2(n-4)-4ac(n-2)\right)\right.}{2an(n+2)\left(4ac-b^2\right)\sqrt{a+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(-4*(2 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-4 + n) - 4*a*c*(-2 + n))*
(2 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n,
1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + 8*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])]*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(2*a*(-b^2 + 4*a*c)*n*(2 + n)
*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int x\left(a+bx^n+cx^{2n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.592 $\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rubi [A] time = 0.0822772, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1348, 429}

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] time = 0.973431, size = 384, normalized size = 2.7

$$\frac{x\left(2bcx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-(n+1)\left(\left(b^2(n-2)-4ac(n-\right.\right.\right.}{an(n+1)\left(4ac-b^2\right)\sqrt{a+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]
)]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1
[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (1 + n)*(2*(b^2 - 2*a*c + b*c*x^n) + (b^2
*(-2 + n) - 4*a*c*(-1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*n*(1 + n)*Sq
rt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.012, size = 0, normalized size = 0.

$$\int \left(a+bx^n+cx^{2n}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^{n}+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n}+bx^{n}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

3.593 $\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{2\left(-2ac+b^2+bcx^n\right)}{an\left(b^2-4ac\right)\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*n*Sqrt[a + b*x^n + c*x^(2*n)])
 - ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(a^(3/2)*
n)
```

Rubi [A] time = 0.0729641, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1357, 740, 12, 724, 206}

$$\frac{2\left(-2ac+b^2+bcx^n\right)}{an\left(b^2-4ac\right)\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*n*Sqrt[a + b*x^n + c*x^(2*n)])
 - ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(a^(3/2)*
n)
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^n\right)}{n}$$

$$= \frac{2\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}} - \frac{2\,\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx,x,x^n\right)}{a\left(b^2-4ac\right)n}$$

$$= \frac{2\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^n\right)}{an}$$

$$= \frac{2\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}} - \frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{an}$$

$$= \frac{2\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}} - \frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

Mathematica [A] time = 0.314725, size = 94, normalized size = 0.96

$$\frac{\frac{2\left(-2ac+b^2+bcx^n\right)}{a\left(b^2-4ac\right)\sqrt{a+x^n(b+cx^n)}} - \frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+x^n(b+cx^n)}}\right)}{a^{3/2}}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] ((2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*Sqrt[a + x^n*(b + c*x^n)]) -
ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + x^n*(b + c*x^n)])]/a^(3/2))/n
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+bx^n+cx^{2\,n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

[Out] int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2\,n} + bx^{n} + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

Fricas [B] time = 2.56634, size = 984, normalized size = 10.04

$$\left[\frac{\left(\left(b^{2} c - 4\,ac^{2}\right)\sqrt{a}x^{2\,n} + \left(b^{3} - 4\,abc\right)\sqrt{a}x^{n} + \left(ab^{2} - 4\,a^{2} c\right)\sqrt{a}\right)\log\left(-\frac{8\,abx^{n} + 8\,a^{2} + \left(b^{2} + 4\,ac\right)x^{2\,n} - 4\,\left(\sqrt{a}bx^{n} + 2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n} + bx^{n} + a}}{x^{2\,n}}\right) + 4}{2\,\left(\left(a^{2} b^{2} c - 4\,a^{3} c^{2}\right)nx^{2\,n} + \left(a^{2} b^{3} - 4\,a^{3} bc\right)nx^{n} + \left(a^{3} b^{2} - 4\,a^{4} c\right)n\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*sqrt(a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(a)*x^n + (a
*b^2 - 4*a^2*c)*sqrt(a))*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) -
4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 4*(a*
b*c*x^n + a*b^2 - 2*a^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4*a^3
*c^2)*n*x^(2*n) + (a^2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n), (((
b^2*c - 4*a*c^2)*sqrt(-a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(-a)*x^n + (a*b^2 -
 4*a^2*c)*sqrt(-a))*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*
n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) + 2*(a*b*c*x^n + a*b^2 - 2*a
^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4*a^3*c^2)*n*x^(2*n) + (a^
2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n} + bx^{n} + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

3.594 $\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{n};\frac{3}{2},\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-n^(-1), 3/2, 3/2, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*x^n
 + c*x^(2*n)]))
```

Rubi [A] time = 0.157029, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{n};\frac{3}{2},\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-n^(-1), 3/2, 3/2, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*x^n
 + c*x^(2*n)]))
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{n};\frac{3}{2},\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] time = 0.805645, size = 395, normalized size = 2.6

$$\frac{(n-1)\left(b^2(n+2)-4ac(n+1)\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};\frac{n-1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-2\left(bcx^n\sqrt{\frac{-\sqrt{}}{}}\right.}{a(n-1)nx\left(4ac-b^2\right)\sqrt{a+x^n}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] ((-1 + n)*(-4*a*c*(1 + n) + b^2*(2 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 2*((-1 + n)*(b^2
- 2*a*c + b*c*x^n) + b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c])]*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(a*(-b^2 + 4*a*c)*(-1 + n)
*n*x*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+bx^n+cx^{2n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2\,n} + bx^n + a\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

3.595 $\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{n};\frac{3}{2},\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[-2/n, 3/2, 3/2, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*x^2*Sqrt[a + b*x^n
 + c*x^(2*n)])
```

Rubi [A] time = 0.154147, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{n};\frac{3}{2},\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[-2/n, 3/2, 3/2, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*x^2*Sqrt[a + b*x^n
 + c*x^(2*n)])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{2}{n};\frac{3}{2},\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] time = 0.843329, size = 399, normalized size = 2.59

$$\frac{(n-2)\left(b^2(n+4)-4ac(n+2)\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n-2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-4\left(2bcx^n \ldots\right.}{2a(n-2)nx^2\left(4ac-b^2\right)\sqrt{a \ldots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] ((-2 + n)*(-4*a*c*(2 + n) + b^2*(4 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*((-2 + n)*(b^2 - 2
*a*c + b*c*x^n) + 2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(2*a*(-b^2 + 4*a*c)*(-2 + n)*n
*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+bx^n+cx^{2n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

3.596 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx$

Optimal. Leaf size=182

$$\frac{3a^2bx^{n+1}(dx)^m}{m+n+1}+\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3ax^{2n+1}\left(ac+b^2\right)(dx)^m}{m+2n+1}+\frac{bx^{3n+1}\left(6ac+b^2\right)(dx)^m}{m+3n+1}+\frac{3cx^{4n+1}\left(ac+b^2\right)(dx)^m}{m+4n+1}+\frac{3bc^2x^{5n+1}(dx)^m}{m+5n+1}$$

```
[Out] (3*a^2*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(1 + 2*n)*(d*x
)^m)/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) +
(3*c*(b^2 + a*c)*x^(1 + 4*n)*(d*x)^m)/(1 + m + 4*n) + (3*b*c^2*x^(1 + 5*n)*
(d*x)^m)/(1 + m + 5*n) + (c^3*x^(1 + 6*n)*(d*x)^m)/(1 + m + 6*n) + (a^3*(d*
x)^(1 + m))/(d*(1 + m))
```

Rubi [A] time = 0.155761, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1353, 20, 30}

$$\frac{3a^2bx^{n+1}(dx)^m}{m+n+1}+\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3ax^{2n+1}\left(ac+b^2\right)(dx)^m}{m+2n+1}+\frac{bx^{3n+1}\left(6ac+b^2\right)(dx)^m}{m+3n+1}+\frac{3cx^{4n+1}\left(ac+b^2\right)(dx)^m}{m+4n+1}+\frac{3bc^2x^{5n+1}(dx)^m}{m+5n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (3*a^2*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(1 + 2*n)*(d*x
)^m)/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) +
(3*c*(b^2 + a*c)*x^(1 + 4*n)*(d*x)^m)/(1 + m + 4*n) + (3*b*c^2*x^(1 + 5*n)*
(d*x)^m)/(1 + m + 5*n) + (c^3*x^(1 + 6*n)*(d*x)^m)/(1 + m + 6*n) + (a^3*(d*
x)^(1 + m))/(d*(1 + m))
```

Rule 1353

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^3 dx = \int \left(a^3(dx)^m + 3a^2bx^n(dx)^m + 3ab^2\left(1 + \frac{ac}{b^2}\right)x^{2n}(dx)^m + b^3\left(1 + \frac{6ac}{b^2}\right)x^{3n}(dx)^m + 3b^2c\left(1\right.\right.$$
$$= \frac{a^3(dx)^{1+m}}{d(1+m)} + \left(3a^2b\right)\int x^n(dx)^m\,dx + \left(3bc^2\right)\int x^{5n}(dx)^m\,dx + c^3\int x^{6n}(dx)^m\,dx + \left(3a\left(b^2\right.\right.$$
$$= \frac{a^3(dx)^{1+m}}{d(1+m)} + \left(3a^2bx^{-m}(dx)^m\right)\int x^{m+n}\,dx + \left(3bc^2x^{-m}(dx)^m\right)\int x^{m+5n}\,dx + \left(c^3x^{-m}(dx)^m\right)$$
$$= \frac{3a^2bx^{1+n}(dx)^m}{1+m+n} + \frac{3a\left(b^2+ac\right)x^{1+2n}(dx)^m}{1+m+2n} + \frac{b\left(b^2+6ac\right)x^{1+3n}(dx)^m}{1+m+3n} + \frac{3c\left(b^2+ac\right)x^{1+4}}{1+m+4}$$

Mathematica [A] time = 0.32275, size = 137, normalized size = 0.75

$$x(dx)^m\left(\frac{3a^2bx^n}{m+n+1} + \frac{a^3}{m+1} + \frac{3ax^{2n}\left(ac+b^2\right)}{m+2n+1} + \frac{bx^{3n}\left(6ac+b^2\right)}{m+3n+1} + \frac{3cx^{4n}\left(ac+b^2\right)}{m+4n+1} + \frac{3bc^2x^{5n}}{m+5n+1} + \frac{c^3x^{6n}}{m+6n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] x*(d*x)^m*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(2*
n))/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(3*n))/(1 + m + 3*n) + (3*c*(b^2 + a
*c)*x^(4*n))/(1 + m + 4*n) + (3*b*c^2*x^(5*n))/(1 + m + 5*n) + (c^3*x^(6*n)
)/(1 + m + 6*n))
```

Maple [C] time = 0.105, size = 3798, normalized size = 20.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] x*(3*a^2*c*(x^n)^2+a^3+972*b*c^2*n^4*(x^n)^5+150*c^3*m^2*n*(x^n)^6+340*c^3*
m*n^2*(x^n)^6+675*c^3*m*n^3*(x^n)^6+18*a*c^2*m^5*(x^n)^4+540*a*c^2*n^5*(x^n
)^4+18*b^3*m^5*n*(x^n)^3+121*b^3*m^4*n^2*(x^n)^3+372*b^3*m^3*n^3*(x^n)^3+50
8*b^3*m^2*n^4*(x^n)^3+240*b^3*m*n^5*(x^n)^3+18*b^2*c*m^5*(x^n)^4+540*b^2*c*
n^5*(x^n)^4+45*b*c^2*m^4*(x^n)^5+85*c^3*m^4*n^2*(x^n)^6+225*c^3*m^3*n^3*(x^
n)^6+274*c^3*m^2*n^4*(x^n)^6+120*c^3*m*n^5*(x^n)^6+3*b*c^2*m^6*(x^n)^5+75*c
^3*m^4*n*(x^n)^6+340*c^3*m^3*n^2*(x^n)^6+675*c^3*m^2*n^3*(x^n)^6+548*c^3*m*
n^4*(x^n)^6+3*a*c^2*m^6*(x^n)^4+3*b^2*c*m^6*(x^n)^4+18*b*c^2*m^5*(x^n)^5+43
2*b*c^2*n^5*(x^n)^5+150*c^3*m^3*n*(x^n)^6+510*c^3*m^2*n^2*(x^n)^6+2160*a^2*
b*n^5*x^n+45*a^2*c*m^4*(x^n)^2+2106*a^2*c*n^4*(x^n)^2+45*a*b^2*m^4*(x^n)^2+
2106*a*b^2*n^4*(x^n)^2+45*a*c^2*m^2*(x^n)^4+321*a*c^2*n^2*(x^n)^4+45*b^2*c*
m^2*(x^n)^4+321*b^2*c*n^2*(x^n)^4+18*m*b*c^2*(x^n)^5+48*b*c^2*(x^n)^5*n+45*
a^2*b*m^4*x^n+3132*a^2*b*n^4*x^n+60*a^2*c*m^3*(x^n)^2+15*c^3*m^5*n*(x^n)^6+
1080*a^2*c*n^5*(x^n)^2+18*a*b^2*m^5*(x^n)^2+1080*a*b^2*n^5*(x^n)^2+60*a*c^2
*m^3*(x^n)^4+921*a*c^2*n^3*(x^n)^4+180*b^3*m^3*n*(x^n)^3+726*b^3*m^2*n^2*(x
^n)^3+1116*b^3*m*n^3*(x^n)^3+60*b^2*c*m^3*(x^n)^4+921*b^2*c*n^3*(x^n)^4+45*
b*c^2*m^2*(x^n)^5+285*b*c^2*n^2*(x^n)^5+18*a^2*b*m^5*x^n+6*a*b*c*(x^n)^3+3*
a^2*c*m^6*(x^n)^2+3*a*b^2*m^6*(x^n)^2+45*a*c^2*m^4*(x^n)^4+1188*a*c^2*n^4*(
x^n)^4+90*b^3*m^4*n*(x^n)^3+484*b^3*m^3*n^2*(x^n)^3+1116*b^3*m^2*n^3*(x^n)^
3+1016*b^3*m*n^4*(x^n)^3+45*b^2*c*m^4*(x^n)^4+1188*b^2*c*n^4*(x^n)^4+60*b*c
^2*m^3*(x^n)^5+780*b*c^2*n^3*(x^n)^5+75*c^3*m*n*(x^n)^6+3*a^2*b*m^6*x^n+18*
```

a^2*c*m^5*(x^n)^2+1383*a^2*c*n^3*(x^n)^2+1383*a*b^2*n^3*(x^n)^2+18*a*c^2*(x^n)^4*m+51*a*c^2*(x^n)^4*n+18*b^2*c*(x^n)^4*m+51*b^2*c*(x^n)^4*n+1740*a^2*b*n^3*x^n+45*a^2*c*m^2*(x^n)^2+411*a^2*c*n^2*(x^n)^2+18*a^2*c*(x^n)^2*m+57*a^2*c*(x^n)^2*n+180*b^3*m^2*n*(x^n)^3+484*b^3*m*n^2*(x^n)^3+60*a*b^2*m^3*(x^n)^2+90*b^3*m*n*(x^n)^3+60*a^2*b*m^3*x^n+45*a*b^2*m^2*(x^n)^2+411*a*b^2*n^2*(x^n)^2+45*a^2*b*m^2*x^n+465*a^2*b*n^2*x^n+18*m*a*b^2*(x^n)^2+57*a*b^2*(x^n)^2*n+18*m*a^2*b*x^n+60*a^2*b*n*x^n+c^3*(x^n)^6+a^3*m^6+6*a^3*m^5+1764*a^3*n^5+15*a^3*m^4+1624*a^3*n^4+720*a^3*n^6+b^3*(x^n)^3+20*a^3*m^3+15*a^3*m^2+175*a^3*n^2+21*a^3*n+735*a^3*n^3+540*a*b*c*m^4*n*(x^n)^3+2904*a*b*c*m^3*n^2*(x^n)^3+6696*a*b*c*m^2*n^3*(x^n)^3+4356*a*b*c*m^2*n^2*(x^n)^3+6696*a*b*c*m*n^3*(x^n)^3+1080*a*b*c*m^2*n*(x^n)^3+2904*a*b*c*m*n^2*(x^n)^3+540*a*b*c*m*n*(x^n)^3+108*a*b*c*m^5*n*(x^n)^3+726*a*b*c*m^4*n^2*(x^n)^3+2232*a*b*c*m^3*n^3*(x^n)^3+3048*a*b*c*m^2*n^4*(x^n)^3+1440*a*b*c*m*n^5*(x^n)^3+6096*a*b*c*m*n^4*(x^n)^3+1080*a*b*c*m^3*n*(x^n)^3+600*a^2*b*m^3*n*x^n+2790*a^2*b*m^2*n^2*x^n+5220*a^2*b*m*n^3*x^n+570*a^2*c*m^2*n*(x^n)^2+1644*a^2*c*m*n^2*(x^n)^2+90*a*b*c*m^2*(x^n)^3+726*a*b*c*n^2*(x^n)^3+285*a^2*c*m*n*(x^n)^2+36*a*b*c*(x^n)^3*m+108*a*b*c*(x^n)^3*n+1284*b^2*c*m*n^2*(x^n)^4+240*b*c^2*m*n*(x^n)^5+300*a^2*b*m^4*n*x^n+1860*a^2*b*m^3*n^2*x^n+5220*a^2*b*m^2*n^3*x^n+6264*a^2*b*m*n^4*x^n+570*a^2*c*m^3*n*(x^n)^2+2466*a^2*c*m^2*n^2*(x^n)^2+4149*a^2*c*m*n^3*(x^n)^2+570*a*b^2*m^3*n*(x^n)^2+2466*a*b^2*m^2*n^2*(x^n)^2+4149*a*b^2*m*n^3*(x^n)^2+120*a*b*c*m^3*(x^n)^3+2232*a*b*c*n^3*(x^n)^3+255*a*c^2*m*n*(x^n)^4+255*b^2*c*m*n*(x^n)^4+3132*a^2*b*m^2*n^4*x^n+2160*a^2*b*m*n^5*x^n+285*a^2*c*m^4*n*(x^n)^2+1644*a^2*c*m^3*n^2*(x^n)^2+4149*a^2*c*m^2*n^3*(x^n)^2+4212*a^2*c*m*n^4*(x^n)^2+285*a*b^2*m^4*n*(x^n)^2+1644*a*b^2*m^3*n^2*(x^n)^2+4149*a*b^2*m^2*n^3*(x^n)^2+4212*a*b^2*m*n^4*(x^n)^2+90*a*b*c*m^4*(x^n)^3+3048*a*b*c*n^4*(x^n)^3+510*a*c^2*m^2*n*(x^n)^4+1284*a*c^2*m*n^2*(x^n)^4+510*b^2*c*m^2*n*(x^n)^4+2106*a*b^2*m^2*n^4*(x^n)^2+1080*a*b^2*m*n^5*(x^n)^2+36*a*b*c*m^5*(x^n)^3+1440*a*b*c*n^5*(x^n)^3+510*a*c^2*m^3*n*(x^n)^4+1926*a*c^2*m^2*n^2*(x^n)^4+2763*a*c^2*m*n^3*(x^n)^4+510*b^2*c*m^3*n*(x^n)^4+1926*b^2*c*m^2*n^2*(x^n)^4+2763*b^2*c*m*n^3*(x^n)^4+480*b*c^2*m^2*n*(x^n)^5+1140*b*c^2*m*n^2*(x^n)^5+60*a^2*b*m^5*n*x^n+465*a^2*b*m^4*n^2*x^n+1740*a^2*b*m^3*n^3*x^n+2376*a*c^2*m*n^4*(x^n)^4+255*b^2*c*m^4*n*(x^n)^4+1284*b^2*c*m^3*n^2*(x^n)^4+2763*b^2*c*m^2*n^3*(x^n)^4+2376*b^2*c*m*n^4*(x^n)^4+480*b*c^2*m^3*n*(x^n)^5+1710*b*c^2*m^2*n^2*(x^n)^5+2340*b*c^2*m*n^3*(x^n)^5+57*a^2*c*m^5*n*(x^n)^2+411*a^2*c*m^4*n^2*(x^n)^2+1383*a^2*c*m^3*n^3*(x^n)^2+2106*a^2*c*m^2*n^4*(x^n)^2+1080*a^2*c*m*n^5*(x^n)^2+57*a*b^2*m^5*n*(x^n)^2+411*a*b^2*m^4*n^2*(x^n)^2+1383*a*b^2*m^3*n^3*(x^n)^2+1188*a*c^2*m^2*n^4*(x^n)^4+540*a*c^2*m*n^5*(x^n)^4+51*b^2*c*m^5*n*(x^n)^4+321*b^2*c*m^4*n^2*(x^n)^4+921*b^2*c*m^3*n^3*(x^n)^4+1188*b^2*c*m^2*n^4*(x^n)^4+540*b^2*c*m*n^5*(x^n)^4+240*b*c^2*m^4*n*(x^n)^5+1140*b*c^2*m^3*n^2*(x^n)^5+2340*b*c^2*m^2*n^3*(x^n)^5+1944*b*c^2*m*n^4*(x^n)^5+6*a*b*c*m^6*(x^n)^3+255*a*c^2*m^4*n*(x^n)^4+1284*a*c^2*m^3*n^2*(x^n)^4+2763*a*c^2*m^2*n^3*(x^n)^4+48*b*c^2*m^5*n*(x^n)^5+285*b*c^2*m^4*n^2*(x^n)^5+780*b*c^2*m^3*n^3*(x^n)^5+972*b*c^2*m^2*n^4*(x^n)^5+432*b*c^2*m*n^5*(x^n)^5+51*a*c^2*m^5*n*(x^n)^4+321*a*c^2*m^4*n^2*(x^n)^4+921*a*c^2*m^3*n^3*(x^n)^4+6*m*a^3+210*a^3*m^2*n+700*a^3*m*n^2+105*a^3*m*n+21*a^3*m^5*n+175*a^3*m^4*n^2+735*a^3*m^3*n^3+1624*a^3*m^2*n^4+1764*a^3*m*n^5+105*a^3*m^4*n+700*a^3*m^3*n^2+2205*a^3*m^2*n^3+3248*a^3*m*n^4+210*a^3*m^3*n+1050*a^3*m^2*n^2+2205*a^3*m*n^3+20*b^3*m^3*(x^n)^3+15*b^3*m^2*(x^n)^3+121*b^3*n^2*(x^n)^3+6*m*b^3*(x^n)^3+18*b^3*(x^n)^3*n+3*a^2*b*x^n+3*a*b^2*(x^n)^2+508*b^3*n^4*(x^n)^3+6*m*c^3*(x^n)^6+3*c^2*a*(x^n)^4+3*b^2*c*(x^n)^4+3*b*c^2*(x^n)^5+6*c^3*m^5*(x^n)^6+120*c^3*n^5*(x^n)^6+15*c^3*m^4*(x^n)^6+274*c^3*n^4*(x^n)^6+b^3*m^6*(x^n)^3+20*c^3*m^3*(x^n)^6+225*c^3*n^3*(x^n)^6+6*b^3*m^5*(x^n)^3+240*b^3*n^5*(x^n)^3+15*c^3*m^2*(x^n)^6+c^3*m^6*(x^n)^6+15*c^3*(x^n)^6*n+372*b^3*n^3*(x^n)^3+85*c^3*n^2*(x^n)^6+15*b^3*m^4*(x^n)^3+570*a*b^2*m^2*n*(x^n)^2+1644*a*b^2*m*n^2*(x^n)^2+600*a^2*b*m^2*n*x^n+1860*a^2*b*m*n^2*x^n+285*a*b^2*m*n*(x^n)^2+300*a^2*b*m*n*x^n)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)/(1+m+4*n)/(1+m+5*n)/(1+m+6*n)*exp(1/2*m*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*csgn(I*d)*Pi+I*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I*d)*csgn(I*x)*Pi+2*ln(x)+2*ln(d)))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.26512, size = 5146, normalized size = 28.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^6 + 6*c^3*m^5 + 15*c^3*m^4 + 20*c^3*m^3 + 120*(c^3*m + c^3)*n^5 + 1
5*c^3*m^2 + 274*(c^3*m^2 + 2*c^3*m + c^3)*n^4 + 6*c^3*m + 225*(c^3*m^3 + 3*
c^3*m^2 + 3*c^3*m + c^3)*n^3 + c^3 + 85*(c^3*m^4 + 4*c^3*m^3 + 6*c^3*m^2 +
4*c^3*m + c^3)*n^2 + 15*(c^3*m^5 + 5*c^3*m^4 + 10*c^3*m^3 + 10*c^3*m^2 + 5*
c^3*m + c^3)*n)*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 3*(b*c^2*m^6 + 6*b*c^2*
m^5 + 15*b*c^2*m^4 + 20*b*c^2*m^3 + 144*(b*c^2*m + b*c^2)*n^5 + 15*b*c^2*m^
2 + 324*(b*c^2*m^2 + 2*b*c^2*m + b*c^2)*n^4 + 6*b*c^2*m + 260*(b*c^2*m^3 +
3*b*c^2*m^2 + 3*b*c^2*m + b*c^2)*n^3 + b*c^2 + 95*(b*c^2*m^4 + 4*b*c^2*m^3
+ 6*b*c^2*m^2 + 4*b*c^2*m + b*c^2)*n^2 + 16*(b*c^2*m^5 + 5*b*c^2*m^4 + 10*b
*c^2*m^3 + 10*b*c^2*m^2 + 5*b*c^2*m + b*c^2)*n)*x*x^(5*n)*e^(m*log(d) + m*l
og(x)) + 3*((b^2*c + a*c^2)*m^6 + 6*(b^2*c + a*c^2)*m^5 + 180*(b^2*c + a*c^
2 + (b^2*c + a*c^2)*m)*n^5 + 15*(b^2*c + a*c^2)*m^4 + 396*(b^2*c + a*c^2 +
(b^2*c + a*c^2)*m^2 + 2*(b^2*c + a*c^2)*m)*n^4 + 20*(b^2*c + a*c^2)*m^3 + 3
07*((b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 3*(b^2*c + a*c^2)*m^2 + 3*(b^2*c
+ a*c^2)*m)*n^3 + b^2*c + a*c^2 + 15*(b^2*c + a*c^2)*m^2 + 107*((b^2*c + a*
c^2)*m^4 + 4*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 6*(b^2*c + a*c^2)*m^2 +
4*(b^2*c + a*c^2)*m)*n^2 + 6*(b^2*c + a*c^2)*m + 17*((b^2*c + a*c^2)*m^5 +
5*(b^2*c + a*c^2)*m^4 + 10*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 10*(b^2*c
+ a*c^2)*m^2 + 5*(b^2*c + a*c^2)*m)*n)*x*x^(4*n)*e^(m*log(d) + m*log(x)) +
((b^3 + 6*a*b*c)*m^6 + 6*(b^3 + 6*a*b*c)*m^5 + 240*(b^3 + 6*a*b*c + (b^3 +
6*a*b*c)*m)*n^5 + 15*(b^3 + 6*a*b*c)*m^4 + 508*(b^3 + 6*a*b*c + (b^3 + 6*a*
b*c)*m^2 + 2*(b^3 + 6*a*b*c)*m)*n^4 + 20*(b^3 + 6*a*b*c)*m^3 + 372*((b^3 +
6*a*b*c)*m^3 + b^3 + 6*a*b*c + 3*(b^3 + 6*a*b*c)*m^2 + 3*(b^3 + 6*a*b*c)*m)
*n^3 + b^3 + 6*a*b*c + 15*(b^3 + 6*a*b*c)*m^2 + 121*((b^3 + 6*a*b*c)*m^4 +
4*(b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 6*(b^3 + 6*a*b*c)*m^2 + 4*(b^3 + 6*
a*b*c)*m)*n^2 + 6*(b^3 + 6*a*b*c)*m + 18*((b^3 + 6*a*b*c)*m^5 + 5*(b^3 + 6*
a*b*c)*m^4 + 10*(b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 10*(b^3 + 6*a*b*c)*m^
2 + 5*(b^3 + 6*a*b*c)*m)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*((a*b^2 +
 a^2*c)*m^6 + 6*(a*b^2 + a^2*c)*m^5 + 360*(a*b^2 + a^2*c + (a*b^2 + a^2*c)*
m)*n^5 + 15*(a*b^2 + a^2*c)*m^4 + 702*(a*b^2 + a^2*c + (a*b^2 + a^2*c)*m^2
+ 2*(a*b^2 + a^2*c)*m)*n^4 + 20*(a*b^2 + a^2*c)*m^3 + 461*((a*b^2 + a^2*c)*
m^3 + a*b^2 + a^2*c + 3*(a*b^2 + a^2*c)*m^2 + 3*(a*b^2 + a^2*c)*m)*n^3 + a*
b^2 + a^2*c + 15*(a*b^2 + a^2*c)*m^2 + 137*((a*b^2 + a^2*c)*m^4 + 4*(a*b^2
+ a^2*c)*m^3 + a*b^2 + a^2*c + 6*(a*b^2 + a^2*c)*m^2 + 4*(a*b^2 + a^2*c)*m)
*n^2 + 6*(a*b^2 + a^2*c)*m + 19*((a*b^2 + a^2*c)*m^5 + 5*(a*b^2 + a^2*c)*m^
4 + 10*(a*b^2 + a^2*c)*m^3 + a*b^2 + a^2*c + 10*(a*b^2 + a^2*c)*m^2 + 5*(a*
```

```
b^2 + a^2*c)*m)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^6 + 6*a^2
*b*m^5 + 15*a^2*b*m^4 + 20*a^2*b*m^3 + 720*(a^2*b*m + a^2*b)*n^5 + 15*a^2*b
*m^2 + 1044*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n^4 + 6*a^2*b*m + 580*(a^2*b*m^
3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b)*n^3 + a^2*b + 155*(a^2*b*m^4 + 4*a^2*b
*m^3 + 6*a^2*b*m^2 + 4*a^2*b*m + a^2*b)*n^2 + 20*(a^2*b*m^5 + 5*a^2*b*m^4 +
 10*a^2*b*m^3 + 10*a^2*b*m^2 + 5*a^2*b*m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*
log(x)) + (a^3*m^6 + 720*a^3*n^6 + 6*a^3*m^5 + 15*a^3*m^4 + 20*a^3*m^3 + 17
64*(a^3*m + a^3)*n^5 + 15*a^3*m^2 + 1624*(a^3*m^2 + 2*a^3*m + a^3)*n^4 + 6*
a^3*m + 735*(a^3*m^3 + 3*a^3*m^2 + 3*a^3*m + a^3)*n^3 + a^3 + 175*(a^3*m^4
+ 4*a^3*m^3 + 6*a^3*m^2 + 4*a^3*m + a^3)*n^2 + 21*(a^3*m^5 + 5*a^3*m^4 + 10
*a^3*m^3 + 10*a^3*m^2 + 5*a^3*m + a^3)*n)*x*e^(m*log(d) + m*log(x)))/(m^7 +
 720*(m + 1)*n^6 + 7*m^6 + 1764*(m^2 + 2*m + 1)*n^5 + 21*m^5 + 1624*(m^3 +
3*m^2 + 3*m + 1)*n^4 + 35*m^4 + 735*(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*n^3 + 3
5*m^3 + 175*(m^5 + 5*m^4 + 10*m^3 + 10*m^2 + 5*m + 1)*n^2 + 21*m^2 + 21*(m^
6 + 6*m^5 + 15*m^4 + 20*m^3 + 15*m^2 + 6*m + 1)*n + 7*m + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.597 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 dx$

Optimal. Leaf size=117

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{x^{2n+1}\left(2ac+b^2\right)(dx)^m}{m+2n+1}+\frac{2abx^{n+1}(dx)^m}{m+n+1}+\frac{2bcx^{3n+1}(dx)^m}{m+3n+1}+\frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

```
[Out] (2*a*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + ((b^2 + 2*a*c)*x^(1 + 2*n)*(d*x)^m)
/(1 + m + 2*n) + (2*b*c*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) + (c^2*x^(1 + 4*
n)*(d*x)^m)/(1 + m + 4*n) + (a^2*(d*x)^(1 + m))/(d*(1 + m))
```

Rubi [A] time = 0.0690006, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1353, 20, 30}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{x^{2n+1}\left(2ac+b^2\right)(dx)^m}{m+2n+1}+\frac{2abx^{n+1}(dx)^m}{m+n+1}+\frac{2bcx^{3n+1}(dx)^m}{m+3n+1}+\frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (2*a*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + ((b^2 + 2*a*c)*x^(1 + 2*n)*(d*x)^m)
/(1 + m + 2*n) + (2*b*c*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) + (c^2*x^(1 + 4*
n)*(d*x)^m)/(1 + m + 4*n) + (a^2*(d*x)^(1 + m))/(d*(1 + m))
```

Rule 1353

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 dx = \int \left(a^2(dx)^m + 2abx^n(dx)^m + b^2\left(1+\frac{2ac}{b^2}\right)x^{2n}(dx)^m + 2bcx^{3n}(dx)^m + c^2x^{4n}(dx)^m\right) dx$$

$$= \frac{a^2(dx)^{1+m}}{d(1+m)} + (2ab)\int x^n(dx)^m\,dx + (2bc)\int x^{3n}(dx)^m\,dx + c^2\int x^{4n}(dx)^m\,dx + \left(b^2 + 2\right.$$

$$= \frac{a^2(dx)^{1+m}}{d(1+m)} + \left(2abx^{-m}(dx)^m\right)\int x^{m+n}\,dx + \left(2bcx^{-m}(dx)^m\right)\int x^{m+3n}\,dx + \left(c^2x^{-m}(dx)^m\right)$$

$$= \frac{2abx^{1+n}(dx)^m}{1+m+n} + \frac{\left(b^2+2ac\right)x^{1+2n}(dx)^m}{1+m+2n} + \frac{2bcx^{1+3n}(dx)^m}{1+m+3n} + \frac{c^2x^{1+4n}(dx)^m}{1+m+4n} + \frac{a^2(dx)^{1+}}{d(1+m}$$

Mathematica [A] time = 0.136293, size = 86, normalized size = 0.74

$$x(dx)^m\left(\frac{a^2}{m+1} + \frac{x^{2n}\left(2ac+b^2\right)}{m+2n+1} + \frac{2abx^n}{m+n+1} + \frac{2bcx^{3n}}{m+3n+1} + \frac{c^2x^{4n}}{m+4n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^n)/(1 + m + n) + ((b^2 + 2*a*c)*x^(2*n))/
(1 + m + 2*n) + (2*b*c*x^(3*n))/(1 + m + 3*n) + (c^2*x^(4*n))/(1 + m + 4*n)
)
```

Maple [C] time = 0.067, size = 1065, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] x*(2*a*b*x^n+14*b*c*m^3*n*(x^n)^3+28*b*c*m^2*n^2*(x^n)^3+16*b*c*m*n^3*(x^n)
^3+16*a*c*m^3*n*(x^n)^2+38*a*c*m^2*n^2*(x^n)^2+24*a*c*m*n^3*(x^n)^2+42*b*c*
m^2*n*(x^n)^3+56*b*c*m*n^2*(x^n)^3+18*a*b*m^3*n*x^n+52*a*b*m^2*n^2*x^n+48*a
*b*m*n^3*x^n+48*a*c*m^2*n*(x^n)^2+76*a*c*m*n^2*(x^n)^2+b^2*(x^n)^2+4*b^2*(x
^n)^2*m+8*b^2*(x^n)^2*n+10*a^2*m^3*n+35*a^2*m^2*n^2+50*a^2*m*n^3+30*a^2*m^2
*n+70*a^2*m*n^2+30*a^2*m*n+28*b*c*n^2*(x^n)^3+8*a*b*m^3*x^n+48*a*b*n^3*x^n+
12*a*c*m^2*(x^n)^2+18*c^2*m^2*n*(x^n)^4+22*c^2*m*n^2*(x^n)^4+2*a*c*m^4*(x^n
)^2+8*b^2*m^3*n*(x^n)^2+19*b^2*m^2*n^2*(x^n)^2+12*b^2*m*n^3*(x^n)^2+8*b*c*m
^3*(x^n)^3+16*b*c*n^3*(x^n)^3+18*c^2*m*n*(x^n)^4+2*a*b*m^4*x^n+8*a*c*m^3*(x
^n)^2+24*a*c*n^3*(x^n)^2+24*b^2*m^2*n*(x^n)^2+38*b^2*m*n^2*(x^n)^2+12*b*c*m
^2*(x^n)^3+14*b*c*(x^n)^3*n+12*a*b*m^2*x^n+52*a*b*n^2*x^n+8*a*c*(x^n)^2*m+1
6*a*c*(x^n)^2*n+8*a*b*x^n*m+18*a*b*x^n*n+6*c^2*m^3*n*(x^n)^4+11*c^2*m^2*n^2
*(x^n)^4+6*c^2*m*n^3*(x^n)^4+2*b*c*m^4*(x^n)^3+8*m*b*c*(x^n)^3+38*a*c*n^2*(
x^n)^2+24*b^2*m*n*(x^n)^2+c^2*(x^n)^4+a^2*m^4+4*a^2*m^3+50*a^2*n^3+6*a^2*m^
2+35*a^2*n^2+24*a^2*n^4+4*a^2*m+10*a^2*n+a^2+42*b*c*m*n*(x^n)^3+54*a*b*m^2*
n*x^n+104*a*b*m*n^2*x^n+48*a*c*m*n*(x^n)^2+54*a*b*m*n*x^n+2*b*c*(x^n)^3+2*a
*c*(x^n)^2+c^2*m^4*(x^n)^4+4*c^2*m^3*(x^n)^4+6*c^2*n^3*(x^n)^4+b^2*m^4*(x^n
)^2+6*c^2*m^2*(x^n)^4+11*c^2*n^2*(x^n)^4+4*b^2*m^3*(x^n)^2+12*b^2*n^3*(x^n)
^2+4*m*c^2*(x^n)^4+6*c^2*(x^n)^4*n+6*b^2*m^2*(x^n)^2+19*b^2*n^2*(x^n)^2)/(1
+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)/(1+m+4*n)*exp(1/2*m*(-I*csgn(I*d*x)^3*Pi+I*
csgn(I*d*x)^2*csgn(I*d)*Pi+I*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(
I*d)*csgn(I*x)*Pi+2*ln(x)+2*ln(d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.84898, size = 1665, normalized size = 14.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 4*c^2*m^3 + 6*c^2*m^2 + 6*(c^2*m + c^2)*n^3 + 4*c^2*m + 11*(c^2
*m^2 + 2*c^2*m + c^2)*n^2 + c^2 + 6*(c^2*m^3 + 3*c^2*m^2 + 3*c^2*m + c^2)*n
)*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*(b*c*m^4 + 4*b*c*m^3 + 6*b*c*m^2 +
8*(b*c*m + b*c)*n^3 + 4*b*c*m + 14*(b*c*m^2 + 2*b*c*m + b*c)*n^2 + b*c + 7*
(b*c*m^3 + 3*b*c*m^2 + 3*b*c*m + b*c)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x))
+ ((b^2 + 2*a*c)*m^4 + 4*(b^2 + 2*a*c)*m^3 + 12*(b^2 + 2*a*c + (b^2 + 2*a*c
)*m)*n^3 + 6*(b^2 + 2*a*c)*m^2 + 19*((b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 2*(b
^2 + 2*a*c)*m)*n^2 + b^2 + 2*a*c + 4*(b^2 + 2*a*c)*m + 8*((b^2 + 2*a*c)*m^3
 + 3*(b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 3*(b^2 + 2*a*c)*m)*n)*x*x^(2*n)*e^(m
*log(d) + m*log(x)) + 2*(a*b*m^4 + 4*a*b*m^3 + 6*a*b*m^2 + 24*(a*b*m + a*b)
*n^3 + 4*a*b*m + 26*(a*b*m^2 + 2*a*b*m + a*b)*n^2 + a*b + 9*(a*b*m^3 + 3*a*
b*m^2 + 3*a*b*m + a*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^2*m^4 + 24*a^2
*n^4 + 4*a^2*m^3 + 6*a^2*m^2 + 50*(a^2*m + a^2)*n^3 + 4*a^2*m + 35*(a^2*m^2
 + 2*a^2*m + a^2)*n^2 + a^2 + 10*(a^2*m^3 + 3*a^2*m^2 + 3*a^2*m + a^2)*n)*x
*e^(m*log(d) + m*log(x)))/(m^5 + 24*(m + 1)*n^4 + 5*m^4 + 50*(m^2 + 2*m + 1
)*n^3 + 10*m^3 + 35*(m^3 + 3*m^2 + 3*m + 1)*n^2 + 10*m^2 + 10*(m^4 + 4*m^3
+ 6*m^2 + 4*m + 1)*n + 5*m + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2227, size = 7363, normalized size = 62.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(4*n)*e^(m*log
(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*
m*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*x^(3*n)*e^(m*log(d) +
 m*log(x)) + c^2*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x)
) + 28*b*c*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x
)) + 6*c^2*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^(2*n)*e^(m
*log(d) + m*log(x)) + 2*a*c*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*c*m
^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c^2*m^4*x*x^(2*n)*e^(m*log(d) + m*lo
g(x)) + 8*b^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*a*c*m^3*n*x*x^(2
*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)
) + 6*c^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 19*b^2*m^2*n^2*x*x^(2*n
)*e^(m*log(d) + m*log(x)) + 38*a*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)
) + 28*b*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(
2*n)*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x
)) + 24*a*c*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^(2*n
)*e^(m*log(d) + m*log(x)) + 6*c^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 2*a*b*m^4*x*x^n*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^n*e^(m*log(d) + m*lo
g(x)) + 2*a*c*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*x^n*e^(m*log(
d) + m*log(x)) + c^2*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 18*a*b*m^3*n*x*x^n
*e^(m*log(d) + m*log(x)) + 8*b^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 16*a
*c*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^n*e^(m*log(d) + m
*log(x)) + 6*c^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 52*a*b*m^2*n^2*x*x^n
*e^(m*log(d) + m*log(x)) + 19*b^2*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 3
8*a*c*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 28*b*c*m^2*n^2*x*x^n*e^(m*log
(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*b*m*n
^3*x*x^n*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*x^n*e^(m*log(d) + m*log(x
)) + 24*a*c*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^n*e^(m*l
og(d) + m*log(x)) + 6*c^2*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + a^2*m^4*x*e
^(m*log(d) + m*log(x)) + 2*a*b*m^4*x*e^(m*log(d) + m*log(x)) + b^2*m^4*x*e^
(m*log(d) + m*log(x)) + 2*a*c*m^4*x*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*e
^(m*log(d) + m*log(x)) + c^2*m^4*x*e^(m*log(d) + m*log(x)) + 10*a^2*m^3*n*x
*e^(m*log(d) + m*log(x)) + 18*a*b*m^3*n*x*e^(m*log(d) + m*log(x)) + 8*b^2*m
^3*n*x*e^(m*log(d) + m*log(x)) + 16*a*c*m^3*n*x*e^(m*log(d) + m*log(x)) + 1
4*b*c*m^3*n*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*e^(m*log(d) + m*log(x
)) + 35*a^2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 52*a*b*m^2*n^2*x*e^(m*log(d
) + m*log(x)) + 19*b^2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 38*a*c*m^2*n^2*x
*e^(m*log(d) + m*log(x)) + 28*b*c*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 11*c^
2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 50*a^2*m*n^3*x*e^(m*log(d) + m*log(x)
) + 48*a*b*m*n^3*x*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*e^(m*log(d) + m
*log(x)) + 24*a*c*m*n^3*x*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*e^(m*log
(d) + m*log(x)) + 6*c^2*m*n^3*x*e^(m*log(d) + m*log(x)) + 24*a^2*n^4*x*e^(m
*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 18*c^2*
m^2*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 22*c^2*m*n^2*x*x^(4*n)*e^(m*log(d
) + m*log(x)) + 6*c^2*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*x
^(3*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x)
) + 42*b*c*m^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^(3*n)
*e^(m*log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
 22*c^2*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*x^(3*n)*e^(m
*log(d) + m*log(x)) + 6*c^2*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 4*b^2*m
^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*c*m^3*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + 8*b*c*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(2*n)
*e^(m*log(d) + m*log(x)) + 24*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 48*a*c*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m^2*n*x*x^(2*n)*e^
(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 38
*b^2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 76*a*c*m*n^2*x*x^(2*n)*e^(m*
```

log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 22*c^
2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*x^(2*n)*e^(m*log(d
) + m*log(x)) + 24*a*c*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x
*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(
x)) + 8*a*b*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 4*b^2*m^3*x*x^n*e^(m*log(d)
 + m*log(x)) + 8*a*c*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*x^n*e^
(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 54*a*b*m^
2*n*x*x^n*e^(m*log(d) + m*log(x)) + 24*b^2*m^2*n*x*x^n*e^(m*log(d) + m*log(
x)) + 48*a*c*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 42*b*c*m^2*n*x*x^n*e^(m*
log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 104*a*b*m
*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 38*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log
(x)) + 76*a*c*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^n*e^(m
*log(d) + m*log(x)) + 22*c^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*b*n
^3*x*x^n*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*x^n*e^(m*log(d) + m*log(x))
 + 24*a*c*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*x^n*e^(m*log(d)
+ m*log(x)) + 6*c^2*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 4*a^2*m^3*x*e^(m*lo
g(d) + m*log(x)) + 8*a*b*m^3*x*e^(m*log(d) + m*log(x)) + 4*b^2*m^3*x*e^(m*l
og(d) + m*log(x)) + 8*a*c*m^3*x*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*e^(m*
log(d) + m*log(x)) + 4*c^2*m^3*x*e^(m*log(d) + m*log(x)) + 30*a^2*m^2*n*x*e
^(m*log(d) + m*log(x)) + 54*a*b*m^2*n*x*e^(m*log(d) + m*log(x)) + 24*b^2*m^
2*n*x*e^(m*log(d) + m*log(x)) + 48*a*c*m^2*n*x*e^(m*log(d) + m*log(x)) + 42
*b*c*m^2*n*x*e^(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*e^(m*log(d) + m*log(x
)) + 70*a^2*m*n^2*x*e^(m*log(d) + m*log(x)) + 104*a*b*m*n^2*x*e^(m*log(d) +
 m*log(x)) + 38*b^2*m*n^2*x*e^(m*log(d) + m*log(x)) + 76*a*c*m*n^2*x*e^(m*l
og(d) + m*log(x)) + 56*b*c*m*n^2*x*e^(m*log(d) + m*log(x)) + 22*c^2*m*n^2*x
*e^(m*log(d) + m*log(x)) + 50*a^2*n^3*x*e^(m*log(d) + m*log(x)) + 48*a*b*n^
3*x*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*e^(m*log(d) + m*log(x)) + 24*a*c
*n^3*x*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*e^(m*log(d) + m*log(x)) + 6*c
^2*n^3*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x*x^(4*n)*e^(m*log(d) + m*log(
x)) + 18*c^2*m*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^(4*n)*e
^(m*log(d) + m*log(x)) + 12*b*c*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 6*c
^2*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m*n*x*x^(3*n)*e^(m*log(d)
 + m*log(x)) + 18*c^2*m*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 28*b*c*n^2*x*
x^(3*n)*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(
x)) + 6*b^2*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*a*c*m^2*x*x^(2*n)*e^
(m*log(d) + m*log(x)) + 12*b*c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^
2*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 24*b^2*m*n*x*x^(2*n)*e^(m*log(d)
+ m*log(x)) + 48*a*c*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m*n*x*x
^(2*n)*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x
)) + 19*b^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 38*a*c*n^2*x*x^(2*n)*e^
(m*log(d) + m*log(x)) + 28*b*c*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 11*c
^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*a*b*m^2*x*x^n*e^(m*log(d) + m
*log(x)) + 6*b^2*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 12*a*c*m^2*x*x^n*e^(m*
log(d) + m*log(x)) + 12*b*c*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x
*x^n*e^(m*log(d) + m*log(x)) + 54*a*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 2
4*b^2*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*c*m*n*x*x^n*e^(m*log(d) + m*
log(x)) + 42*b*c*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*x^n*e^(m*
log(d) + m*log(x)) + 52*a*b*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 19*b^2*n^2*
x*x^n*e^(m*log(d) + m*log(x)) + 38*a*c*n^2*x*x^n*e^(m*log(d) + m*log(x)) +
28*b*c*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^n*e^(m*log(d) + m
*log(x)) + 6*a^2*m^2*x*e^(m*log(d) + m*log(x)) + 12*a*b*m^2*x*e^(m*log(d) +
 m*log(x)) + 6*b^2*m^2*x*e^(m*log(d) + m*log(x)) + 12*a*c*m^2*x*e^(m*log(d)
 + m*log(x)) + 12*b*c*m^2*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x*e^(m*log(
d) + m*log(x)) + 30*a^2*m*n*x*e^(m*log(d) + m*log(x)) + 54*a*b*m*n*x*e^(m*l
og(d) + m*log(x)) + 24*b^2*m*n*x*e^(m*log(d) + m*log(x)) + 48*a*c*m*n*x*e^(
m*log(d) + m*log(x)) + 42*b*c*m*n*x*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*
e^(m*log(d) + m*log(x)) + 35*a^2*n^2*x*e^(m*log(d) + m*log(x)) + 52*a*b*n^2
*x*e^(m*log(d) + m*log(x)) + 19*b^2*n^2*x*e^(m*log(d) + m*log(x)) + 38*a*c*
n^2*x*e^(m*log(d) + m*log(x)) + 28*b*c*n^2*x*e^(m*log(d) + m*log(x)) + 11*c

```
^2*n^2*x*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x^(4*n)*e^(m*log(d) + m*log(x)
) + 6*c^2*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 8*b*c*m*x*x^(3*n)*e^(m*log(
d) + m*log(x)) + 4*c^2*m*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 14*b*c*n*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
4*b^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*c*m*x*x^(2*n)*e^(m*log(d) +
 m*log(x)) + 8*b*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x^(2*n)*
e^(m*log(d) + m*log(x)) + 8*b^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*a*
c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 14*b*c*n*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + 6*c^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*b*m*x*x^n*e^(m*lo
g(d) + m*log(x)) + 4*b^2*m*x*x^n*e^(m*log(d) + m*log(x)) + 8*a*c*m*x*x^n*e^
(m*log(d) + m*log(x)) + 8*b*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x
^n*e^(m*log(d) + m*log(x)) + 18*a*b*n*x*x^n*e^(m*log(d) + m*log(x)) + 8*b^2
*n*x*x^n*e^(m*log(d) + m*log(x)) + 16*a*c*n*x*x^n*e^(m*log(d) + m*log(x)) +
 14*b*c*n*x*x^n*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*x^n*e^(m*log(d) + m*log
(x)) + 4*a^2*m*x*e^(m*log(d) + m*log(x)) + 8*a*b*m*x*e^(m*log(d) + m*log(x)
) + 4*b^2*m*x*e^(m*log(d) + m*log(x)) + 8*a*c*m*x*e^(m*log(d) + m*log(x)) +
 8*b*c*m*x*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*e^(m*log(d) + m*log(x)) + 10
*a^2*n*x*e^(m*log(d) + m*log(x)) + 18*a*b*n*x*e^(m*log(d) + m*log(x)) + 8*b
^2*n*x*e^(m*log(d) + m*log(x)) + 16*a*c*n*x*e^(m*log(d) + m*log(x)) + 14*b*
c*n*x*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*e^(m*log(d) + m*log(x)) + c^2*x*x
^(4*n)*e^(m*log(d) + m*log(x)) + 2*b*c*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
c^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + b^2*x*x^(2*n)*e^(m*log(d) + m*log(x
)) + 2*a*c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*c*x*x^(2*n)*e^(m*log(d)
+ m*log(x)) + c^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*a*b*x*x^n*e^(m*log(
d) + m*log(x)) + b^2*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*c*x*x^n*e^(m*log(d
) + m*log(x)) + 2*b*c*x*x^n*e^(m*log(d) + m*log(x)) + c^2*x*x^n*e^(m*log(d)
 + m*log(x)) + a^2*x*e^(m*log(d) + m*log(x)) + 2*a*b*x*e^(m*log(d) + m*log(
x)) + b^2*x*e^(m*log(d) + m*log(x)) + 2*a*c*x*e^(m*log(d) + m*log(x)) + 2*b
*c*x*e^(m*log(d) + m*log(x)) + c^2*x*e^(m*log(d) + m*log(x)))/(m^5 + 10*m^4
*n + 35*m^3*n^2 + 50*m^2*n^3 + 24*m*n^4 + 5*m^4 + 40*m^3*n + 105*m^2*n^2 +
100*m*n^3 + 24*n^4 + 10*m^3 + 60*m^2*n + 105*m*n^2 + 50*n^3 + 10*m^2 + 40*m
*n + 35*n^2 + 5*m + 10*n + 1)
```

3.598 $\int (dx)^m \left(a+bx^n+cx^{2n}\right) dx$

Optimal. Leaf size=58

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{bx^{n+1}(dx)^m}{m+n+1}+\frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

```
[Out] (b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (c*x^(1 + 2*n)*(d*x)^m)/(1 + m + 2*n) +
 (a*(d*x)^(1 + m))/(d*(1 + m))
```

Rubi [A] time = 0.0242398, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {14, 20, 30}

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{bx^{n+1}(dx)^m}{m+n+1}+\frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (c*x^(1 + 2*n)*(d*x)^m)/(1 + m + 2*n) +
 (a*(d*x)^(1 + m))/(d*(1 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+bx^n+cx^{2n}\right)dx&=\int\left(a(dx)^m+bx^n(dx)^m+cx^{2n}(dx)^m\right)dx\\&=\frac{a(dx)^{1+m}}{d(1+m)}+b\int x^n(dx)^m\,dx+c\int x^{2n}(dx)^m\,dx\\&=\frac{a(dx)^{1+m}}{d(1+m)}+\left(bx^{-m}(dx)^m\right)\int x^{m+n}\,dx+\left(cx^{-m}(dx)^m\right)\int x^{m+2n}\,dx\\&=\frac{bx^{1+n}(dx)^m}{1+m+n}+\frac{cx^{1+2n}(dx)^m}{1+m+2n}+\frac{a(dx)^{1+m}}{d(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0615337, size = 41, normalized size = 0.71

$$x(dx)^m\left(\frac{a}{m+1}+x^n\left(\frac{b}{m+n+1}+\frac{cx^n}{m+2n+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x*(d*x)^m*(a/(1 + m) + x^n*(b/(1 + m + n) + (c*x^n)/(1 + m + 2*n)))
```

Maple [C] time = 0.042, size = 205, normalized size = 3.5

$$\frac{x\left(cm^2(x^n)^2+cmn(x^n)^2+bm^2x^n+2\,bmnx^n+2\,mc(x^n)^2+c(x^n)^2n+am^2+3\,amn+2\,an^2+2\,mbx^n+2\,bx^nn+c(\ldots\right.}{(1+m)(1+m+n)(1+m+2\,n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] x*(c*m^2*(x^n)^2+c*m*n*(x^n)^2+b*m^2*x^n+2*b*m*n*x^n+2*m*c*(x^n)^2+c*(x^n)^
2*n+a*m^2+3*a*m*n+2*a*n^2+2*m*b*x^n+2*b*x^n*n+c*(x^n)^2+2*a*m+3*a*n+b*x^n+a
)/(1+m)/(1+m+n)/(1+m+2*n)*exp(1/2*m*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*cs
gn(I*d)*Pi+I*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I*d)*csgn(I*x)*P
i+2*ln(x)+2*ln(d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.89088, size = 371, normalized size = 6.4

$$\frac{\left(cm^2+2\,cm+(cm+c)n+c\right)xx^{2\,n}e^{(m\log(d)+m\log(x))}+\left(bm^2+2\,bm+2\,(bm+b)n+b\right)xx^ne^{(m\log(d)+m\log(x))}+\left(am^2+2\ldots\right.}{m^3+2\,(m+1)n^2+3\,m^2+3\left(m^2+2\,m+1\right)n+3\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 2*c*m + (c*m + c)*n + c)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + (b*m
^2 + 2*b*m + 2*(b*m + b)*n + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m^2 + 2*
a*n^2 + 2*a*m + 3*(a*m + a)*n + a)*x*e^(m*log(d) + m*log(x)))/(m^3 + 2*(m +
 1)*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n + 3*m + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.10858, size = 752, normalized size = 12.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] (c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c*m*n*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + b*m^2*x*x^n*e^(m*log(d) + m*log(x)) + c*m^2*x*x^n*e^(m*log(d) + m
*log(x)) + 2*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + c*m*n*x*x^n*e^(m*log(d)
+ m*log(x)) + a*m^2*x*e^(m*log(d) + m*log(x)) + b*m^2*x*e^(m*log(d) + m*log
(x)) + c*m^2*x*e^(m*log(d) + m*log(x)) + 3*a*m*n*x*e^(m*log(d) + m*log(x))
+ 2*b*m*n*x*e^(m*log(d) + m*log(x)) + c*m*n*x*e^(m*log(d) + m*log(x)) + 2*a
*n^2*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*m*x*x^n*e^(m*log(d) + m*log(x))
 + 2*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*n*x*x^n*e^(m*log(d) + m*log(x)
) + c*n*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*m*x*e^(m*log(d) + m*log(x)) + 2
*b*m*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*e^(m*log(d) + m*log(x)) + 3*a*n*x*
e^(m*log(d) + m*log(x)) + 2*b*n*x*e^(m*log(d) + m*log(x)) + c*n*x*e^(m*log(
d) + m*log(x)) + c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + b*x*x^n*e^(m*log(d)
+ m*log(x)) + c*x*x^n*e^(m*log(d) + m*log(x)) + a*x*e^(m*log(d) + m*log(x))
 + b*x*e^(m*log(d) + m*log(x)) + c*x*e^(m*log(d) + m*log(x)))/(m^3 + 3*m^2*
n + 2*m*n^2 + 3*m^2 + 6*m*n + 2*n^2 + 3*m + 3*n + 1)
```

3.599 $\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=175

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1
 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a
*c])*d*(1 + m))
```

Rubi [A] time = 0.185346, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1383, 364}

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1
 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a
*c])*d*(1 + m))
```

Rule 1383

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[(2*c)/q, Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx &= \frac{(2c)\int \frac{(dx)^m}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{(dx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\ &= \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)}-\frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}\end{aligned}$$

Mathematica [A] time = 0.148448, size = 143, normalized size = 0.82

$$\frac{x(dx)^m\left(\left(\sqrt{b^2-4ac}+b\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+\left(\sqrt{b^2-4ac}-b\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)\right)}{2a(m+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x*(d*x)^m*((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m
+ n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (-b + Sqrt[b^2 - 4*a*c])*Hype
rgeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
])]))/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{cx^{2n}+bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.600 $\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^2}\,dx$

Optimal. Leaf size=328

$$\frac{c(dx)^{m+1}\left(\frac{4ac(m-2n+1)-b^2(m-n+1)}{\sqrt{b^2-4ac}}-b(m-n+1)\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{ad(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(b(m-n+1)\sqrt{b^2-4ac}+4a\right.}{ad(m}$$

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*d*n*(a + b*x^n + c
*x^(2*n))) + (c*((4*a*c*(1 + m - 2*n) - b^2*(1 + m - n))/Sqrt[b^2 - 4*a*c]
- b*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/
n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*
a*c])*d*(1 + m)*n) - (c*(4*a*c*(1 + m - 2*n) - b^2*(1 + m - n) + b*Sqrt[b^2
 - 4*a*c]*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m
 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)^(3/2)*(b + S
qrt[b^2 - 4*a*c])*d*(1 + m)*n)
```

Rubi [A] time = 0.961283, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1384, 1560, 364}

$$\frac{c(dx)^{m+1}\left(\frac{4ac(m-2n+1)-b^2(m-n+1)}{\sqrt{b^2-4ac}}-b(m-n+1)\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{ad(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(b(m-n+1)\sqrt{b^2-4ac}+4a\right.}{ad(m}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*d*n*(a + b*x^n + c
*x^(2*n))) + (c*((4*a*c*(1 + m - 2*n) - b^2*(1 + m - n))/Sqrt[b^2 - 4*a*c]
- b*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/
n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*
a*c])*d*(1 + m)*n) - (c*(4*a*c*(1 + m - 2*n) - b^2*(1 + m - n) + b*Sqrt[b^2
 - 4*a*c]*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m
 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)^(3/2)*(b + S
qrt[b^2 - 4*a*c])*d*(1 + m)*n)
```

Rule 1384

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p + 1)
 + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[p + 1, 0]
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
```

```
[q, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^2}\,dx = \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} - \frac{\int \frac{(dx)^m\left(-2ac(1+m-2n)+b^2(1+m-n)+bc(1+m-n)x^n\right)}{a+bx^n+cx^{2n}}\,dx}{a\left(b^2-4ac\right)n}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} - \frac{\int \left(\frac{\left(bc(1+m-n)+\frac{c\left(b^2-4ac+b^2m-4acm-b^2n+8acn\right)}{\sqrt{b^2-4ac}}\right)(dx)^m}{b-\sqrt{b^2-4ac}+2cx^n} + \frac{\left(bc(1+m-n)-\frac{c}{}\right.}{} \right.}{a\left(b^2-4ac\right)n}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} + \frac{\left(c\left(4ac(1+m-2n)-b^2(1+m-n)-b\sqrt{b^2-4ac}(1+\right.\right.}{a\left(b^2-4ac\right)^{3/2}n}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} + \frac{c\left(4ac(1+m-2n)-b^2(1+m-n)-b\sqrt{b^2-4ac}(1+m\right.}{a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-\right.}}$$

Mathematica [B] time = 2.28882, size = 1511, normalized size = 4.61

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] -((x*(d*x)^m*(-((b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*
c])*(1 + m)*(1 + m + n)*(b^2 - 2*a*c + b*c*x^n)) + 2*b^2*c*Sqrt[b^2 - 4*a*c
]*(1 + m + n)*(a + x^n*(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometr
ic2F1[1, (1 + m)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (
b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])]) - 4*a*c^2*Sqrt[b^2 - 4*a*c]*(1 + m + n)*(a
+ x^n*(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/
n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*
a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]) + 2*b^2*c*Sqrt[b^2 - 4*a*c]*m*(1 + m + n)*(a + x^n*(b + c*x^
n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/
n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])*Hypergeom
etric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
- 4*a*c^2*Sqrt[b^2 - 4*a*c]*m*(1 + m + n)*(a + x^n*(b + c*x^n))*(-((b + Sqr
t[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (2*c*x^n)/(-
b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1
+ m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) - 2*b^2*c*Sqrt[
b^2 - 4*a*c]*n*(1 + m + n)*(a + x^n*(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])
*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m
+ n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) + 8*a*c^2*Sqrt[b^2 - 4*a*c]*n*
```

```
(1 + m + n)*(a + x^n*(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric
2F1[1, (1 + m)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b
- Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]) + 2*b*c^2*Sqrt[b^2 - 4*a*c]*(1 + m)*x^n*(a +
x^n*(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m + n
)/n, 2 + (1 + m)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 -
4*a*c])*Hypergeometric2F1[1, (1 + m + n)/n, 2 + (1 + m)/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c])]) + 2*b*c^2*Sqrt[b^2 - 4*a*c]*m*(1 + m)*x^n*(a + x^n*(b
+ c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m + n)/n, 2
+ (1 + m)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])
*Hypergeometric2F1[1, (1 + m + n)/n, 2 + (1 + m)/n, (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c])]) - 2*b*c^2*Sqrt[b^2 - 4*a*c]*(1 + m)*n*x^n*(a + x^n*(b + c*x^n
))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m + n)/n, 2 + (1 +
m)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])*Hyperg
eometric2F1[1, (1 + m + n)/n, 2 + (1 + m)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])))/(a*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c]
)*(1 + m)*n*(1 + m + n)*(a + x^n*(b + c*x^n))))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + bx^n + cx^{2n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcd^m xe^{(m\log(x)+n\log(x))} + \left(b^2d^m - 2\,acd^m\right)xx^m}{a^2b^2n - 4\,a^3cn + \left(ab^2cn - 4\,a^2c^2n\right)x^{2\,n} + \left(ab^3n - 4\,a^2bcn\right)x^n} + \int -\frac{bcd^m(m-n+1)e^{(m\log(x)+n\log(x))} + \left(b^2d^m(m-n+1)\right.}{a^2b^2n - 4\,a^3cn + \left(ab^2cn - 4\,a^2c^2n\right)x^{2\,n} + \left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] (b*c*d^m*x*e^(m*log(x) + n*log(x)) + (b^2*d^m - 2*a*c*d^m)*x*x^m)/(a^2*b^2*
n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)
*x^n) + integrate(-(b*c*d^m*(m - n + 1)*e^(m*log(x) + n*log(x)) + (b^2*d^m*
(m - n + 1) - 2*a*c*d^m*(m - 2*n + 1))*x^m)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2
*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx)^m}{c^2x^{4\,n} + b^2x^{2\,n} + 2\,abx^n + a^2 + 2\,(bcx^n + ac)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*x^n
+ a*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^{2\,n}+bx^n+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

3.601 $\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^3}\,dx$

Optimal. Leaf size=615

$$-\frac{c(dx)^{m+1}\left(-8a^2c^2\left(m^2+m(2-6n)+8n^2-6n+1\right)+6ab^2c\left(m^2+m(2-4n)+3n^2-4n+1\right)+b(m-n+1)\sqrt{b^2-4ac}\right.}{2a^2d(m+1)n^2\left(b^2-4ac\right)}$$

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(2*a*(b^2 - 4*a*c)*d*n*(a + b*x^n +
 c*x^(2*n))^2) - ((d*x)^(1 + m)*(4*a^2*c^2*(1 + m - 4*n) - 5*a*b^2*c*(1 + m
 - 3*n) + b^4*(1 + m - 2*n) - b*c*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n
))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*d*n^2*(a + b*x^n + c*x^(2*n))) - (c*(b*Sqrt
[b^2 - 4*a*c]*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n))*(1 + m - n) - b^4
*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*(1 + m^2 + m*(2 - 4*n) -
 4*n + 3*n^2) - 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))*(d*x)^(1 +
 m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)*n
^2) - (c*(b*Sqrt[b^2 - 4*a*c]*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n))*(
1 + m - n) + b^4*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) - 6*a*b^2*c*(1 + m^2
 + m*(2 - 4*n) - 4*n + 3*n^2) + 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*
n^2))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b + Sqrt[b^2 - 4*a
*c])*d*(1 + m)*n^2)
```

Rubi [A] time = 10.7238, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1384, 1558, 1560, 364}

$$-\frac{c(dx)^{m+1}\left(-8a^2c^2\left(m^2+m(2-6n)+8n^2-6n+1\right)+6ab^2c\left(m^2+m(2-4n)+3n^2-4n+1\right)+b(m-n+1)\sqrt{b^2-4ac}\right.}{2a^2d(m+1)n^2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^3,x]
```


```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(2*a*(b^2 - 4*a*c)*d*n*(a + b*x^n +
 c*x^(2*n))^2) - ((d*x)^(1 + m)*(4*a^2*c^2*(1 + m - 4*n) - 5*a*b^2*c*(1 + m
 - 3*n) + b^4*(1 + m - 2*n) - b*c*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n
))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*d*n^2*(a + b*x^n + c*x^(2*n))) - (c*(b*Sqrt
[b^2 - 4*a*c]*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n))*(1 + m - n) - b^4
*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*(1 + m^2 + m*(2 - 4*n) -
 4*n + 3*n^2) - 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))*(d*x)^(1 +
 m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)*n
^2) - (c*(b*Sqrt[b^2 - 4*a*c]*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n))*(
1 + m - n) + b^4*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) - 6*a*b^2*c*(1 + m^2
 + m*(2 - 4*n) - 4*n + 3*n^2) + 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*
n^2))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b + Sqrt[b^2 - 4*a
*c])*d*(1 + m)*n^2)
```


Rule 1384

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p + 1)
 + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[p + 1, 0]
```

Rule 1558

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && ILtQ[p + 1, 0]
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^3}\,dx = \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2} - \frac{\int \frac{(dx)^m\left(-2ac(1+m-4n)+b^2(1+m-2n)+bc(1+m-3n)x^n\right)}{\left(a+bx^n+cx^{2n}\right)^2}\,dx}{2a\left(b^2-4ac\right)n}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2} - \frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-3n)+b^4\right.}{2a^2\left(b^2-4ac\right)^2}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2} - \frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-3n)+b^4\right.}{2a^2\left(b^2-4ac\right)^2}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2} - \frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-3n)+b^4\right.}{2a^2\left(b^2-4ac\right)^2}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2} - \frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-3n)+b^4\right.}{2a^2\left(b^2-4ac\right)^2}$$

Mathematica [B] time = 6.72233, size = 7827, normalized size = 12.73

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + bx^n + cx^{2n}\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*b^2*c*d^m*(5*m - 21*n + 5) - a*b^4*d^m*(m - 3*n + 1) - 4*a^3*c^2*
d^m*(m - 6*n + 1))*x*x^m + (2*a*b*c^3*d^m*(2*m - 7*n + 2) - b^3*c^2*d^m*(m
- 2*n + 1))*x*e^(m*log(x) + 3*n*log(x)) + (a*b^2*c^2*d^m*(9*m - 29*n + 9) -
 2*b^4*c*d^m*(m - 2*n + 1) - 4*a^2*c^3*d^m*(m - 4*n + 1))*x*e^(m*log(x) + 2
*n*log(x)) - (b^5*d^m*(m - 2*n + 1) - 4*a*b^3*c*d^m*(m - 3*n + 1) + 2*a^2*b
*c^2*d^m*n)*x*e^(m*log(x) + n*log(x)))/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*
a^6*c^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n
) + 2*(a^2*b^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2
*b^6*n^2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a
^4*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) - integrate(-1/2*(((m^2 - m*(3*n - 2)
 + 2*n^2 - 3*n + 1)*b^4*d^m - (5*m^2 - m*(21*n - 10) + 16*n^2 - 21*n + 5)*a
*b^2*c*d^m + 4*(m^2 - 2*m*(3*n - 1) + 8*n^2 - 6*n + 1)*a^2*c^2*d^m)*x^m + (
(m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c*d^m - 2*(2*m^2 - m*(9*n - 4) +
7*n^2 - 9*n + 2)*a*b*c^2*d^m)*e^(m*log(x) + n*log(x)))/(a^3*b^4*n^2 - 8*a^4
*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c
^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{c^3x^{6n} + b^3x^{3n} + 3\,ab^2x^{2n} + 3\,a^2bx^n + a^3 + 3\left(bc^2x^n + ac^2\right)x^{4n} + 3\left(b^2cx^{2n} + 2\,abcx^n + a^2c\right)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n
 + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n + a
^2*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^{2n} + bx^n + a\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)
```

3.602 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=161

$$\frac{a(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{m+1}{n};-\frac{3}{2},-\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -3/2, -3/2
, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.17926, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 510}

$$\frac{a(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{m+1}{n};-\frac{3}{2},-\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -3/2, -3/2
, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int (dx)^m \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2} dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{a(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1+m}{n};-\frac{3}{2},-\frac{3}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 3.31845, size = 618, normalized size = 3.84

$$\frac{x(dx)^m\left((m+1)\left(2(m+n+1)\left(4a^2c\left(m^2+m(6n+2)+8n^2+6n+1\right)+a\left(3b^2n^2+2bc\left(4m^2+m(21n+8)+23n^2+\right.\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*(-6*a*n^2*(1 + m + n)*(b^2*(1 + m) - 4*a*c*(1 + m + 2*n))*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2
, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])] + (1 + m)*(2*(1 + m + n)*(4*a^2*c*(1 + m^2 + 6*n + 8*n^2 + m*
(2 + 6*n)) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*b*c*(2 + 2*m^2 + 9*n + 7*n^2 +
m*(4 + 9*n))*x^n + 4*c^2*(1 + m^2 + 3*n + 2*n^2 + m*(2 + 3*n))*x^(2*n)) + a
*(3*b^2*n^2 + 2*b*c*(4 + 4*m^2 + 21*n + 23*n^2 + m*(8 + 21*n))*x^n + 4*c^2*
(2 + 2*m^2 + 9*n + 10*n^2 + m*(4 + 9*n))*x^(2*n))) - 3*b*n^2*(b^2*(2 + 2*m
+ n) - 4*a*c*(2 + 2*m + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[(1 + m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(8*c*(1 + m)
*(1 + m + n)^2*(1 + m + 2*n)*(1 + m + 3*n)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.253, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}(dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

3.603 $\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=160

$$\frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{m+1}{n};-\frac{1}{2},-\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -1/2, -1/2,
(1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.167816, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{m+1}{n};-\frac{1}{2},-\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```


```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -1/2, -1/2,
(1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```


Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}\int (dx)^m\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1+m}{n};-\frac{1}{2},-\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.770621, size = 388, normalized size = 2.42

$$\frac{x(dx)^m\left(2an(m+n+1)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+(m+1)\left(bnx^n\sqrt{\right.\right.}{2(m+1)(m+n+1)^2\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*(2*a*n*(1 + m + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*(2*(1 + m + n)*
(a + x^n*(b + c*x^n)) + b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[(1 + m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(2*(1 + m)*(1
+ m + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.14, size = 0, normalized size = 0.

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n}+bx^n+a}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \sqrt{a + bx^n + cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2\,n} + bx^n + a}\,(dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

3.604 $\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=160

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(d*(
1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.169189, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(d*(
1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}$$
$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{n};\frac{1}{2},\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A] time = 0.19068, size = 183, normalized size = 1.14

$$\frac{x(dx)^m\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1
 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{a + bx^n + cx^{2n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^{2n} + bx^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.605 $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=163

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{n};\frac{3}{2},\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 3/2, 3/2, (1 + m + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*d
*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.173355, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{n};\frac{3}{2},\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 3/2, 3/2, (1 + m + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*d
*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int\frac{(dx)^m}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{n};\frac{3}{2},\frac{3}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] time = 1.52138, size = 428, normalized size = 2.63

$$\frac{x(dx)^m\left((m+n+1)\left(b^2(2m-n+2)-4ac(m-n+1)\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4a}}\right.\right.}{a(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*((-4*a*c*(1 + m - n) + b^2*(2 + 2*m - n))*(1 + m + n)*Sqrt[(b -
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1
 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
 4*a*c])] - 2*(1 + m)*((1 + m + n)*(b^2 - 2*a*c + b*c*x^n) - b*c*(1 + m)*x^
n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m + n)
/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])]))))/(a*(-b^2 + 4*a*c)*(1 + m)*n*(1 + m + n)*Sqrt
[a + x^n*(b + c*x^n)])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+bx^n+cx^{2\,n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.606 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=158

$$\frac{(dx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m
 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.129094, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m
 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (dx)^m\left(a+bx^n+cx^{2n}\right)^p\,dx=\left(\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int (dx)^m\left(1+\frac{2cx}{b-\sqrt{b^2}}\right.$$

$$=\frac{(dx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\right.}{d(1+m)}$$

Mathematica [A] time = 0.377937, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\left(\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}\right)^{-p}(a+x^n(b+cx^n))^pF_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(d*x)^m*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int (dx)^m\left(a+bx^n+cx^{2n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n}+bx^n+a\right)^p(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^{2n}+bx^n+a\right)^p(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

3.607 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$

Optimal. Leaf size=46

$$\frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

```
[Out] (a*(d + e*x)^4)/(4*e) + (b*(d + e*x)^6)/(6*e) + (c*(d + e*x)^8)/(8*e)
```

Rubi [A] time = 0.0536108, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1142, 14}

$$\frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (a*(d + e*x)^4)/(4*e) + (b*(d + e*x)^6)/(6*e) + (c*(d + e*x)^8)/(8*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx &= \frac{\text{Subst}\left(\int x^3 \left(a+bx^2+cx^4\right) dx, x, d+ex\right)}{e}\\ &= \frac{\text{Subst}\left(\int \left(ax^3+bx^5+cx^7\right) dx, x, d+ex\right)}{e}\\ &= \frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}\end{aligned}$$

Mathematica [B] time = 0.0383303, size = 150, normalized size = 3.26

$$\frac{1}{4}e^3x^4\left(a+10bd^2+35cd^4\right)+\frac{1}{3}de^2x^3\left(3a+10bd^2+21cd^4\right)+\frac{1}{2}d^2ex^2\left(3a+5bd^2+7cd^4\right)+d^3x\left(a+bd^2+cd^4\right)+\frac{1}{6}e^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 + (d*(3
*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*x^4)/
```

```
4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7 + (c
*e^7*x^8)/8
```

Maple [B] time = 0.001, size = 298, normalized size = 6.5

$$\frac{e^7cx^8}{8}+de^6cx^7+\frac{\left(15\,d^2e^5c+e^3\left(6\,cd^2e^2+be^2\right)\right)x^6}{6}+\frac{\left(13\,d^3ce^4+3\,de^2\left(6\,cd^2e^2+be^2\right)+e^3\left(4\,cd^3e+2\,bde\right)\right)x^5}{5}+\frac{\left(4\,d^4ce\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/8*e^7*c*x^8+d*e^6*c*x^7+1/6*(15*d^2*e^5*c+e^3*(6*c*d^2*e^2+b*e^2))*x^6+1/
5*(13*d^3*c*e^4+3*d*e^2*(6*c*d^2*e^2+b*e^2)+e^3*(4*c*d^3*e+2*b*d*e))*x^5+1/
4*(4*d^4*c*e^3+3*d^2*e*(6*c*d^2*e^2+b*e^2)+3*d*e^2*(4*c*d^3*e+2*b*d*e)+e^3*
(c*d^4+b*d^2+a))*x^4+1/3*(d^3*(6*c*d^2*e^2+b*e^2)+3*d^2*e*(4*c*d^3*e+2*b*d*
e)+3*d*e^2*(c*d^4+b*d^2+a))*x^3+1/2*(d^3*(4*c*d^3*e+2*b*d*e)+3*d^2*e*(c*d^4
+b*d^2+a))*x^2+d^3*(c*d^4+b*d^2+a)*x
```

Maxima [B] time = 1.03065, size = 192, normalized size = 4.17

$$\frac{1}{8}\,ce^7x^8+cde^6x^7+\frac{1}{6}\left(21\,cd^2+b\right)e^5x^6+\left(7\,cd^3+bd\right)e^4x^5+\frac{1}{4}\left(35\,cd^4+10\,bd^2+a\right)e^3x^4+\frac{1}{3}\left(21\,cd^5+10\,bd^3+3\,ad\right)e^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] 1/8*c*e^7*x^8 + c*d*e^6*x^7 + 1/6*(21*c*d^2 + b)*e^5*x^6 + (7*c*d^3 + b*d)*
e^4*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*x^4 + 1/3*(21*c*d^5 + 10*b*d^3
+ 3*a*d)*e^2*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2)*e*x^2 + (c*d^7 + b*d^5
 + a*d^3)*x
```

Fricas [B] time = 1.48395, size = 393, normalized size = 8.54

$$\frac{1}{8}x^8e^7c+x^7e^6dc+\frac{7}{2}x^6e^5d^2c+7x^5e^4d^3c+\frac{35}{4}x^4e^3d^4c+\frac{1}{6}x^6e^5b+7x^3e^2d^5c+x^5e^4db+\frac{7}{2}x^2ed^6c+\frac{5}{2}x^4e^3d^2b+xd^7c+\frac{10}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e^7*c + x^7*e^6*d*c + 7/2*x^6*e^5*d^2*c + 7*x^5*e^4*d^3*c + 35/4*x^
4*e^3*d^4*c + 1/6*x^6*e^5*b + 7*x^3*e^2*d^5*c + x^5*e^4*d*b + 7/2*x^2*e*d^6
*c + 5/2*x^4*e^3*d^2*b + x*d^7*c + 10/3*x^3*e^2*d^3*b + 5/2*x^2*e*d^4*b + 1
/4*x^4*e^3*a + x*d^5*b + x^3*e^2*d*a + 3/2*x^2*e*d^2*a + x*d^3*a
```

Sympy [B] time = 0.093475, size = 178, normalized size = 3.87

$$cde^6x^7+\frac{ce^7x^8}{8}+x^6\left(\frac{be^5}{6}+\frac{7cd^2e^5}{2}\right)+x^5\left(bde^4+7cd^3e^4\right)+x^4\left(\frac{ae^3}{4}+\frac{5bd^2e^3}{2}+\frac{35cd^4e^3}{4}\right)+x^3\left(ade^2+\frac{10bd^3e^2}{3}+7cd^5e^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*x**7 + c*e**7*x**8/8 + x**6*(b*e**5/6 + 7*c*d**2*e**5/2) + x**5*(b
*d*e**4 + 7*c*d**3*e**4) + x**4*(a*e**3/4 + 5*b*d**2*e**3/2 + 35*c*d**4*e**
3/4) + x**3*(a*d*e**2 + 10*b*d**3*e**2/3 + 7*c*d**5*e**2) + x**2*(3*a*d**2*
e/2 + 5*b*d**4*e/2 + 7*c*d**6*e/2) + x*(a*d**3 + b*d**5 + c*d**7)
```

Giac [B] time = 1.07922, size = 224, normalized size = 4.87

$$\frac{1}{8}cx^8e^7 + cdx^7e^6 + \frac{7}{2}cd^2x^6e^5 + 7\,cd^3x^5e^4 + \frac{35}{4}cd^4x^4e^3 + 7\,cd^5x^3e^2 + \frac{7}{2}cd^6x^2e + cd^7x + \frac{1}{6}bx^6e^5 + bdx^5e^4 + \frac{5}{2}bd^2x^4e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/8*c*x^8*e^7 + c*d*x^7*e^6 + 7/2*c*d^2*x^6*e^5 + 7*c*d^3*x^5*e^4 + 35/4*c*
d^4*x^4*e^3 + 7*c*d^5*x^3*e^2 + 7/2*c*d^6*x^2*e + c*d^7*x + 1/6*b*x^6*e^5 +
 b*d*x^5*e^4 + 5/2*b*d^2*x^4*e^3 + 10/3*b*d^3*x^3*e^2 + 5/2*b*d^4*x^2*e + b
*d^5*x + 1/4*a*x^4*e^3 + a*d*x^3*e^2 + 3/2*a*d^2*x^2*e + a*d^3*x
```

3.608 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx$

Optimal. Leaf size=89

$$\frac{a^2(d+ex)^4}{4e}+\frac{\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{ab(d+ex)^6}{3e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

```
[Out] (a^2*(d + e*x)^4)/(4*e) + (a*b*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*(d + e*x
)^8)/(8*e) + (b*c*(d + e*x)^10)/(5*e) + (c^2*(d + e*x)^12)/(12*e)
```

Rubi [A] time = 0.187073, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1142, 1114, 631}

$$\frac{a^2(d+ex)^4}{4e}+\frac{\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{ab(d+ex)^6}{3e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (a^2*(d + e*x)^4)/(4*e) + (a*b*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*(d + e*x
)^8)/(8*e) + (b*c*(d + e*x)^10)/(5*e) + (c^2*(d + e*x)^12)/(12*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx &= \frac{\text{Subst}\left(\int x^3\left(a+bx^2+cx^4\right)^2 dx, x, d+ex\right)}{e}\\ &= \frac{\text{Subst}\left(\int x\left(a+bx+cx^2\right)^2 dx, x, (d+ex)^2\right)}{2e}\\ &= \frac{\text{Subst}\left(\int \left(a^2x+2abx^2+\left(b^2+2ac\right)x^3+2bcx^4+c^2x^5\right) dx, x, (d+ex)^2\right)}{2e}\\ &= \frac{a^2(d+ex)^4}{4e}+\frac{ab(d+ex)^6}{3e}+\frac{\left(b^2+2ac\right)(d+ex)^8}{8e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d-}{1}\end{aligned}$$

Mathematica [B] time = 0.115766, size = 401, normalized size = 4.51

$$\frac{1}{4}e^3x^4\left(a^2+20abd^2+70acd^4+35b^2d^4+168bcd^6+165c^2d^8\right)+\frac{1}{3}de^2x^3\left(3a^2+20abd^2+42acd^4+21b^2d^4+72bcd^6+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 14*a*c
*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21*b^2*
d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20*a*b*d^2
 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (d*(10
*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5 + ((
2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)/6 + d
*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 72*b*c*d
^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/3 + (c*(2*b +
 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12
```

Maple [B] time = 0.002, size = 1314, normalized size = 14.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 1/12*e^11*c^2*x^12+d*e^10*c^2*x^11+1/10*(27*d^2*e^9*c^2+e^3*(2*(6*c*d^2*e^2
+b*e^2)*c*e^4+16*c^2*d^2*e^6))*x^10+1/9*(25*d^3*c^2*e^8+3*d*e^2*(2*(6*c*d^2
*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+e^3*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d
^2*e^2+b*e^2)*c*d*e^3))*x^9+1/8*(8*d^4*c^2*e^7+3*d^2*e*(2*(6*c*d^2*e^2+b*e^
2)*c*e^4+16*c^2*d^2*e^6)+3*d*e^2*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^
2+b*e^2)*c*d*e^3)+e^3*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^
3+(6*c*d^2*e^2+b*e^2)^2))*x^8+1/7*(d^3*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*
d^2*e^6)+3*d^2*e*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3
)+3*d*e^2*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e
^2+b*e^2)^2)+e^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*
e^2+b*e^2)))*x^7+1/6*(d^3*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2
)*c*d*e^3)+3*d^2*e*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(
6*c*d^2*e^2+b*e^2)^2)+3*d*e^2*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d
*e)*(6*c*d^2*e^2+b*e^2))+e^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^
3*e+2*b*d*e)^2))*x^6+1/5*(d^3*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e
)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2)+3*d^2*e*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*
d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d*e^2*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2
+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+2*e^3*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e))*x
^5+1/4*(d^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b
*e^2))+3*d^2*e*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2
)+6*d*e^2*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+e^3*(c*d^4+b*d^2+a)^2)*x^4+1/
3*(d^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d^2*
e*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+3*d*e^2*(c*d^4+b*d^2+a)^2)*x^3+1/2*(2
*d^3*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+3*d^2*e*(c*d^4+b*d^2+a)^2)*x^2+d^3
*(c*d^4+b*d^2+a)^2*x
```

Maxima [B] time = 1.02789, size = 544, normalized size = 6.11

$$\frac{1}{12}c^2e^{11}x^{12}+c^2de^{10}x^{11}+\frac{1}{10}\left(55c^2d^2+2bc\right)e^9x^{10}+\frac{1}{3}\left(55c^2d^3+6bcd\right)e^8x^9+\frac{1}{8}\left(330c^2d^4+72bcd^2+b^2+2ac\right)e^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/12*c^2*e^11*x^12 + c^2*d*e^10*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^9*x^10 +
 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*x^9 + 1/8*(330*c^2*d^4 + 72*b*c*d^2 + b^2 +
 2*a*c)*e^7*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a*c)*d)*e^6*x^7 + 1/6
*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2 + 2*a*b)*e^5*x^6 + 1/5*(
330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^3 + 10*a*b*d)*e^4*x^5 + 1/4*
(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*c)*d^4 + 20*a*b*d^2 + a^2)*e^3*x
^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a
^2*d)*e^2*x^3 + 1/2*(11*c^2*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*
b*d^4 + 3*a^2*d^2)*e*x^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*
b*d^5 + a^2*d^3)*x
```

Fricas [B] time = 1.45956, size = 1258, normalized size = 14.13

$$\frac{1}{12}x^{12}e^{11}c^2 + x^{11}e^{10}dc^2 + \frac{11}{2}x^{10}e^9d^2c^2 + \frac{55}{3}x^9e^8d^3c^2 + \frac{165}{4}x^8e^7d^4c^2 + \frac{1}{5}x^{10}e^9cb + 66x^7e^6d^5c^2 + 2x^9e^8dcb + 77x^6e^5d^6c^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^11*c^2 + x^11*e^10*d*c^2 + 11/2*x^10*e^9*d^2*c^2 + 55/3*x^9*e^8
*d^3*c^2 + 165/4*x^8*e^7*d^4*c^2 + 1/5*x^10*e^9*c*b + 66*x^7*e^6*d^5*c^2 +
2*x^9*e^8*d*c*b + 77*x^6*e^5*d^6*c^2 + 9*x^8*e^7*d^2*c*b + 66*x^5*e^4*d^7*c
^2 + 24*x^7*e^6*d^3*c*b + 165/4*x^4*e^3*d^8*c^2 + 42*x^6*e^5*d^4*c*b + 1/8*
x^8*e^7*b^2 + 1/4*x^8*e^7*c*a + 55/3*x^3*e^2*d^9*c^2 + 252/5*x^5*e^4*d^5*c*
b + x^7*e^6*d*b^2 + 2*x^7*e^6*d*c*a + 11/2*x^2*e*d^10*c^2 + 42*x^4*e^3*d^6*
c*b + 7/2*x^6*e^5*d^2*b^2 + 7*x^6*e^5*d^2*c*a + x*d^11*c^2 + 24*x^3*e^2*d^7
*c*b + 7*x^5*e^4*d^3*b^2 + 14*x^5*e^4*d^3*c*a + 9*x^2*e*d^8*c*b + 35/4*x^4*
e^3*d^4*b^2 + 35/2*x^4*e^3*d^4*c*a + 1/3*x^6*e^5*b*a + 2*x*d^9*c*b + 7*x^3*
e^2*d^5*b^2 + 14*x^3*e^2*d^5*c*a + 2*x^5*e^4*d*b*a + 7/2*x^2*e*d^6*b^2 + 7*
x^2*e*d^6*c*a + 5*x^4*e^3*d^2*b*a + x*d^7*b^2 + 2*x*d^7*c*a + 20/3*x^3*e^2*
d^3*b*a + 5*x^2*e*d^4*b*a + 1/4*x^4*e^3*a^2 + 2*x*d^5*b*a + x^3*e^2*d*a^2 +
 3/2*x^2*e*d^2*a^2 + x*d^3*a^2
```

Sympy [B] time = 0.16445, size = 559, normalized size = 6.28

$$c^2de^{10}x^{11} + \frac{c^2e^{11}x^{12}}{12} + x^{10}\left(\frac{bce^9}{5} + \frac{11c^2d^2e^9}{2}\right) + x^9\left(2bcde^8 + \frac{55c^2d^3e^8}{3}\right) + x^8\left(\frac{ace^7}{4} + \frac{b^2e^7}{8} + 9bcd^2e^7 + \frac{165c^2d^4e^7}{4}\right) + x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] c**2*d*e**10*x**11 + c**2*e**11*x**12/12 + x**10*(b*c*e**9/5 + 11*c**2*d**2
*e**9/2) + x**9*(2*b*c*d*e**8 + 55*c**2*d**3*e**8/3) + x**8*(a*c*e**7/4 + b
**2*e**7/8 + 9*b*c*d**2*e**7 + 165*c**2*d**4*e**7/4) + x**7*(2*a*c*d*e**6 +
 b**2*d*e**6 + 24*b*c*d**3*e**6 + 66*c**2*d**5*e**6) + x**6*(a*b*e**5/3 + 7
*a*c*d**2*e**5 + 7*b**2*d**2*e**5/2 + 42*b*c*d**4*e**5 + 77*c**2*d**6*e**5)
 + x**5*(2*a*b*d*e**4 + 14*a*c*d**3*e**4 + 7*b**2*d**3*e**4 + 252*b*c*d**5*
e**4/5 + 66*c**2*d**7*e**4) + x**4*(a**2*e**3/4 + 5*a*b*d**2*e**3 + 35*a*c*
```

```
d**4*e**3/2 + 35*b**2*d**4*e**3/4 + 42*b*c*d**6*e**3 + 165*c**2*d**8*e**3/4
) + x**3*(a**2*d*e**2 + 20*a*b*d**3*e**2/3 + 14*a*c*d**5*e**2 + 7*b**2*d**5
*e**2 + 24*b*c*d**7*e**2 + 55*c**2*d**9*e**2/3) + x**2*(3*a**2*d**2*e/2 + 5
*a*b*d**4*e + 7*a*c*d**6*e + 7*b**2*d**6*e/2 + 9*b*c*d**8*e + 11*c**2*d**10
*e/2) + x*(a**2*d**3 + 2*a*b*d**5 + 2*a*c*d**7 + b**2*d**7 + 2*b*c*d**9 + c
**2*d**11)
```

Giac [B] time = 1.07931, size = 730, normalized size = 8.2

$$\frac{1}{12}\,c^2x^{12}e^{11} + c^2dx^{11}e^{10} + \frac{11}{2}\,c^2d^2x^{10}e^9 + \frac{55}{3}\,c^2d^3x^9e^8 + \frac{165}{4}\,c^2d^4x^8e^7 + 66\,c^2d^5x^7e^6 + 77\,c^2d^6x^6e^5 + 66\,c^2d^7x^5e^4 + \frac{16}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/12*c^2*x^12*e^11 + c^2*d*x^11*e^10 + 11/2*c^2*d^2*x^10*e^9 + 55/3*c^2*d^3
*x^9*e^8 + 165/4*c^2*d^4*x^8*e^7 + 66*c^2*d^5*x^7*e^6 + 77*c^2*d^6*x^6*e^5
+ 66*c^2*d^7*x^5*e^4 + 165/4*c^2*d^8*x^4*e^3 + 55/3*c^2*d^9*x^3*e^2 + 11/2*
c^2*d^10*x^2*e + c^2*d^11*x + 1/5*b*c*x^10*e^9 + 2*b*c*d*x^9*e^8 + 9*b*c*d^
2*x^8*e^7 + 24*b*c*d^3*x^7*e^6 + 42*b*c*d^4*x^6*e^5 + 252/5*b*c*d^5*x^5*e^4
 + 42*b*c*d^6*x^4*e^3 + 24*b*c*d^7*x^3*e^2 + 9*b*c*d^8*x^2*e + 2*b*c*d^9*x
+ 1/8*b^2*x^8*e^7 + 1/4*a*c*x^8*e^7 + b^2*d*x^7*e^6 + 2*a*c*d*x^7*e^6 + 7/2
*b^2*d^2*x^6*e^5 + 7*a*c*d^2*x^6*e^5 + 7*b^2*d^3*x^5*e^4 + 14*a*c*d^3*x^5*e
^4 + 35/4*b^2*d^4*x^4*e^3 + 35/2*a*c*d^4*x^4*e^3 + 7*b^2*d^5*x^3*e^2 + 14*a
*c*d^5*x^3*e^2 + 7/2*b^2*d^6*x^2*e + 7*a*c*d^6*x^2*e + b^2*d^7*x + 2*a*c*d^
7*x + 1/3*a*b*x^6*e^5 + 2*a*b*d*x^5*e^4 + 5*a*b*d^2*x^4*e^3 + 20/3*a*b*d^3*
x^3*e^2 + 5*a*b*d^4*x^2*e + 2*a*b*d^5*x + 1/4*a^2*x^4*e^3 + a^2*d*x^3*e^2 +
 3/2*a^2*d^2*x^2*e + a^2*d^3*x
```

3.609 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx$

Optimal. Leaf size=138

$$\frac{a^2b(d+ex)^6}{2e}+\frac{a^3(d+ex)^4}{4e}+\frac{c\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{b\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3a\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2(d+ex)^{14}}{14e}+$$

```
[Out] (a^3*(d + e*x)^4)/(4*e) + (a^2*b*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a*c)*(d +
 e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*(d + e*x)^10)/(10*e) + (c*(b^2 + a*c)*(d
+ e*x)^12)/(4*e) + (3*b*c^2*(d + e*x)^14)/(14*e) + (c^3*(d + e*x)^16)/(16*e
)
```

Rubi [A] time = 0.373254, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1142, 1114, 631}

$$\frac{a^2b(d+ex)^6}{2e}+\frac{a^3(d+ex)^4}{4e}+\frac{c\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{b\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3a\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2(d+ex)^{14}}{14e}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```


```
[Out] (a^3*(d + e*x)^4)/(4*e) + (a^2*b*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a*c)*(d +
 e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*(d + e*x)^10)/(10*e) + (c*(b^2 + a*c)*(d
+ e*x)^12)/(4*e) + (3*b*c^2*(d + e*x)^14)/(14*e) + (c^3*(d + e*x)^16)/(16*e
)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx = \frac{\text{Subst}\left(\int x^3 \left(a+bx^2+cx^4\right)^3 dx, x, d+ex\right)}{e}$$
$$= \frac{\text{Subst}\left(\int x \left(a+bx+cx^2\right)^3 dx, x, (d+ex)^2\right)}{2e}$$
$$= \frac{\text{Subst}\left(\int \left(a^3x+3a^2bx^2+3a\left(b^2+ac\right)x^3+b\left(b^2+6ac\right)x^4+3c\left(b^2+\right.\right.}{2e}$$
$$= \frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{3a\left(b^2+ac\right)(d+ex)^8}{8e}+\frac{b\left(b^2+6ac\right)(d}{10e}$$

Mathematica [B] time = 0.290201, size = 797, normalized size = 5.78

$$\frac{1}{16}c^3e^{15}x^{16}+c^3de^{14}x^{15}+\frac{3}{14}c^2\left(35cd^2+b\right)e^{13}x^{14}+c^2d\left(35cd^2+3b\right)e^{12}x^{13}+\frac{1}{4}c\left(455c^2d^4+78bcd^2+b^2+ac\right)e^{11}x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5
*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 12
*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^10 + 35*
c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 +
84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2*d^10
 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*d^2 +
 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^2*d^8
 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 42*b^
3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2*d^8 + 10
01*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a*b*c*
d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)*e^6*
x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4 + 33
0*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a*b*c
+ 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*x^9 + ((b^
3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4 + 5005*c^3*d^6
)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4)*e^10*x^11 + (c
*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35*c*d^
2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 + (c^3
*e^15*x^16)/16
```

Maple [B] time = 0.002, size = 7550, normalized size = 54.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

Maxima [B] time = 1.04916, size = 1177, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*c^3*e^15*x^16 + c^3*d*e^14*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^13*x^14
+ (35*c^3*d^3 + 3*b*c^2*d)*e^12*x^13 + 1/4*(455*c^3*d^4 + 78*b*c^2*d^2 + b^
2*c + a*c^2)*e^11*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 + (b^2*c + a*c^2)*d)*
e^10*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b^3 + 6*a*b*c + 165*(b^2*
c + a*c^2)*d^2)*e^9*x^10 + (715*c^3*d^7 + 429*b*c^2*d^5 + 55*(b^2*c + a*c^2
)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*x^9 + 3/8*(2145*c^3*d^8 + 1716*b*c^2*d^6 + 3
30*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7*x^8 +
1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6
*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c
^2*d^8 + 462*(b^2*c + a*c^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b
^2 + a^2*c)*d^2)*e^5*x^6 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c +
 a*c^2)*d^7 + 42*(b^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*
e^4*x^5 + 1/4*(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84
*(b^3 + 6*a*b*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*x^
4 + (35*c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b
*c)*d^7 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*x^3 + 3/2*(5*c
^3*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 +
 5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*e*x^2 + (c^3*d^15 + 3*b*c^2
*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b
^2 + a^2*c)*d^7 + a^3*d^3)*x
```

Fricas [B] time = 1.54233, size = 3015, normalized size = 21.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*e^15*c^3 + x^15*e^14*d*c^3 + 15/2*x^14*e^13*d^2*c^3 + 35*x^13*e^1
2*d^3*c^3 + 455/4*x^12*e^11*d^4*c^3 + 3/14*x^14*e^13*c^2*b + 273*x^11*e^10*
d^5*c^3 + 3*x^13*e^12*d*c^2*b + 1001/2*x^10*e^9*d^6*c^3 + 39/2*x^12*e^11*d^
2*c^2*b + 715*x^9*e^8*d^7*c^3 + 78*x^11*e^10*d^3*c^2*b + 6435/8*x^8*e^7*d^8
*c^3 + 429/2*x^10*e^9*d^4*c^2*b + 1/4*x^12*e^11*c*b^2 + 1/4*x^12*e^11*c^2*a
 + 715*x^7*e^6*d^9*c^3 + 429*x^9*e^8*d^5*c^2*b + 3*x^11*e^10*d*c*b^2 + 3*x^
11*e^10*d*c^2*a + 1001/2*x^6*e^5*d^10*c^3 + 1287/2*x^8*e^7*d^6*c^2*b + 33/2
*x^10*e^9*d^2*c*b^2 + 33/2*x^10*e^9*d^2*c^2*a + 273*x^5*e^4*d^11*c^3 + 5148
/7*x^7*e^6*d^7*c^2*b + 55*x^9*e^8*d^3*c*b^2 + 55*x^9*e^8*d^3*c^2*a + 455/4*
x^4*e^3*d^12*c^3 + 1287/2*x^6*e^5*d^8*c^2*b + 495/4*x^8*e^7*d^4*c*b^2 + 1/1
0*x^10*e^9*b^3 + 495/4*x^8*e^7*d^4*c^2*a + 3/5*x^10*e^9*c*b*a + 35*x^3*e^2*
d^13*c^3 + 429*x^5*e^4*d^9*c^2*b + 198*x^7*e^6*d^5*c*b^2 + x^9*e^8*d*b^3 +
198*x^7*e^6*d^5*c^2*a + 6*x^9*e^8*d*c*b*a + 15/2*x^2*e*d^14*c^3 + 429/2*x^4
*e^3*d^10*c^2*b + 231*x^6*e^5*d^6*c*b^2 + 9/2*x^8*e^7*d^2*b^3 + 231*x^6*e^5
*d^6*c^2*a + 27*x^8*e^7*d^2*c*b*a + x*d^15*c^3 + 78*x^3*e^2*d^11*c^2*b + 19
8*x^5*e^4*d^7*c*b^2 + 12*x^7*e^6*d^3*b^3 + 198*x^5*e^4*d^7*c^2*a + 72*x^7*e
^6*d^3*c*b*a + 39/2*x^2*e*d^12*c^2*b + 495/4*x^4*e^3*d^8*c*b^2 + 21*x^6*e^5
*d^4*b^3 + 495/4*x^4*e^3*d^8*c^2*a + 126*x^6*e^5*d^4*c*b*a + 3/8*x^8*e^7*b^
2*a + 3/8*x^8*e^7*c*a^2 + 3*x*d^13*c^2*b + 55*x^3*e^2*d^9*c*b^2 + 126/5*x^5
*e^4*d^5*b^3 + 55*x^3*e^2*d^9*c^2*a + 756/5*x^5*e^4*d^5*c*b*a + 3*x^7*e^6*d
*b^2*a + 3*x^7*e^6*d*c*a^2 + 33/2*x^2*e*d^10*c*b^2 + 21*x^4*e^3*d^6*b^3 + 3
3/2*x^2*e*d^10*c^2*a + 126*x^4*e^3*d^6*c*b*a + 21/2*x^6*e^5*d^2*b^2*a + 21/
2*x^6*e^5*d^2*c*a^2 + 3*x*d^11*c*b^2 + 12*x^3*e^2*d^7*b^3 + 3*x*d^11*c^2*a
+ 72*x^3*e^2*d^7*c*b*a + 21*x^5*e^4*d^3*b^2*a + 21*x^5*e^4*d^3*c*a^2 + 9/2*
```

```
x^2*e*d^8*b^3 + 27*x^2*e*d^8*c*b*a + 105/4*x^4*e^3*d^4*b^2*a + 105/4*x^4*e^
3*d^4*c*a^2 + 1/2*x^6*e^5*b*a^2 + x*d^9*b^3 + 6*x*d^9*c*b*a + 21*x^3*e^2*d^
5*b^2*a + 21*x^3*e^2*d^5*c*a^2 + 3*x^5*e^4*d*b*a^2 + 21/2*x^2*e*d^6*b^2*a +
 21/2*x^2*e*d^6*c*a^2 + 15/2*x^4*e^3*d^2*b*a^2 + 3*x*d^7*b^2*a + 3*x*d^7*c*
a^2 + 10*x^3*e^2*d^3*b*a^2 + 15/2*x^2*e*d^4*b*a^2 + 1/4*x^4*e^3*a^3 + 3*x*d
^5*b*a^2 + x^3*e^2*d*a^3 + 3/2*x^2*e*d^2*a^3 + x*d^3*a^3
```

Sympy [B] time = 0.2887, size = 1314, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] c**3*d*e**14*x**15 + c**3*e**15*x**16/16 + x**14*(3*b*c**2*e**13/14 + 15*c*
*3*d**2*e**13/2) + x**13*(3*b*c**2*d*e**12 + 35*c**3*d**3*e**12) + x**12*(a
*c**2*e**11/4 + b**2*c*e**11/4 + 39*b*c**2*d**2*e**11/2 + 455*c**3*d**4*e**
11/4) + x**11*(3*a*c**2*d*e**10 + 3*b**2*c*d*e**10 + 78*b*c**2*d**3*e**10 +
 273*c**3*d**5*e**10) + x**10*(3*a*b*c*e**9/5 + 33*a*c**2*d**2*e**9/2 + b**
3*e**9/10 + 33*b**2*c*d**2*e**9/2 + 429*b*c**2*d**4*e**9/2 + 1001*c**3*d**6
*e**9/2) + x**9*(6*a*b*c*d*e**8 + 55*a*c**2*d**3*e**8 + b**3*d*e**8 + 55*b*
*2*c*d**3*e**8 + 429*b*c**2*d**5*e**8 + 715*c**3*d**7*e**8) + x**8*(3*a**2*
c*e**7/8 + 3*a*b**2*e**7/8 + 27*a*b*c*d**2*e**7 + 495*a*c**2*d**4*e**7/4 +
9*b**3*d**2*e**7/2 + 495*b**2*c*d**4*e**7/4 + 1287*b*c**2*d**6*e**7/2 + 643
5*c**3*d**8*e**7/8) + x**7*(3*a**2*c*d*e**6 + 3*a*b**2*d*e**6 + 72*a*b*c*d*
*3*e**6 + 198*a*c**2*d**5*e**6 + 12*b**3*d**3*e**6 + 198*b**2*c*d**5*e**6 +
 5148*b*c**2*d**7*e**6/7 + 715*c**3*d**9*e**6) + x**6*(a**2*b*e**5/2 + 21*a
**2*c*d**2*e**5/2 + 21*a*b**2*d**2*e**5/2 + 126*a*b*c*d**4*e**5 + 231*a*c**
2*d**6*e**5 + 21*b**3*d**4*e**5 + 231*b**2*c*d**6*e**5 + 1287*b*c**2*d**8*e
**5/2 + 1001*c**3*d**10*e**5/2) + x**5*(3*a**2*b*d*e**4 + 21*a**2*c*d**3*e*
*4 + 21*a*b**2*d**3*e**4 + 756*a*b*c*d**5*e**4/5 + 198*a*c**2*d**7*e**4 + 1
26*b**3*d**5*e**4/5 + 198*b**2*c*d**7*e**4 + 429*b*c**2*d**9*e**4 + 273*c**
3*d**11*e**4) + x**4*(a**3*e**3/4 + 15*a**2*b*d**2*e**3/2 + 105*a**2*c*d**4
*e**3/4 + 105*a*b**2*d**4*e**3/4 + 126*a*b*c*d**6*e**3 + 495*a*c**2*d**8*e*
*3/4 + 21*b**3*d**6*e**3 + 495*b**2*c*d**8*e**3/4 + 429*b*c**2*d**10*e**3/2
 + 455*c**3*d**12*e**3/4) + x**3*(a**3*d*e**2 + 10*a**2*b*d**3*e**2 + 21*a*
*2*c*d**5*e**2 + 21*a*b**2*d**5*e**2 + 72*a*b*c*d**7*e**2 + 55*a*c**2*d**9*
e**2 + 12*b**3*d**7*e**2 + 55*b**2*c*d**9*e**2 + 78*b*c**2*d**11*e**2 + 35*
c**3*d**13*e**2) + x**2*(3*a**3*d**2*e/2 + 15*a**2*b*d**4*e/2 + 21*a**2*c*d
**6*e/2 + 21*a*b**2*d**6*e/2 + 27*a*b*c*d**8*e + 33*a*c**2*d**10*e/2 + 9*b*
*3*d**8*e/2 + 33*b**2*c*d**10*e/2 + 39*b*c**2*d**12*e/2 + 15*c**3*d**14*e/2
) + x*(a**3*d**3 + 3*a**2*b*d**5 + 3*a**2*c*d**7 + 3*a*b**2*d**7 + 6*a*b*c*
d**9 + 3*a*c**2*d**11 + b**3*d**9 + 3*b**2*c*d**11 + 3*b*c**2*d**13 + c**3*
d**15)
```

Giac [B] time = 1.11603, size = 1708, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 1/16*c^3*x^16*e^15 + c^3*d*x^15*e^14 + 15/2*c^3*d^2*x^14*e^13 + 35*c^3*d^3*
x^13*e^12 + 455/4*c^3*d^4*x^12*e^11 + 273*c^3*d^5*x^11*e^10 + 1001/2*c^3*d^
6*x^10*e^9 + 715*c^3*d^7*x^9*e^8 + 6435/8*c^3*d^8*x^8*e^7 + 715*c^3*d^9*x^7
*e^6 + 1001/2*c^3*d^10*x^6*e^5 + 273*c^3*d^11*x^5*e^4 + 455/4*c^3*d^12*x^4*
e^3 + 35*c^3*d^13*x^3*e^2 + 15/2*c^3*d^14*x^2*e + c^3*d^15*x + 3/14*b*c^2*x
^14*e^13 + 3*b*c^2*d*x^13*e^12 + 39/2*b*c^2*d^2*x^12*e^11 + 78*b*c^2*d^3*x^
11*e^10 + 429/2*b*c^2*d^4*x^10*e^9 + 429*b*c^2*d^5*x^9*e^8 + 1287/2*b*c^2*d
^6*x^8*e^7 + 5148/7*b*c^2*d^7*x^7*e^6 + 1287/2*b*c^2*d^8*x^6*e^5 + 429*b*c^
2*d^9*x^5*e^4 + 429/2*b*c^2*d^10*x^4*e^3 + 78*b*c^2*d^11*x^3*e^2 + 39/2*b*c
^2*d^12*x^2*e + 3*b*c^2*d^13*x + 1/4*b^2*c*x^12*e^11 + 1/4*a*c^2*x^12*e^11
+ 3*b^2*c*d*x^11*e^10 + 3*a*c^2*d*x^11*e^10 + 33/2*b^2*c*d^2*x^10*e^9 + 33/
2*a*c^2*d^2*x^10*e^9 + 55*b^2*c*d^3*x^9*e^8 + 55*a*c^2*d^3*x^9*e^8 + 495/4*
b^2*c*d^4*x^8*e^7 + 495/4*a*c^2*d^4*x^8*e^7 + 198*b^2*c*d^5*x^7*e^6 + 198*a
*c^2*d^5*x^7*e^6 + 231*b^2*c*d^6*x^6*e^5 + 231*a*c^2*d^6*x^6*e^5 + 198*b^2*
c*d^7*x^5*e^4 + 198*a*c^2*d^7*x^5*e^4 + 495/4*b^2*c*d^8*x^4*e^3 + 495/4*a*c
^2*d^8*x^4*e^3 + 55*b^2*c*d^9*x^3*e^2 + 55*a*c^2*d^9*x^3*e^2 + 33/2*b^2*c*d
^10*x^2*e + 33/2*a*c^2*d^10*x^2*e + 3*b^2*c*d^11*x + 3*a*c^2*d^11*x + 1/10*
b^3*x^10*e^9 + 3/5*a*b*c*x^10*e^9 + b^3*d*x^9*e^8 + 6*a*b*c*d*x^9*e^8 + 9/2
*b^3*d^2*x^8*e^7 + 27*a*b*c*d^2*x^8*e^7 + 12*b^3*d^3*x^7*e^6 + 72*a*b*c*d^3
*x^7*e^6 + 21*b^3*d^4*x^6*e^5 + 126*a*b*c*d^4*x^6*e^5 + 126/5*b^3*d^5*x^5*e
^4 + 756/5*a*b*c*d^5*x^5*e^4 + 21*b^3*d^6*x^4*e^3 + 126*a*b*c*d^6*x^4*e^3 +
 12*b^3*d^7*x^3*e^2 + 72*a*b*c*d^7*x^3*e^2 + 9/2*b^3*d^8*x^2*e + 27*a*b*c*d
^8*x^2*e + b^3*d^9*x + 6*a*b*c*d^9*x + 3/8*a*b^2*x^8*e^7 + 3/8*a^2*c*x^8*e^
7 + 3*a*b^2*d*x^7*e^6 + 3*a^2*c*d*x^7*e^6 + 21/2*a*b^2*d^2*x^6*e^5 + 21/2*a
^2*c*d^2*x^6*e^5 + 21*a*b^2*d^3*x^5*e^4 + 21*a^2*c*d^3*x^5*e^4 + 105/4*a*b^
2*d^4*x^4*e^3 + 105/4*a^2*c*d^4*x^4*e^3 + 21*a*b^2*d^5*x^3*e^2 + 21*a^2*c*d
^5*x^3*e^2 + 21/2*a*b^2*d^6*x^2*e + 21/2*a^2*c*d^6*x^2*e + 3*a*b^2*d^7*x +
3*a^2*c*d^7*x + 1/2*a^2*b*x^6*e^5 + 3*a^2*b*d*x^5*e^4 + 15/2*a^2*b*d^2*x^4*
e^3 + 10*a^2*b*d^3*x^3*e^2 + 15/2*a^2*b*d^4*x^2*e + 3*a^2*b*d^5*x + 1/4*a^3
*x^4*e^3 + a^3*d*x^3*e^2 + 3/2*a^3*d^2*x^2*e + a^3*d^3*x
```

3.610 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$

Optimal. Leaf size=55

$$\frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

```
[Out] (a*f^3*(d + e*x)^4)/(4*e) + (b*f^3*(d + e*x)^6)/(6*e) + (c*f^3*(d + e*x)^8)
/(8*e)
```

Rubi [A] time = 0.0530198, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1142, 14}

$$\frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (a*f^3*(d + e*x)^4)/(4*e) + (b*f^3*(d + e*x)^6)/(6*e) + (c*f^3*(d + e*x)^8)
/(8*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx &= \frac{f^3 \operatorname{Subst}\left(\int x^3 \left(a+bx^2+cx^4\right) dx, x, d+ex\right)}{e}\\ &= \frac{f^3 \operatorname{Subst}\left(\int \left(ax^3+bx^5+cx^7\right) dx, x, d+ex\right)}{e}\\ &= \frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}\end{aligned}$$

Mathematica [B] time = 0.0213215, size = 154, normalized size = 2.8

$$f^3 \left(\frac{1}{4}e^3x^4\left(a+10bd^2+35cd^4\right)+\frac{1}{3}de^2x^3\left(3a+10bd^2+21cd^4\right)+\frac{1}{2}d^2ex^2\left(3a+5bd^2+7cd^4\right)+d^3x\left(a+bd^2+cd^4\right)+\frac{1}{6}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 +
(d*(3*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*
x^4)/4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7
 + (c*e^7*x^8)/8)
```

Maple [B] time = 0., size = 349, normalized size = 6.4

$$\frac{e^7 f^3 c x^8}{8} + d f^3 e^6 c x^7 + \frac{\left(15 d^2 f^3 e^5 c + e^3 f^3 \left(6 c d^2 e^2 + b e^2\right)\right) x^6}{6} + \frac{\left(13 d^3 f^3 c e^4 + 3 d f^3 e^2 \left(6 c d^2 e^2 + b e^2\right) + e^3 f^3 \left(4 c d^3 e + 2 b\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/8*e^7*f^3*c*x^8+d*f^3*e^6*c*x^7+1/6*(15*d^2*f^3*e^5*c+e^3*f^3*(6*c*d^2*e^
2+b*e^2))*x^6+1/5*(13*d^3*f^3*c*e^4+3*d*f^3*e^2*(6*c*d^2*e^2+b*e^2)+e^3*f^3
*(4*c*d^3*e+2*b*d*e))*x^5+1/4*(4*d^4*f^3*c*e^3+3*d^2*f^3*e*(6*c*d^2*e^2+b*e
^2)+3*d*f^3*e^2*(4*c*d^3*e+2*b*d*e)+e^3*f^3*(c*d^4+b*d^2+a))*x^4+1/3*(d^3*f
^3*(6*c*d^2*e^2+b*e^2)+3*d^2*f^3*e*(4*c*d^3*e+2*b*d*e)+3*d*f^3*e^2*(c*d^4+b
*d^2+a))*x^3+1/2*(d^3*f^3*(4*c*d^3*e+2*b*d*e)+3*d^2*f^3*e*(c*d^4+b*d^2+a))*
x^2+d^3*f^3*(c*d^4+b*d^2+a)*x
```

Maxima [B] time = 1.0143, size = 224, normalized size = 4.07

$$\frac{1}{8} c e^7 f^3 x^8 + c d e^6 f^3 x^7 + \frac{1}{6} \left(21 c d^2 + b\right) e^5 f^3 x^6 + \left(7 c d^3 + b d\right) e^4 f^3 x^5 + \frac{1}{4} \left(35 c d^4 + 10 b d^2 + a\right) e^3 f^3 x^4 + \frac{1}{3} \left(21 c d^5 + 10 b d^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] 1/8*c*e^7*f^3*x^8 + c*d*e^6*f^3*x^7 + 1/6*(21*c*d^2 + b)*e^5*f^3*x^6 + (7*c
*d^3 + b*d)*e^4*f^3*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*f^3*x^4 + 1/3*(
21*c*d^5 + 10*b*d^3 + 3*a*d)*e^2*f^3*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2
)*e*f^3*x^2 + (c*d^7 + b*d^5 + a*d^3)*f^3*x
```

Fricas [B] time = 1.45419, size = 490, normalized size = 8.91

$$\frac{1}{8} x^8 f^3 e^7 c + x^7 f^3 e^6 d c + \frac{7}{2} x^6 f^3 e^5 d^2 c + 7 x^5 f^3 e^4 d^3 c + \frac{35}{4} x^4 f^3 e^3 d^4 c + \frac{1}{6} x^6 f^3 e^5 b + 7 x^3 f^3 e^2 d^5 c + x^5 f^3 e^4 d b + \frac{7}{2} x^2 f^3 e d^6 c +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*f^3*e^7*c + x^7*f^3*e^6*d*c + 7/2*x^6*f^3*e^5*d^2*c + 7*x^5*f^3*e^4
*d^3*c + 35/4*x^4*f^3*e^3*d^4*c + 1/6*x^6*f^3*e^5*b + 7*x^3*f^3*e^2*d^5*c +
 x^5*f^3*e^4*d*b + 7/2*x^2*f^3*e*d^6*c + 5/2*x^4*f^3*e^3*d^2*b + x*f^3*d^7*
c + 10/3*x^3*f^3*e^2*d^3*b + 5/2*x^2*f^3*e*d^4*b + 1/4*x^4*f^3*e^3*a + x*f^
3*d^5*b + x^3*f^3*e^2*d*a + 3/2*x^2*f^3*e*d^2*a + x*f^3*d^3*a
```

Sympy [B] time = 0.100178, size = 240, normalized size = 4.36

$$cde^6f^3x^7 + \frac{ce^7f^3x^8}{8} + x^6\left(\frac{be^5f^3}{6} + \frac{7cd^2e^5f^3}{2}\right) + x^5\left(bde^4f^3 + 7cd^3e^4f^3\right) + x^4\left(\frac{ae^3f^3}{4} + \frac{5bd^2e^3f^3}{2} + \frac{35cd^4e^3f^3}{4}\right) + x^3\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*f**3*x**7 + c*e**7*f**3*x**8/8 + x**6*(b*e**5*f**3/6 + 7*c*d**2*e*
*5*f**3/2) + x**5*(b*d*e**4*f**3 + 7*c*d**3*e**4*f**3) + x**4*(a*e**3*f**3/
4 + 5*b*d**2*e**3*f**3/2 + 35*c*d**4*e**3*f**3/4) + x**3*(a*d*e**2*f**3 + 1
0*b*d**3*e**2*f**3/3 + 7*c*d**5*e**2*f**3) + x**2*(3*a*d**2*e*f**3/2 + 5*b*
d**4*e*f**3/2 + 7*c*d**6*e*f**3/2) + x*(a*d**3*f**3 + b*d**5*f**3 + c*d**7*
f**3)
```

Giac [B] time = 1.09214, size = 297, normalized size = 5.4

$$\frac{1}{8}cf^3x^8e^7 + cdf^3x^7e^6 + \frac{7}{2}cd^2f^3x^6e^5 + 7cd^3f^3x^5e^4 + \frac{35}{4}cd^4f^3x^4e^3 + 7cd^5f^3x^3e^2 + \frac{7}{2}cd^6f^3x^2e + cd^7f^3x + \frac{1}{6}bf^3x^6e^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/8*c*f^3*x^8*e^7 + c*d*f^3*x^7*e^6 + 7/2*c*d^2*f^3*x^6*e^5 + 7*c*d^3*f^3*x
^5*e^4 + 35/4*c*d^4*f^3*x^4*e^3 + 7*c*d^5*f^3*x^3*e^2 + 7/2*c*d^6*f^3*x^2*e
 + c*d^7*f^3*x + 1/6*b*f^3*x^6*e^5 + b*d*f^3*x^5*e^4 + 5/2*b*d^2*f^3*x^4*e^
3 + 10/3*b*d^3*f^3*x^3*e^2 + 5/2*b*d^4*f^3*x^2*e + b*d^5*f^3*x + 1/4*a*f^3*
x^4*e^3 + a*d*f^3*x^3*e^2 + 3/2*a*d^2*f^3*x^2*e + a*d^3*f^3*x
```

3.611 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

Optimal. Leaf size=104

$$\frac{a^2f^3(d+ex)^4}{4e}+\frac{f^3\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{abf^3(d+ex)^6}{3e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}$$

```
[Out] (a^2*f^3*(d + e*x)^4)/(4*e) + (a*b*f^3*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*
f^3*(d + e*x)^8)/(8*e) + (b*c*f^3*(d + e*x)^10)/(5*e) + (c^2*f^3*(d + e*x)^
12)/(12*e)
```

Rubi [A] time = 0.164176, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1142, 1114, 631}

$$\frac{a^2f^3(d+ex)^4}{4e}+\frac{f^3\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{abf^3(d+ex)^6}{3e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (a^2*f^3*(d + e*x)^4)/(4*e) + (a*b*f^3*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*
f^3*(d + e*x)^8)/(8*e) + (b*c*f^3*(d + e*x)^10)/(5*e) + (c^2*f^3*(d + e*x)^
12)/(12*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx = \frac{f^3\,\mathrm{Subst}\left(\int x^3\left(a+bx^2+cx^4\right)^2\,dx,x,d+ex\right)}{e}$$
$$= \frac{f^3\,\mathrm{Subst}\left(\int x\left(a+bx+cx^2\right)^2\,dx,x,(d+ex)^2\right)}{2e}$$
$$= \frac{f^3\,\mathrm{Subst}\left(\int \left(a^2x+2abx^2+\left(b^2+2ac\right)x^3+2bcx^4+c^2x^5\right)\,dx,x,(d$$
$$= \frac{a^2f^3(d+ex)^4}{4e}+\frac{abf^3(d+ex)^6}{3e}+\frac{\left(b^2+2ac\right)f^3(d+ex)^8}{8e}+\frac{bcf^3(d}{5$$

Mathematica [B] time = 0.0758172, size = 405, normalized size = 3.89

$$f^3\left(\frac{1}{4}e^3x^4\left(a^2+20abd^2+70acd^4+35b^2d^4+168bcd^6+165c^2d^8\right)+\frac{1}{3}de^2x^3\left(3a^2+20abd^2+42acd^4+21b^2d^4+72bcd\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 1
4*a*c*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21
*b^2*d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20*a*
b*d^2 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (
d*(10*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5
 + ((2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)/
6 + d*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 72*
b*c*d^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/3 + (c*(
2*b + 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12)
```

Maple [B] time = 0.001, size = 1413, normalized size = 13.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 1/12*e^11*f^3*c^2*x^12+d*f^3*e^10*c^2*x^11+1/10*(27*d^2*f^3*e^9*c^2+e^3*f^3
*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6))*x^10+1/9*(25*d^3*f^3*c^2*e^8
+3*d*f^3*e^2*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+e^3*f^3*(2*(4*c*d
^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3))*x^9+1/8*(8*d^4*f^3*c^2*
e^7+3*d^2*f^3*e*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+3*d*f^3*e^2*(2
*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3)+e^3*f^3*(2*(c*d^4
+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2))*x^8+1
/7*(d^3*f^3*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+3*d^2*f^3*e*(2*(4*
c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3)+3*d*f^3*e^2*(2*(c*d^4
+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2)+e^3*f^
3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2)))*x^
7+1/6*(d^3*f^3*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3)+
3*d^2*f^3*e*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2
*e^2+b*e^2)^2)+3*d*f^3*e^2*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)
*(6*c*d^2*e^2+b*e^2))+e^3*f^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d
^3*e+2*b*d*e)^2))*x^6+1/5*(d^3*f^3*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*
```

```
b*d*e)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2)+3*d^2*f^3*e*(8*(c*d^4+b*d^2+a)*c*d*e^
3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d*f^3*e^2*(2*(c*d^4+b*d^2+a)
*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+2*e^3*f^3*(c*d^4+b*d^2+a)*(4*c*
d^3*e+2*b*d*e))*x^5+1/4*(d^3*f^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*
b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d^2*f^3*e*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e
^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d*f^3*e^2*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+
e^3*f^3*(c*d^4+b*d^2+a)^2)*x^4+1/3*(d^3*f^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2
+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d^2*f^3*e*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d
*e)+3*d*f^3*e^2*(c*d^4+b*d^2+a)^2)*x^3+1/2*(2*d^3*f^3*(c*d^4+b*d^2+a)*(4*c*
d^3*e+2*b*d*e)+3*d^2*f^3*e*(c*d^4+b*d^2+a)^2)*x^2+d^3*f^3*(c*d^4+b*d^2+a)^2
*x
```

Maxima [B] time = 0.982613, size = 593, normalized size = 5.7

$$\frac{1}{12}c^2e^{11}f^3x^{12}+c^2de^{10}f^3x^{11}+\frac{1}{10}\left(55\,c^2d^2+2\,bc\right)e^9f^3x^{10}+\frac{1}{3}\left(55\,c^2d^3+6\,bcd\right)e^8f^3x^9+\frac{1}{8}\left(330\,c^2d^4+72\,bcd^2+b^2+2\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] 1/12*c^2*e^11*f^3*x^12 + c^2*d*e^10*f^3*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^
9*f^3*x^10 + 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*f^3*x^9 + 1/8*(330*c^2*d^4 + 72
*b*c*d^2 + b^2 + 2*a*c)*e^7*f^3*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a
*c)*d)*e^6*f^3*x^7 + 1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2
+ 2*a*b)*e^5*f^3*x^6 + 1/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^
3 + 10*a*b*d)*e^4*f^3*x^5 + 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*
c)*d^4 + 20*a*b*d^2 + a^2)*e^3*f^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*
(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*e^2*f^3*x^3 + 1/2*(11*c^2*d^10 +
18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*e*f^3*x^2 + (c^2
*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*f^3*x
```

Fricas [B] time = 1.55417, size = 1517, normalized size = 14.59

$$\frac{1}{12}x^{12}f^3e^{11}c^2+x^{11}f^3e^{10}dc^2+\frac{11}{2}x^{10}f^3e^9d^2c^2+\frac{55}{3}x^9f^3e^8d^3c^2+\frac{165}{4}x^8f^3e^7d^4c^2+\frac{1}{5}x^{10}f^3e^9cb+66x^7f^3e^6d^5c^2+2x^9f^3e^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] 1/12*x^12*f^3*e^11*c^2 + x^11*f^3*e^10*d*c^2 + 11/2*x^10*f^3*e^9*d^2*c^2 +
55/3*x^9*f^3*e^8*d^3*c^2 + 165/4*x^8*f^3*e^7*d^4*c^2 + 1/5*x^10*f^3*e^9*c*b
 + 66*x^7*f^3*e^6*d^5*c^2 + 2*x^9*f^3*e^8*d*c*b + 77*x^6*f^3*e^5*d^6*c^2 +
9*x^8*f^3*e^7*d^2*c*b + 66*x^5*f^3*e^4*d^7*c^2 + 24*x^7*f^3*e^6*d^3*c*b + 1
65/4*x^4*f^3*e^3*d^8*c^2 + 42*x^6*f^3*e^5*d^4*c*b + 1/8*x^8*f^3*e^7*b^2 + 1
/4*x^8*f^3*e^7*c*a + 55/3*x^3*f^3*e^2*d^9*c^2 + 252/5*x^5*f^3*e^4*d^5*c*b +
 x^7*f^3*e^6*d*b^2 + 2*x^7*f^3*e^6*d*c*a + 11/2*x^2*f^3*e*d^10*c^2 + 42*x^4
*f^3*e^3*d^6*c*b + 7/2*x^6*f^3*e^5*d^2*b^2 + 7*x^6*f^3*e^5*d^2*c*a + x*f^3*
d^11*c^2 + 24*x^3*f^3*e^2*d^7*c*b + 7*x^5*f^3*e^4*d^3*b^2 + 14*x^5*f^3*e^4*
d^3*c*a + 9*x^2*f^3*e*d^8*c*b + 35/4*x^4*f^3*e^3*d^4*b^2 + 35/2*x^4*f^3*e^3
*d^4*c*a + 1/3*x^6*f^3*e^5*b*a + 2*x*f^3*d^9*c*b + 7*x^3*f^3*e^2*d^5*b^2 +
14*x^3*f^3*e^2*d^5*c*a + 2*x^5*f^3*e^4*d*b*a + 7/2*x^2*f^3*e*d^6*b^2 + 7*x^
```

```
2*f^3*e*d^6*c*a + 5*x^4*f^3*e^3*d^2*b*a + x*f^3*d^7*b^2 + 2*x*f^3*d^7*c*a +
 20/3*x^3*f^3*e^2*d^3*b*a + 5*x^2*f^3*e*d^4*b*a + 1/4*x^4*f^3*e^3*a^2 + 2*x
*f^3*d^5*b*a + x^3*f^3*e^2*d*a^2 + 3/2*x^2*f^3*e*d^2*a^2 + x*f^3*d^3*a^2
```

Sympy [B] time = 0.182221, size = 722, normalized size = 6.94

$$c^2de^{10}f^3x^{11} + \frac{c^2e^{11}f^3x^{12}}{12} + x^{10}\left(\frac{bce^9f^3}{5} + \frac{11c^2d^2e^9f^3}{2}\right) + x^9\left(2bcde^8f^3 + \frac{55c^2d^3e^8f^3}{3}\right) + x^8\left(\frac{ace^7f^3}{4} + \frac{b^2e^7f^3}{8} + 9bcd\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] c**2*d*e**10*f**3*x**11 + c**2*e**11*f**3*x**12/12 + x**10*(b*c*e**9*f**3/5
 + 11*c**2*d**2*e**9*f**3/2) + x**9*(2*b*c*d*e**8*f**3 + 55*c**2*d**3*e**8*
f**3/3) + x**8*(a*c*e**7*f**3/4 + b**2*e**7*f**3/8 + 9*b*c*d**2*e**7*f**3 +
 165*c**2*d**4*e**7*f**3/4) + x**7*(2*a*c*d*e**6*f**3 + b**2*d*e**6*f**3 +
24*b*c*d**3*e**6*f**3 + 66*c**2*d**5*e**6*f**3) + x**6*(a*b*e**5*f**3/3 + 7
*a*c*d**2*e**5*f**3 + 7*b**2*d**2*e**5*f**3/2 + 42*b*c*d**4*e**5*f**3 + 77*
c**2*d**6*e**5*f**3) + x**5*(2*a*b*d*e**4*f**3 + 14*a*c*d**3*e**4*f**3 + 7*
b**2*d**3*e**4*f**3 + 252*b*c*d**5*e**4*f**3/5 + 66*c**2*d**7*e**4*f**3) +
x**4*(a**2*e**3*f**3/4 + 5*a*b*d**2*e**3*f**3 + 35*a*c*d**4*e**3*f**3/2 + 3
5*b**2*d**4*e**3*f**3/4 + 42*b*c*d**6*e**3*f**3 + 165*c**2*d**8*e**3*f**3/4
) + x**3*(a**2*d*e**2*f**3 + 20*a*b*d**3*e**2*f**3/3 + 14*a*c*d**5*e**2*f**
3 + 7*b**2*d**5*e**2*f**3 + 24*b*c*d**7*e**2*f**3 + 55*c**2*d**9*e**2*f**3/
3) + x**2*(3*a**2*d**2*e*f**3/2 + 5*a*b*d**4*e*f**3 + 7*a*c*d**6*e*f**3 + 7
*b**2*d**6*e*f**3/2 + 9*b*c*d**8*e*f**3 + 11*c**2*d**10*e*f**3/2) + x*(a**2
*d**3*f**3 + 2*a*b*d**5*f**3 + 2*a*c*d**7*f**3 + b**2*d**7*f**3 + 2*b*c*d**
9*f**3 + c**2*d**11*f**3)
```

Giac [B] time = 1.1143, size = 925, normalized size = 8.89

$$\frac{1}{12}c^2f^3x^{12}e^{11} + c^2df^3x^{11}e^{10} + \frac{11}{2}c^2d^2f^3x^{10}e^9 + \frac{55}{3}c^2d^3f^3x^9e^8 + \frac{165}{4}c^2d^4f^3x^8e^7 + 66\,c^2d^5f^3x^7e^6 + 77\,c^2d^6f^3x^6e^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/12*c^2*f^3*x^12*e^11 + c^2*d*f^3*x^11*e^10 + 11/2*c^2*d^2*f^3*x^10*e^9 +
55/3*c^2*d^3*f^3*x^9*e^8 + 165/4*c^2*d^4*f^3*x^8*e^7 + 66*c^2*d^5*f^3*x^7*e
^6 + 77*c^2*d^6*f^3*x^6*e^5 + 66*c^2*d^7*f^3*x^5*e^4 + 165/4*c^2*d^8*f^3*x^
4*e^3 + 55/3*c^2*d^9*f^3*x^3*e^2 + 11/2*c^2*d^10*f^3*x^2*e + c^2*d^11*f^3*x
 + 1/5*b*c*f^3*x^10*e^9 + 2*b*c*d*f^3*x^9*e^8 + 9*b*c*d^2*f^3*x^8*e^7 + 24*
b*c*d^3*f^3*x^7*e^6 + 42*b*c*d^4*f^3*x^6*e^5 + 252/5*b*c*d^5*f^3*x^5*e^4 +
42*b*c*d^6*f^3*x^4*e^3 + 24*b*c*d^7*f^3*x^3*e^2 + 9*b*c*d^8*f^3*x^2*e + 2*b
*c*d^9*f^3*x + 1/8*b^2*f^3*x^8*e^7 + 1/4*a*c*f^3*x^8*e^7 + b^2*d*f^3*x^7*e^
6 + 2*a*c*d*f^3*x^7*e^6 + 7/2*b^2*d^2*f^3*x^6*e^5 + 7*a*c*d^2*f^3*x^6*e^5 +
 7*b^2*d^3*f^3*x^5*e^4 + 14*a*c*d^3*f^3*x^5*e^4 + 35/4*b^2*d^4*f^3*x^4*e^3
+ 35/2*a*c*d^4*f^3*x^4*e^3 + 7*b^2*d^5*f^3*x^3*e^2 + 14*a*c*d^5*f^3*x^3*e^2
 + 7/2*b^2*d^6*f^3*x^2*e + 7*a*c*d^6*f^3*x^2*e + b^2*d^7*f^3*x + 2*a*c*d^7*
f^3*x + 1/3*a*b*f^3*x^6*e^5 + 2*a*b*d*f^3*x^5*e^4 + 5*a*b*d^2*f^3*x^4*e^3 +
 20/3*a*b*d^3*f^3*x^3*e^2 + 5*a*b*d^4*f^3*x^2*e + 2*a*b*d^5*f^3*x + 1/4*a^2
*f^3*x^4*e^3 + a^2*d*f^3*x^3*e^2 + 3/2*a^2*d^2*f^3*x^2*e + a^2*d^3*f^3*x
```

3.612 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

Optimal. Leaf size=159

$$\frac{a^2bf^3(d+ex)^6}{2e}+\frac{a^3f^3(d+ex)^4}{4e}+\frac{cf^3\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{bf^3\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3af^3\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2}{}$$

```
[Out] (a^3*f^3*(d + e*x)^4)/(4*e) + (a^2*b*f^3*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a
*c)*f^3*(d + e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*f^3*(d + e*x)^10)/(10*e) + (c
*(b^2 + a*c)*f^3*(d + e*x)^12)/(4*e) + (3*b*c^2*f^3*(d + e*x)^14)/(14*e) +
(c^3*f^3*(d + e*x)^16)/(16*e)
```

Rubi [A] time = 0.315301, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1142, 1114, 631}

$$\frac{a^2bf^3(d+ex)^6}{2e}+\frac{a^3f^3(d+ex)^4}{4e}+\frac{cf^3\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{bf^3\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3af^3\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (a^3*f^3*(d + e*x)^4)/(4*e) + (a^2*b*f^3*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a
*c)*f^3*(d + e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*f^3*(d + e*x)^10)/(10*e) + (c
*(b^2 + a*c)*f^3*(d + e*x)^12)/(4*e) + (3*b*c^2*f^3*(d + e*x)^14)/(14*e) +
(c^3*f^3*(d + e*x)^16)/(16*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx = \frac{f^3\,\text{Subst}\left(\int x^3\left(a+bx^2+cx^4\right)^3\,dx, x, d+ex\right)}{e}$$
$$= \frac{f^3\,\text{Subst}\left(\int x\left(a+bx+cx^2\right)^3\,dx, x, (d+ex)^2\right)}{2e}$$
$$= \frac{f^3\,\text{Subst}\left(\int \left(a^3x+3a^2bx^2+3a\left(b^2+ac\right)x^3+b\left(b^2+6ac\right)x^4+3c\right.\right.}{2e}$$
$$= \frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{3a\left(b^2+ac\right)f^3(d+ex)^8}{8e}+\frac{b\left(b^2\right.}{}$$

Mathematica [B] time = 0.0408879, size = 801, normalized size = 5.04

$$f^3\left(\frac{1}{16}c^3e^{15}x^{16}+c^3de^{14}x^{15}+\frac{3}{14}c^2\left(35cd^2+b\right)e^{13}x^{14}+c^2d\left(35cd^2+3b\right)e^{12}x^{13}+\frac{1}{4}c\left(455c^2d^4+78bcd^2+b^2+ac\right)e^{11}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^
2 + 5*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4
 + 12*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^10
+ 35*c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d
^4 + 84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2
*d^10 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*
d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^
2*d^8 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2 +
42*b^3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2*d^8
 + 1001*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a
*b*c*d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)
*e^6*x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4
 + 330*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a
*b*c + 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*x^9 +
 ((b^3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4 + 5005*c^
3*d^6)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4)*e^10*x^11
 + (c*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35
*c*d^2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 +
 (c^3*e^15*x^16)/16)
```

Maple [B] time = 0.001, size = 7697, normalized size = 48.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

Maxima [B] time = 1.00452, size = 1242, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] 1/16*c^3*e^15*f^3*x^16 + c^3*d*e^14*f^3*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^
13*f^3*x^14 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*f^3*x^13 + 1/4*(455*c^3*d^4 + 7
8*b*c^2*d^2 + b^2*c + a*c^2)*e^11*f^3*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 +
 (b^2*c + a*c^2)*d)*e^10*f^3*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b
^3 + 6*a*b*c + 165*(b^2*c + a*c^2)*d^2)*e^9*f^3*x^10 + (715*c^3*d^7 + 429*b
*c^2*d^5 + 55*(b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*f^3*x^9 + 3/8*(2
145*c^3*d^8 + 1716*b*c^2*d^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12
*(b^3 + 6*a*b*c)*d^2)*e^7*f^3*x^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 13
86*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6
*f^3*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c^2*d^8 + 462*(b^2*c + a*c^2)*d^6 +
42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b^2 + a^2*c)*d^2)*e^5*f^3*x^6 + 3/5*
(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b^3 + 6*a*b*c
)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*e^4*f^3*x^5 + 1/4*(455*c^3*d^12
 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84*(b^3 + 6*a*b*c)*d^6 + 30*a
^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*f^3*x^4 + (35*c^3*d^13 + 78*b
*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b*c)*d^7 + 10*a^2*b*d^3
+ 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*f^3*x^3 + 3/2*(5*c^3*d^14 + 13*b*c^2*
d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 + 5*a^2*b*d^4 + 7*(a
*b^2 + a^2*c)*d^6 + a^3*d^2)*e*f^3*x^2 + (c^3*d^15 + 3*b*c^2*d^13 + 3*(b^2*
c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b^2 + a^2*c)*d^7
 + a^3*d^3)*f^3*x
```

Fricas [B] time = 1.56755, size = 3555, normalized size = 22.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] 1/16*x^16*f^3*e^15*c^3 + x^15*f^3*e^14*d*c^3 + 15/2*x^14*f^3*e^13*d^2*c^3 +
 35*x^13*f^3*e^12*d^3*c^3 + 455/4*x^12*f^3*e^11*d^4*c^3 + 3/14*x^14*f^3*e^1
3*c^2*b + 273*x^11*f^3*e^10*d^5*c^3 + 3*x^13*f^3*e^12*d*c^2*b + 1001/2*x^10
*f^3*e^9*d^6*c^3 + 39/2*x^12*f^3*e^11*d^2*c^2*b + 715*x^9*f^3*e^8*d^7*c^3 +
 78*x^11*f^3*e^10*d^3*c^2*b + 6435/8*x^8*f^3*e^7*d^8*c^3 + 429/2*x^10*f^3*e
^9*d^4*c^2*b + 1/4*x^12*f^3*e^11*c*b^2 + 1/4*x^12*f^3*e^11*c^2*a + 715*x^7*
f^3*e^6*d^9*c^3 + 429*x^9*f^3*e^8*d^5*c^2*b + 3*x^11*f^3*e^10*d*c*b^2 + 3*x
^11*f^3*e^10*d*c^2*a + 1001/2*x^6*f^3*e^5*d^10*c^3 + 1287/2*x^8*f^3*e^7*d^6
*c^2*b + 33/2*x^10*f^3*e^9*d^2*c*b^2 + 33/2*x^10*f^3*e^9*d^2*c^2*a + 273*x^
5*f^3*e^4*d^11*c^3 + 5148/7*x^7*f^3*e^6*d^7*c^2*b + 55*x^9*f^3*e^8*d^3*c*b^
2 + 55*x^9*f^3*e^8*d^3*c^2*a + 455/4*x^4*f^3*e^3*d^12*c^3 + 1287/2*x^6*f^3*
e^5*d^8*c^2*b + 495/4*x^8*f^3*e^7*d^4*c*b^2 + 1/10*x^10*f^3*e^9*b^3 + 495/4
*x^8*f^3*e^7*d^4*c^2*a + 3/5*x^10*f^3*e^9*c*b*a + 35*x^3*f^3*e^2*d^13*c^3 +
 429*x^5*f^3*e^4*d^9*c^2*b + 198*x^7*f^3*e^6*d^5*c*b^2 + x^9*f^3*e^8*d*b^3
+ 198*x^7*f^3*e^6*d^5*c^2*a + 6*x^9*f^3*e^8*d*c*b*a + 15/2*x^2*f^3*e*d^14*c
^3 + 429/2*x^4*f^3*e^3*d^10*c^2*b + 231*x^6*f^3*e^5*d^6*c*b^2 + 9/2*x^8*f^3
*e^7*d^2*b^3 + 231*x^6*f^3*e^5*d^6*c^2*a + 27*x^8*f^3*e^7*d^2*c*b*a + x*f^3
*d^15*c^3 + 78*x^3*f^3*e^2*d^11*c^2*b + 198*x^5*f^3*e^4*d^7*c*b^2 + 12*x^7*
f^3*e^6*d^3*b^3 + 198*x^5*f^3*e^4*d^7*c^2*a + 72*x^7*f^3*e^6*d^3*c*b*a + 39
/2*x^2*f^3*e*d^12*c^2*b + 495/4*x^4*f^3*e^3*d^8*c*b^2 + 21*x^6*f^3*e^5*d^4*
b^3 + 495/4*x^4*f^3*e^3*d^8*c^2*a + 126*x^6*f^3*e^5*d^4*c*b*a + 3/8*x^8*f^3
```

```
*e^7*b^2*a + 3/8*x^8*f^3*e^7*c*a^2 + 3*x*f^3*d^13*c^2*b + 55*x^3*f^3*e^2*d^
9*c*b^2 + 126/5*x^5*f^3*e^4*d^5*b^3 + 55*x^3*f^3*e^2*d^9*c^2*a + 756/5*x^5*
f^3*e^4*d^5*c*b*a + 3*x^7*f^3*e^6*d*b^2*a + 3*x^7*f^3*e^6*d*c*a^2 + 33/2*x^
2*f^3*e*d^10*c*b^2 + 21*x^4*f^3*e^3*d^6*b^3 + 33/2*x^2*f^3*e*d^10*c^2*a + 1
26*x^4*f^3*e^3*d^6*c*b*a + 21/2*x^6*f^3*e^5*d^2*b^2*a + 21/2*x^6*f^3*e^5*d^
2*c*a^2 + 3*x*f^3*d^11*c*b^2 + 12*x^3*f^3*e^2*d^7*b^3 + 3*x*f^3*d^11*c^2*a
+ 72*x^3*f^3*e^2*d^7*c*b*a + 21*x^5*f^3*e^4*d^3*b^2*a + 21*x^5*f^3*e^4*d^3*
c*a^2 + 9/2*x^2*f^3*e*d^8*b^3 + 27*x^2*f^3*e*d^8*c*b*a + 105/4*x^4*f^3*e^3*
d^4*b^2*a + 105/4*x^4*f^3*e^3*d^4*c*a^2 + 1/2*x^6*f^3*e^5*b*a^2 + x*f^3*d^9
*b^3 + 6*x*f^3*d^9*c*b*a + 21*x^3*f^3*e^2*d^5*b^2*a + 21*x^3*f^3*e^2*d^5*c*
a^2 + 3*x^5*f^3*e^4*d*b*a^2 + 21/2*x^2*f^3*e*d^6*b^2*a + 21/2*x^2*f^3*e*d^6
*c*a^2 + 15/2*x^4*f^3*e^3*d^2*b*a^2 + 3*x*f^3*d^7*b^2*a + 3*x*f^3*d^7*c*a^2
 + 10*x^3*f^3*e^2*d^3*b*a^2 + 15/2*x^2*f^3*e*d^4*b*a^2 + 1/4*x^4*f^3*e^3*a^
3 + 3*x*f^3*d^5*b*a^2 + x^3*f^3*e^2*d*a^3 + 3/2*x^2*f^3*e*d^2*a^3 + x*f^3*d
^3*a^3
```

Sympy [B] time = 0.317605, size = 1654, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] c**3*d*e**14*f**3*x**15 + c**3*e**15*f**3*x**16/16 + x**14*(3*b*c**2*e**13*
f**3/14 + 15*c**3*d**2*e**13*f**3/2) + x**13*(3*b*c**2*d*e**12*f**3 + 35*c*
*3*d**3*e**12*f**3) + x**12*(a*c**2*e**11*f**3/4 + b**2*c*e**11*f**3/4 + 39
*b*c**2*d**2*e**11*f**3/2 + 455*c**3*d**4*e**11*f**3/4) + x**11*(3*a*c**2*d
*e**10*f**3 + 3*b**2*c*d*e**10*f**3 + 78*b*c**2*d**3*e**10*f**3 + 273*c**3*
d**5*e**10*f**3) + x**10*(3*a*b*c*e**9*f**3/5 + 33*a*c**2*d**2*e**9*f**3/2
+ b**3*e**9*f**3/10 + 33*b**2*c*d**2*e**9*f**3/2 + 429*b*c**2*d**4*e**9*f**
3/2 + 1001*c**3*d**6*e**9*f**3/2) + x**9*(6*a*b*c*d*e**8*f**3 + 55*a*c**2*d
**3*e**8*f**3 + b**3*d*e**8*f**3 + 55*b**2*c*d**3*e**8*f**3 + 429*b*c**2*d*
*5*e**8*f**3 + 715*c**3*d**7*e**8*f**3) + x**8*(3*a**2*c*e**7*f**3/8 + 3*a*
b**2*e**7*f**3/8 + 27*a*b*c*d**2*e**7*f**3 + 495*a*c**2*d**4*e**7*f**3/4 +
9*b**3*d**2*e**7*f**3/2 + 495*b**2*c*d**4*e**7*f**3/4 + 1287*b*c**2*d**6*e*
*7*f**3/2 + 6435*c**3*d**8*e**7*f**3/8) + x**7*(3*a**2*c*d*e**6*f**3 + 3*a*
b**2*d*e**6*f**3 + 72*a*b*c*d**3*e**6*f**3 + 198*a*c**2*d**5*e**6*f**3 + 12
*b**3*d**3*e**6*f**3 + 198*b**2*c*d**5*e**6*f**3 + 5148*b*c**2*d**7*e**6*f*
*3/7 + 715*c**3*d**9*e**6*f**3) + x**6*(a**2*b*e**5*f**3/2 + 21*a**2*c*d**2
*e**5*f**3/2 + 21*a*b**2*d**2*e**5*f**3/2 + 126*a*b*c*d**4*e**5*f**3 + 231*
a*c**2*d**6*e**5*f**3 + 21*b**3*d**4*e**5*f**3 + 231*b**2*c*d**6*e**5*f**3
+ 1287*b*c**2*d**8*e**5*f**3/2 + 1001*c**3*d**10*e**5*f**3/2) + x**5*(3*a**
2*b*d*e**4*f**3 + 21*a**2*c*d**3*e**4*f**3 + 21*a*b**2*d**3*e**4*f**3 + 756
*a*b*c*d**5*e**4*f**3/5 + 198*a*c**2*d**7*e**4*f**3 + 126*b**3*d**5*e**4*f*
*3/5 + 198*b**2*c*d**7*e**4*f**3 + 429*b*c**2*d**9*e**4*f**3 + 273*c**3*d**
11*e**4*f**3) + x**4*(a**3*e**3*f**3/4 + 15*a**2*b*d**2*e**3*f**3/2 + 105*a
**2*c*d**4*e**3*f**3/4 + 105*a*b**2*d**4*e**3*f**3/4 + 126*a*b*c*d**6*e**3*
f**3 + 495*a*c**2*d**8*e**3*f**3/4 + 21*b**3*d**6*e**3*f**3 + 495*b**2*c*d*
*8*e**3*f**3/4 + 429*b*c**2*d**10*e**3*f**3/2 + 455*c**3*d**12*e**3*f**3/4)
 + x**3*(a**3*d*e**2*f**3 + 10*a**2*b*d**3*e**2*f**3 + 21*a**2*c*d**5*e**2*
f**3 + 21*a*b**2*d**5*e**2*f**3 + 72*a*b*c*d**7*e**2*f**3 + 55*a*c**2*d**9*
e**2*f**3 + 12*b**3*d**7*e**2*f**3 + 55*b**2*c*d**9*e**2*f**3 + 78*b*c**2*d
**11*e**2*f**3 + 35*c**3*d**13*e**2*f**3) + x**2*(3*a**3*d**2*e*f**3/2 + 15
*a**2*b*d**4*e*f**3/2 + 21*a**2*c*d**6*e*f**3/2 + 21*a*b**2*d**6*e*f**3/2 +
 27*a*b*c*d**8*e*f**3 + 33*a*c**2*d**10*e*f**3/2 + 9*b**3*d**8*e*f**3/2 + 3
3*b**2*c*d**10*e*f**3/2 + 39*b*c**2*d**12*e*f**3/2 + 15*c**3*d**14*e*f**3/2
```

```
) + x*(a**3*d**3*f**3 + 3*a**2*b*d**5*f**3 + 3*a**2*c*d**7*f**3 + 3*a*b**2*
d**7*f**3 + 6*a*b*c*d**9*f**3 + 3*a*c**2*d**11*f**3 + b**3*d**9*f**3 + 3*b*
*2*c*d**11*f**3 + 3*b*c**2*d**13*f**3 + c**3*d**15*f**3)
```

Giac [B] time = 1.15179, size = 2113, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 1/16*c^3*f^3*x^16*e^15 + c^3*d*f^3*x^15*e^14 + 15/2*c^3*d^2*f^3*x^14*e^13 +
 35*c^3*d^3*f^3*x^13*e^12 + 455/4*c^3*d^4*f^3*x^12*e^11 + 273*c^3*d^5*f^3*x
^11*e^10 + 1001/2*c^3*d^6*f^3*x^10*e^9 + 715*c^3*d^7*f^3*x^9*e^8 + 6435/8*c
^3*d^8*f^3*x^8*e^7 + 715*c^3*d^9*f^3*x^7*e^6 + 1001/2*c^3*d^10*f^3*x^6*e^5
+ 273*c^3*d^11*f^3*x^5*e^4 + 455/4*c^3*d^12*f^3*x^4*e^3 + 35*c^3*d^13*f^3*x
^3*e^2 + 15/2*c^3*d^14*f^3*x^2*e + c^3*d^15*f^3*x + 3/14*b*c^2*f^3*x^14*e^1
3 + 3*b*c^2*d*f^3*x^13*e^12 + 39/2*b*c^2*d^2*f^3*x^12*e^11 + 78*b*c^2*d^3*f
^3*x^11*e^10 + 429/2*b*c^2*d^4*f^3*x^10*e^9 + 429*b*c^2*d^5*f^3*x^9*e^8 + 1
287/2*b*c^2*d^6*f^3*x^8*e^7 + 5148/7*b*c^2*d^7*f^3*x^7*e^6 + 1287/2*b*c^2*d
^8*f^3*x^6*e^5 + 429*b*c^2*d^9*f^3*x^5*e^4 + 429/2*b*c^2*d^10*f^3*x^4*e^3 +
 78*b*c^2*d^11*f^3*x^3*e^2 + 39/2*b*c^2*d^12*f^3*x^2*e + 3*b*c^2*d^13*f^3*x
 + 1/4*b^2*c*f^3*x^12*e^11 + 1/4*a*c^2*f^3*x^12*e^11 + 3*b^2*c*d*f^3*x^11*e
^10 + 3*a*c^2*d*f^3*x^11*e^10 + 33/2*b^2*c*d^2*f^3*x^10*e^9 + 33/2*a*c^2*d^
2*f^3*x^10*e^9 + 55*b^2*c*d^3*f^3*x^9*e^8 + 55*a*c^2*d^3*f^3*x^9*e^8 + 495/
4*b^2*c*d^4*f^3*x^8*e^7 + 495/4*a*c^2*d^4*f^3*x^8*e^7 + 198*b^2*c*d^5*f^3*x
^7*e^6 + 198*a*c^2*d^5*f^3*x^7*e^6 + 231*b^2*c*d^6*f^3*x^6*e^5 + 231*a*c^2*
d^6*f^3*x^6*e^5 + 198*b^2*c*d^7*f^3*x^5*e^4 + 198*a*c^2*d^7*f^3*x^5*e^4 + 4
95/4*b^2*c*d^8*f^3*x^4*e^3 + 495/4*a*c^2*d^8*f^3*x^4*e^3 + 55*b^2*c*d^9*f^3
*x^3*e^2 + 55*a*c^2*d^9*f^3*x^3*e^2 + 33/2*b^2*c*d^10*f^3*x^2*e + 33/2*a*c^
2*d^10*f^3*x^2*e + 3*b^2*c*d^11*f^3*x + 3*a*c^2*d^11*f^3*x + 1/10*b^3*f^3*x
^10*e^9 + 3/5*a*b*c*f^3*x^10*e^9 + b^3*d*f^3*x^9*e^8 + 6*a*b*c*d*f^3*x^9*e^
8 + 9/2*b^3*d^2*f^3*x^8*e^7 + 27*a*b*c*d^2*f^3*x^8*e^7 + 12*b^3*d^3*f^3*x^7
*e^6 + 72*a*b*c*d^3*f^3*x^7*e^6 + 21*b^3*d^4*f^3*x^6*e^5 + 126*a*b*c*d^4*f^
3*x^6*e^5 + 126/5*b^3*d^5*f^3*x^5*e^4 + 756/5*a*b*c*d^5*f^3*x^5*e^4 + 21*b^
3*d^6*f^3*x^4*e^3 + 126*a*b*c*d^6*f^3*x^4*e^3 + 12*b^3*d^7*f^3*x^3*e^2 + 72
*a*b*c*d^7*f^3*x^3*e^2 + 9/2*b^3*d^8*f^3*x^2*e + 27*a*b*c*d^8*f^3*x^2*e + b
^3*d^9*f^3*x + 6*a*b*c*d^9*f^3*x + 3/8*a*b^2*f^3*x^8*e^7 + 3/8*a^2*c*f^3*x^
8*e^7 + 3*a*b^2*d*f^3*x^7*e^6 + 3*a^2*c*d*f^3*x^7*e^6 + 21/2*a*b^2*d^2*f^3*
x^6*e^5 + 21/2*a^2*c*d^2*f^3*x^6*e^5 + 21*a*b^2*d^3*f^3*x^5*e^4 + 21*a^2*c*
d^3*f^3*x^5*e^4 + 105/4*a*b^2*d^4*f^3*x^4*e^3 + 105/4*a^2*c*d^4*f^3*x^4*e^3
 + 21*a*b^2*d^5*f^3*x^3*e^2 + 21*a^2*c*d^5*f^3*x^3*e^2 + 21/2*a*b^2*d^6*f^3
*x^2*e + 21/2*a^2*c*d^6*f^3*x^2*e + 3*a*b^2*d^7*f^3*x + 3*a^2*c*d^7*f^3*x +
 1/2*a^2*b*f^3*x^6*e^5 + 3*a^2*b*d*f^3*x^5*e^4 + 15/2*a^2*b*d^2*f^3*x^4*e^3
 + 10*a^2*b*d^3*f^3*x^3*e^2 + 15/2*a^2*b*d^4*f^3*x^2*e + 3*a^2*b*d^5*f^3*x
+ 1/4*a^3*f^3*x^4*e^3 + a^3*d*f^3*x^3*e^2 + 3/2*a^3*d^2*f^3*x^2*e + a^3*d^3
*f^3*x
```

3.613 $\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=193

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e
*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*e)
```

Rubi [A] time = 0.438332, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1142, 1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e
*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{e}$$
$$= \frac{x}{c} - \frac{\text{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{ce}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{2ce} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}\ldots}\right.}{2\ldots}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

Mathematica [A] time = 0.153751, size = 219, normalized size = 1.13

$$\frac{-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + 2\sqrt{c}(d+ex)}{2c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*
c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2)*e)
```

Maple [C] time = 0.065, size = 158, normalized size = 0.8

$$\frac{x}{c} + \frac{1}{2\,ce}\sum_{\_R=\text{RootOf}\left(ce^4\_Z^4+4\,cde^3\_Z^3+\left(6\,cd^2e^2+be^2\right)\_Z^2+\left(4\,cd^3e+2\,bde\right)\_Z+cd^4+bd^2+a\right)} \frac{\left(-\_R^2be^2-2\_R\,bde-bd^2-a\right)\ln(x-\_R)}{2\,ce^3\_R^3+6\,cde^2\_R^2+6\,cd^2e\_R+2\,cd^3+be\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] x/c+1/2/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.00144, size = 2539, normalized size = 13.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2)
)*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2 - a^2*c)*d + sqrt(1/2)*((b^3*c^3 -
4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4))
- (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c
^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c
^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2 - a^2*c)*d - sqrt
(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6
 - 4*a*c^7)*e^4)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(-((b^2*c^3 - 4*a*
c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3
- 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(((b^2*c^3 - 4*a*c
^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) - b^3 +
 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2
- a^2*c)*d + sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(
((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c
^7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) + sqrt(1/2)*c*sqrt((
(b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^
7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)
*e*x - 2*(a*b^2 - a^2*c)*d - sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e)*sqrt(((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(
(b^2*c^6 - 4*a*c^7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) + 2*
x)/c
```

Sympy [A] time = 2.7413, size = 178, normalized size = 0.92

$$\operatorname{RootSum}\left(t^{4}\left(256a^{2}c^{5}e^{4}-128ab^{2}c^{4}e^{4}+16b^{4}c^{3}e^{4}\right)+t^{2}\left(48a^{2}bc^{2}e^{2}-28ab^{3}ce^{2}+4b^{5}e^{2}\right)+a^{3},\left(t\mapsto t\log\left(x+\frac{32t^{3}abc}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e**
4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + a**3, L
ambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t**3*b**3*c**3*e**3 - 4*_
t*a**2*c**2*e + 8*_t*a*b**2*c*e - 2*_t*b**4*e + a**2*c*d - a*b**2*d)/(a**2*
c*e - a*b**2*e)))) + x/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^4}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.614 $\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=81

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]
*e) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*c*e)
```

Rubi [A] time = 0.128536, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1142, 1114, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]
*e) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*c*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce} \\
&= \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{2ce} \\
&= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ace}} + \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}
\end{aligned}$$

Mathematica [A] time = 0.0442972, size = 77, normalized size = 0.95

$$\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right) - \frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

Maple [C] time = 0.006, size = 151, normalized size = 1.9

$$\frac{1}{2e}\sum_{\_R=\text{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2bde\right)\_Z+cd^4+bd^2+a\right)} \frac{\left(\_R^3e^3+3\_R^2de^2+3\_R\,d^2e+d^3\right)\ln(x-\_R)}{2ce^3\_R^3+6cde^2\_R^2+6cd^2e\_R+2cd^3+be\_R+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Fricas [B] time = 1.84879, size = 956, normalized size = 11.8

$$\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\left(6\,c^2d^2+bc\right)e^2x^2+2\,bcd^2+4\left(2\,c^2d^3+bcd\right)ex+b^2-2\,ac+\left(2\,ce^2x^2+4\,cdex+2\,cd^2+b\right)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\left(2\,cd^3+bd\right)ex+a}\right)+\left(b^2-4\right.}{4\left(b^2c-4\,ac^2\right)e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
+ 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*e*x + a)) + (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6
*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c - 4*a*c^2
)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*
c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x
 + a))/((b^2*c - 4*a*c^2)*e)]
```

Sympy [B] time = 1.81221, size = 280, normalized size = 3.46

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{1}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{b\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{1}{4ce}\right)+2a+2b^2e\left(-\frac{b\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{1}{4ce}\right)+bd^2}{be^2}\right)+\left(\frac{b\sqrt{-4a}}{4ce\left(4a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))*log(2*d*x/e + x
**2 + (-8*a*c*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))
 + 2*a + 2*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e
)) + b*d**2)/(b*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/
(4*c*e))*log(2*d*x/e + x**2 + (-8*a*c*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*
c - b**2)) + 1/(4*c*e)) + 2*a + 2*b**2*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a
*c - b**2)) + 1/(4*c*e)) + b*d**2)/(b*e**2))
```

Giac [B] time = 1.40668, size = 375, normalized size = 4.63

$$-\frac{\sqrt{b^2-4\,ac}bce\log\left(\left|2\left(b+\sqrt{b^2-4\,ac}\right)x^2e^6+4\left(b+\sqrt{b^2-4\,ac}\right)dxe^5+2\left(b+\sqrt{b^2-4\,ac}\right)d^2e^4+4\,ae^4\right|\right)}{4\left(b^2c^2e^2-4\,ac^3e^2\right)}+\frac{\sqrt{b^2-4\,ac}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(b^2 - 4*a*c)*b*c*e*log(abs(2*(b + sqrt(b^2 - 4*a*c))*x^2*e^6 + 4*
(b + sqrt(b^2 - 4*a*c))*d*x*e^5 + 2*(b + sqrt(b^2 - 4*a*c))*d^2*e^4 + 4*a*e
^4))/(b^2*c^2*e^2 - 4*a*c^3*e^2) + 1/4*sqrt(b^2 - 4*a*c)*b*c*e*log(abs(-2*(
b - sqrt(b^2 - 4*a*c))*x^2*e^6 - 4*(b - sqrt(b^2 - 4*a*c))*d*x*e^5 - 2*(b -
 sqrt(b^2 - 4*a*c))*d^2*e^4 - 4*a*e^4))/(b^2*c^2*e^2 - 4*a*c^3*e^2) + 1/4*e
^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c
*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/c
```

3.615 $\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4} dx$

Optimal. Leaf size=164

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[b + Sqr
t[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)
```

Rubi [A] time = 0.154839, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1142, 1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[b + Sqr
t[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{e}$$
$$= \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{2e} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{2e}$$
$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e}$$

Mathematica [A] time = 0.100724, size = 175, normalized size = 1.07

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[
b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Ar
cTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

Maple [C] time = 0.004, size = 140, normalized size = 0.9

$$\frac{1}{2e}\sum_{\_R=\text{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2bde\right)\_Z+cd^4+bd^2+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln(x-\_R)}{2ce^3\_R^3+6cde^2\_R^2+6cd^2e\_R+2cd^3+be\_R+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e
+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*
e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Fricas [B] time = 1.75622, size = 1524, normalized size = 9.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)
) + b)/((b^2*c - 4*a*c^2)*e^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(-(
(b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*
c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sqr
t(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c -
4*a*c^2)*e^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(-((b^2*c - 4*a*c^2
)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(
1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sqrt(((b^2*c - 4*a*
c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*lo
g(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*
c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c
^3)*e^4)) + e*x + d) + 1/2*sqrt(1/2)*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b
^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*log(-sqrt(1/2)*(b^2*c
 - 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4
)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x +
d)
```

Sympy [A] time = 1.56952, size = 104, normalized size = 0.63

$$\operatorname{RootSum}\left(t^4\left(256a^2c^3e^4 - 128ab^2c^2e^4 + 16b^4ce^4\right) + t^2\left(-16abce^2 + 4b^3e^2\right) + a, \left(t \mapsto t\log\left(x + \frac{64t^3ac^2e^3 - 16t^3b^2ce^3}{e}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
+ _t**2*(-16*a*b*c*e**2 + 4*b**3*e**2) + a, Lambda(_t, _t*log(x + (64*_t**3
*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e + d)/e)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{(ex+d)^4c + (ex+d)^2b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.616 $\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

```
[Out] -(ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(Sqrt[b^2 - 4*a*c]*e))
```

Rubi [A] time = 0.0612972, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1142, 1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -(ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(Sqrt[b^2 - 4*a*c]*e))
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{x}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{e}$$
$$= -\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ace}}$$

Mathematica [A] time = 0.0165225, size = 46, normalized size = 1.07

$$\frac{\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{e\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]]/(Sqrt[-b^2 + 4*a*c]*e)
```

Maple [C] time = 0.005, size = 129, normalized size = 3.

$$\frac{1}{2e}\sum_{\_R=\text{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2bde\right)\_Z+cd^4+bd^2+a\right)}\frac{(\_Re+d)\ln(x-\_R)}{2ce^3\_R^3+6cde^2\_R^2+6cd^2e\_R+2cd^3+be\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e
+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2
+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Fricas [A] time = 1.75535, size = 598, normalized size = 13.91

$$\left[\frac{\log\left(\frac{2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2\left(6c^2d^2+bc\right)e^2x^2+2bcd^2+4\left(2c^2d^3+bcd\right)ex+b^2-2ac-\left(2ce^2x^2+4cdex+2cd^2+b\right)\sqrt{b^2-4ac}}{ce^4x^4+4cde^3x^3+cd^4+\left(6cd^2+b\right)e^2x^2+bd^2+2\left(2cd^3+bd\right)ex+a}\right)}{2\sqrt{b^2-4ace}}, -\frac{\sqrt{-b^2+4ac}\arctan\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)
*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x
^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3
 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/(sqr
t(b^2 - 4*a*c)*e), -sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2
*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/((b^2 - 4*a*c)*e)]
```

Sympy [B] time = 1.02467, size = 168, normalized size = 3.91

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2
)) + b**2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e) + sqrt(
-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b*
*2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e)
```

Giac [B] time = 1.38266, size = 247, normalized size = 5.74

$$\frac{\sqrt{b^2-4ace}\log\left(\left|\left(b+\sqrt{b^2-4ac}\right)x^2e^2+2\left(b+\sqrt{b^2-4ac}\right)dxe+\left(b+\sqrt{b^2-4ac}\right)d^2+2a\right|\right)}{2\left(b^2e^2-4ace^2\right)}-\frac{\sqrt{b^2-4ace}\log\left(\left|-\left(b-\sqrt{\cdots}\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b^2 - 4*a*c)*e*log(abs((b + sqrt(b^2 - 4*a*c))*x^2*e^2 + 2*(b + sq
rt(b^2 - 4*a*c))*d*x*e + (b + sqrt(b^2 - 4*a*c))*d^2 + 2*a))/(b^2*e^2 - 4*a
*c*e^2) - 1/2*sqrt(b^2 - 4*a*c)*e*log(abs(-(b - sqrt(b^2 - 4*a*c))*x^2*e^2
- 2*(b - sqrt(b^2 - 4*a*c))*d*x*e - (b - sqrt(b^2 - 4*a*c))*d^2 - 2*a))/(b^
2*e^2 - 4*a*c*e^2)
```

3.617 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=94

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ae\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}+\frac{\log(d+ex)}{ae}$$

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]
*e) + Log[d + e*x]/(a*e) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*e)
```

Rubi [A] time = 0.132291, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1142, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ae\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}+\frac{\log(d+ex)}{ae}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]
*e) + Log[d + e*x]/(a*e) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx, x, d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, (d+ex)^2\right)}{2e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (d+ex)^2\right)}{2ae} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2ae}\\
&= \frac{\log(d+ex)}{ae} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ae} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx\right.}{4ae}\\
&= \frac{\log(d+ex)}{ae} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,\right.}{2ae}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}e} + \frac{\log(d+ex)}{ae} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}
\end{aligned}
$$

Mathematica [A] time = 0.0802059, size = 128, normalized size = 1.36

$$
\frac{4\sqrt{b^2-4ac}\log(d+ex)-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2c\right.}{4ae\sqrt{b^2-4ac}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 -
4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e)
```

Maple [C] time = 0.008, size = 184, normalized size = 2.

$$\frac{\ln(ex+d)}{ae}+\frac{1}{2ae}\sum_{\_R=\mathrm{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2bde\right)\_Z+cd^4+bd^2+a\right)}\frac{\left(-ce^3\_R^3-3cde^2\_R^2+e\left(-3cd^2\right.\right.}{2ce^3\_R^3+6cde^2\_R^2+6cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] ln(e*x+d)/a/e+1/2/a/e*sum((-c*e^3*_R^3-3*c*d*e^2*_R^2+e*(-3*c*d^2-b)*_R-c*d
^3-b*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-
_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e
+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.99223, size = 1040, normalized size = 11.06

$$\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\left(6\,c^2d^2+bc\right)e^2x^2+2\,bcd^2+4\left(2\,c^2d^3+bcd\right)ex+b^2-2\,ac+\left(2\,ce^2x^2+4\,cdex+2\,cd^2+b\right)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\left(2\,cd^3+bd\right)ex+a}\right)-\left(b^2-4\right.}{4\left(ab^2-4\,a^2c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
+ 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*e*x + a)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6
*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*
log(e*x + d))/((a*b^2 - 4*a^2*c)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2
*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (
b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2
+ b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(e*x + d))/((a*b^
2 - 4*a^2*c)*e)]
```

Sympy [B] time = 4.86643, size = 320, normalized size = 3.4

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4ae\left(4ac-b^2\right)}-\frac{1}{4ae}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2ce\left(-\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)+2ab^2e\left(-\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)-2ac+b^2+bcd^2}{bce^2}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e))*log(2*d*x/e + x
**2 + (-8*a**2*c*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*
e)) + 2*a*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)
) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*a*e*(4
*a*c - b**2)) - 1/(4*a*e))*log(2*d*x/e + x**2 + (-8*a**2*c*e*(b*sqrt(-4*a*c
 + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) + 2*a*b**2*e*(b*sqrt(-4*a*c +
b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e
**2)) + log(d/e + x)/(a*e)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.618 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=195

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}ae\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}ae\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ae(d+ex)}$$

```
[Out] -(1/(a*e*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*e) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*e)
```

Rubi [A] time = 0.286361, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1142, 1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}ae\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}ae\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ae(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(1/(a*e*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*e) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)}dx, x, d+ex\right)}{e}$$

$$= -\frac{1}{ae(d+ex)} + \frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}dx, x, d+ex\right)}{ae}$$

$$= -\frac{1}{ae(d+ex)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx, x, d+ex\right)}{2ae} -$$

$$= -\frac{1}{ae(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ta}}{\sqrt{2}a\sqrt{b+\sqrt{}}}$$

Mathematica [A] time = 0.367265, size = 206, normalized size = 1.06

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{d+ex}}{2ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*e)
```

Maple [C] time = 0.007, size = 168, normalized size = 0.9

$$\frac{1}{2\,ae}\sum_{\_R=\text{RootOf}\left(ce^4\_Z^4+4\,cde^3\_Z^3+\left(6\,cd^2e^2+be^2\right)\_Z^2+\left(4\,cd^3e+2\,bde\right)\_Z+cd^4+bd^2+a\right)}\frac{\left(-\_R^2ce^2-2\,\_R\,cde-cd^2-b\right)\ln\left(x-\_R\right)}{2\,ce^3\_R^3+6\,cde^2\_R^2+6\,cd^2e\_R+2\,cd^3+be\_R+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6
*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^
3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))-1/a/e/(e*
x+d)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.27964, size = 2753, normalized size = 14.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2
 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sq
rt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt
(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a
^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - sqrt(1/2)*(a*e^2
*x + a*d*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))*log
(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6
*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*
a^7*c)*e^4)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(-((a^3*b^2 - 4*a^4*c
)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3
*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(((a^
3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*
e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*
e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2
)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + (b^5 -
5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a
^4*c)*e^2))) + sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/
((a^3*b^2 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^
3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2
)*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b
^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - 2)/(a*e^
2*x + a*d*e)
```

Sympy [A] time = 3.66293, size = 211, normalized size = 1.08

$$\mathrm{RootSum}\left(t^4\left(256a^5c^2e^4 - 128a^4b^2ce^4 + 16a^3b^4e^4\right) + t^2\left(48a^2bc^2e^2 - 28ab^3ce^2 + 4b^5e^2\right) + c^3, \left(t \mapsto t\log\left(x + \frac{-64t^3a^5\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2*e**4 - 128*a**4*b**2*c*e**4 + 16*a**3*b**4*e**
4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + c**3, L
ambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**3 + 48*_t**3*a**4*b**2*c*e**3
- 8*_t**3*a**3*b**4*e**3 - 10*_t*a**2*b*c**2*e + 10*_t*a*b**3*c*e - 2*_t*b*
*5*e + a*c**3*d - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e + a*e
**2*x)
```

Giac [C] time = 3.04139, size = 4884, normalized size = 25.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -2*(3*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs
(a)/(a^2*c))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))
)^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) - (
a^3*c)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^
3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 - 9
*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(
a^2*c))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*s
in(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) + 3*(a^3*c)^(3/4)*b*cos
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sin(5/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sinh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) + 9*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*cosh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*a
bs(a)/(a^2*c))))^2 - 3*(a^3*c)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b*abs(a)/(a^2*c))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*a
bs(a)/(a^2*c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)
)))^2 - 3*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
*abs(a)/(a^2*c))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(
a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3
 + (a^3*c)^(3/4)*b*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)
/(a^2*c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3
 + (a^3*c)^(1/4)*a^2*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^
2*c))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))
 - (a^3*c)^(1/4)*a^2*c*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*ab
s(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))
)*arctan(-((c/a)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*
c)))*e^(-1) + e^(-1)/(x*e + d))*e/((c/a)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*
sqrt(a*c)*b*abs(a)/(a^2*c)))))/(sqrt(b^2 - 4*a*c)*a^2*b*abs(a)*e - (a*b^2*e
 - 4*a^2*c*e)*a^2) - 2*(3*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b*abs(a)/(a^2*c))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*a
bs(a)/(a^2*c)))) - (a^3*c)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b*abs(a)/(a^2*c))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs
(a)/(a^2*c))))^3 - 9*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*
sqrt(a*c)*b*abs(a)/(a^2*c))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*a
bs(a)/(a^2*c))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)
/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) + 3
*(a^3*c)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))
```

```
)^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) + 9*(a^3*c)^(3/4
)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*c
osh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(1/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - 3*(a^3*c)^(3/4)*b*cosh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(1/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b*abs(a)/(a^2*c))))^2 - 3*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sin(1/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
*abs(a)/(a^2*c))))^3 + (a^3*c)^(3/4)*b*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
*abs(a)/(a^2*c))))^3 + (a^3*c)^(1/4)*a^2*c*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b*abs(a)/(a^2*c))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)
*b*abs(a)/(a^2*c)))) - (a^3*c)^(1/4)*a^2*c*sin(1/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b*abs(a)/(a^2*c)))))*arctan(-((c/a)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt
(a*c)*b*abs(a)/(a^2*c)))*e^(-1) + e^(-1)/(x*e + d))*e/((c/a)^(1/4)*sin(1/4*
pi + 1/2*arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))))/(sqrt(b^2 - 4*a*c)*a^2*b
*abs(a)*e - (a*b^2*e - 4*a^2*c*e)*a^2) + ((a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*cosh(1/2*imag_part(arc
sin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 - 3*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part(arc
sin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - 3*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b*abs(a)/(a^2*c)))) + 9*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b*abs(a)/(a^2*c))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs
(a)/(a^2*c))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))
) + 3*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs
(a)/(a^2*c))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))
)*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - 9*(a^3*c)
^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))
)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(5/4*pi +
1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sinh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - (a^3*c)^(3/4)*b*cos(5/4*pi +
1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sinh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 + 3*(a^3*c)^(3/4)*b*cos(5/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(5/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 + (a^3*c)^(1/4)*a^2*c*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) - (a^3*c)^(1/4)*a^2*c*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))))*log(sqrt(c/a)*e^(-2) + 2*(c/a)^(1/4
)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))*e^(-2)/(x*e + d)
 + e^(-2)/(x*e + d)^2)/(sqrt(b^2 - 4*a*c)*a^2*b*abs(a)*e - (a*b^2*e - 4*a^2
*c*e)*a^2) + ((a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b*abs(a)/(a^2*c))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(
a^2*c))))^3 - 3*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(
a^2*c))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c
))))^2 - 3*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b*abs(a)/(a^2*c))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2
*c))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) + 9*(a
^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2
```

```
*c))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sin(1/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sinh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) + 3*(a^3*c)^(3/4)*b*cos(1/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*cosh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - 9*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b*abs(a)/(a^2*c))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(
a)/(a^2*c))))^2 - (a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b*abs(a)/(a^2*c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(
a)/(a^2*c))))^3 + 3*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b*abs(a)/(a^2*c))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b*abs(a)/(a^2*c))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a
^2*c))))^3 + (a^3*c)^(1/4)*a^2*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/
(a^2*c)))) - (a^3*c)^(1/4)*a^2*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/
(a^2*c)))))*log(sqrt(c/a)*e^(-2) + 2*(c/a)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/
2*sqrt(a*c)*b*abs(a)/(a^2*c)))*e^(-2)/(x*e + d) + e^(-2)/(x*e + d)^2)/(sqrt
(b^2 - 4*a*c)*a^2*b*abs(a)*e - (a*b^2*e - 4*a^2*c*e)*a^2) - e^(-1)/((x*e +
d)*a)
```

3.619 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=121

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}-\frac{b\log(d+ex)}{a^2e}-\frac{1}{2ae(d+ex)^2}$$

```
[Out] -1/(2*a*e*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e) - (b*Log[d + e*x])/(a^2*e) + (b*
Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e)
```

Rubi [A] time = 0.198484, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1142, 1114, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}-\frac{b\log(d+ex)}{a^2e}-\frac{1}{2ae(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(2*a*e*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e) - (b*Log[d + e*x])/(a^2*e) + (b*
Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx,x,(d+ex)^2\right)}{2e}\\
&= -\frac{1}{2ae(d+ex)^2}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx,x,(d+ex)^2\right)}{2ae}\\
&= -\frac{1}{2ae(d+ex)^2}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx,x,(d+ex)^2\right)}{2ae}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{b\log(d+ex)}{a^2e}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{2a^2e}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{b\log(d+ex)}{a^2e}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{4a^2e}+\cdots\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{b\log(d+ex)}{a^2e}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}-\cdots\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,e}-\frac{b\log(d+ex)}{a^2e}+\frac{b\log\left(\cdots\right.}{\cdots}
\end{aligned}$$

Mathematica [A] time = 0.129821, size = 154, normalized size = 1.27

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{(d+ex)^2}-4b\log(d+ex)}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c
])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b^2
 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2*e)
```

Maple [C] time = 0.012, size = 213, normalized size = 1.8

$$-\frac{1}{2\,ae\,(ex+d)^{2}}-\frac{b\ln (ex+d)}{a^{2}e}+\frac{1}{2\,a^{2}e}\sum_{\_R=\mathrm{RootOf}\left(ce^{4}\_Z^{4}+4\,cde^{3}\_Z^{3}+\left(6\,cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4\,cd^{3}e+2\,bde\right)\_Z+cd^{4}+bd^{2}+a\right)}\frac{\left(\_R^{3}bce^{3}+3\_\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] -1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/a^2/e+1/2/a^2/e*sum((_R^3*b*c*e^3+3*_R^2*b*c
*d*e^2+e*(3*b*c*d^2-a*c+b^2)*_R+b*c*d^3-a*c*d+b^2*d)/(2*_R^3*c*e^3+6*_R^2*c
*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d
*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.46894, size = 1782, normalized size = 14.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x +
 (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3
+ 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*
d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4
*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)
*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*
d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*
e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^
2*b^2 - 4*a^3*c)*e^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*x + (a^2*b^2 - 4*a^3
```

```
*c)*d^2*e), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2
*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 +
4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*
c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 +
 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*
e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*
a*b*c)*d^2)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*e^3*x^2 + 2*(a^2*b^2 - 4*a^3
*c)*d*e^2*x + (a^2*b^2 - 4*a^3*c)*d^2*e)]
```

Sympy [B] time = 14.4246, size = 464, normalized size = 3.83

$$\left(\frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2e\left(4ac-b^2\right)}\right)\log\left(\frac{2dx}{e} + x^2 + \frac{-8a^3ce\left(\frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2e(4ac-b^2)}\right) + 2a^2b^2e\left(\frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2e(4ac-b^2)}\right) + 3ab}{2ac^2e^2 - b^2ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (b/(4*a**2*e) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2)
))*log(2*d*x/e + x**2 + (-8*a**3*c*e*(b/(4*a**2*e) - sqrt(-4*a*c + b**2)*(2
*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) + 2*a**2*b**2*e*(b/(4*a**2*e) - sqr
t(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) + 3*a*b*c + 2*a*
c**2*d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) + (b/(4*a**2
*e) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2)))*log(2*d
*x/e + x**2 + (-8*a**3*c*e*(b/(4*a**2*e) + sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(4*a**2*e*(4*a*c - b**2))) + 2*a**2*b**2*e*(b/(4*a**2*e) + sqrt(-4*a*c +
 b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2*d**2
- b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) - 1/(2*a*d**2*e + 4*a*
d*e**2*x + 2*a*e**3*x**2) - b*log(d/e + x)/(a**2*e)
```

Giac [A] time = 1.23402, size = 138, normalized size = 1.14

$$\frac{be^{(-1)}\log\left(c + \frac{b}{(xe+d)^2} + \frac{a}{(xe+d)^4}\right)}{4\,a^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^2}}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{2\,\sqrt{-b^2+4\,ac}a^2} - \frac{e^{(-1)}}{2\,(xe+d)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/4*b*e^(-1)*log(c + b/(x*e + d)^2 + a/(x*e + d)^4)/a^2 + 1/2*(b^2 - 2*a*c)
*arctan(-(b + 2*a/(x*e + d)^2)/sqrt(-b^2 + 4*a*c))*e^(-1)/(sqrt(-b^2 + 4*a*
c)*a^2) - 1/2*e^(-1)/((x*e + d)^2*a)
```

3.620 $\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=224

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2e\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2e(d+ex)}-\frac{1}{3ae(d+ex)^3}$$

```
[Out] -1/(3*a*e*(d + e*x)^3) + b/(a^2*e*(d + e*x)) + (Sqrt[c]*(b + (b^2 - 2*a*c)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b - (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.496765, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1123, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2e\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2e(d+ex)}-\frac{1}{3ae(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```


```
[Out] -1/(3*a*e*(d + e*x)^3) + b/(a^2*e*(d + e*x)) + (Sqrt[c]*(b + (b^2 - 2*a*c)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b - (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
```

```
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{e}\\ &= -\frac{1}{3ae(d+ex)^3}+\frac{\text{Subst}\left(\int \frac{-3b-3cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{3ae}\\ &= -\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)}-\frac{\text{Subst}\left(\int \frac{-3\left(b^2-ac\right)-3bcx^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{3a^2e}\\ &= -\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)}+\frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\right.}{2a^2e}\\ &= -\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\,e}+\ldots\end{aligned}$$

Mathematica [A] time = 0.216544, size = 235, normalized size = 1.05

$$\frac{\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}}{6a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt
[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(6*a^2*e)
```

Maple [C] time = 0.01, size = 188, normalized size = 0.8

$$-\frac{1}{3\,ae\,(ex+d)^3}+\frac{b}{a^2e\,(ex+d)}+\frac{1}{2\,a^2e}\sum_{\_R=\mathrm{RootOf}\left(ce^4\_Z^4+4\,cde^3\_Z^3+\left(6\,cd^2e^2+be^2\right)\_Z^2+\left(4\,cd^3e+2\,bde\right)\_Z+cd^4+bd^2+a\right)}\frac{\left(\_R^2bce^2+2\right.}{2\,ce^3\_R^3+6\,cc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] -1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d)+1/2/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e
+b*c*d^2-a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+
b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+
(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.17542, size = 4215, normalized size = 18.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a^2*
d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c
^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2)
)*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 +
a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt((b^
8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^
11*c)*e^4)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^
4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*
a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a
^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*
b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e
^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4
 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt(
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4
*a^11*c)*e^4)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4
*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 -
 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 3*sqrt(1/2)*(a^2*e^4*x^3 +
```

```
3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a
^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c
)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*
c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2
- 4*a^11*c)*e^4)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*
a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^
2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) + 3*sqrt(1/2)*(a^2*e^4*x^3
 + 3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^
6*c)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b
^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b
^2 - 4*a^11*c)*e^4)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 +
 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e
^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10
*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 2*a)/(a^2*e^4*x^3 + 3
*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)
```

Sympy [A] time = 12.0593, size = 347, normalized size = 1.55

$$\frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3e+9a^2d^2e^2x+9a^2de^3x^2+3a^2e^4x^3}+\operatorname{RootSum}\left(t^4\left(256a^7c^2e^4-128a^6b^2ce^4+16a^5b^4e^4\right)+t^2\left(-80a^3bc^3e^2+100a^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e + 9*a**2*d**2*e*
*2*x + 9*a**2*d*e**3*x**2 + 3*a**2*e**4*x**3) + RootSum(_t**4*(256*a**7*c**
2*e**4 - 128*a**6*b**2*c*e**4 + 16*a**5*b**4*e**4) + _t**2*(-80*a**3*b*c**3
*e**2 + 100*a**2*b**3*c**2*e**2 - 36*a*b**5*c*e**2 + 4*b**7*e**2) + c**5, L
ambda(_t, _t*log(x + (-96*_t**3*a**7*b*c**2*e**3 + 56*_t**3*a**6*b**3*c*e**
3 - 8*_t**3*a**5*b**5*e**3 - 4*_t*a**4*c**4*e + 32*_t*a**3*b**2*c**3*e - 40
*_t*a**2*b**4*c**2*e + 16*_t*a*b**6*c*e - 2*_t*b**8*e + a**2*c**5*d - 3*a*b
**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2*c**4*e + b**4*c**3*e))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^4), x)
```

3.621 $\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=270

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\ldots\right.}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-\ldots}}}$$

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a
*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.579225, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1142, 1120, 1166, 205}

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\ldots\right.}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-\ldots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a
*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{2\left(b^2-4ac\right)e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \ldots\right.}{4\left(b^2-4ac\right)\ldots}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt{b-\ldots}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-\ldots}}}$$

Mathematica [A] time = 0.549425, size = 263, normalized size = 0.97

$$\frac{-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*e)
```

Maple [C] time = 0.017, size = 323, normalized size = 1.2

$$\frac{1}{ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a}\left(-\frac{be^2x^3}{8\,ac-2\,b^2}-\frac{3\,bdex^2}{8\,ac-2\,b^2}-\frac{\left(3\,bd^2+2\,a\right)x}{8\,ac-2\,b^2}-\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (-1/2*b*e^2/(4*a*c-b^2)*x^3-3/2*d*b*e/(4*a*c-b^2)*x^2-1/2*(3*b*d^2+2*a)/(4*
a*c-b^2)*x-1/2*d/e*(b*d^2+2*a)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*
sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c
*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{be^3x^3 + 3\,bde^2x^2 + bd^3 + \left(3\,bd^2 + 2\,a\right)ex + 2\,ad}{2\left(\left(b^2c - 4\,ac^2\right)e^5x^4 + 4\left(b^2c - 4\,ac^2\right)de^4x^3 + \left(b^3 - 4\,abc + 6\left(b^2c - 4\,ac^2\right)d^2\right)e^3x^2 + 2\left(2\left(b^2c - 4\,ac^2\right)d^3 + \left(b^3 - 4\,abc\right)d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*e^3*x^3 + 3*b*d*e^2*x^2 + b*d^3 + (3*b*d^2 + 2*a)*e*x + 2*a*d)/((b^2
*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*
(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*
c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^
2)*e) - 1/2*integrate(-(b*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c - 4*a*
c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)
```

Fricas [B] time = 2.13479, size = 5245, normalized size = 19.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*e^3*x^3 + 6*b*d*e^2*x^2 + 2*b*d^3 + 2*(3*b*d^2 + 2*a)*e*x + sqrt(1
/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*
e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) +
b^3 + 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))
*log((3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d + sqrt(1/2)*(2*(b^7*c - 12*a*b
^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3
 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*s
qrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/
((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((
b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c +
 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a
*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)
*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sq
rt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 +
12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((
3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d - sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2
 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*
```

```
a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(-(
(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/((b^6*
c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c
- 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^
2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*
d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*
e)*sqrt(((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*
c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 +
 4*a*c)*e*x + (3*b^2 + 4*a*c)*d + sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2 + 48*a
^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*e^4)) - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(((b^6*c -
 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6*c - 12*a
*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) + sqrt(1/2)*((b^2*c - 4*a*c^
2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*
a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x
 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(
((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6
*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*
e*x + (3*b^2 + 4*a*c)*d - sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c
^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5)*e^4)) - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(((b^6*c - 12*a*b^
4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6*c - 12*a*b^4*c^2
 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) + 4*a*d)/((b^2*c - 4*a*c^2)*e^5*x^4
+ 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)
*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c
- 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Sympy [B] time = 48.5438, size = 571, normalized size = 2.11

$$-\frac{2ad + bd^3 + 3bde^2x^2 + be^3x^3 + x\left(2ae + 3b\right.}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2\left(8abce^3 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -(2*a*d + b*d**3 + 3*b*d*e**2*x**2 + b*e**3*x**3 + x*(2*a*e + 3*b*d**2*e))/
(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e
 - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2
*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b
**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2
- 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**6*c**7*e
**4 - 1572864*a**5*b**2*c**6*e**4 + 983040*a**4*b**4*c**5*e**4 - 327680*a**
3*b**6*c**4*e**4 + 61440*a**2*b**8*c**3*e**4 - 6144*a*b**10*c**2*e**4 + 256
*b**12*c*e**4) + _t**2*(-12288*a**4*b*c**4*e**2 + 8192*a**3*b**3*c**3*e**2
- 1536*a**2*b**5*c**2*e**2 + 16*b**9*e**2) + 16*a**3*c**2 + 24*a**2*b**2*c
+ 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 12288*_t*
*3*a**2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**7*c*e**
3 + 64*_t*a**2*c**2*e - 128*_t*a*b**2*c*e - 4*_t*b**4*e + 4*a*c*d + 3*b**2*
d)/(4*a*c*e + 3*b**2*e))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^2, x)
```

3.622 $\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx$

Optimal. Leaf size=97

$$\frac{2a+b(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (2*a + b*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) - (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3
/2)*e)
```

Rubi [A] time = 0.135144, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1114, 638, 618, 206}

$$\frac{2a+b(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (2*a + b*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) - (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3
/2)*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2e}$$
$$= \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex\right.}{2\left(b^2-4ac\right)e}$$
$$= \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c\right.}{\left(b^2-4ac\right)e}$$
$$= \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A] time = 0.139106, size = 100, normalized size = 1.03

$$\frac{\frac{2a+b(d+ex)^2}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))
) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(
3/2))/(2*e)
```

Maple [C] time = 0.02, size = 276, normalized size = 2.9

$$\frac{1}{ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a}\left(-\frac{bex^2}{8\,ac-2\,b^2}-\frac{bdx}{4\,ac-b^2}-\frac{bd^2+2\,a}{2\,e\left(4\,ac-b^2\right)}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (-1/2*b*e/(4*a*c-b^2)*x^2-b*d/(4*a*c-b^2)*x-1/2/e*(b*d^2+2*a)/(4*a*c-b^2))/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)+1/2*b/(4*a*c-b^2)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2
+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_
Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-b\int -\frac{ex+d}{\left(b^2c-4\,ac^2\right)e^4x^4+4\left(b^2c-4\,ac^2\right)de^3x^3+\left(b^2c-4\,ac^2\right)d^4+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^2x^2+ab^2-4\,a^2c+(\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -b*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*
e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)
*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d
^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*x^2 + 2*b*d*e*x + b*d^2 + 2*a
)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b
*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 -
 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*d^2)*e)
```

Fricas [B] time = 1.96927, size = 2192, normalized size = 22.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + 2*a*b^2 - 8*a^2*c
 + (b^3 - 4*a*b*c)*d^2 - (b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*c*d^4 + (6*b*c*
d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a*b)*sqrt(b^2 -
4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*
c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2
*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x
^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/(
(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2
 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + 2*a*
b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*d^2 - 2*(b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*
c*d^4 + (6*b*c*d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a
*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4
+ 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^
4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

Sympy [B] time = 38.4456, size = 493, normalized size = 5.08

$$\frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2+2bcd^2}{2bce^2}\right)}{2e}-\frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*sqrt(-1/
(4*a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(
4*a*c - b**2)**3) + b**2 + 2*b*c*d**2)/(2*b*c*e**2))/(2*e) - b*sqrt(-1/(4*a
*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)*
*3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c - b**2)**
3) + b**2 + 2*b*c*d**2)/(2*b*c*e**2))/(2*e) - (2*a + b*d**2 + 2*b*d*e*x + b
*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2
*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**
3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**
2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**
2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

Giac [B] time = 3.21884, size = 490, normalized size = 5.05

$$\frac{\left(b^3e-4\,abce\right)\sqrt{b^2-4\,ac}\log\left(\left|\left(b+\sqrt{b^2-4\,ac}\right)x^2e^2+2\left(b+\sqrt{b^2-4\,ac}\right)dxe+\left(b+\sqrt{b^2-4\,ac}\right)d^2+2\,a\right|\right)}{2\left(b^6e^2-12\,ab^4ce^2+48\,a^2b^2c^2e^2-64\,a^3c^3e^2\right)}-\frac{\left(b^3e-4\,a\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*e - 4*a*b*c*e)*sqrt(b^2 - 4*a*c)*log(abs((b + sqrt(b^2 - 4*a*c))*x
^2*e^2 + 2*(b + sqrt(b^2 - 4*a*c))*d*x*e + (b + sqrt(b^2 - 4*a*c))*d^2 + 2*
a))/(b^6*e^2 - 12*a*b^4*c*e^2 + 48*a^2*b^2*c^2*e^2 - 64*a^3*c^3*e^2) - 1/2*
(b^3*e - 4*a*b*c*e)*sqrt(b^2 - 4*a*c)*log(abs(-(b - sqrt(b^2 - 4*a*c))*x^2*
e^2 - 2*(b - sqrt(b^2 - 4*a*c))*d*x*e - (b - sqrt(b^2 - 4*a*c))*d^2 - 2*a))
/(b^6*e^2 - 12*a*b^4*c*e^2 + 48*a^2*b^2*c^2*e^2 - 64*a^3*c^3*e^2) + 1/2*(b*
x^2*e^2 + 2*b*d*x*e + b*d^2 + 2*a)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^
2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4
*a*c*e))
```

3.623 $\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=254

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\cdots\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-\cdots}}}$$

```
[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*
c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.388064, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1142, 1119, 1166, 205}

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\cdots\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*
c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx,x,d+ex\right)}{e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\text{Subst}\left(\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx,x,d+ex\right.}{2\left(b^2-4ac\right)e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(c\left(2b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int\right.}{2\left(b^2-4a\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{b}}\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-}}}$$

Mathematica [A] time = 1.08438, size = 247, normalized size = 0.97

$$-\frac{\frac{b(d+ex)+2c(d+ex)^3}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d
+ e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*e)
```

Maple [C] time = 0.018, size = 319, normalized size = 1.3

$$\frac{1}{ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a}\left(\frac{ce^2x^3}{4\,ac-b^2}+3\,\frac{cdex^2}{4\,ac-b^2}+\frac{\left(6\,cd^2+b\right)x}{8\,ac-2\,b^2}+\frac{d}{2\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (c*e^2/(4*a*c-b^2)*x^3+3*d*c*e/(4*a*c-b^2)*x^2+1/2*(6*c*d^2+b)/(4*a*c-b^2)*
x+1/2*d/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((2*_R
^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*
c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2
+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,ce^3x^3+6\,cde^2x^2+2\,cd^3+\left(6\,cd^2+b\right)ex+bd}{2\left(\left(b^2c-4\,ac^2\right)e^5x^4+4\left(b^2c-4\,ac^2\right)de^4x^3+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^3x^2+2\left(2\left(b^2c-4\,ac^2\right)d^3+\left(b^3-4\,abc\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*e^3*x^3 + 6*c*d*e^2*x^2 + 2*c*d^3 + (6*c*d^2 + b)*e*x + b*d)/((b^
2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6
*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b
*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d
^2)*e) + 1/2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4
*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (
b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)
```

Fricas [B] time = 2.2738, size = 5304, normalized size = 20.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*e^3*x^3 + 12*c*d*e^2*x^2 + 4*c*d^3 + 2*(6*c*d^2 + b)*e*x - sqrt(1
/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) +
b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))
*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/2)*((a*b^
8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*s
qrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + b^3 +
 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) + s
qrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
 (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
 - 4*a*b*c)*d^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*
c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4
)) + b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d - 1/2*sqrt(1/2)*(
(a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*b^6
```

```
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) +
b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))
) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +
(b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*
d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c +
 (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
*e^4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c
^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/
2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))
 - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^
2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3
 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*
c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 -
64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3)*e^4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^
4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d - 1/2*sqrt
(1/2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a
^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + (b^5 - 8*a*b^3
*c + 16*a^2*b*c^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4
*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^
4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)
*e^2))) + 2*b*d)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3
 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*
c + (b^3 - 4*a*b*c)*d^2)*e)
```

Sympy [B] time = 35.2598, size = 578, normalized size = 2.28

$$\frac{bd + 2cd^3 + 6cde^2x^2 + 2ce^3x^3 + x\left(be + 6\right.}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2\left(8abce^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (b*d + 2*c*d**3 + 6*c*d*e**2*x**2 + 2*c*e**3*x**3 + x*(b*e + 6*c*d**2*e))/(
8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e
- 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*
d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b*
*3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 -
 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**7*c**6*e*
*4 - 1572864*a**6*b**2*c**5*e**4 + 983040*a**5*b**4*c**4*e**4 - 327680*a**4
*b**6*c**3*e**4 + 61440*a**3*b**8*c**2*e**4 - 6144*a**2*b**10*c*e**4 + 256*
a*b**12*e**4) + _t**2*(-12288*a**4*b*c**4*e**2 + 8192*a**3*b**3*c**3*e**2 -
 1536*a**2*b**5*c**2*e**2 + 16*b**9*e**2) + 16*a**2*c**3 + 24*a*b**2*c**2 +
 9*b**4*c, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4*e**3 - 8192*_t**3*a
**4*b**2*c**3*e**3 + 512*_t**3*a**2*b**6*c*e**3 - 64*_t**3*a*b**8*e**3 - 12
8*_t*a**2*b*c**2*e - 16*_t*a*b**3*c*e - 4*_t*b**5*e + 4*a*c**2*d + 3*b**2*c
*d)/(4*a*c**2*e + 3*b**2*c*e))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^2, x)
```

3.624 $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=96

$$\frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{b+2c(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[Out] -(b + 2*c*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)) + (2*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)
^(3/2)*e)
```

Rubi [A] time = 0.122484, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1142, 1107, 614, 618, 206}

$$\frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{b+2c(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(b + 2*c*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)) + (2*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)
^(3/2)*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{c\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{\left(b^2-4ac\right)e}$$

$$= -\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2\right.}{\left(b^2-4ac\right)e}$$

$$= -\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A] time = 0.133052, size = 98, normalized size = 1.02

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}}{2e\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*ArcTan[(
b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a
*c)*e)
```

Maple [C] time = 0.02, size = 270, normalized size = 2.8

$$\frac{1}{ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a}\left(\frac{cex^2}{4\,ac-b^2}+2\,\frac{cdx}{4\,ac-b^2}+\frac{2\,cd^2+b}{2\,e\left(4\,ac-b^2\right)}\right)+\frac{}{e(4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (c*e/(4*a*c-b^2)*x^2+2*c*d/(4*a*c-b^2)*x+1/2/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)+c/(4*a*c-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c
```

```
*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2c\int -\frac{ex+d}{(b^2c-4\,ac^2)e^4x^4+4\,(b^2c-4\,ac^2)de^3x^3+(b^2c-4\,ac^2)d^4+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^2x^2+ab^2-4\,a^2c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 2*c*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d
*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2
)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*
d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2
+ b)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*
a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^
3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4
*a*b*c)*d^2)*e)
```

Fricas [B] time = 2.02356, size = 2214, normalized size = 23.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*a*
b*c + 2*(b^2*c - 4*a*c^2)*d^2 + 2*(c^2*e^4*x^4 + 4*c^2*d*e^3*x^3 + c^2*d^4
+ (6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3 + b*c*d)*e*x + a*c)*sq
rt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2
*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c -
 (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*
c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x
 + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c +
16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c
^2)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 - 4*(c^2*e^4*x^4 + 4*c^
2*d*e^3*x^3 + c^2*d^4 + (6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3
+ b*c*d)*e*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2
*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8
*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2
 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

Sympy [B] time = 25.6904, size = 495, normalized size = 5.16

$$-\frac{c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc+2c^2d^2}{2c^2e^2}\right)}{e}+\frac{c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*sqrt(-1/(
4*a*c - b**2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(
-1/(4*a*c - b**2)**3) + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + c*sqrt(-1/(4*
a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*sqrt(-1/(4*a*c - b**2)**
3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*sqrt(-1/(4*a*c - b**
2)**3) + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + (b + 2*c*d**2 + 4*c*d*e*x +
2*c*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e
- 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) +
x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*
d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*
c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

Giac [B] time = 3.04122, size = 494, normalized size = 5.15

$$-\frac{(b^2ce-4ac^2e)\sqrt{b^2-4ac}\log\left(\left|\left(b+\sqrt{b^2-4ac}\right)x^2e^2+2\left(b+\sqrt{b^2-4ac}\right)dxe+\left(b+\sqrt{b^2-4ac}\right)d^2+2a\right|\right)}{b^6e^2-12ab^4ce^2+48a^2b^2c^2e^2-64a^3c^3e^2}+\frac{(b^2ce-4a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -(b^2*c*e - 4*a*c^2*e)*sqrt(b^2 - 4*a*c)*log(abs((b + sqrt(b^2 - 4*a*c))*x^
2*e^2 + 2*(b + sqrt(b^2 - 4*a*c))*d*x*e + (b + sqrt(b^2 - 4*a*c))*d^2 + 2*a
))/(b^6*e^2 - 12*a*b^4*c*e^2 + 48*a^2*b^2*c^2*e^2 - 64*a^3*c^3*e^2) + (b^2*
c*e - 4*a*c^2*e)*sqrt(b^2 - 4*a*c)*log(abs(-(b - sqrt(b^2 - 4*a*c))*x^2*e^2
 - 2*(b - sqrt(b^2 - 4*a*c))*d*x*e - (b - sqrt(b^2 - 4*a*c))*d^2 - 2*a))/(b
^6*e^2 - 12*a*b^4*c*e^2 + 48*a^2*b^2*c^2*e^2 - 64*a^3*c^3*e^2) - 1/2*(2*c*x
^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2
*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*
a*c*e))
```

3.625 $\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=299

$$\frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}ae\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4a\ldots}\right.}{2\sqrt{2}ae\left(b\ldots\right.}$$

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(2*a*(b^2 - 4*a*c)*(a + b*e
^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*
(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.701916, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1106, 1092, 1166, 205}

$$\frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}ae\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4a\ldots}\right.}{2\sqrt{2}ae\left(b\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(2*a*(b^2 - 4*a*c)*(a + b*e
^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*
(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]*e)
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+be^2x^2+ce^4x^4\right)^2}\,dx, x, \frac{d}{e}+x\right)$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{b^2e^4-2ace^4-2\left(b^2e^4-4ace^4\right)-\cdots}{a+be^2x^2+ce^4x^4}\cdots\right.}{2a\left(b^2-4ac\right)e\cdots}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)e^2\right)\cdots}{4a\cdots}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\text{ta}\cdots}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\cdots}}$$

Mathematica [A] time = 1.00214, size = 271, normalized size = 0.91

$$\frac{\frac{2(d+ex)\left(-2ac+b^2+bc(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] ((2*(d + e*x)*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x
)^2*(b + c*(d + e*x)^2))) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b
^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 1
2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a*
e)
```

Maple [C] time = 0.017, size = 364, normalized size = 1.2

$$\frac{1}{ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a}\left(-\frac{bce^2x^3}{2\,a\left(4\,ac-b^2\right)}-\frac{3\,bcdex^2}{2\,a\left(4\,ac-b^2\right)}+\frac{\left(-3\,cd^2b\right.}{2\,a\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (-1/2*b*c*e^2/a/(4*a*c-b^2)*x^3-3/2*d*b*c*e/a/(4*a*c-b^2)*x^2+1/2*(-3*b*c*d
^2+2*a*c-b^2)/a/(4*a*c-b^2)*x+1/2*d/e*(-b*c*d^2+2*a*c-b^2)/a/(4*a*c-b^2))/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)+1/4/a/(4*a*c-b^2)/e*sum((-_R^2*b*c*e^2-2*_R*b*c*d*e-b*c*d^2+6*a
*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-
_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e
+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bce^3x^3+3\,bcde^2x^2+bcd^3+\left(3\,bcd^2+b^2-2\,ac\right)ex+\left(\right.}{2\left(\left(ab^2c-4\,a^2c^2\right)e^5x^4+4\left(ab^2c-4\,a^2c^2\right)de^4x^3+\left(ab^3-4\,a^2bc+6\left(ab^2c-4\,a^2c^2\right)d^2\right)e^3x^2+2\left(2\left(ab^2c-4\,a^2c^2\right)d^3+\left(\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*e^3*x^3 + 3*b*c*d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - 2*a*c)*e*
x + (b^2 - 2*a*c)*d)/((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^
2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 +
2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c -
 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e) - 1/2*int
egrate(-(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 6*a*c)/((b^2*c - 4*a*c
^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a
*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a
```

Fricas [B] time = 2.62106, size = 6835, normalized size = 22.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*e^3*x^3 + 6*b*c*d*e^2*x^2 + 2*b*c*d^3 + 2*(3*b*c*d^2 + b^2 - 2*a
*c)*e*x - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2
)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2
*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c -
4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5
 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 6
4*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*
c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
```

```
^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x
 + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d + 1/2*sqrt(1/2)*((a^3*b^9 - 2
0*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((
b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
64*a^9*c^3)*e^4)) - (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 +
 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c
^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^
6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) + sqrt(1/2)*((a*b^2*
c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2
*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^
3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4
*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2
 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 1
8*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*
c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*l
og((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3
 + 324*a^2*c^4)*d - 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^
2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^
2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)) - (b^8 - 2
3*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5
- 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64
*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3)*e^2))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(
a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^
2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e
^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)
*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a
^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3
 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d + 1/2*sqrt
(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^
7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)) + (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2
- 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2
- (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18
*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c
^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) -
 sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x
^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^
2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2
)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b
^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3
)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4
*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d - 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18*
a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)*e^4)) + (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*
c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^
4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) + 2*(b^2 - 2*a*c)*d)/((a*b^2*
c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2
*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^
3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4
```

```
*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)
```

Sympy [B] time = 24.5303, size = 740, normalized size = 2.47

$$-\frac{-2acd+b^2d+bcd^3+3bcde^2x^2+bce^3x^3}{8a^3ce-2a^2b^2e+8a^2bcd^2e+8a^2c^2d^4e-2ab^3d^2e-2ab^2cd^4e+x^4\left(8a^2c^2e^5-2ab^2ce^5\right)+x^3\left(32a^2c^2de^4-8ab^2cde^4\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -(-2*a*c*d + b**2*d + b*c*d**3 + 3*b*c*d*e**2*x**2 + b*c*e**3*x**3 + x*(-2*
a*c*e + b**2*e + 3*b*c*d**2*e))/(8*a**3*c*e - 2*a**2*b**2*e + 8*a**2*b*c*d*
*2*e + 8*a**2*c**2*d**4*e - 2*a*b**3*d**2*e - 2*a*b**2*c*d**4*e + x**4*(8*a
**2*c**2*e**5 - 2*a*b**2*c*e**5) + x**3*(32*a**2*c**2*d*e**4 - 8*a*b**2*c*d
*e**4) + x**2*(8*a**2*b*c*e**3 + 48*a**2*c**2*d**2*e**3 - 2*a*b**3*e**3 - 1
2*a*b**2*c*d**2*e**3) + x*(16*a**2*b*c*d*e**2 + 32*a**2*c**2*d**3*e**2 - 4*
a*b**3*d*e**2 - 8*a*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**9*c**6*e
**4 - 1572864*a**8*b**2*c**5*e**4 + 983040*a**7*b**4*c**4*e**4 - 327680*a**
6*b**6*c**3*e**4 + 61440*a**5*b**8*c**2*e**4 - 6144*a**4*b**10*c*e**4 + 256
*a**3*b**12*e**4) + _t**2*(-61440*a**5*b*c**5*e**2 + 61440*a**4*b**3*c**4*e
**2 - 24064*a**3*b**5*c**3*e**2 + 4608*a**2*b**7*c**2*e**2 - 432*a*b**9*c*e
**2 + 16*b**11*e**2) + 1296*a**2*c**5 - 360*a*b**2*c**4 + 25*b**4*c**3, Lam
bda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4*e**3 - 28672*_t**3*a**6*b**3*c*
*3*e**3 + 9216*_t**3*a**5*b**5*c**2*e**3 - 1280*_t**3*a**4*b**7*c*e**3 + 64
*_t**3*a**3*b**9*e**3 + 1728*_t*a**4*c**4*e - 2304*_t*a**3*b**2*c**3*e + 74
0*_t*a**2*b**4*c**2*e - 92*_t*a*b**6*c*e + 4*_t*b**8*e + 324*a**2*c**4*d -
81*a*b**2*c**3*d + 5*b**4*c**2*d)/(324*a**2*c**4*e - 81*a*b**2*c**3*e + 5*b
**4*c**2*e))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^(-2), x)
```

3.626 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=162

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}+\frac{\log(d+ex)}{a^2e}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d\right.}$$

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 -
4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e) + Log[d + e*x]/(a^2*e) - Log[a + b*(
d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e)
```

Rubi [A] time = 0.294215, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}+\frac{\log(d+ex)}{a^2e}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 -
4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e) + Log[d + e*x]/(a^2*e) - Log[a + b*(
d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^2}dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^2}dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac-bcx}{x(a+bx+cx^2)}dx, x, \right.}{2a\left(b^2-4ac\right)e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-b^2+4ac}{ax} + \frac{b(b^2-5}{a(}\right.\right.}{2a\left(b^2\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2e} - \frac{\text{Subst}\left(\int \frac{b(}{}\right.}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2e} - \frac{\text{Subst}\left(\int \frac{}{a+}\right.}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2e} - \frac{\log\left(a+b(d\right.}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d}{\sqrt{b^2-}}\right.}{2a^2\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A] time = 0.428707, size = 235, normalized size = 1.45

$$\frac{\frac{2a\left(-2ac+b^2+bc(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(\sqrt{b^2-4}\right.}{\left(b^2-4ac\right)^{3/2}}}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b +
 c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c
] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/
(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^2*e)
```

Maple [C] time = 0.03, size = 693, normalized size = 4.3

$$\frac{\ln(ex+d)}{a^2e} - \frac{bcex^2}{2a\left(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a\right)\left(4ac-b^2\right)} - \frac{}{a\left(ce^4x^4+4cde^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] ln(e*x+d)/a^2/e-1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*c*e/(4*a*c-b^2)*x^2-1/a/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b
*c*d/(4*a*c-b^2)*x-1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c*d^2*b+1/(c*e^4*x^4+4*
c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/
e/(4*a*c-b^2)*c-1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2-1/2/a^2/(4*a*c-b^2)/e*
sum((c*e^3*(4*a*c-b^2)*_R^3+3*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*c^2*d^2-3*b^
2*c*d^2+5*a*b*c-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d)/(2*_R^3*c*e^
3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*
_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b
*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.37138, size = 5200, normalized size = 32.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*x
^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((b^
3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*a*b
*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b^3 -
 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 -
6*a*b^2*c)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 +
 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d
)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*
a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
2*(2*c*d^3 + b*d)*e*x + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 +
 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d
*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c -
```

```
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*l
og(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*x^4 + 4*(a^2*b^4
*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^
4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*x^2 + 2*(2*(a
^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^
4*b*c^2)*d)*e^2*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^
3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e
), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2
*x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + 2*
((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6
*a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b
^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^
4 - 6*a*b^2*c)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x
+ 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^
2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 +
4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e
*x + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*d)*e*x)*log(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)
*e^5*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*x^3 + (a^2*b^5
- 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d
^2)*e^3*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5
- 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c +
 16*a^4*b*c^2)*d^2)*e)]
```

Sympy [B] time = 167.586, size = 1091, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*e*(64*a**3*c**3 - 48*a*
*2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2*e))*log(2*d*x/e + x**2 + (-
32*a**4*c**2*e*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*e*(64*a*
*3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2*e)) + 16*a**
3*b**2*c*e*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*e*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2*e)) - 2*a**2*b**
4*e*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*e*(64*a**3*c**3 - 4
8*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2*e)) - 8*a**2*c**2 + 7*a
*b**2*c + 6*a*b*c**2*d**2 - b**4 - b**3*c*d**2)/(6*a*b*c**2*e**2 - b**3*c*e
**2)) + (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*e*(64*a**3*c**3
- 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2*e))*log(2*d*x/e + x*
*2 + (-32*a**4*c**2*e*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*e*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2*e)) +
16*a**3*b**2*c*e*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*e*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2*e)) - 2*a**
2*b**4*e*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*e*(64*a**3*c**3
```

```
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2*e)) - 8*a**2*c**2 +
 7*a*b**2*c + 6*a*b*c**2*d**2 - b**4 - b**3*c*d**2)/(6*a*b*c**2*e**2 - b**3
*c*e**2)) - (-2*a*c + b**2 + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2)/(8*a**
3*c*e - 2*a**2*b**2*e + 8*a**2*b*c*d**2*e + 8*a**2*c**2*d**4*e - 2*a*b**3*d
**2*e - 2*a*b**2*c*d**4*e + x**4*(8*a**2*c**2*e**5 - 2*a*b**2*c*e**5) + x**
3*(32*a**2*c**2*d*e**4 - 8*a*b**2*c*d*e**4) + x**2*(8*a**2*b*c*e**3 + 48*a*
*2*c**2*d**2*e**3 - 2*a*b**3*e**3 - 12*a*b**2*c*d**2*e**3) + x*(16*a**2*b*c
*d*e**2 + 32*a**2*c**2*d**3*e**2 - 4*a*b**3*d*e**2 - 8*a*b**2*c*d**3*e**2))
 + log(d/e + x)/(a**2*e)
```

Giac [B] time = 3.25002, size = 784, normalized size = 4.84

$$-\frac{\left(a^2b^5e-10\,a^3b^3ce+24\,a^4bc^2e\right)\sqrt{b^2-4\,ac}\log\left(\left|4\,a^3ce^4+2\left(a^2bc+\sqrt{b^2-4\,ac}a^2c\right)x^2e^6+4\left(a^2bc+\sqrt{b^2-4\,ac}a^2c\right)dx\right.\right.}{4\left(a^4b^6e^2-12\,a^5b^4ce^2+48\,a^6b^2c^2e^2-64\,a^7c^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(a^2*b^5*e - 10*a^3*b^3*c*e + 24*a^4*b*c^2*e)*sqrt(b^2 - 4*a*c)*log(ab
s(4*a^3*c*e^4 + 2*(a^2*b*c + sqrt(b^2 - 4*a*c)*a^2*c)*x^2*e^6 + 4*(a^2*b*c
+ sqrt(b^2 - 4*a*c)*a^2*c)*d*x*e^5 + 2*(a^2*b*c + sqrt(b^2 - 4*a*c)*a^2*c)*
d^2*e^4))/(a^4*b^6*e^2 - 12*a^5*b^4*c*e^2 + 48*a^6*b^2*c^2*e^2 - 64*a^7*c^3
*e^2) + 1/4*(a^2*b^5*e - 10*a^3*b^3*c*e + 24*a^4*b*c^2*e)*sqrt(b^2 - 4*a*c)
*log(abs(-4*a^3*c*e^4 - 2*(a^2*b*c - sqrt(b^2 - 4*a*c)*a^2*c)*x^2*e^6 - 4*(
a^2*b*c - sqrt(b^2 - 4*a*c)*a^2*c)*d*x*e^5 - 2*(a^2*b*c - sqrt(b^2 - 4*a*c)
*a^2*c)*d^2*e^4))/(a^4*b^6*e^2 - 12*a^5*b^4*c*e^2 + 48*a^6*b^2*c^2*e^2 - 64
*a^7*c^3*e^2) - 1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*
e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/a^2 + e^(-1
)*log(abs(x*e + d))/a^2 + 1/2*(a*b*c*x^2*e^2 + 2*a*b*c*d*x*e + a*b*c*d^2 +
a*b^2 - 2*a^2*c)*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c
*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2 - 4*a*c)*a^2)
```

3.627 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=348

$$-\frac{3b^2-10ac}{2a^2e\left(b^2-4ac\right)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4\right.}{2\sqrt{2}a^2e\left(b^2-\right.}$$

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d
 + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2
*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(3*b^3 - 16*
a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*
x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 1.65286, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1121, 1281, 1166, 205}

$$-\frac{3b^2-10ac}{2a^2e\left(b^2-4ac\right)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4\right.}{2\sqrt{2}a^2e\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```


```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d
 + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2
*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(3*b^3 - 16*
a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*
x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*e)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{-3b^2\ldots}{x^2(\ldots}\right.}{2a\ldots}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+\ldots\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+\ldots\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+\ldots\right.}$$

Mathematica [A] time = 1.7561, size = 339, normalized size = 0.97

$$\frac{\frac{2(d+ex)\left(-3abc-2ac^2(d+ex)^2+b^2c(d+ex)^2+b^3\right)}{\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-\ldots\right.}{\left(b^2-4ac\right)^{3\ldots}}}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2
```

```
]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e)
```

Maple [C] time = 0.027, size = 1304, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/a^2/e/(e*x+d)-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*e^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)*c*e^2/(4*a*c-b^2)*x^3*b^2-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+
4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c^2*e/(4*a*c-b^2)*x^2+3/2/
a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)*d*c*e/(4*a*c-b^2)*x^2*b^2-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c
^2*d^2+3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c*d^2-3/2/a/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a
*c-b^2)*x*b*c+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3-1/a/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e
/(4*a*c-b^2)*c^2+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b^2*c-3/2/a/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)*d/e/(4*a*c-b^2)*b*c+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+
4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^3-1/4/a^2/
(4*a*c-b^2)/e*sum((c*e^2*(10*a*c-3*b^2)*_R^2+2*c*d*e*(10*a*c-3*b^2)*_R+10*a
*c^2*d^2-3*b^2*c*d^2+13*a*b*c-3*b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^
2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d
^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.3395, size = 9384, normalized size = 26.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + 2
*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^
2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2*c
- 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x + sqrt(1/2)*((a^2*b^2*c - 4*a^3
*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c
+ 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3
 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c -
 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*
c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b
^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*
b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
 - 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5
- 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 25
00*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2
 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 -
918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) - (27*b^11 - 486*a*
b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5
*b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b
^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*
c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c +
 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3
+ 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c -
4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c
^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^
7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c
 + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b
^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 -
 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 250
0*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2
- 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6
- 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) - (27*b^11 - 486*a*b
^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*
b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918
*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^
4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c
^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c +
10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 +
 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4
*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^
2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^7
 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c
+ 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^
4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^1
2*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
```

```
 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 -
2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500
*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 -
 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) + (27*b^11 - 486*a*b^
9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b
*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^
5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*
a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 1
2*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^
2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 1
0*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 +
3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*
a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2
)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^7
- 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4
*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12
*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2
500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*
a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 -
1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 918
*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) + (27*b^11 - 486*a*b^9
*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*
c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5
*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12
*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))))/((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 +
 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c
 - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 -
 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4
 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2
*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.628 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=213

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3e\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2e\left(b^2-4ac\right)(d+ex)^2}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3e}-\frac{2b\log(d+}{a^3e}$$

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)^2)) + (b^2 - 2*a*c + b*c*(d
+ e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)
^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^
2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e) - (2*b*Log[d + e*x])/(a^3*e) + (b*
Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e)
```

Rubi [A] time = 0.390107, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3e\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2e\left(b^2-4ac\right)(d+ex)^2}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3e}-\frac{2b\log(d+}{a^3e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```


```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)^2)) + (b^2 - 2*a*c + b*c*(d
+ e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)
^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^
2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e) - (2*b*Log[d + e*x])/(a^3*e) + (b*
Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntegerQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{-2(b^2-3}{x^2(a+b}\right.}{2a(}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int\left(\frac{2(-b^2+}{ax}\right.\right.}{}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)\,e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,e(d+ex)^2\left(a+b(d+ex)^2+c(d\right.}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)\,e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,e(d+ex)^2\left(a+b(d+ex)^2+c(d\right.}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)\,e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,e(d+ex)^2\left(a+b(d+ex)^2+c(d\right.}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)\,e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,e(d+ex)^2\left(a+b(d+ex)^2+c(d\right.}$$

Mathematica [A] time = 0.509979, size = 284, normalized size = 1.33

$$\frac{\frac{\left(6a^2c^2+b^3\sqrt{b^2-4ac}-6ab^2c-4abc\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}} + \frac{\left(-6a^2c^2+b^3\sqrt{b^2-4ac}+6ab^2c-4abc\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}} +}{2a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*
x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d + e
*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[
b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)
^(3/2))/(2*a^3*e)
```

Maple [C] time = 0.036, size = 1014, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2/a^2/e/(e*x+d)^2-2*b*ln(e*x+d)/a^3/e-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*e/(4*a*c-b^2)
*x^2+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)*c*e/(4*a*c-b^2)*x^2*b^2-2/a/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*d/(4
*a*c-b^2)*x+1/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*d/(4*a*c-b^2)*x*b^2-1/a/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*
a*c-b^2)*c^2*d^2+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2*c*d^2-3/2/a/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)/e/(4*a*c-b^2)*b*c+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^3+1/a^3/(4*a*c
-b^2)/e*sum((b*e^3*c*(4*a*c-b^2)*_R^3+3*b*d*e^2*c*(4*a*c-b^2)*_R^2+e*(12*a*
b*c^2*d^2-3*b^3*c*d^2-3*a^2*c^2+5*a*b^2*c-b^4)*_R+4*a*b*c^2*d^3-b^3*c*d^3-3
*a^2*c^2*d+5*a*b^2*c*d-b^4*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c
*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+
b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 5.09061, size = 9488, normalized size = 44.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^
2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*
(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a
^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^
5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a
^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + ((b^4*c - 6*
a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*
x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b
^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5
 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)
*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^
2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2
*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*
e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^
3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
```

b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*x^6 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*x^5 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*x^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*e^2*x + ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*e), -1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 +

```
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a
*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2
)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8
*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*x^6 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*
c^3)*d*e^6*x^5 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*
a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 +
16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*x^3 + (a^4*
b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3
)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*x^2 + 2*(3*(a^3*b
^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*
b*c^2)*d^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*e^2*x + ((a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^
4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.83423, size = 302, normalized size = 1.42

$$\frac{\left(b^4-6\,ab^2c+6\,a^2c^2\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^2}}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{\left(a^3b^2-4\,a^4c\right)\sqrt{-b^2+4\,ac}}+\frac{be^{(-1)}\log\left(c+\frac{b}{(xe+d)^2}+\frac{a}{(xe+d)^4}\right)}{2\,a^3}+\frac{\frac{b^3c-3\,abc^2}{a}+\frac{\left(b^4e-4\,ab^2ce+2\,a^2c^2e\right)e^{(-1)}}{(xe+d)^2a}}{2\left(b^2-4\,ac\right)a^2\left(c+\frac{b}{(xe+d)^2}+\frac{a}{(xe+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan(-(b + 2*a/(x*e + d)^2)/sqrt(-b^2 + 4*a
*c))*e^(-1)/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) + 1/2*b*e^(-1)*log(c +
 b/(x*e + d)^2 + a/(x*e + d)^4)/a^3 + 1/2*((b^3*c - 3*a*b*c^2)/a + (b^4*e -
 4*a*b^2*c*e + 2*a^2*c^2*e)*e^(-1)/((x*e + d)^2*a))*e^(-1)/((b^2 - 4*a*c)*a
^2*(c + b/(x*e + d)^2 + a/(x*e + d)^4)) - 1/2*e^(-1)/((x*e + d)^2*a^2)
```

3.629 $\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=408

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2\cdots}\right.}{2\sqrt{2}a^3e\left(b^2-4ac\right)^{3/2}\sqrt{\cdots}}$$

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*(d + e*x)^3) + (b*(5*b^2 - 19*a*c)
)/(2*a^3*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*
(b^2 - 4*a*c)*e*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]
*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(5*b^4 -
29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 -
 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 3.6755, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1121, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2\cdots}\right.}{2\sqrt{2}a^3e\left(b^2-4ac\right)^{3/2}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```


```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*(d + e*x)^3) + (b*(5*b^2 - 19*a*c)
)/(2*a^3*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*
(b^2 - 4*a*c)*e*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]
*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(5*b^4 -
29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 -
 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{-5b}{x^4}\right.}{2a}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^3\left(a+b(d+ex)^2+\right.}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)e(d+ex)} + \frac{}{2a\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)e(d+ex)} + \frac{}{2a\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)e(d+ex)} + \frac{}{2a\left(b^2-4ac\right)}$$

Mathematica [A] time = 3.25593, size = 384, normalized size = 0.94

$$\frac{\frac{6(d+ex)\left(2a^2c^2-4ab^2c-3abc^2(d+ex)^2+b^3c(d+ex)^2+b^4\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} + \frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2+5b^3\sqrt{b^2-4ac}-29ab^2c-19abc\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(-28a^2\right.}{}}{12a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2*
a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (
d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c +
 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c
- 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e)
```

Maple [C] time = 0.034, size = 1518, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/3/a^2/e/(e*x+d)^3+2/a^3*b/e/(e*x+d)+3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*e^2*c^2/(4*a*
c-b^2)*x^3-1/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b^3*e^2*c/(4*a*c-b^2)*x^3+9/2/a^2/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)*d*b*e*c^2/(4*a*c-b^2)*x^2-3/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b^3*e*c/(4*a*c-b^2)*x^
2+9/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*c^2*d^2-3/2/a^3/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-
b^2)*x*b^3*c*d^2-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c^2+2/a^2/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a
*c-b^2)*x*b^2*c-1/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^4+3/2/a^2/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
*d^3/e/(4*a*c-b^2)*b*c^2-1/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b^3*c-1/a/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)*d/e/(4*a*c-b^2)*c^2+2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^2*c-1/
2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^4+1/4/a^3/(4*a*c-b^2)/e*sum((b*e^2*c*(
19*a*c-5*b^2)*_R^2+2*b*d*e*c*(19*a*c-5*b^2)*_R+19*a*b*c^2*d^2-5*b^3*c*d^2-1
4*a^2*c^2+24*a*b^2*c-5*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d
^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*
e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 4.71913, size = 12725, normalized size = 31.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 36*(5*b^3*c - 19*a*b*c^2)*d*e^5*x^
5 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19*a*b*c^2)*d^2)*e^
4*x^4 + 6*(5*b^3*c - 19*a*b*c^2)*d^6 + 8*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (
15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e^3*x^3 + 2*(15*b^4 - 62*a*b^2*c + 14*
a^2*c^2)*d^4 + 2*(45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6
*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 4*a^2*b^2 + 16*a^3*c + 2
0*(a*b^3 - 4*a^2*b*c)*d^2 + 4*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 6
2*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x - 3*sqrt(1/2)*(
(a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^
3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2
*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c
 + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 +
 (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^
2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a
^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d
^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9
- 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7
*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 82
50*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 2
4108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c
^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^1
0*c^3)*e^2))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 5042
1*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410
*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7*
b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c
^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^
2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^
6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)) - (125
*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4
*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(
-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c
^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*
b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a
^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
 - 64*a^10*c^3)*e^2))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(
a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4
*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^
4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 1
0*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10
*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^2
*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d
^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4
*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 24
15*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
```

- 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 836
30*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^
14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 -
12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))*log((1125*b^8*c^4 - 1232
5*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (
1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 96
04*a^4*c^8)*d - 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2
 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*b
^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4
*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^
16*b^2*c^2 - 64*a^17*c^3)*e^4)) - (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^1
0*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408
*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5
*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*
c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*
c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a
^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))) + 3*sqrt(1/2)*
((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a
^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^
2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*
c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3
+ (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b
^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*
a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*
d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9
 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^
7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8
250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 -
24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*
c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^
10*c^3)*e^2))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 504
21*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 4341
0*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7
*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*
c^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c
^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c
^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)) + (12
5*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^
4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt
(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*
c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625
*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b
^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*
a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^
2 - 64*a^10*c^3)*e^2))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*
(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c -
4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a
^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 +
10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 1
0*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^
2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*
d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^
4*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2
415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83
630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a
^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 -
 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))*log((1125*b^8*c^4 - 123

```
25*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x +
(1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9
604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^
2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*
b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a
^16*b^2*c^2 - 64*a^17*c^3)*e^4)) + (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^
10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 7940
8*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^
5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^
9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8
*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6
*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((
a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))))/((a^3*b^2*c
- 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a
^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*
c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*
b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b
^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c
)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4
 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b
^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^4), x)
```

3.630 $\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx$

Optimal. Leaf size=341

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+\right.}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}}$$

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) - ((d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))/(8*(b^2
 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(3*b^2 + 4*
a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(
5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.950579, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1120, 1178, 1166, 205}

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+\right.}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```


```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) - ((d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))/(8*(b^2
 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(3*b^2 + 4*
a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(
5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{2a-5bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ \cdots\right)}{4\left(b^2-4ac\right)e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4ac+12bc(d \cdots\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+ \cdots\right.}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4ac+12bc(d \cdots\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+ \cdots\right.}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4ac+12bc(d \cdots\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+ \cdots\right.}$$

Mathematica [A] time = 4.78153, size = 328, normalized size = 0.96

$$\frac{\frac{(d+ex)\left(4ac-7b^2-12bc(d+ex)^2\right)}{\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3\sqrt{2}\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{2}\sqrt{c}\left(2b\sqrt{b^2-4ac}+4a \cdots\right.}{\left(b^2-4ac\right)^{5 \cdots}}}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/((b^2
 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*(3*b^
2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
- (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]))/(8*e)
```

Maple [C] time = 0.042, size = 704, normalized size = 2.1

$$\frac{1}{\left(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a\right)^2}\left(-\frac{3\,c^2e^6bx^7}{32\,a^2c^2-16\,ab^2c+2\,b^4}-\frac{21\,c^2de^5bx}{32\,a^2c^2-16\,ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (-3/2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-21/2*c^2*d*e^5*b/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^6+1/8*(-252*b*c*d^2+4*a*c-19*b^2)*c*e^4/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^5+5/8*c*d*e^3*(-84*b*c*d^2+4*a*c-19*b^2)/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^4-1/8*e^2*(420*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+16*a*b*c+5*b^3)/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(252*b*c^2*d^4-40*a*c^2*d^2+190*b^2*
c*d^2+48*a*b*c+15*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(84*b*c^2*d^6-20*
a*c^2*d^4+95*b^2*c*d^4+48*a*b*c*d^2+15*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x-1/8*d/e*(12*b*c^2*d^6-4*a*c^2*d^4+19*b^2*c*d^4+16*a*b*c*
d^2+5*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+
3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*d*e-4*b*c*d
^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)
*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c
*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.93006, size = 14348, normalized size = 42.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*b*c^2*e^7*x^7 + 168*b*c^2*d*e^6*x^6 + 2*(252*b*c^2*d^2 + 19*b^2*c
 - 4*a*c^2)*e^5*x^5 + 24*b*c^2*d^7 + 10*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2
)*d)*e^4*x^4 + 2*(420*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^2
)*d^2)*e^3*x^3 + 2*(19*b^2*c - 4*a*c^2)*d^5 + 2*(252*b*c^2*d^5 + 10*(19*b^2
*c - 4*a*c^2)*d^3 + 3*(5*b^3 + 16*a*b*c)*d)*e^2*x^2 + 2*(5*b^3 + 16*a*b*c)*
d^3 + 2*(84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 + 12*a^2*c + 3
*(5*b^3 + 16*a*b*c)*d^2)*e*x - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2
)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*
e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 -
 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c
+ 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*
d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*
c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c
^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c
+ 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)
))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b
^2*c^4 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*
x + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d + 3/2*sqrt(1/2)*((a*b^13 - 8*
a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a
^6*b^3*c^5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^
4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) - (b^8
 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c +
80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 +
 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 16
0*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/(
(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c
^4 - 1024*a^6*c^5)*e^2))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e
^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^
3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^
3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4
*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c
 + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*
a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 3
2*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^
6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4
 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c +
 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3
+ 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 1
60*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/
((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*
c^4 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*x +
 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d - 3/2*sqrt(1/2)*((a*b^13 - 8*a^2
*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*
```

b^3*c^5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) - (b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*x + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d + 3/2*sqrt(1/2)*((a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) + (b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*x + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d - 3/2*sqrt(1/2)*((a*b^13 - 8*a^2*b^11*

```
c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^
5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2
 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) + (b^8 - 8*a*b^
6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*
c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5
*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6
*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 -
 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024
*a^6*c^5)*e^2))) + 6*(a*b^2 + 4*a^2*c)*d)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)
*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e
^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 -
6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c +
 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d
^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^3, x)
```

3.631 $\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=150

$$\frac{2a+b(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3b\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}$$

```
[Out] (2*a + b*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)^2) - (3*b*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (3*b*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]
)/((b^2 - 4*a*c)^(5/2)*e)
```

Rubi [A] time = 0.196141, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1142, 1114, 638, 614, 618, 206}

$$\frac{2a+b(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3b\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (2*a + b*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)^2) - (3*b*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (3*b*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]
)/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, (\right.}{4\left(b^2-4ac\right)e}$$

$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)^2 e\left(a+b(d+ex)^2+\right.}$$

$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)^2 e\left(a+b(d+ex)^2+\right.}$$

$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)^2 e\left(a+b(d+ex)^2+\right.}$$

Mathematica [A] time = 0.223177, size = 146, normalized size = 0.97

$$\frac{\frac{\left(b^2-4ac\right)\left(2a+b(d+ex)^2\right)}{\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)^2} - \frac{12bc\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{3b\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((b^2 -
 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2 - (1
2*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])
/(4*(b^2 - 4*a*c)^2*e)
```

Maple [C] time = 0.038, size = 544, normalized size = 3.6

$$\frac{1}{\left(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a\right)^2}\left(-\frac{3\,c^2e^5bx^6}{32\,a^2c^2-16\,ab^2c+2\,b^4}-9\,\frac{c^2de^4bx^5}{16\,a^2c^2-8\,ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (-3/2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*e^4*d*b*c^2/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^5-9/4*b*c*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-3*
c*d*e^2*b*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*b*e*(45*c^2*d^4
+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-d*b*(9*c^2*d^4+9*b*c*
d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/4/e*(6*b*c^2*d^6+9*b^2*c*d^4+
10*a*b*c*d^2+2*b^3*d^2+8*a^2*c+a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2+3/2*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R
^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4
*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a
))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.2829, size = 7896, normalized size = 52.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^5*x^5
 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*x^4 + 6*(b^3*
c^2 - 4*a*b*c^3)*d^6 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^
2*c^2)*d)*e^3*x^3 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^2*c
^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 +
27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*x^2 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^
2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a
*b^3*c - 20*a^2*b*c^2)*d)*e*x - 6*(b*c^3*e^8*x^8 + 8*b*c^3*d*e^7*x^7 + 2*(1
4*b*c^3*d^2 + b^2*c^2)*e^6*x^6 + b*c^3*d^8 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)
*e^5*x^5 + 2*b^2*c^2*d^6 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c
^2)*e^4*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3
*x^3 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + 2*(14*b*c^3*d^6 + 15*b^2*c
^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*x^2 + a^2*b*c + 4*(2*b*c^
```

```
3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*x)*sqrt(b^2
- 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 +
b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e
^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3
*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))
/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
- 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^
2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e
^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 +
 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*
a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*
b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*
e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 +
 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 +
32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^
2 - 64*a^4*b*c^3)*d^2)*e), -1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*x^6 + 36*(b^3*
c^2 - 4*a*b*c^3)*d*e^5*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c
^3)*d^2)*e^4*x^4 + 6*(b^3*c^2 - 4*a*b*c^3)*d^6 + 12*(10*(b^3*c^2 - 4*a*b*c^
3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*x^3 + a*b^4 + 4*a^2*b^2*c - 32*a^3*
c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b
^3*c^2 - 4*a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*x^2 + 2*(b^5 +
a*b^3*c - 20*a^2*b*c^2)*d^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4
*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*x - 12*(b*c^3*e^8*x^8
 + 8*b*c^3*d*e^7*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*x^6 + b*c^3*d^8 + 4*(
14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*x^5 + 2*b^2*c^2*d^6 + (70*b*c^3*d^4 + 30*b^
2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 +
 (b^3*c + 2*a*b*c^2)*d)*e^3*x^3 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 +
 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^
2*x^2 + a^2*b*c + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a
*b*c^2)*d^3)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c
*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)
*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3
 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b
^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5
*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a
^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 +
3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e
^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*
```

```
b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d
^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 17.3666, size = 873, normalized size = 5.82

$$-\frac{3\left(b^5ce - 8\,ab^3c^2e + 16\,a^2bc^3e\right)\sqrt{b^2-4\,ac}\log\left(\left|\left(b+\sqrt{b^2-4\,ac}\right)x^2e^2 + 2\left(b+\sqrt{b^2-4\,ac}\right)dxe + \left(b+\sqrt{b^2-4\,ac}\right)d^2 + 2\,a\right.\right.}{2\left(b^{10}e^2 - 20\,ab^8ce^2 + 160\,a^2b^6c^2e^2 - 640\,a^3b^4c^3e^2 + 1280\,a^4b^2c^4e^2 - 1024\,a^5c^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3/2*(b^5*c*e - 8*a*b^3*c^2*e + 16*a^2*b*c^3*e)*sqrt(b^2 - 4*a*c)*log(abs((
b + sqrt(b^2 - 4*a*c))*x^2*e^2 + 2*(b + sqrt(b^2 - 4*a*c))*d*x*e + (b + sqr
t(b^2 - 4*a*c))*d^2 + 2*a))/(b^10*e^2 - 20*a*b^8*c*e^2 + 160*a^2*b^6*c^2*e^
2 - 640*a^3*b^4*c^3*e^2 + 1280*a^4*b^2*c^4*e^2 - 1024*a^5*c^5*e^2) + 3/2*(b
^5*c*e - 8*a*b^3*c^2*e + 16*a^2*b*c^3*e)*sqrt(b^2 - 4*a*c)*log(abs(-(b - sq
rt(b^2 - 4*a*c))*x^2*e^2 - 2*(b - sqrt(b^2 - 4*a*c))*d*x*e - (b - sqrt(b^2
- 4*a*c))*d^2 - 2*a))/(b^10*e^2 - 20*a*b^8*c*e^2 + 160*a^2*b^6*c^2*e^2 - 64
0*a^3*b^4*c^3*e^2 + 1280*a^4*b^2*c^4*e^2 - 1024*a^5*c^5*e^2) - 1/4*(6*b*c^2
*x^6*e^6 + 36*b*c^2*d*x^5*e^5 + 90*b*c^2*d^2*x^4*e^4 + 120*b*c^2*d^3*x^3*e^
3 + 90*b*c^2*d^4*x^2*e^2 + 36*b*c^2*d^5*x*e + 6*b*c^2*d^6 + 9*b^2*c*x^4*e^4
 + 36*b^2*c*d*x^3*e^3 + 54*b^2*c*d^2*x^2*e^2 + 36*b^2*c*d^3*x*e + 9*b^2*c*d
^4 + 2*b^3*x^2*e^2 + 10*a*b*c*x^2*e^2 + 4*b^3*d*x*e + 20*a*b*c*d*x*e + 2*b^
3*d^2 + 10*a*b*c*d^2 + a*b^2 + 8*a^2*c)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d
^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^
4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

3.632 $\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=363

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20\right.}{8\sqrt{2}ae\left(b^2-\right.}$$

```
[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)^2) + ((d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*(d + e*x
)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c
]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 1.03918, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1119, 1178, 1166, 205}

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20\right.}{8\sqrt{2}ae\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)^2) + ((d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*(d + e*x
)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c
]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{\text{Subst}\left(\int \frac{b-10cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right.}{4\left(b^2-4ac\right)e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b\left(b^2+8ac\right)+c\left(b^2+20\right.\right.}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b\left(b^2+8ac\right)+c\left(b^2+20\right.\right.}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b\left(b^2+8ac\right)+c\left(b^2+20\right.\right.}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2\right.}$$

Mathematica [A] time = 5.05696, size = 382, normalized size = 1.05

$$\frac{\frac{2(d+ex)\left(8abc+20ac^2(d+ex)^2+b^2c(d+ex)^2+b^3\right)}{a\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{4\left(b(d+ex)+2c(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \cdots}{16e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 20*a*c
^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
(Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^
3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*e)
```

Maple [C] time = 0.043, size = 885, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (1/8*c^2*e^6*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7+7/8*c^2*d*e^5*(2
0*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6+1/8*(420*a*c^2*d^2+21*b^2*c*d^2
+28*a*b*c+2*b^3)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5+5/8*c*d*e^3*(140*a*
c^2*d^2+7*b^2*c*d^2+28*a*b*c+2*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4+1/8*e^
2*(700*a*c^3*d^4+35*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+36*a^2*c^2+5*a
*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3+1/8*d*e*(420*a*c^3*d^4+21*b^2*
c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+108*a^2*c^2+15*a*b^2*c+3*b^4)/(16*a^2*
c^2-8*a*b^2*c+b^4)/a*x^2+1/8*(140*a*c^3*d^6+7*b^2*c^2*d^6+140*a*b*c^2*d^4+1
0*b^3*c*d^4+108*a^2*c^2*d^2+15*a*b^2*c*d^2+3*b^4*d^2+16*a^2*b*c-a*b^3)/(16*
a^2*c^2-8*a*b^2*c+b^4)/a*x+1/8*d/e*(20*a*c^3*d^6+b^2*c^2*d^6+28*a*b*c^2*d^4
+2*b^3*c*d^4+36*a^2*c^2*d^2+5*a*b^2*c*d^2+b^4*d^2+16*a^2*b*c-a*b^3)/(16*a^2
*c^2-8*a*b^2*c+b^4)/a)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/a/e*s
um((c*e^2*(20*a*c+b^2)*_R^2+2*c*d*e*(20*a*c+b^2)*_R+20*a*c^2*d^2+b^2*c*d^2-
16*a*b*c+b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)
*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c
*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 4.94417, size = 16612, normalized size = 45.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*e^7*x^7 + 14*(b^2*c^2 + 20*a*c^3)*d*e^6*x^6 +
2*(2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*e^5*x^5 + 10*(7*(b^2
*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)*d)*e^4*x^4 + 2*(b^2*c^2 + 20*
a*c^3)*d^7 + 2*(35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*c + 36*a^2*c^2
+ 20*(b^3*c + 14*a*b*c^2)*d^2)*e^3*x^3 + 4*(b^3*c + 14*a*b*c^2)*d^5 + 2*(21
*(b^2*c^2 + 20*a*c^3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*
c + 36*a^2*c^2)*d)*e^2*x^2 + 2*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 + 2*(7*(b
^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c +
 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*x - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*
b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*
d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a
^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (
a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c
^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*
(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2
+ 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3
*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*
a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16
*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6
*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^
2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6
*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2
)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10
 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 10
24*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b
^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^
5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1
280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15
000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000
*a^2*b^2*c^4 + 10000*a^3*c^5)*d + 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*b^12*c
+ 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5
 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*sqrt((b^4 - 50*a*b^2*c + 625*a^2
*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*
a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)) - (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2
- 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 3
5*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 1
60*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqr
t((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*
c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^1
0 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1
024*a^8*c^5)*e^2))) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e
^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c
- 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 3
2*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c -
 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*
c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (
a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 1
6*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c
 - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)
*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3
)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a
```

^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3
)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-(b^7 - 35*a*
b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a
^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b
^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2
- 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 -
20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*
a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000
*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^
3*c^5)*d - 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 27
20*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 +
40960*a^10*c^7)*e^3*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a
^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^1
1*c^5)*e^4)) - (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21
824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3
*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^
6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 6
25*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 +
1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*
a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) + s
qrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^
3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*
(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 +
16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c
^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3
*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 +
10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 3
2*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4
*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a
^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a
*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 1
6*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^
3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 1
6*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*
a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2
+ 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^
4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a
^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 128
0*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*
b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*
b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6
*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d + 1/2*sqrt(1/2
)*((a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a
^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*sqr
t((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*
c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)) + (b^11 -
53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600
*a^5*b*c^5)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 -
(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^
2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10
- 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 10
24*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b
^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - sqrt(1/2)*((a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3
*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2
- 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*

```
c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x
^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16
*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 +
4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^
3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 +
 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^
6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^
3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b
^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)
*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4
*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b
^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^
2)*d^2)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^
3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^
4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20
*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a
^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c
^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^
3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 +
 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d - 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*b
^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b
^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*sqrt((b^4 - 50*a*b^2*c + 6
25*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 +
 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)) + (b^11 - 53*a*b^9*c + 940*a^2*b^
7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b
^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8
*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e
^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^
8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a
^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c
^4 - 1024*a^8*c^5)*e^2))) - 2*(a*b^3 - 16*a^2*b*c)*d)/((a*b^4*c^2 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d
*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^
2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 1
6*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a
*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^
4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(
a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 +
 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2
*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a
^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*
a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*
a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2
*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c
+ 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*
a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)
*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^3, x)
```

3.633 $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=150

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b+2c(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[Out] -(b + 2*c*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)^2) + (3*c*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) - (6*c^2*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]
])/((b^2 - 4*a*c)^(5/2)*e)
```

Rubi [A] time = 0.184865, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1142, 1107, 614, 618, 206}

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b+2c(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```


```
[Out] -(b + 2*c*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)^2) + (3*c*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) - (6*c^2*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]
])/((b^2 - 4*a*c)^(5/2)*e)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x,\right.}{2\left(b^2-4ac\right)e}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c\left(b+2c(d+ex)^2\right.}{2\left(b^2-4ac\right)^2 e\left(a+b(d+ex)^2\right.}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c\left(b+2c(d+ex)^2\right.}{2\left(b^2-4ac\right)^2 e\left(a+b(d+ex)^2\right.}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c\left(b+2c(d+ex)^2\right.}{2\left(b^2-4ac\right)^2 e\left(a+b(d+ex)^2\right.}$$

Mathematica [A] time = 0.183401, size = 147, normalized size = 0.98

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{\left(b^2-4ac\right)\left(-b-2c(d+ex)^2\right)}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{6c\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4)
^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (24*
c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(
4*(b^2 - 4*a*c)^2*e)
```

Maple [C] time = 0.04, size = 541, normalized size = 3.6

$$\frac{1}{\left(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a\right)^2}\left(3\,\frac{c^3e^5x^6}{16\,a^2c^2-8\,ab^2c+b^4}+18\,\frac{c^3de^4x}{16\,a^2c^2-8\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (3*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*e^4*d*c^3/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^5+9/2*c^2*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+6*c^2*d*
e^2*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+c*e*(45*c^2*d^4+27*b*c*d^
2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+2*c*d*(9*c^2*d^4+9*b*c*d^2+5*a*
c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/4/e*(12*c^3*d^6+18*b*c^2*d^4+20*a*c^2
*d^2+4*b^2*c*d^2+10*a*b*c-b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3
*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2
+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_
Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.31392, size = 7795, normalized size = 51.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*x^5 + 1
8*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*x^4 + 12*(b^2*c^3
- 4*a*c^4)*d^6 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d
)*e^3*x^3 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4
+ 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*
c^2 - 4*a*b*c^3)*d^2)*e^2*x^2 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2 + 8*
(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c
^2 - 20*a^2*c^3)*d)*e*x + 12*(c^4*e^8*x^8 + 8*c^4*d*e^7*x^7 + 2*(14*c^4*d^2
 + b*c^3)*e^6*x^6 + c^4*d^8 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*x^5 + 2*b*c^3*
d^6 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*x^4 + 4*(14*c^4*d
^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*x^3 + 2*a*b*c^2*d^2 + (b^2*c
^2 + 2*a*c^3)*d^4 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2
*a*c^3)*d^2)*e^2*x^2 + a^2*c^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (
b^2*c^2 + 2*a*c^3)*d^3)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d
*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d
^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqr
t(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2
+ b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x
^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 1
```

```
0*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 +
 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^
3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2
*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 -
 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d
^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2
 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*
c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 +
 (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2
*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e), 1/4*(12*(b
^2*c^3 - 4*a*c^4)*e^6*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*x^5 + 18*(b^3*c^2
- 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*x^4 + 12*(b^2*c^3 - 4*a*c^4)*
d^6 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*e^3*x^3 -
 b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c
+ a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b
*c^3)*d^2)*e^2*x^2 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2 + 8*(9*(b^2*c^3
 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2
*c^3)*d)*e*x - 24*(c^4*e^8*x^8 + 8*c^4*d*e^7*x^7 + 2*(14*c^4*d^2 + b*c^3)*e
^6*x^6 + c^4*d^8 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*x^5 + 2*b*c^3*d^6 + (70*c
^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*x^4 + 4*(14*c^4*d^5 + 10*b*c
^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*x^3 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^
3)*d^4 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2
)*e^2*x^2 + a^2*c^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2
*a*c^3)*d^3)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c
*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)
*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3
 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b
^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5
*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a
^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 +
3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e
^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*
b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d
^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 17.1234, size = 873, normalized size = 5.82

$$\frac{3\left(b^4c^2e-8\,ab^2c^3e+16\,a^2c^4e\right)\sqrt{b^2-4\,ac}\log\left(\left|\left(b+\sqrt{b^2-4\,ac}\right)x^2e^2+2\left(b+\sqrt{b^2-4\,ac}\right)dxe+\left(b+\sqrt{b^2-4\,ac}\right)d^2+2\,a\right|\right)}{b^{10}e^2-20\,ab^8ce^2+160\,a^2b^6c^2e^2-640\,a^3b^4c^3e^2+1280\,a^4b^2c^4e^2-1024\,a^5c^5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3*(b^4*c^2*e - 8*a*b^2*c^3*e + 16*a^2*c^4*e)*sqrt(b^2 - 4*a*c)*log(abs((b +
 sqrt(b^2 - 4*a*c))*x^2*e^2 + 2*(b + sqrt(b^2 - 4*a*c))*d*x*e + (b + sqrt(b
^2 - 4*a*c))*d^2 + 2*a))/(b^10*e^2 - 20*a*b^8*c*e^2 + 160*a^2*b^6*c^2*e^2 -
 640*a^3*b^4*c^3*e^2 + 1280*a^4*b^2*c^4*e^2 - 1024*a^5*c^5*e^2) - 3*(b^4*c^
2*e - 8*a*b^2*c^3*e + 16*a^2*c^4*e)*sqrt(b^2 - 4*a*c)*log(abs(-(b - sqrt(b^
2 - 4*a*c))*x^2*e^2 - 2*(b - sqrt(b^2 - 4*a*c))*d*x*e - (b - sqrt(b^2 - 4*a
*c))*d^2 - 2*a))/(b^10*e^2 - 20*a*b^8*c*e^2 + 160*a^2*b^6*c^2*e^2 - 640*a^3
*b^4*c^3*e^2 + 1280*a^4*b^2*c^4*e^2 - 1024*a^5*c^5*e^2) + 1/4*(12*c^3*x^6*e
^6 + 72*c^3*d*x^5*e^5 + 180*c^3*d^2*x^4*e^4 + 240*c^3*d^3*x^3*e^3 + 180*c^3
*d^4*x^2*e^2 + 72*c^3*d^5*x*e + 12*c^3*d^6 + 18*b*c^2*x^4*e^4 + 72*b*c^2*d*
x^3*e^3 + 108*b*c^2*d^2*x^2*e^2 + 72*b*c^2*d^3*x*e + 18*b*c^2*d^4 + 4*b^2*c
*x^2*e^2 + 20*a*c^2*x^2*e^2 + 8*b^2*c*d*x*e + 40*a*c^2*d*x*e + 4*b^2*c*d^2
+ 20*a*c^2*d^2 - b^3 + 10*a*b*c)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*
e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^4*e - 8
*a*b^2*c*e + 16*a^2*c^2*e))
```

3.634 $\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=437

$$\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c-b\left(b^2-8ac\right)\sqrt{b^2-\right.}{8\sqrt{2}a^2e\left(b^2-4ac\right)^{5/2}\sqrt{\right.}$$

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(4*a*(b^2 - 4*a*c)*(a + b*e
^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)^2) + ((d/e + x)*((b^2 - 7*a*c)*(3*b^2 -
 4*a*c) + 3*b*c*(b^2 - 8*a*c)*e^2*(d/e + x)^2))/(8*a^2*(b^2 - 4*a*c)^2*(a +
 b*e^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 5
6*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
 e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b
*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*e)
```

Rubi [A] time = 5.36105, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1106, 1092, 1178, 1166, 205}

$$\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c-b\left(b^2-8ac\right)\sqrt{b^2-\right.}{8\sqrt{2}a^2e\left(b^2-4ac\right)^{5/2}\sqrt{\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(4*a*(b^2 - 4*a*c)*(a + b*e
^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)^2) + ((d/e + x)*((b^2 - 7*a*c)*(3*b^2 -
 4*a*c) + 3*b*c*(b^2 - 8*a*c)*e^2*(d/e + x)^2))/(8*a^2*(b^2 - 4*a*c)^2*(a +
 b*e^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 5
6*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
 e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b
*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*e)
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+be^2x^2+ce^4x^4\right)^3}\,dx, x, \frac{d}{e}+x\right)$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{b^2e^4-2ace^4-4\left(b^2e^4-4ace^4\right)\cdots}{\left(a+be^2x^2+ce^4x^4\right)^2}\cdots\right.}{4a\left(b^2-4ac\right)\cdots}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)\left(\left(b^2-7ac\right)\left(3b^2-4ac\cdots\right.\right.}{8a^2\left(b^2-4ac\right)^2e\left(a+b(\cdots\right.}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)\left(\left(b^2-7ac\right)\left(3b^2-4ac\cdots\right.\right.}{8a^2\left(b^2-4ac\right)^2e\left(a+b(\cdots\right.}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)\left(\left(b^2-7ac\right)\left(3b^2-4ac\cdots\right.\right.}{8a^2\left(b^2-4ac\right)^2e\left(a+b(\cdots\right.}$$

Mathematica [A] time = 6.16749, size = 463, normalized size = 1.06

$$\frac{28a^2c^2(d+ex)-25ab^2c(d+ex)-24abc^2(d+ex)^3+3b^3c(d+ex)^3+3b^4(d+ex)}{8a^2e\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(56a^2c^2+b^3\sqrt{b^2-4ac}-10a\right.}{8\sqrt{2}a^2e\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] (-(b^2*(d + e*x)) + 2*a*c*(d + e*x) - b*c*(d + e*x)^3)/(4*a*(-b^2 + 4*a*c)*
e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4*(d + e*x) - 25*a*b^2*c*(d
 + e*x) + 28*a^2*c^2*(d + e*x) + 3*b^3*c*(d + e*x)^3 - 24*a*b*c^2*(d + e*x)
^3)/(8*a^2*(-b^2 + 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqr
t[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (3*
Sqrt[c]*(-b^4 + 10*a*b^2*c - 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Maple [C] time = 0.043, size = 1010, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (-3/8*c^2*e^6*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^7-21/8*c^2*d*e
^5*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^6+1/8*(-504*a*b*c^2*d^2+6
3*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b^4)*e^4*c/(16*a^2*c^2-8*a*b^2*c+b^4)/a
^2*x^5+5/8*c*d*e^3*(-168*a*b*c^2*d^2+21*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b
^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^4-1/8*e^2*(840*a*b*c^3*d^4-105*b^3*c^2
*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+4*a^2*b*c^2+20*a*b^3*c-
3*b^5)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(504*a*b*c^3*d^4-63*b^3*c
^2*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+12*a^2*b*c^2+60*a*b^3
*c-9*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^2+1/8*(-168*a*b*c^3*d^6+21*b^3*c
^2*d^6+140*a^2*c^3*d^4-245*a*b^2*c^2*d^4+30*b^4*c*d^4-12*a^2*b*c^2*d^2-60*a
*b^3*c*d^2+9*b^5*d^2+44*a^3*c^2-37*a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c
+b^4)/a^2*x+1/8*d/e*(-24*a*b*c^3*d^6+3*b^3*c^2*d^6+28*a^2*c^3*d^4-49*a*b^2*
c^2*d^4+6*b^4*c*d^4-4*a^2*b*c^2*d^2-20*a*b^3*c*d^2+3*b^5*d^2+44*a^3*c^2-37*
a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2)/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/16/(16*
a^2*c^2-8*a*b^2*c+b^4)/a^2/e*sum((b*c*e^2*(-8*a*c+b^2)*_R^2+2*e*c*b*d*(-8*a
*c+b^2)*_R-8*a*b*c^2*d^2+b^3*c*d^2+28*a^2*c^2-9*a*b^2*c+b^4)/(2*_R^3*c*e^3+
6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z
^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d
^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^3*c^2 - 8*a*b*c^3)*e^7*x^7 + 21*(b^3*c^2 - 8*a*b*c^3)*d*e^6*x^6 +
 (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63*(b^3*c^2 - 8*a*b*c^3)*d^2)*e^5*x
^5 + 5*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3
)*d)*e^4*x^4 + 3*(b^3*c^2 - 8*a*b*c^3)*d^7 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*
c^2 + 105*(b^3*c^2 - 8*a*b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c
^3)*d^2)*e^3*x^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^5 + (63*(b^3*c^2
 - 8*a*b*c^3)*d^5 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^3 + 3*(3*b^5
 - 20*a*b^3*c - 4*a^2*b*c^2)*d)*e^2*x^2 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2
)*d^3 + (21*(b^3*c^2 - 8*a*b*c^3)*d^6 + 5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2
 + 5*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^4 + 3*(3*b^5 - 20*a*b^3*c - 4*
a^2*b*c^2)*d^2)*e*x + (5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d)/((a^2*b^4*c^
2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a
^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2
- 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b
*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 -
8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 +
10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c
+ 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^
2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2
+ 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^
3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 +
(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2
*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c
- 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*
d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e) - 3/8*integrate(-((b
^3*c - 8*a*b*c^2)*e^2*x^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2 + 2*(b^3*c - 8*a*b
*c^2)*d*e*x + (b^3*c - 8*a*b*c^2)*d^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 +
 (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [B] time = 6.20492, size = 18502, normalized size = 42.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^3*c^2 - 8*a*b*c^3)*e^7*x^7 + 42*(b^3*c^2 - 8*a*b*c^3)*d*e^6*x^6
+ 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63*(b^3*c^2 - 8*a*b*c^3)*d^2)*e^
5*x^5 + 10*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2
*c^3)*d)*e^4*x^4 + 6*(b^3*c^2 - 8*a*b*c^3)*d^7 + 2*(3*b^5 - 20*a*b^3*c - 4*
a^2*b*c^2 + 105*(b^3*c^2 - 8*a*b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28
*a^2*c^3)*d^2)*e^3*x^3 + 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^5 + 2*(6
3*(b^3*c^2 - 8*a*b*c^3)*d^5 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^3
+ 3*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d)*e^2*x^2 + 2*(3*b^5 - 20*a*b^3*c -
 4*a^2*b*c^2)*d^3 + 2*(21*(b^3*c^2 - 8*a*b*c^3)*d^6 + 5*a*b^4 - 37*a^2*b^2*
c + 44*a^3*c^2 + 5*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^4 + 3*(3*b^5 - 2
0*a*b^3*c - 4*a^2*b*c^2)*d^2)*e*x + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c
^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e
```

^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3
 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^
5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x
^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d
)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a
^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4
*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*
a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^
4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c
^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(a^3*b^
5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b
^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*
a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt(
(b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^
10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b
^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2
 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))*log(27*(21*b^8*
c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)
*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^
6 + 38416*a^4*c^7)*d + 27/2*sqrt(1/2)*((a^5*b^15 - 31*a^6*b^13*c + 424*a^7*
b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 6758
4*a^11*b^3*c^6 - 45056*a^12*b*c^7)*e^3*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4
*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a
^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)) -
 (b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*
c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*sqrt(-(b^9
 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b
^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 -
 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2
*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a
^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6
*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c
^5)*e^2))) - 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^
8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^
4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*
(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^
4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(
a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4
*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5
 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*
a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b
^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)
*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a
^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 +
(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^
5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^
3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b
^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219
*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c
 + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)

*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 128
0*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4
189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*e*x + 27*(21*b^8*c^3 -
 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*d -
27/2*sqrt(1/2)*((a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9
*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*
a^12*b*c^7)*e^3*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3
 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*
b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)) - (b^14 - 32*a*b^12*c +
464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5
 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2
*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 16
0*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqr
t((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((
a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14
*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^
2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))) - 3*sqrt(1/2
)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4
*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14
*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*
c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(
a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c
^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a
^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6
 - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*
b*c^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)
*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^
2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a
^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c
^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 +
 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c
+ 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-
(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a
^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c
^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3
*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 6
40*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20
*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^
10*c^5)*e^2))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208
*a^3*b^2*c^6 + 38416*a^4*c^7)*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a
^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*d + 27/2*sqrt(1/2)*((a^5*b^
15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^
4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*e^3*sqrt((b
^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10
*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2
*c^4 - 1024*a^15*c^5)*e^4)) + (b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885
*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 -
 87808*a^7*c^7)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*
c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8
*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 2
19*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8
*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^
5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1
280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))) + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3
*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^
4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^
2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b

```
^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*
e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2
*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)
*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b
^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*
c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4
*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a
^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16
*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b
^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189
*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c
+ 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2
*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4
)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*
a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^
6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))*log(27*(2
1*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^
4*c^7)*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*
b^2*c^6 + 38416*a^4*c^7)*d - 27/2*sqrt(1/2)*((a^5*b^15 - 31*a^6*b^13*c + 42
4*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5
+ 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*e^3*sqrt((b^8 - 22*a*b^6*c + 219*a
^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c +
 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e
^4)) + (b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^
4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*sqrt
(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 -
(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2
*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a
^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 -
 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 -
20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*
a^10*c^5)*e^2))) + 2*(5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d)/((a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 1
6*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^
2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 -
 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8
*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c
^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 1
0*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c +
 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2
*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2
+ 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 +
 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3
*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*
c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d
^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^(-3), x)
```

3.635 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=255

$$\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3e\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+b(d+ex)\right.}{4a^3}$$

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*(
d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
 (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e) + Log[d + e*x]/(a^3*e) - Log[a + b
*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e)
```

Rubi [A] time = 0.485574, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3e\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+b(d+ex)\right.}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```


```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*(
d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
 (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e) + Log[d + e*x]/(a^3*e) - Log[a + b
*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-2\left(b^2-4ac\right)-3b\ldots}{x\left(a+bx+cx^2\right)^2} \ldots\right)}{4a\left(b^2-4\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2\ldots}{4a^2\left(b^2-4ac\right)^2e\left(a+\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2\ldots}{4a^2\left(b^2-4ac\right)^2e\left(a+\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2\ldots}{4a^2\left(b^2-4ac\right)^2e\left(a+\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2\ldots}{4a^2\left(b^2-4ac\right)^2e\left(a+\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2\ldots}{4a^2\left(b^2-4ac\right)^2e\left(a+\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2\ldots}{4a^2\left(b^2-4ac\right)^2e\left(a+\ldots\right.}$$

Mathematica [A] time = 3.99071, size = 391, normalized size = 1.53

$$\frac{\frac{a\left(16a^2c^2-15ab^2c-14abc^2(d+ex)^2+2b^3c(d+ex)^2+2b^4\right)}{\left(b^2-4ac\right)^2\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2+b^4\sqrt{b^2-4ac}-10ab^3c-8ab^2c\sqrt{b^2-4ac}+b^5\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{5/2}} \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] ((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d + e*
x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c*(d
 + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqr
t[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) + ((b^5
 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 -
 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d
+ e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e)
```

Maple [C] time = 0.068, size = 4477, normalized size = 17.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)

[Out] 16/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3+4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*d^4-21/4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c+6*a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2+3/4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4+ln(e*x+d)/a^3/e-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((c*e^3*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*c*d*e^2*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*c^2*d^2+3*b^4*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*a*b^2*c^2*d^3+b^4*c*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^5+1/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^5+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^5*d^2-1/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^2-1/2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2*d^2+16/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+24/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^3*d^2-1/2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2+10/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^3-29/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2+4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4-105/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^3*d^4+15/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c^2*d^4-87/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c^2*d^2+6/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^4*c*d^2-21/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*c^3*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^3*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-105/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b*d^2+15/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^3*d^2-70/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^

```
3*d^3*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b-21/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/(16*a^2*
c^2-8*a*b^2*c+b^4)*x*b*c^3+3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/(16*a^2*c^2-8*a*b^2*c+b
^4)*x*b^3*c^2-29/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2+
4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4*c-6/a/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c-7/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2
-8*a*b^2*c+b^4)*b*c^3*d^6-7/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^5*b/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^6+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^5*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^6+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c^2*d^6-29/4/a/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c^2*d^4+1/a^2/(c*e^4*x^4+4*
c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^
2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4*c*d^4-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8
*a*b^2*c+b^4)*b^3*c*d^2-29/4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c
*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^4*b^2+1/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^4
-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c+4/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*e^3*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")

[Out] Timed out
```

Fricas [B] time = 11.1274, size = 20893, normalized size = 81.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out] [1/4*(2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2
 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d*e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 +
 132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c
^4)*d^2)*e^4*x^4 + 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3
+ 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11
*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^
```

2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*
c^3 - 64*a^4*c^4)*d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*
c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 4
5*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*
a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^
3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 -
64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)
*e*x + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a
*b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c
^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c
^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^
2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*
a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a
^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a
^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2
*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c -
10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 6
0*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)
d^4 + 2(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^
3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*
(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 -
10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2
*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^
5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b
^2*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*
c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e
*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c
))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*e*x + a)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a
*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2
+ 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 +
2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^
5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4
*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^
2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 4
8*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^
4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*
d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6
*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3
+ c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*

c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*e^9*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d*e^8*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^2)*e^7*x^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^3 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d)*e^6*x^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^4 + 30*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2)*e^5*x^4 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^5 + 10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d)*e^4*x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^6 + 15*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^4 + 3*(a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^2)*e^3*x^2 + 4*(2*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^3 + (a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d)*e^2*x + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d^2)*e), 1/4*(2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d*e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^2)*e^4*x^4 + 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)*e*x + 2*((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b

^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c
^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*
b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^
3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8
*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 3
0*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c
- 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2
+ 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^
2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*
c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^
3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)*sqrt(-b^2 + 4*a*c)
*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
4*a*c)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 +
8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
- 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^
4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^
3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2
*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 6
4*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6
+ 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 -
10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 +
2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c
^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d
^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^6*c^2 - 12*a*b
^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*
e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8
- 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c
^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c +
48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 +
(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x
^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4
+ 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 -
12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2
+ 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2
+ 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 4
8*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^
4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)
*log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5

```
)*e^9*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*
d*e^8*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4 +
 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^2)*e^7*x
^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^3
 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d)*e^6*x^
5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4
 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^4 + 30
*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2)*e^5*x^4
+ 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^5 +
10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^3 + (a^3*
b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d)*e^4*
x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3 + 14*(a^3*b
^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^6 + 15*(a^3*b^7*c
- 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^4 + 3*(a^3*b^8 - 10*a^4
*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^2)*e^3*x^2 + 4*(2
*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*
b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^5 + (a^3*b^8 - 10
*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^3 + (a^4*b^7
- 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d)*e^2*x + (a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*
a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*
c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6
*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 -
64*a^7*b*c^3)*d^2)*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out
```

Giac [B] time = 17.4483, size = 1802, normalized size = 7.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] -1/4*(a^3*b^9*e - 18*a^4*b^7*c*e + 126*a^5*b^5*c^2*e - 400*a^6*b^3*c^3*e +
480*a^7*b*c^4*e)*sqrt(b^2 - 4*a*c)*log(abs(2*(a^3*b^3*c - 4*a^4*b*c^2 + (a^
3*b^2*c - 4*a^4*c^2)*sqrt(b^2 - 4*a*c))*x^2*e^6 + 4*(a^3*b^3*c - 4*a^4*b*c^
2 + (a^3*b^2*c - 4*a^4*c^2)*sqrt(b^2 - 4*a*c))*d*x*e^5 + 2*(a^3*b^3*c - 4*a
^4*b*c^2 + (a^3*b^2*c - 4*a^4*c^2)*sqrt(b^2 - 4*a*c))*d^2*e^4 + 4*(a^4*b^2*
c - 4*a^5*c^2)*e^4))/(a^6*b^10*e^2 - 20*a^7*b^8*c*e^2 + 160*a^8*b^6*c^2*e^2
 - 640*a^9*b^4*c^3*e^2 + 1280*a^10*b^2*c^4*e^2 - 1024*a^11*c^5*e^2) + 1/4*(
a^3*b^9*e - 18*a^4*b^7*c*e + 126*a^5*b^5*c^2*e - 400*a^6*b^3*c^3*e + 480*a^
7*b*c^4*e)*sqrt(b^2 - 4*a*c)*log(abs(-2*(a^3*b^3*c - 4*a^4*b*c^2 - (a^3*b^2
*c - 4*a^4*c^2)*sqrt(b^2 - 4*a*c))*x^2*e^6 - 4*(a^3*b^3*c - 4*a^4*b*c^2 - (
a^3*b^2*c - 4*a^4*c^2)*sqrt(b^2 - 4*a*c))*d*x*e^5 - 2*(a^3*b^3*c - 4*a^4*b*
```

```
c^2 - (a^3*b^2*c - 4*a^4*c^2)*sqrt(b^2 - 4*a*c))*d^2*e^4 - 4*(a^4*b^2*c - 4
*a^5*c^2)*e^4))/(a^6*b^10*e^2 - 20*a^7*b^8*c*e^2 + 160*a^8*b^6*c^2*e^2 - 64
0*a^9*b^4*c^3*e^2 + 1280*a^10*b^2*c^4*e^2 - 1024*a^11*c^5*e^2) - 1/4*e^(-1)
*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4
+ b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/a^3 + e^(-1)*log(abs(x*e + d))/a^3 +
1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3*d^6 + 4*a*b^4*c*d^4 - 29*a^2*b^2*c^2*d^
4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 - 12*a^2*b^3*c*d^2 - 2*a^3*b*c^2*d^2 + 2*(
a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^6 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^
2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c^3*d*e^5)*x^5 + (30*a*b^3*c^2*d^2*e^4 -
210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e^4 - 29*a^2*b^2*c^2*e^4 + 16*a^3*c^3*e^4
)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 70*a^2*b*c^3*d^3*e^3 + 4*a*b^4*c*d*e^3 -
29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d*e^3)*x^3 + 2*(15*a*b^3*c^2*d^4*e^2 - 10
5*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^2*e^2 - 87*a^2*b^2*c^2*d^2*e^2 + 48*a^3*
c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3*c*e^2 - a^3*b*c^2*e^2)*x^2 + 4*(3*a*b^3
*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4*a*b^4*c*d^3*e - 29*a^2*b^2*c^2*d^3*e +
16*a^3*c^3*d^3*e + a*b^5*d*e - 6*a^2*b^3*c*d*e - a^3*b*c^2*d*e)*x)*e^(-1)/(
(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*
e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^2 - 4*a*c)^2*a^3)
```

3.636 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=484

$$\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2e\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}\left(\cdots\right.}{8\sqrt{2}a^3e\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)) + (
b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e
*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 -
 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4
 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[
b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5
*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*
Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 1.22676, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1142, 1121, 1277, 1281, 1166, 205}

$$\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2e\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}\left(\cdots\right.}{8\sqrt{2}a^3e\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```


```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)) + (
b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e
*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 -
 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4
 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[
b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5
*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*
Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
```

```
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-5b\cdots}{x^2(\cdots}\right.}{4\cdots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{5b^4-35ab\cdots}{8a^2\left(b^2-4ac\cdots\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+\cdots\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+\cdots\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+\cdots\right.}$$

Mathematica [A] time = 6.23231, size = 560, normalized size = 1.16

$$\frac{-3abc(d+ex)-2ac^2(d+ex)^3+b^2c(d+ex)^3+b^3(d+ex)}{4a^2e\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{-84a^2bc^2(d+ex)-52a^2c^3(d+ex)^3+47ab^2c^2(d+ex)^3\cdots}{8a^3e\left(4ac-b^2\right)^2\left(a+b(d+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -(1/(a^3*e*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d + e*
x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*a^2*b*c^2*(
d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*c^3*(d +
 e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (
3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 3
7*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a
*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(-5*b^5 + 47*a*b^3*c
- 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] +
60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]*e)
```

Maple [C] time = 0.062, size = 6821, normalized size = 14.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 8.97768, size = 22920, normalized size = 47.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 48*(5*b^4*c^2 -
37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^
2*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 12*(28*
(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 3
92*a^2*b*c^3)*d)*e^5*x^5 + 6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 +
2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37
*a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3
)*d^2)*e^4*x^4 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 8*(42*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 +
392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d
)*e^3*x^3 + 16*a^2*b^4 - 128*a^3*b^2*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4
*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + 2*(84*(5*b^4*c^2 - 37*a*b^2*c^3 +
60*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + 2*(25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^
2)*d^2 + 4*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^
2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x -
3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c
^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*
x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8
*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c
^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8
*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 +
5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(
a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c +
32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)
```

```
*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a
^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^
3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^
3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5
*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a
^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*
(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 3
2*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 -
8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7
*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10
- 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1
024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351
310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/
((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^
18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*
c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-27*(
4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7
+ 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b
^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 8
10000*a^5*c^9)*d + 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*
b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 +
 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt((625
*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^
4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c
 + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)
*e^4)) - (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c
^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684
672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894
*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a
^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2
*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c
^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a
^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 +
 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*
a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2)))
+ 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3
*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3
*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^
8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c -
 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6
*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c -
 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2
*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3
+ 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^
3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35
*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c
+ 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^
4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6
*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*
e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*
c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 +
 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16
*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 +
2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c +
 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b
^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^1
0 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 -
 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 3
```

51310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d - 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)) - (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d + 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b

^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)) + (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d - 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)) + (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))))/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^

```
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 +
16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*x^7 +
 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 +
126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^
3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 1
6*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^
6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7
+ 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^
4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2
 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 1
6*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3
*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*
c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*
b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6
*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6
*b^2*c + 16*a^7*c^2)*d)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.637 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=325

$$\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{4a^2e\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4e\left(b^2-4ac\right)^{5/2}}-$$

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)^2) + (b
^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2
- 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)^2*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*
ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2
)*e) - (3*b*Log[d + e*x])/(a^4*e) + (3*b*Log[a + b*(d + e*x)^2 + c*(d + e*x
)^4])/(4*a^4*e)
```

Rubi [A] time = 0.58483, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{4a^2e\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4e\left(b^2-4ac\right)^{5/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)^2) + (b
^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2
- 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)^2*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*
ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2
)*e) - (3*b*Log[d + e*x])/(a^4*e) + (3*b*Log[a + b*(d + e*x)^2 + c*(d + e*x
)^4])/(4*a^4*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
```

```
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-3}{x^2} \cdots\right.}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3b^4-20a\cdots}{4a^2\left(b^2-4a\cdots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3b^4-20a\cdots}{4a^2\left(b^2-4a\cdots\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2\cdots\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2\cdots\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2\cdots\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2\cdots\right.}$$

Mathematica [A] time = 6.18376, size = 491, normalized size = 1.51

$$\frac{-3abc-2ac^2(d+ex)^2+b^2c(d+ex)^2+b^3}{4a^2e\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{-46a^2bc^2-28a^2c^3(d+ex)^2+26ab^2c^2(d+ex)^2+29ab^3c-4b^4c(d+\cdots}{4a^3e\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -1/(2*a^3*e*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d
 + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) +
(-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26*a*b^2*c^2*(d
 + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d + e
*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e) + (3*(b^6 - 10*a*b^4*c
 + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2
 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c
*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e) + (3*(-b^6 + 10*a*b^4*c - 30*a
^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*
c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e
*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e)
```

Maple [C] time = 0.077, size = 5575, normalized size = 17.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 17.5968, size = 32173, normalized size = 98.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*e^8*x^8
 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*e^7*x^7
+ 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b
^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*e^6*x^6 + 6*(56
*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^
7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d)*e^5*x^5 + 2*a^3*
b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b
^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60*a^2*b^6*c + 158*a
^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3
 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*
a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 +
 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^
3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 7
9*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^2*b^7 - 104*a^3*b^
5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 3
8*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*
b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7 - 104*a^3*b^5*c +
 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3
*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*
```

b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c
^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 -
20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b
c^4 + 45(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x
^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(
b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8
*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10
*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 +
30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*
a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^
3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^
4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4
)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*
c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*
b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10
*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 - 10*a*b^4
*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2
*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3
*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*
a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b
*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8
+ a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*
b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*
a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4
*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
- 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4
)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*
d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b
^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*sqrt(b^2 - 4*a*c)*
log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*
x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 +
4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*
d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - 3*((b^7
*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^
2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24
*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3
+ 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*
b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 +
32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*
c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^
3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3
*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c
^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 +
3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*
e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 +
(2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 -
12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*
c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*
b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c
+ 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 1
2*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2
+ 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*
c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^
3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b
^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6

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4*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d
^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(
b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 1
2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (
a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2
 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*
c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b
^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48
*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c
^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2
 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12*a*b^
5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c
^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^
3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a
^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^
3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b
*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2
)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 +
 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(
b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a
*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b
^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*
a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3
+ 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b
^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a
^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c
^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 +
48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*
c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b
^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64
*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^
4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 +
45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c
 - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7
*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*
a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a
^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^
3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b
^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^
3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 -
 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*
c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
- 64*a^7*c^5)*e^11*x^10 + 10*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
 - 64*a^7*c^5)*d*e^10*x^9 + (2*a^4*b^7*c - 24*a^5*b^5*c^2 + 96*a^6*b^3*c^3
- 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^
7*c^5)*d^2)*e^9*x^8 + 8*(15*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
- 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7
*b*c^4)*d)*e^8*x^7 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*
c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64
*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*
c^4)*d^2)*e^7*x^6 + 2*(126*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 -
 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7
*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 -
 128*a^8*c^4)*d)*e^6*x^5 + (2*a^5*b^7 - 24*a^6*b^5*c + 96*a^7*b^3*c^2 - 128
*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^
5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d
^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8
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*c^4)*d^2)*e^5*x^4 + 4*(30*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 -
 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7
*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 -
 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c
^3)*d)*e^4*x^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45
*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4
*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8
- 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^4 + 12*(a
^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^2)*e^3*x^2 + 2*(5*
(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b
^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 1
0*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b
^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d)*e^2*x + ((a^4*b^6*c^2 - 12*a^5*b^4*
c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 4
8*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^
2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*
b^3*c^2 - 64*a^8*b*c^3)*d^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64
*a^9*c^3)*d^2)*e), -1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 4
0*a^4*c^5)*e^8*x^8 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a
^4*c^5)*d*e^7*x^7 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a
^4*b*c^4 + 56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^
2)*e^6*x^6 + 6*(56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^
5)*d^3 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d
)*e^5*x^5 + 2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*
b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60
*a^2*b^6*c + 158*a^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^
2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*
a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c
- 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2
- 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^
2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*
c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8
- 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^
2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 -
11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*
b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c
 + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7
 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2
- 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2
*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c
 + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^
3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 6*((b^6*c^2 - 10*a*b^4*
c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 +
 30*a^2*b^2*c^4 - 20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*
b^3*c^3 - 40*a^3*b*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a
^3*c^5)*d^2)*e^8*x^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*
a^3*c^5)*d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*
e^7*x^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 +
 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c
 - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 -
10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b
^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a
^2*b^3*c^3 - 20*a^3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a
^3*b^2*c^3 - 40*a^4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*
c^3 - 20*a^3*b*c^4)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4
*b*c^3 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 1
40*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8
*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^
```

8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(
b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*
a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^
2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30
*a^3*b^3*c^2 - 20*a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*
b^3*c^2 - 20*a^4*b*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
 - 20*a^3*c^5)*d^8 + a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 +
56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8
*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 -
10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*
a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3
 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*
c^3 - 20*a^3*b*c^4)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*
c^3 - 40*a^4*c^4)*d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b
*c^3)*d^3 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*
sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^
2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 6
4*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a
^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^
2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^
8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3
+ 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (
b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b
^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12
*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*
a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^
5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*
a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 +
32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a
^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2
 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*
b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d
^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4
)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*
a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*
c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5
+ 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^
3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 +
 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7
- 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3
 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b
^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a
^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2
 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2
- 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^
3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d
^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)
d^5 + 4(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^
2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4
 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d
)*e*x + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e
^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^
9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^
7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*
(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 1
2*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7
*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*
b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 4
8*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48

*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2
*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 -
64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3
 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 6
4*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2
c^3 + 210(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 1
40*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 -
 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4
 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6
 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*
(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*
a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 -
 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12
*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c
 + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*
c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^
3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 1
28*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*
c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3
)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 +
 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 -
10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^
8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3
*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2
 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*e^11*x^10 + 10*(a^4*b^6*c^
2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d*e^10*x^9 + (2*a^4*b^7*c
 - 24*a^5*b^5*c^2 + 96*a^6*b^3*c^3 - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a
^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^2)*e^9*x^8 + 8*(15*(a^4*b^6*c^2
 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^
5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d)*e^8*x^7 + (a^4*b^8 - 10*a^5*b
^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 1
2*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b
^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^2)*e^7*x^6 + 2*(126*(a^4*b^6*c^2
- 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^
5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c
+ 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d)*e^6*x^5 + (2*a^5*b^7 -
24*a^6*b^5*c + 96*a^7*b^3*c^2 - 128*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b
^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2
 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6
*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^2)*e^5*x^4 + 4*(30*(a^4*b^6*c^2
- 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^
5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c
+ 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*
b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d)*e^4*x^3 + (a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*
b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3
 - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7
*b^2*c^3 - 128*a^8*c^4)*d^4 + 12*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 -
 64*a^8*b*c^3)*d^2)*e^3*x^2 + 2*(5*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b
^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 -
 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^
2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*
a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d)*
e^2*x + ((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^10
+ 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^8 + (a^4
*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^6 +
2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^4 + (a^6*b^6 -
 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^2)*e)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 17.0446, size = 509, normalized size = 1.57

$$\frac{3\left(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^2}}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{2\left(a^4b^4-8\,a^5b^2c+16\,a^6c^2\right)\sqrt{-b^2+4\,ac}}+\frac{3\,be^{(-1)}\log\left(c+\frac{b}{(xe+d)^2}+\frac{a}{(xe+d)^4}\right)}{4\,a^4}-\frac{e^{(-1)}}{2\,(xe+d)^2a^3}+\frac{(5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan(-(b + 2*a/(x*e
+ d)^2)/sqrt(-b^2 + 4*a*c))*e^(-1)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sq
rt(-b^2 + 4*a*c)) + 3/4*b*e^(-1)*log(c + b/(x*e + d)^2 + a/(x*e + d)^4)/a^4
 - 1/2*e^(-1)/((x*e + d)^2*a^3) + 1/4*(5*b^5*c^2 - 36*a*b^3*c^3 + 58*a^2*b*
c^4 + 2*(5*b^6*c*e - 38*a*b^4*c^2*e + 71*a^2*b^2*c^3*e - 14*a^3*c^4*e)*e^(-
1)/(x*e + d)^2 + (5*b^7*e^2 - 34*a*b^5*c*e^2 + 41*a^2*b^3*c^2*e^2 + 42*a^3*
b*c^3*e^2)*e^(-2)/(x*e + d)^4 + 6*(a*b^6*e^3 - 8*a^2*b^4*c*e^3 + 17*a^3*b^2
*c^2*e^3 - 6*a^4*c^3*e^3)*e^(-3)/(x*e + d)^6)*e^(-1)/((b^2 - 4*a*c)^2*a^4*(
c + b/(x*e + d)^2 + a/(x*e + d)^4)^2)
```

3.638 $\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=202

$$-\frac{f^4\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{f^4\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{f^4x}{c}$$

```
[Out] (f^4*x)/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt
[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt
[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.361944, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1142, 1122, 1166, 205}

$$-\frac{f^4\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{f^4\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{f^4x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^4*x)/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt
[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt
[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^4\,\text{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$
$$= \frac{f^4x}{c} - \frac{f^4\,\text{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{ce}$$
$$= \frac{f^4x}{c} - \frac{\left(\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2ce} - \frac{\left(\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\right)}{}$$
$$= \frac{f^4x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

Mathematica [A] time = 0.14723, size = 222, normalized size = 1.1

$$\frac{f^4\left(-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + 2\sqrt{c}(d+ex)\right)}{2c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^4*(2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(2*c^(3/2)*e)
```

Maple [C] time = 0.003, size = 164, normalized size = 0.8

$$\frac{f^4x}{c} + \frac{f^4}{2\,ce}\sum_{\_R=\text{RootOf}\left(ce^4\_Z^4+4\,cde^3\_Z^3+\left(6\,cd^2e^2+be^2\right)\_Z^2+\left(4\,cd^3e+2\,bde\right)\_Z+cd^4+bd^2+a\right)} \frac{\left(-\_R^2be^2-2\_R\,bde-bd^2-a\right)\ln(x-}{2\,ce^3\_R^3+6\,cde^2\_R^2+6\,cd^2e\_R+2\,cd^3+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] f^4*x/c+1/2*f^4/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R
^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4
*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a
```

```
))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.65602, size = 2772, normalized size = 13.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*f^4*x - sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^
2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*
d*f^12 + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*
sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^
6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) + s
qrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^
16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)
*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 - sqrt(1/
2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*
f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*
a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4
))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(
-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4
*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a
*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 + sqrt(1/2)*((b^4 - 5*a*b
^2*c + 4*a^2*c^2)*e*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 -
 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*
a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) + sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c
)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b
^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*
f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2
)*e*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*
(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^
2*c^3 - 4*a*c^4)*e^2))))/c
```

Sympy [A] time = 3.70435, size = 219, normalized size = 1.08

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5e^4 - 128ab^2c^4e^4 + 16b^4c^3e^4\right) + t^2\left(48a^2bc^2e^2f^8 - 28ab^3ce^2f^8 + 4b^5e^2f^8\right) + a^3f^{16}, \left(t \mapsto t\log\left(x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e**
4) + _t**2*(48*a**2*b*c**2*e**2*f**8 - 28*a*b**3*c*e**2*f**8 + 4*b**5*e**2*
f**8) + a**3*f**16, Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t**3
*b**3*c**3*e**3 - 4*_t*a**2*c**2*e*f**8 + 8*_t*a*b**2*c*e*f**8 - 2*_t*b**4*
e*f**8 + a**2*c*d*f**12 - a*b**2*d*f**12)/(a**2*c*e*f**12 - a*b**2*e*f**12)
))) + f**4*x/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(efx + df\right)^4}{(ex + d)^4 c + (ex + d)^2 b + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.639 $\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=87

$$\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[Out] (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*
a*c]*e) + (f^3*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

Rubi [A] time = 0.127477, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1142, 1114, 634, 618, 206, 628}

$$\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*
a*c]*e) + (f^3*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx &= \frac{f^3\,\mathrm{Subst}\left(\int \frac{x^3}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}\\ &= \frac{f^3\,\mathrm{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e}\\ &= \frac{f^3\,\mathrm{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce} - \frac{\left(bf^3\right)\mathrm{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce}\\ &= \frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce} + \frac{\left(bf^3\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right.}{2ce}\\ &= \frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ace}} + \frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}\end{aligned}$$

Mathematica [A] time = 0.0418863, size = 80, normalized size = 0.92

$$\frac{f^3\left(\log\left(a+b(d+ex)^2+c(d+ex)^4\right) - \frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^3*((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]))/(4*c*e)
```

Maple [C] time = 0.004, size = 154, normalized size = 1.8

$$\frac{f^3}{2e}\sum_{\_R=\mathrm{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2bde\right)\_Z+cd^4+bd^2+a\right)} \frac{\left(\_R^3e^3+3\_R^2de^2+3\_Rd^2e+d^3\right)\ln(x-\_R)}{2ce^3\_R^3+6cde^2\_R^2+6cd^2e\_R+2cd^3+be\_R+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2*f^3/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c
*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d
*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(efx+df)^3}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Fricas [A] time = 1.56381, size = 977, normalized size = 11.23

$$\left[\frac{\sqrt{b^2-4\,ac}bf^3\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac+(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right)+(b^2 -}{4\,(b^2c-4\,ac^2)e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*f^3*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*
d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x +
 b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(
c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*
d^3 + b*d)*e*x + a)) + (b^2 - 4*a*c)*f^3*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*
d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c -
 4*a*c^2)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*f^3*arctan(-(2*c*e^2*x^2 + 4*c*d*
e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*f^3*lo
g(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*
c*d^3 + b*d)*e*x + a))/((b^2*c - 4*a*c^2)*e)]
```

Sympy [B] time = 2.15366, size = 332, normalized size = 3.82

$$\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d
*x/e + x**2 + (-8*a*c*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2))
 + f**3/(4*c*e)) + 2*a*f**3 + 2*b**2*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*
(4*a*c - b**2)) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3)) + (b*f**3*sqr
t(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d*x/e + x**2
+ (-8*a*c*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*
e)) + 2*a*f**3 + 2*b**2*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)
) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3))
```

Giac [B] time = 1.47233, size = 387, normalized size = 4.45

$$-\frac{\sqrt{b^2-4\,ac}bcf^3e\log\left(\left|2\left(b+\sqrt{b^2-4\,ac}\right)x^2e^6+4\left(b+\sqrt{b^2-4\,ac}\right)dxe^5+2\left(b+\sqrt{b^2-4\,ac}\right)d^2e^4+4\,ae^4\right|\right)}{4\left(b^2c^2e^2-4\,ac^3e^2\right)}+\frac{\sqrt{b^2-4\,acl}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(b^2 - 4*a*c)*b*c*f^3*e*log(abs(2*(b + sqrt(b^2 - 4*a*c))*x^2*e^6
+ 4*(b + sqrt(b^2 - 4*a*c))*d*x*e^5 + 2*(b + sqrt(b^2 - 4*a*c))*d^2*e^4 + 4
*a*e^4))/(b^2*c^2*e^2 - 4*a*c^3*e^2) + 1/4*sqrt(b^2 - 4*a*c)*b*c*f^3*e*log(
abs(-2*(b - sqrt(b^2 - 4*a*c))*x^2*e^6 - 4*(b - sqrt(b^2 - 4*a*c))*d*x*e^5
- 2*(b - sqrt(b^2 - 4*a*c))*d^2*e^4 - 4*a*e^4))/(b^2*c^2*e^2 - 4*a*c^3*e^2)
 + 1/4*f^3*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c
*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/c
```

3.640 $\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=170

$$\frac{f^2\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}-\frac{f^2\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)
```

Rubi [A] time = 0.152551, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1142, 1130, 205}

$$\frac{f^2\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}-\frac{f^2\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^2\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\left(\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)f^2\right)\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2e} + \frac{\left(\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)f^2\right)\mathrm{Subst}}{}$$
$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\,f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\,f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e}$$

Mathematica [A] time = 0.094886, size = 178, normalized size = 1.05

$$\frac{f^2\left(\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^2*((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]))/(Sqrt[
2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

Maple [C] time = 0.003, size = 143, normalized size = 0.8

$$\frac{f^2}{2e}\sum_{\_R=\mathrm{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2bde\right)\_Z+cd^4+bd^2+a\right)}\frac{\left(\_R^2e^2+2\_R\,de+d^2\right)\ln(x-\_R)}{2ce^3\_R^3+6cde^2\_R^2+6cd^2e\_R+2cd^3+be\_R+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2*f^2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d
^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*
d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(efx+df\right)^2}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Fricas [B] time = 1.6106, size = 1646, normalized size = 9.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3
)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 + sqrt(1/2)*(b^2*
c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4 + (b^2*c
- 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2)
)) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*
a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 - sqrt(1/2)*
(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4 + (b
^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)
*e^2))) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2
 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 + sqrt(
1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4
 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a
*c^2)*e^2))) + 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^
2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 -
sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(
b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c
- 4*a*c^2)*e^2)))
```

Sympy [A] time = 1.91699, size = 124, normalized size = 0.73

$$\mathrm{RootSum}\left(t^4\left(256a^2c^3e^4 - 128ab^2c^2e^4 + 16b^4ce^4\right) + t^2\left(-16abce^2f^4 + 4b^3e^2f^4\right) + af^8, \left(t \mapsto t\log\left(x + \frac{64t^3ac^2e^3 - 16\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
+ _t**2*(-16*a*b*c*e**2*f**4 + 4*b**3*e**2*f**4) + a*f**8, Lambda(_t, _t*lo
g(x + (64*_t**3*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e*f**4 + d*f**6
)/(e*f**6))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(efx + df\right)^2}{(ex + d)^4 c + (ex + d)^2 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.641 $\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=44

$$-\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

```
[Out] -((f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*e
))
```

Rubi [A] time = 0.0631629, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1142, 1107, 618, 206}

$$-\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*e
))
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4} dx = \frac{f \operatorname{Subst}\left(\int \frac{x}{a+bx^2+cx^4} dx, x, d + ex\right)}{e}$$
$$= \frac{f \operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, (d + ex)^2\right)}{2e}$$
$$= -\frac{f \operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2c(d + ex)^2\right)}{e}$$
$$= -\frac{f \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2 - 4ace}}$$

Mathematica [A] time = 0.0172062, size = 47, normalized size = 1.07

$$\frac{f \tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{e\sqrt{4ac - b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*e)
```

Maple [C] time = 0.005, size = 130, normalized size = 3.

$$\frac{f}{2e} \sum_{\_R=\operatorname{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2bde\right)\_Z+cd^4+bd^2+a\right)} \frac{(\_Re + d)\ln(x - \_R)}{2ce^3\_R^3 + 6cde^2\_R^2 + 6cd^2e\_R + 2cd^3 + be\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2*f/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z
^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{efx + df}{(ex + d)^4 c + (ex + d)^2 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Fricas [A] time = 1.55573, size = 603, normalized size = 13.7

$$\left[\frac{f\log\left(\frac{2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2(6c^2d^2+bc)e^2x^2+2bcd^2+4(2c^2d^3+bcd)ex+b^2-2ac-(2ce^2x^2+4cdex+2cd^2+b)\sqrt{b^2-4ac}}{ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a}\right)}{2\sqrt{b^2-4ace}}, -\frac{\sqrt{-b^2+4ac}f\arctan\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/2*f*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*
c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2
*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x
^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/(s
qrt(b^2 - 4*a*c)*e), -sqrt(-b^2 + 4*a*c)*f*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x
 + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/((b^2 - 4*a*c)*e)]
```

Sympy [B] time = 1.3556, size = 189, normalized size = 4.3

$$-\frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4acf\sqrt{-\frac{1}{4ac-b^2}}+b^2f\sqrt{-\frac{1}{4ac-b^2}}+bf+2cd^2f}{2ce^2f}\right)}{2e}+\frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4acf\sqrt{-\frac{1}{4ac-b^2}}-b^2f\sqrt{-\frac{1}{4ac-b^2}}+bf\ldots}{2ce^2f}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*f*sqrt(-1/(4*a*c -
b**2)) + b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*f))/(
2*e) + f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*f*sqrt(-1/(4*a
*c - b**2)) - b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*
f))/(2*e)
```

Giac [B] time = 1.43048, size = 250, normalized size = 5.68

$$\frac{\sqrt{b^2-4ac}fe\log\left(\left|\left(b+\sqrt{b^2-4ac}\right)x^2e^2+2\left(b+\sqrt{b^2-4ac}\right)dxe+\left(b+\sqrt{b^2-4ac}\right)d^2+2a\right|\right)}{2\left(b^2e^2-4ace^2\right)}-\frac{\sqrt{b^2-4ac}fe\log\left(\left|-\left(b-\ldots\right.\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b^2 - 4*a*c)*f*e*log(abs((b + sqrt(b^2 - 4*a*c))*x^2*e^2 + 2*(b +
sqrt(b^2 - 4*a*c))*d*x*e + (b + sqrt(b^2 - 4*a*c))*d^2 + 2*a))/(b^2*e^2 - 4
*a*c*e^2) - 1/2*sqrt(b^2 - 4*a*c)*f*e*log(abs(-(b - sqrt(b^2 - 4*a*c))*x^2*
e^2 - 2*(b - sqrt(b^2 - 4*a*c))*d*x*e - (b - sqrt(b^2 - 4*a*c))*d^2 - 2*a))
/(b^2*e^2 - 4*a*c*e^2)
```

3.642 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=103

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2aef\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}+\frac{\log(d+ex)}{aef}$$

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]
*e*f) + Log[d + e*x]/(a*e*f) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*
e*f)
```

Rubi [A] time = 0.138291, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1142, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2aef\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}+\frac{\log(d+ex)}{aef}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]
*e*f) + Log[d + e*x]/(a*e*f) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*
e*f)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx, x, d+ex\right)}{ef}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, (d+ex)^2\right)}{2ef}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (d+ex)^2\right)}{2aef} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2aef}\\
&= \frac{\log(d+ex)}{aef} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4aef} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\right.}{4ae}\\
&= \frac{\log(d+ex)}{aef} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\right.}{2a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}ef} + \frac{\log(d+ex)}{aef} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}
\end{aligned}
$$

Mathematica [A] time = 0.0749589, size = 131, normalized size = 1.27

$$
\frac{4\sqrt{b^2-4ac}\log(d+ex) - \left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right) + \left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2c\right.}{4aef\sqrt{b^2-4ac}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 -
4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e*f)
```

Maple [C] time = 0.007, size = 190, normalized size = 1.8

$$\frac{\ln(ex+d)}{aef}+\frac{1}{2aef}\sum_{\_R=\mathrm{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2bde\right)\_Z+cd^4+bd^2+a\right)}\frac{\left(-ce^3\_R^3-3cde^2\_R^2+e\left(-3cd^2\right.\right.}{2ce^3\_R^3+6cde^2\_R^2+6cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] ln(e*x+d)/a/e/f+1/2/f/a/e*sum((-c*e^3*_R^3-3*c*d*e^2*_R^2+e*(-3*c*d^2-b)*_R
-c*d^3-b*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*l
n(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d
^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.71827, size = 1045, normalized size = 10.15

$$\left[\frac{\sqrt{b^2-4acb}\log\left(\frac{2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2\left(6c^2d^2+bc\right)e^2x^2+2bcd^2+4\left(2c^2d^3+bcd\right)ex+b^2-2ac+\left(2ce^2x^2+4cdex+2cd^2+b\right)\sqrt{b^2-4ac}}{ce^4x^4+4cde^3x^3+cd^4+\left(6cd^2+b\right)e^2x^2+bd^2+2\left(2cd^3+bd\right)ex+a}\right)-\left(b^2-4\right.}{4\left(ab^2-4a^2c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
+ 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*e*x + a)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6
*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*
log(e*x + d))/((a*b^2 - 4*a^2*c)*e*f), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-
(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) -
 (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^
2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(e*x + d))/((a*
b^2 - 4*a^2*c)*e*f)]
```

Sympy [B] time = 6.01436, size = 348, normalized size = 3.38

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)}-\frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)}-\frac{1}{4aef}\right)+2ab^2ef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)}-\frac{1}{4aef}\right)-2ac+b^2+b}{bce^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/e
 + x**2 + (-8*a**2*c*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) -
 1/(4*a*e*f)) + 2*a*b**2*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2
)) - 1/(4*a*e*f)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a*c +
 b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/e + x**2 + (-8*a**
2*c*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)) + 2*
a*b**2*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)) -
 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + log(d/e + x)/(a*e*f)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.643 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=204

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}aef^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}aef^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{aef^2(d+ex)}$$

```
[Out] -(1/(a*e*f^2*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqr
t[b^2 - 4*a*c]]*e*f^2) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*e*f^2)
```

Rubi [A] time = 0.273281, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1142, 1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}aef^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}aef^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{aef^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(1/(a*e*f^2*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqr
t[b^2 - 4*a*c]]*e*f^2) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*e*f^2)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)}dx, x, d+ex\right)}{ef^2}$$

$$= -\frac{1}{aef^2(d+ex)} + \frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}dx, x, d+ex\right)}{aef^2}$$

$$= -\frac{1}{aef^2(d+ex)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx, x, d+e$$

$$= -\frac{1}{aef^2(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}ef^2} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4a}}\right.}{\sqrt{2}a\sqrt{b}}$$

Mathematica [A] time = 0.34671, size = 209, normalized size = 1.02

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{d+ex}}{2aef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*e*f^2)
```

Maple [C] time = 0.004, size = 174, normalized size = 0.9

$$\frac{1}{2f^2ae}\sum_{\_R=\text{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2bde\right)\_Z+cd^4+bd^2+a\right)}\frac{\left(-\_R^2ce^2-2\_R\,cde-cd^2-b\right)\ln(x-\_R)}{2ce^3\_R^3+6cde^2\_R^2+6cd^2e\_R+2cd^3+be\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/f^2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))-1/a/e
/f^2/(e*x+d)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [B] time = 1.72404, size = 3001, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] 1/2*(sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*
a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2
*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - (b^5 - 5*a*
b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*
b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(-((a^3
*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*
c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^
2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c
+ 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*
e^4*f^8)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*
c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8))
+ b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)*(a*e^2*f^2*x +
 a*d*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^
2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((
a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sq
rt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2
- 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) + sqr
t(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((
a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*
c^3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + (b^5 - 5*a*b^3*c + 4
*a^2*b*c^2)*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^
4*c)*e^2*f^4))) - 2)/(a*e^2*f^2*x + a*d*e*f^2)
```

Sympy [A] time = 4.56816, size = 258, normalized size = 1.26

$$\operatorname{RootSum}\left(t^4\left(256a^5c^2e^4f^8-128a^4b^2ce^4f^8+16a^3b^4e^4f^8\right)+t^2\left(48a^2bc^2e^2f^4-28ab^3ce^2f^4+4b^5e^2f^4\right)+c^3,\left(t\mapsto t\,\mathrm{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2*e**4*f**8 - 128*a**4*b**2*c*e**4*f**8 + 16*a**
3*b**4*e**4*f**8) + _t**2*(48*a**2*b*c**2*e**2*f**4 - 28*a*b**3*c*e**2*f**4
 + 4*b**5*e**2*f**4) + c**3, Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**
3*f**6 + 48*_t**3*a**4*b**2*c*e**3*f**6 - 8*_t**3*a**3*b**4*e**3*f**6 - 10*
_t*a**2*b*c**2*e*f**2 + 10*_t*a*b**3*c*e*f**2 - 2*_t*b**5*e*f**2 + a*c**3*d
 - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e*f**2 + a*e**2*f**2*x
)
```

Giac [C] time = 2.99966, size = 5022, normalized size = 24.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -2*(3*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs
(a)/(a^2*c))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))
)^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) - (
a^3*c)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^
3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 - 9
*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(
a^2*c))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*s
in(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) + 3*(a^3*c)^(3/4)*b*cos
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sin(5/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sinh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) + 9*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*cosh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*a
bs(a)/(a^2*c))))^2 - 3*(a^3*c)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b*abs(a)/(a^2*c))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*a
bs(a)/(a^2*c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)
)))^2 - 3*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
*abs(a)/(a^2*c))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(
a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3
 + (a^3*c)^(3/4)*b*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)
/(a^2*c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3
 + (a^3*c)^(1/4)*a^2*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^
2*c))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))
 - (a^3*c)^(1/4)*a^2*c*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*ab
s(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))
)*arctan(-((c/(a*f^8))^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b*abs(a)
/(a^2*c)))*e^(-1) + e^(-1)/((f*x*e + d*f)*f))*e/((c/(a*f^8))^(1/4)*sin(5/4*
pi + 1/2*arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))))/(sqrt(b^2 - 4*a*c)*a^2*b
*f^2*abs(a)*e - (a*b^2*f^2*e - 4*a^2*c*f^2*e)*a^2) - 2*(3*(a^3*c)^(3/4)*b*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*cosh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sin(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) - (a^3*c)^(3/4)*b*cosh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sin(1/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 - 9*(a^3*c)^(3/4)*b*cos(1/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*cosh(1/2*im
```

ag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) + 3*(a^3*c)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) + 9*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - 3*(a^3*c)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - 3*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 + (a^3*c)^(3/4)*b*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 + (a^3*c)^(1/4)*a^2*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) - (a^3*c)^(1/4)*a^2*c*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))))*arctan(-((c/(a*f^8))^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))*e^(-1) + e^(-1)/((f*x*e + d*f)*f))*e/((c/(a*f^8))^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))))/(sqrt(b^2 - 4*a*c)*a^2*b*f^2*abs(a)*e - (a*b^2*f^2*e - 4*a^2*c*f^2*e)*a^2) + ((a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 - 3*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - 3*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) + 9*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) + 3*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - 9*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - (a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 + 3*(a^3*c)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 + (a^3*c)^(1/4)*a^2*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) - (a^3*c)^(1/4)*a^2*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))))*log(sqrt(c/(a*f^8))*e^(-2) + 2*(c/(a*f^8))^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))*e^(-2)/((f*x*e + d*f)*f) + e^(-2)/((f*x*e + d*f)^2*f^2))/(sqrt(b^2 - 4*a*c)*a^2*b*f^2*abs(a)*e - (a*b^2*f^2*e - 4*a^2*c*f^2*e)*a^2) + ((a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 - 3*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sin(1/4*pi + 1/2*real_part(arcsi

```
n(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - 3*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*cosh(1/2*imag_part(arc
sin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b*abs(a)/(a^2*c)))) + 9*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b*abs(a)/(a^2*c))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*
abs(a)/(a^2*c))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c
)))) + 3*(a^3*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*
abs(a)/(a^2*c))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c
))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - 9*(a^3
*c)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(1/4*pi
 + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2 - (a^3*c)^(3/4)*b*cos(1/4*pi
 + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 + 3*(a^3*c)^(3/4)*b*cos(1/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sin(1/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^2*sinh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))^3 + (a^3*c)^(1/4)*a^2*c*cos(1/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*cosh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))) - (a^3*c)^(1/4)*a^2*c*cos(1/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c))))*sinh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))))*log(sqrt(c/(a*f^8))*e^(-2) + 2*(
c/(a*f^8))^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b*abs(a)/(a^2*c)))*e
^(-2)/((f*x*e + d*f)*f) + e^(-2)/((f*x*e + d*f)^2*f^2))/(sqrt(b^2 - 4*a*c)*
a^2*b*f^2*abs(a)*e - (a*b^2*f^2*e - 4*a^2*c*f^2*e)*a^2) - e^(-1)/((f*x*e +
d*f)*a*f)
```

3.644 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=133

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef^3\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}-\frac{b\log(d+ex)}{a^2ef^3}-\frac{1}{2aef^3(d+ex)^2}$$

```
[Out] -1/(2*a*e*f^3*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/S
qrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e*f^3) - (b*Log[d + e*x])/(a^2*
e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e*f^3)
```

Rubi [A] time = 0.19531, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1142, 1114, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef^3\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}-\frac{b\log(d+ex)}{a^2ef^3}-\frac{1}{2aef^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(2*a*e*f^3*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/S
qrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e*f^3) - (b*Log[d + e*x])/(a^2*
e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e*f^3)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{ef^3}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx,x,(d+ex)^2\right)}{2ef^3}\\
&= -\frac{1}{2aef^3(d+ex)^2}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx,x,(d+ex)^2\right)}{2aef^3}\\
&= -\frac{1}{2aef^3(d+ex)^2}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx,x,(d+ex)^2\right)}{2aef^3}\\
&= -\frac{1}{2aef^3(d+ex)^2}-\frac{b\log(d+ex)}{a^2ef^3}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{2a^2ef^3}\\
&= -\frac{1}{2aef^3(d+ex)^2}-\frac{b\log(d+ex)}{a^2ef^3}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,(d+ex)^2\right.}{4a^2ef^3}\\
&= -\frac{1}{2aef^3(d+ex)^2}-\frac{b\log(d+ex)}{a^2ef^3}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}\\
&= -\frac{1}{2aef^3(d+ex)^2}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}ef^3}-\frac{b\log(d+ex)}{a^2ef^3}+\frac{b}{-}
\end{aligned}$$

Mathematica [A] time = 0.133494, size = 157, normalized size = 1.18

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{(d+ex)^2}-4b\log(d+ex)}{4a^2ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c
])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b^2
 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2*e*f^3)
```

Maple [C] time = 0.007, size = 222, normalized size = 1.7

$$-\frac{1}{2\,aef^{3}\,(ex+d)^{2}}-\frac{b\ln(ex+d)}{a^{2}ef^{3}}+\frac{1}{2\,a^{2}ef^{3}}\sum_{\_R=\mathrm{RootOf}\left(ce^{4}\_Z^{4}+4\,cde^{3}\_Z^{3}+\left(6\,cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4\,cd^{3}e+2\,bde\right)\_Z+cd^{4}+bd^{2}+a\right)}\frac{\left(\_R^{3}bce^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] -1/2/a/e/f^3/(e*x+d)^2-b*ln(e*x+d)/a^2/e/f^3+1/2/f^3/a^2/e*sum((_R^3*b*c*e^
3+3*_R^2*b*c*d*e^2+e*(3*b*c*d^2-a*c+b^2)*_R+b*c*d^3-a*c*d+b^2*d)/(2*_R^3*c*
e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^
4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4
+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [B] time = 2.41604, size = 1814, normalized size = 13.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x +
 (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3
+ 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*
d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4
*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)
*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*
```

```
d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*
e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^
2*b^2 - 4*a^3*c)*e^3*f^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x + (a^2*b^2
 - 4*a^3*c)*d^2*e*f^3), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*e^2*x^2
+ 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((
b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log
(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c
*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x
 + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*e^3*f^3*x^2 + 2*
(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x + (a^2*b^2 - 4*a^3*c)*d^2*e*f^3)]
```

Sympy [B] time = 17.9185, size = 532, normalized size = 4.

$$\left(\frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2ef^3\left(4ac-b^2\right)}\right)\log\left(\frac{2dx}{e} + x^2 + \frac{-8a^3cef^3\left(\frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2ef^3\left(4ac-b^2\right)}\right) + 2a^2b^2ef^3\left(\frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}\left(2ac\ldots\right.}{4a^2ef^3\left(4ac-\ldots\right.}\right.}{2ac^2e^2 - b^2ce^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (b/(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a
*c - b**2)))*log(2*d*x/e + x**2 + (-8*a**3*c*e*f**3*(b/(4*a**2*e*f**3) - sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2))) + 2*a**2*b
**2*e*f**3*(b/(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*
e*f**3*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2*d**2 - b**3 - b**2*c*d**2)/(2*
a*c**2*e**2 - b**2*c*e**2)) + (b/(4*a**2*e*f**3) + sqrt(-4*a*c + b**2)*(2*a
*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2)))*log(2*d*x/e + x**2 + (-8*a**3*c*
e*f**3*(b/(4*a**2*e*f**3) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f*
*3*(4*a*c - b**2))) + 2*a**2*b**2*e*f**3*(b/(4*a**2*e*f**3) + sqrt(-4*a*c +
 b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2*
d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) - 1/(2*a*d**2*e*f
**3 + 4*a*d*e**2*f**3*x + 2*a*e**3*f**3*x**2) - b*log(d/e + x)/(a**2*e*f**3
)
```

Giac [B] time = 1.46316, size = 618, normalized size = 4.65

$$\frac{\left(a^2b^2f^3e - 2\,a^3cf^3e\right)\sqrt{b^2-4\,ac}\log\left(\left|4\,a^3cf^3e^4 + 2\left(a^2bc + \sqrt{b^2-4\,ac}a^2c\right)f^3x^2e^6 + 4\left(a^2bc + \sqrt{b^2-4\,ac}a^2c\right)df^3xe^5 + 2\left(\ldots\right.\right.\right.}{4\left(a^4b^2f^6e^2 - 4\,a^5cf^6e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/4*(a^2*b^2*f^3*e - 2*a^3*c*f^3*e)*sqrt(b^2 - 4*a*c)*log(abs(4*a^3*c*f^3*e
^4 + 2*(a^2*b*c + sqrt(b^2 - 4*a*c)*a^2*c)*f^3*x^2*e^6 + 4*(a^2*b*c + sqrt(
b^2 - 4*a*c)*a^2*c)*d*f^3*x*e^5 + 2*(a^2*b*c + sqrt(b^2 - 4*a*c)*a^2*c)*d^2
*f^3*e^4))/(a^4*b^2*f^6*e^2 - 4*a^5*c*f^6*e^2) - 1/4*(a^2*b^2*f^3*e - 2*a^3
*c*f^3*e)*sqrt(b^2 - 4*a*c)*log(abs(-4*a^3*c*f^3*e^4 - 2*(a^2*b*c - sqrt(b^
2 - 4*a*c)*a^2*c)*f^3*x^2*e^6 - 4*(a^2*b*c - sqrt(b^2 - 4*a*c)*a^2*c)*d*f^3
*x*e^5 - 2*(a^2*b*c - sqrt(b^2 - 4*a*c)*a^2*c)*d^2*f^3*e^4))/(a^4*b^2*f^6*e
```

```
^2 - 4*a^5*c*f^6*e^2) + 1/4*b*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*
c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(
a^2*f^3) - b*e^(-1)*log(abs(x*e + d))/(a^2*f^3) - 1/2*e^(-1)/((x*e + d)^2*a
*f^3)
```

3.645 $\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2ef^4\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2ef^4\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2ef^4(d+ex)}-\frac{1}{3aef^4(d+ex)^3}$$

```
[Out] -1/(3*a*e*f^4*(d + e*x)^3) + b/(a^2*e*f^4*(d + e*x)) + (Sqrt[c]*(b + (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) + (Sqrt[c]
*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4
)
```

Rubi [A] time = 0.48815, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1123, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2ef^4\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2ef^4\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2ef^4(d+ex)}-\frac{1}{3aef^4(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(3*a*e*f^4*(d + e*x)^3) + b/(a^2*e*f^4*(d + e*x)) + (Sqrt[c]*(b + (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) + (Sqrt[c]
*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4
)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{ef^4}$$

$$= -\frac{1}{3aef^4(d+ex)^3} + \frac{\text{Subst}\left(\int \frac{-3b-3cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{3aef^4}$$

$$= -\frac{1}{3aef^4(d+ex)^3} + \frac{b}{a^2ef^4(d+ex)} - \frac{\text{Subst}\left(\int \frac{-3\left(b^2-ac\right)-3bcx^2}{a+bx^2+cx^4}\,dx,x,d\ldots\right.}{3a^2ef^4}$$

$$= -\frac{1}{3aef^4(d+ex)^3} + \frac{b}{a^2ef^4(d+ex)} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{\ldots}{\frac{b}{2}+\ldots}\right.}{2a^2ef^4}$$

$$= -\frac{1}{3aef^4(d+ex)^3} + \frac{b}{a^2ef^4(d+ex)} + \frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(\ldots}{\sqrt{b-\sqrt{b^2\ldots}}}\right.}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}ef^4}$$

Mathematica [A] time = 0.212881, size = 238, normalized size = 1.01

$$\frac{\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}}{6a^2ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt
[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(6*a^2*e*f^4)
```

Maple [C] time = 0.009, size = 197, normalized size = 0.8

$$-\frac{1}{3\,aef^4\,(ex+d)^3}+\frac{b}{a^2ef^4\,(ex+d)}+\frac{1}{2\,a^2ef^4}\sum_{\_R=\mathrm{RootOf}\left(ce^4\_Z^4+4\,cde^3\_Z^3+\left(6\,cd^2e^2+be^2\right)\_Z^2+\left(4\,cd^3e+2\,bde\right)\_Z+cd^4+bd^2+a\right)}\frac{\left(\_R^2bce^2\right.}{2\,ce^3\_R^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] -1/3/a/e/f^4/(e*x+d)^3+b/a^2/e/f^4/(e*x+d)+1/2/f^4/a^2/e*sum((_R^2*b*c*e^2+
2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c
*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+
b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [B] time = 1.89719, size = 4533, normalized size = 19.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] 1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*
a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2 -
4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/
((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x
+ 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c
 + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)
*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/
((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^
2 - 4*a^6*c)*e^2*f^8))) - 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^
2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b
^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^
6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3
*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)
*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17
```

```
*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a
^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2
*f^8))) - 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^
2*f^4*x + a^2*d^3*e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^
4*f^16)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*lo
g(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*
c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a
^11*c)*e^4*f^16)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*
a^4*c^4)*e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16))
- b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) + 3*sqrt(1
/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*
e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - b^5 + 5
*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a
*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)
*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16))
+ (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sq
rt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - b^5 + 5*a*b^3*c
- 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) - 2*a)/(a^2*e^4*f^4*x^3 + 3*
a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)
```

Sympy [A] time = 14.6098, size = 411, normalized size = 1.74

$$\frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3ef^4+9a^2d^2e^2f^4x+9a^2de^3f^4x^2+3a^2e^4f^4x^3}+\operatorname{RootSum}\left(t^4\left(256a^7c^2e^4f^{16}-128a^6b^2ce^4f^{16}+16a^5b^4e^4f^{16}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e*f**4 + 9*a**2*d*
*2*e**2*f**4*x + 9*a**2*d*e**3*f**4*x**2 + 3*a**2*e**4*f**4*x**3) + RootSum
(_t**4*(256*a**7*c**2*e**4*f**16 - 128*a**6*b**2*c*e**4*f**16 + 16*a**5*b**
4*e**4*f**16) + _t**2*(-80*a**3*b*c**3*e**2*f**8 + 100*a**2*b**3*c**2*e**2*
f**8 - 36*a*b**5*c*e**2*f**8 + 4*b**7*e**2*f**8) + c**5, Lambda(_t, _t*log(
x + (-96*_t**3*a**7*b*c**2*e**3*f**12 + 56*_t**3*a**6*b**3*c*e**3*f**12 - 8
*_t**3*a**5*b**5*e**3*f**12 - 4*_t*a**4*c**4*e*f**4 + 32*_t*a**3*b**2*c**3*
e*f**4 - 40*_t*a**2*b**4*c**2*e*f**4 + 16*_t*a*b**6*c*e*f**4 - 2*_t*b**8*e*
f**4 + a**2*c**5*d - 3*a*b**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2
*c**4*e + b**4*c**3*e))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)\left(efx+df\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^4), x)
```

3.646 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=279

$$\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{f^4\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2}}}$$

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b
^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4
*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.538598, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1142, 1120, 1166, 205}

$$\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{f^4\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b
^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4
*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx,x,d+ex\right)}{e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{f^4\,\text{Subst}\left(\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{2\left(b^2-4ac\right)e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(\left(b^2+4ac-b\sqrt{b^2-4ac}\right)f^4\right)\text{Subst}}{4\left(b^2-4a\right.}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)f^4\tan^{-1}\left(\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2}}}$$

Mathematica [A] time = 0.535135, size = 266, normalized size = 0.95

$$\frac{f^4\left(-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(4*e)
```

Maple [C] time = 0.016, size = 695, normalized size = 2.5

$$-\frac{f^4be^2x^3}{\left(2\,ce^4x^4+8\,cde^3x^3+12\,cd^2e^2x^2+8\,cd^3ex+2\,be^2x^2+2\,cd^4+4\,bdex+2\,bd^2+2\,a\right)\left(4\,ac-b^2\right)} - \frac{}{\left(2\,ce^4x^4+8\,cde^3x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*b*e^2/(4*a*c-b^2)*x^3-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b*e/(4*a
*c-b^2)*x^2-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*d^2-f^4/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*
c-b^2)*x*a-1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b-f^4/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4
*a*c-b^2)*a+1/4*f^4/(4*a*c-b^2)/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2
*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=Roo
tOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*
_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} f^4 \int -\frac{be^2x^2 + 2\,bdex + bd^2 - 2\,a}{\left(b^2c - 4\,ac^2\right)e^4x^4 + 4\left(b^2c - 4\,ac^2\right)de^3x^3 + \left(b^2c - 4\,ac^2\right)d^4 + \left(b^3 - 4\,abc + 6\left(b^2c - 4\,ac^2\right)d^2\right)e^2x^2 + ab^2 - 4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*f^4*integrate(-(b*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c - 4*a*c^2
)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 -
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b
*e^3*f^4*x^3 + 3*b*d*e^2*f^4*x^2 + (3*b*d^2 + 2*a)*e*f^4*x + (b*d^3 + 2*a*d
)*f^4)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 -
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (
b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2)*e)
```

Fricas [B] time = 2.11815, size = 5463, normalized size = 19.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] 1/4*(2*b*e^3*f^4*x^3 + 6*b*d*e^2*f^4*x^2 + 2*(3*b*d^2 + 2*a)*e*f^4*x + 2*(b
*d^3 + 2*a*d)*f^4 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c
^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b
^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt
(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c
- 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 +
 4*a*c)*d*f^12 + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 + 2*sqrt(f
```

```
^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c -
12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*
f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4
))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a
*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^
2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*
a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x
 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(
-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*
e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2
 + 4*a*c)*e*f^12*x + (3*b^2 + 4*a*c)*d*f^12 - sqrt(1/2)*((b^4 - 8*a*b^2*c +
 16*a^2*c^2)*e*f^8 + 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
 - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*
e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64
*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)))
 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (
b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d
^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c +
(b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 -
 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 + 4*a*c)*d*f^12 +
sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 - 2*sqrt(f^16/((b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48
*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/(
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*
b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2
*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b
^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x
^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*
c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c
)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e
^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12
*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*
x + (3*b^2 + 4*a*c)*d*f^12 - sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^
8 - 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4
))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3
+ 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((
b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))))/((b^2*c - 4*a*c
^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4
*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*
x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Sympy [B] time = 50.0545, size = 639, normalized size = 2.29

$$-\frac{2adf^4 + bd^3f^4 + 3bde^2f^4x^2 + be^3f^4x^3 + x\left(2aef^4\right.}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2\left(8abce^3 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -(2*a*d*f**4 + b*d**3*f**4 + 3*b*d*e**2*f**4*x**2 + b*e**3*f**4*x**3 + x*(2
*a*e*f**4 + 3*b*d**2*e*f**4))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8
```

```
*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*
b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e*
*3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c
*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + Root
Sum(_t**4*(1048576*a**6*c**7*e**4 - 1572864*a**5*b**2*c**6*e**4 + 983040*a*
*4*b**4*c**5*e**4 - 327680*a**3*b**6*c**4*e**4 + 61440*a**2*b**8*c**3*e**4
- 6144*a*b**10*c**2*e**4 + 256*b**12*c*e**4) + _t**2*(-12288*a**4*b*c**4*e*
*2*f**8 + 8192*a**3*b**3*c**3*e**2*f**8 - 1536*a**2*b**5*c**2*e**2*f**8 + 1
6*b**9*e**2*f**8) + 16*a**3*c**2*f**16 + 24*a**2*b**2*c*f**16 + 9*a*b**4*f*
*16, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 12288*_t**3*a**2
*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**7*c*e**3 + 64*
_t*a**2*c**2*e*f**8 - 128*_t*a*b**2*c*e*f**8 - 4*_t*b**4*e*f**8 + 4*a*c*d*f
**12 + 3*b**2*d*f**12)/(4*a*c*e*f**12 + 3*b**2*e*f**12))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(efx+df\right)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^2, x)
```

3.647 $\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{f^3\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)) - (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 -
 4*a*c)^(3/2)*e)
```

Rubi [A] time = 0.137605, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1114, 638, 618, 206}

$$\frac{f^3\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)) - (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 -
 4*a*c)^(3/2)*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx &= \frac{f^3\,\mathrm{Subst}\left(\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}\\ &= \frac{f^3\,\mathrm{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2e}\\ &= \frac{f^3\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(bf^3\right)\mathrm{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d\right.}{2\left(b^2-4ac\right)e}\\ &= \frac{f^3\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\left(bf^3\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b\right.}{\left(b^2-4ac\right)e}\\ &= \frac{f^3\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}\end{aligned}$$

Mathematica [A] time = 0.134716, size = 103, normalized size = 1.

$$\frac{f^3\left(\frac{2a+b(d+ex)^2}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)}-\frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^3*((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x
)^2))) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a
*c)^(3/2)))/(2*e)
```

Maple [C] time = 0.017, size = 500, normalized size = 4.9

$$-\frac{f^3bex^2}{\left(2\,ce^4x^4+8\,cde^3x^3+12\,cd^2e^2x^2+8\,cd^3ex+2\,be^2x^2+2\,cd^4+4\,bdex+2\,bd^2+2\,a\right)\left(4\,ac-b^2\right)}-\frac{}{\left(ce^4x^4+4\,cde^3x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*b*e/(4*a*c-b^2)*x^2-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*d/(4*a*c-b^2)*
x-1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b*d^2-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
```

```
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*a
+1/2*f^3*b/(4*a*c-b^2)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c
*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-bf^3 \int -\frac{ex+d}{\left(b^2c-4\,ac^2\right)e^4x^4+4\left(b^2c-4\,ac^2\right)de^3x^3+\left(b^2c-4\,ac^2\right)d^4+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^2x^2+ab^2-4\,a^2c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] -b*f^3*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2
)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*
d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*f^3*x^2 + 2*b*d*e*f^3*x +
 (b*d^2 + 2*a)*f^3)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*
x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a
*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a
^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Fricas [B] time = 1.67859, size = 2290, normalized size = 22.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*((b^3 - 4*a*b*c)*e^2*f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2
- 8*a^2*c + (b^3 - 4*a*b*c)*d^2)*f^3 - (b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x
^3 + (6*b*c*d^2 + b^2)*e^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d
^4 + b^2*d^2 + a*b)*f^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3
*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 +
 b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^
2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*
x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^
2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3
)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*e^2*
f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c
)*d^2)*f^3 - 2*(b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x^3 + (6*b*c*d^2 + b^2)*e
^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d^4 + b^2*d^2 + a*b)*f^3)
*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b
^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4
*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (
```

```
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

Sympy [B] time = 39.0261, size = 554, normalized size = 5.38

$$\frac{bf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3cf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e}-\frac{bf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*f**
3*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**3*c*f**3*sqrt(-1/(4*a*c - b**2)**3) -
 b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b*c*d**2*f**3)/(2*b*c
*e**2*f**3))/(2*e) - b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 +
 (16*a**2*b*c**2*f**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*f**3*sqrt(-1/
(4*a*c - b**2)**3) + b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b
*c*d**2*f**3)/(2*b*c*e**2*f**3))/(2*e) - (2*a*f**3 + b*d**2*f**3 + 2*b*d*e*
f**3*x + b*e**2*f**3*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*
c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**
2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3
+ 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*
e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

Giac [B] time = 2.37564, size = 522, normalized size = 5.07

$$\frac{\left(b^3f^3e-4abcf^3e\right)\sqrt{b^2-4ac}\log\left(\left|\left(b+\sqrt{b^2-4ac}\right)x^2e^2+2\left(b+\sqrt{b^2-4ac}\right)dxe+\left(b+\sqrt{b^2-4ac}\right)d^2+2a\right|\right)}{2\left(b^6e^2-12ab^4ce^2+48a^2b^2c^2e^2-64a^3c^3e^2\right)}-\frac{\left(b^3f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*f^3*e - 4*a*b*c*f^3*e)*sqrt(b^2 - 4*a*c)*log(abs((b + sqrt(b^2 - 4
*a*c))*x^2*e^2 + 2*(b + sqrt(b^2 - 4*a*c))*d*x*e + (b + sqrt(b^2 - 4*a*c))*
d^2 + 2*a))/(b^6*e^2 - 12*a*b^4*c*e^2 + 48*a^2*b^2*c^2*e^2 - 64*a^3*c^3*e^2
) - 1/2*(b^3*f^3*e - 4*a*b*c*f^3*e)*sqrt(b^2 - 4*a*c)*log(abs(-(b - sqrt(b^
2 - 4*a*c))*x^2*e^2 - 2*(b - sqrt(b^2 - 4*a*c))*d*x*e - (b - sqrt(b^2 - 4*a
*c))*d^2 - 2*a))/(b^6*e^2 - 12*a*b^4*c*e^2 + 48*a^2*b^2*c^2*e^2 - 64*a^3*c^
3*e^2) + 1/2*(b*f^3*x^2*e^2 + 2*b*d*f^3*x*e + b*d^2*f^3 + 2*a*f^3)/((c*x^4*
e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2
*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

3.648 $\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=263

$$-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}f^2\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}f^2\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2}}}$$

```
[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(
3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*f^
2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.367975, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1142, 1119, 1166, 205}

$$-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}f^2\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}f^2\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```


```
[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(
3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*f^
2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{f^2\,\text{Subst}\left(\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx, x, d+\right.}{2\left(b^2-4ac\right)e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(c\left(2b-\sqrt{b^2-4ac}\right)f^2\right)\text{Subst}}{2\left(b^2-\right.}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)f^2\tan^{-1}\left(\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2}}}$$

Mathematica [A] time = 1.03501, size = 250, normalized size = 0.95

$$-\frac{f^2\left(\frac{b(d+ex)+2c(d+ex)^3}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(f^2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(2*e)
```

Maple [C] time = 0.016, size = 693, normalized size = 2.6

$$\frac{f^2ce^2x^3}{\left(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a\right)\left(4\,ac-b^2\right)}+3\,\frac{}{\left(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)*c*e^2/(4*a*c-b^2)*x^3+3*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c*e/(4*a*c-b^2)
*x^2+3*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c*d^2+1/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2
)*x*b+f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*c+1/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-
b^2)*b+1/4*f^2/(4*a*c-b^2)/e*sum((2*_R^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^
3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(
c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c
*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} f^2 \int -\frac{2\,ce^2x^2 + 4\,cdex + 2\,cd^2 - b}{\left(b^2c - 4\,ac^2\right)e^4x^4 + 4\left(b^2c - 4\,ac^2\right)de^3x^3 + \left(b^2c - 4\,ac^2\right)d^4 + \left(b^3 - 4\,abc + 6\left(b^2c - 4\,ac^2\right)d^2\right)e^2x^2 + ab^2 - 4\,a^2c +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*f^2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4*a*c^
2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 -
 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(
2*c*e^3*f^2*x^3 + 6*c*d*e^2*f^2*x^2 + (6*c*d^2 + b)*e*f^2*x + (2*c*d^3 + b*
d)*f^2)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 -
 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
(b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*d^2)*e)
```

Fricas [B] time = 2.06679, size = 5496, normalized size = 20.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] -1/4*(4*c*e^3*f^2*x^3 + 12*c*d*e^2*f^2*x^2 + 2*(6*c*d^2 + b)*e*f^2*x + 2*(2
*c*d^3 + b*d)*f^2 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c
^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b
^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b
^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^
3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2
*c + 4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4
```

```
- (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*
a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/
((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1/2)*((b^2*c -
4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*
c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)
*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e
^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((
3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2*sqrt(1/2)*((b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^
3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*
a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4
*c^3)*e^2))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d
*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c
- 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 -
12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^
3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2*c +
4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 + (a
*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c
)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2
*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*
a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c
^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4
*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*
x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt
(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c
^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*
e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2
*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2*sqrt(1/2)*((b^5 - 8
*a*b^3*c + 16*a^2*b*c^2)*e*f^4 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 2
56*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b
^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)
*e^2))))/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
 (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
 - 4*a*b*c)*d^2)*e)
```

Sympy [B] time = 39.2046, size = 646, normalized size = 2.46

$$\frac{bdf^2 + 2cd^3f^2 + 6cde^2f^2x^2 + 2ce^3f^2x^3 + x\left(bef\right.}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2\left(8abce^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (b*d*f**2 + 2*c*d**3*f**2 + 6*c*d*e**2*f**2*x**2 + 2*c*e**3*f**2*x**3 + x*(
```

```
b*e*f**2 + 6*c*d**2*e*f**2))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*
a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b
**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**
3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*
d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootS
um(_t**4*(1048576*a**7*c**6*e**4 - 1572864*a**6*b**2*c**5*e**4 + 983040*a**
5*b**4*c**4*e**4 - 327680*a**4*b**6*c**3*e**4 + 61440*a**3*b**8*c**2*e**4 -
 6144*a**2*b**10*c*e**4 + 256*a*b**12*e**4) + _t**2*(-12288*a**4*b*c**4*e**
2*f**4 + 8192*a**3*b**3*c**3*e**2*f**4 - 1536*a**2*b**5*c**2*e**2*f**4 + 16
*b**9*e**2*f**4) + 16*a**2*c**3*f**8 + 24*a*b**2*c**2*f**8 + 9*b**4*c*f**8,
 Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4*e**3 - 8192*_t**3*a**4*b**2*c
**3*e**3 + 512*_t**3*a**2*b**6*c*e**3 - 64*_t**3*a*b**8*e**3 - 128*_t*a**2*
b*c**2*e*f**4 - 16*_t*a*b**3*c*e*f**4 - 4*_t*b**5*e*f**4 + 4*a*c**2*d*f**6
+ 3*b**2*c*d*f**6)/(4*a*c**2*e*f**6 + 3*b**2*c*e*f**6))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(efx+df\right)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^2, x)
```

3.649 $\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx$

Optimal. Leaf size=98

$$\frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{f\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[Out] -(f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)) + (2*c*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 -
4*a*c)^(3/2)*e)
```

Rubi [A] time = 0.127071, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1142, 1107, 614, 618, 206}

$$\frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{f\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)) + (2*c*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 -
4*a*c)^(3/2)*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f\,\text{Subst}\left(\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{(cf)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+e\right.}{\left(b^2-4ac\right)e}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{(2cf)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\right.}{\left(b^2-4ac\right)e}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A] time = 0.128798, size = 99, normalized size = 1.01

$$-\frac{f\left(\frac{4c\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\right)}{2e\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(f*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*ArcTa
n[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/(2*(b^2 -
 4*a*c)*e)
```

Maple [C] time = 0.019, size = 484, normalized size = 4.9

$$\frac{fcex^2}{\left(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a\right)\left(4\,ac-b^2\right)}+2\,\frac{}{\left(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)*c*e/(4*a*c-b^2)*x^2+2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*d/(4*a*c-b^2)*x+f/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
```

```
+b*d^2+a)/e/(4*a*c-b^2)*c*d^2+1/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^
2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b+f*c/(4*a*c
-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z
^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,cf\int -\frac{ex+d}{\left(b^2c-4\,ac^2\right)e^4x^4+4\left(b^2c-4\,ac^2\right)de^3x^3+\left(b^2c-4\,ac^2\right)d^4+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^2x^2+ab^2-4\,a^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 2*c*f*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)
*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d
^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2
)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*f*x^2 + 4*c*d*e*f*x + (2
*c*d^2 + b)*f)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +
 (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)
*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*d^2)*e)
```

Fricas [B] time = 1.75624, size = 2268, normalized size = 23.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*e^2*f*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x + 2*(c^2
*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 + (6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3
 + b*c*d)*e*f*x + (c^2*d^4 + b*c*d^2 + a*c)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2
*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*
c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x
+ 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c
*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) + (b^3 - 4*a*b*c +
2*(b^2*c - 4*a*c^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4
*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*e^2*f*
x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x - 4*(c^2*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 +
(6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3 + b*c*d)*e*f*x + (c^2*d^4 + b*c*
d^2 + a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2
 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c
^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^
2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*
```

```
c + 16*a^2*b*c^2)*d^2)*e)]
```

Sympy [B] time = 26.7481, size = 525, normalized size = 5.36

$$-\frac{cf\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2c^3f\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^2c^2f\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^4cf\sqrt{-\frac{1}{(4ac-b^2)^3}}+bcf+2c^2d^2f}{2c^2e^2f}\right)}{e}+\frac{cf\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*f*sqrt(
-1/(4*a*c - b**2)**3) + 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) - b**4*c
*f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e +
 c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*f*sqrt(-
1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*
f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e +
(b*f + 2*c*d**2*f + 4*c*d*e*f*x + 2*c*e**2*f*x**2)/(8*a**2*c*e - 2*a*b**2*e
 + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**
4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**
4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**
2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2
*c*d**3*e**2))
```

Giac [B] time = 2.40469, size = 506, normalized size = 5.16

$$-\frac{\left(b^2cfe-4ac^2fe\right)\sqrt{b^2-4ac}\log\left(\left|\left(b+\sqrt{b^2-4ac}\right)x^2e^2+2\left(b+\sqrt{b^2-4ac}\right)dxe+\left(b+\sqrt{b^2-4ac}\right)d^2+2a\right|\right)}{b^6e^2-12ab^4ce^2+48a^2b^2c^2e^2-64a^3c^3e^2}+\frac{\left(b^2cfe\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -(b^2*c*f*e - 4*a*c^2*f*e)*sqrt(b^2 - 4*a*c)*log(abs((b + sqrt(b^2 - 4*a*c)
)*x^2*e^2 + 2*(b + sqrt(b^2 - 4*a*c))*d*x*e + (b + sqrt(b^2 - 4*a*c))*d^2 +
 2*a))/(b^6*e^2 - 12*a*b^4*c*e^2 + 48*a^2*b^2*c^2*e^2 - 64*a^3*c^3*e^2) + (
b^2*c*f*e - 4*a*c^2*f*e)*sqrt(b^2 - 4*a*c)*log(abs(-(b - sqrt(b^2 - 4*a*c))
*x^2*e^2 - 2*(b - sqrt(b^2 - 4*a*c))*d*x*e - (b - sqrt(b^2 - 4*a*c))*d^2 -
2*a))/(b^6*e^2 - 12*a*b^4*c*e^2 + 48*a^2*b^2*c^2*e^2 - 64*a^3*c^3*e^2) - 1/
2*(2*c*f*x^2*e^2 + 4*c*d*f*x*e + 2*c*d^2*f + b*f)/((c*x^4*e^4 + 4*c*d*x^3*e
^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2
+ a)*(b^2*e - 4*a*c*e))
```

3.650 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=174

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}+\frac{\log(d+ex)}{a^2ef}+\frac{-2ac+b^2+bc(d+ex}{2aef\left(b^2-4ac\right)\left(a+b(d+ex)^2\right.}$$

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e*f) + Log[d + e*x]/(a^2*e*f) - Log[a
 + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e*f)
```

Rubi [A] time = 0.295372, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}+\frac{\log(d+ex)}{a^2ef}+\frac{-2ac+b^2+bc(d+ex}{2aef\left(b^2-4ac\right)\left(a+b(d+ex)^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e*f) + Log[d + e*x]/(a^2*e*f) - Log[a
 + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e*f)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac-b\ldots}{x\left(a+bx+cx\ldots\right.}\ldots\right.}{2a\left(b^2-\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int\left(\frac{-b^2+4ac}{ax}\ldots\right.\right.}{\ldots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2ef} - \frac{\text{Sub}\ldots}{\ldots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2ef} - \frac{\text{Sub}\ldots}{\ldots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2ef} - \frac{\log\left(\ldots\right.}{\ldots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\ldots}{2a^2\left(b^2-4a\ldots\right.}$$

Mathematica [A] time = 0.425243, size = 238, normalized size = 1.37

$$\frac{\frac{2a\left(-2ac+b^2+bc(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(\sqrt{\ldots}\right.}{\left(b^2-4ac\right)^{3/2}}\ldots}{4a^2ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b +
 c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c
] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/
(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^2*e*f)
```

Maple [C] time = 0.027, size = 714, normalized size = 4.1

$$\frac{\ln(ex+d)}{a^2ef} - \frac{bcex^2}{2fa\left(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a\right)\left(4ac-b^2\right)} - \frac{\ldots}{fa\left(ce^4x^4+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] ln(e*x+d)/a^2/e/f-1/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*c*e/(4*a*c-b^2)*x^2-1/f/a/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)*b*c*d/(4*a*c-b^2)*x-1/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c*d^2*b+1/f/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)/e/(4*a*c-b^2)*c-1/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+
4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2-1/2/f/a^2/
(4*a*c-b^2)/e*sum((c*e^3*(4*a*c-b^2)*_R^3+3*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*
a*c^2*d^2-3*b^2*c*d^2+5*a*b*c-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d
)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R
=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d
*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [B] time = 4.08337, size = 5227, normalized size = 30.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*x
^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((b^
3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*a*b
*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b^3 -
 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 -
6*a*b^2*c)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 +
 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d
)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*
a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
2*(2*c*d^3 + b*d)*e*x + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 +
 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d
*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2
```

```
*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*l
og(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*f*x^4 + 4*(a^2*b
^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*f*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 1
6*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*f*x^2 + 2
*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c +
 16*a^4*b*c^2)*d)*e^2*f*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*
c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2
)*d^2)*e*f), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*
b*c^2)*e^2*x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2
)*d^2 + 2*((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 +
(b^3*c - 6*a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2
*x^2 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)
*d^3 + (b^4 - 6*a*b^2*c)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 +
4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*
x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)
*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*
e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^
3 + b*d)*e*x + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*
c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
+ 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*
b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 1
6*a^4*c^3)*e^5*f*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*f*x
^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 +
 16*a^4*c^3)*d^2)*e^3*f*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)
*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*f*x + (a^3*b^4 - 8*a^4
*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b
^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.41473, size = 848, normalized size = 4.87

$$-\frac{\left(a^2b^5fe - 10\,a^3b^3cfe + 24\,a^4bc^2fe\right)\sqrt{b^2-4\,ac}\log\left(\left|4\,a^3cfe^4 + 2\left(a^2bc + \sqrt{b^2-4\,ac}a^2c\right)fx^2e^6 + 4\left(a^2bc + \sqrt{b^2-4\,ac}\right.\right.\right.}{4\left(a^4b^6f^2e^2 - 12\,a^5b^4cf^2e^2 + 48\,a^6b^2c^2f^2e^2 - 64\,a^7c^3f^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(a^2*b^5*f*e - 10*a^3*b^3*c*f*e + 24*a^4*b*c^2*f*e)*sqrt(b^2 - 4*a*c)*
log(abs(4*a^3*c*f*e^4 + 2*(a^2*b*c + sqrt(b^2 - 4*a*c)*a^2*c)*f*x^2*e^6 + 4
*(a^2*b*c + sqrt(b^2 - 4*a*c)*a^2*c)*d*f*x*e^5 + 2*(a^2*b*c + sqrt(b^2 - 4*
a*c)*a^2*c)*d^2*f*e^4))/(a^4*b^6*f^2*e^2 - 12*a^5*b^4*c*f^2*e^2 + 48*a^6*b^
2*c^2*f^2*e^2 - 64*a^7*c^3*f^2*e^2) + 1/4*(a^2*b^5*f*e - 10*a^3*b^3*c*f*e +
 24*a^4*b*c^2*f*e)*sqrt(b^2 - 4*a*c)*log(abs(-4*a^3*c*f*e^4 - 2*(a^2*b*c -
sqrt(b^2 - 4*a*c)*a^2*c)*f*x^2*e^6 - 4*(a^2*b*c - sqrt(b^2 - 4*a*c)*a^2*c)*
d*f*x*e^5 - 2*(a^2*b*c - sqrt(b^2 - 4*a*c)*a^2*c)*d^2*f*e^4))/(a^4*b^6*f^2*
e^2 - 12*a^5*b^4*c*f^2*e^2 + 48*a^6*b^2*c^2*f^2*e^2 - 64*a^7*c^3*f^2*e^2) -
 1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x
*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^2*f) + e^(-1)*log(abs(x
*e + d))/(a^2*f) + 1/2*(a*b*c*x^2*e^2 + 2*a*b*c*d*x*e + a*b*c*d^2 + a*b^2 -
 2*a^2*c)*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*
e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2 - 4*a*c)*a^2*f)
```

3.651 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=360

$$-\frac{3b^2-10ac}{2a^2ef^2\left(b^2-4ac\right)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2ef^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{}\right.}{2\sqrt{2}a^2ef}$$

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*f^2*(d + e*x)) + (b^2 - 2*a*c + b*
c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a + b*(d + e*x)^2 + c*(d
 + e*x)^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqr
t[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) + (Sqrt[c]*
(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rubi [A] time = 1.59926, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1121, 1281, 1166, 205}

$$-\frac{3b^2-10ac}{2a^2ef^2\left(b^2-4ac\right)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2ef^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{}\right.}{2\sqrt{2}a^2ef}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*f^2*(d + e*x)) + (b^2 - 2*a*c + b*
c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a + b*(d + e*x)^2 + c*(d
 + e*x)^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqr
t[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) + (Sqrt[c]*
(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{ef^2}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{-3}{x}\right.}{2a}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)ef^2(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)ef^2(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)ef^2(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex\right.}$$

Mathematica [A] time = 1.71326, size = 342, normalized size = 0.95

$$\frac{\frac{2(d+ex)\left(-3abc-2ac^2(d+ex)^2+b^2c(d+ex)^2+b^3\right)}{\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{}}}{4a^2ef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2
```

```
]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e*f^2)
```

Maple [C] time = 0.026, size = 1346, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/f^2/a^2/e/(e*x+d)-1/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*e^2/(4*a*c-b^2)*x^3+1/2/f^2/a
^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)*c*e^2/(4*a*c-b^2)*x^3*b^2-3/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+
6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c^2*e/(4*a
*c-b^2)*x^2+3/2/f^2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c*e/(4*a*c-b^2)*x^2*b^2-3/f^2/a/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)/(4*a*c-b^2)*x*c^2*d^2+3/2/f^2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2
*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c
*d^2-3/2/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*c+1/2/f^2/a^2/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a
*c-b^2)*x*b^3-1/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*c^2+1/2/f^2/a^2/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)*d^3/e/(4*a*c-b^2)*b^2*c-3/2/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b*c+1
/2/f^2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^3-1/4/f^2/a^2/(4*a*c-b^2)/e*sum((
c*e^2*(10*a*c-3*b^2)*_R^2+2*c*d*e*(10*a*c-3*b^2)*_R+10*a*c^2*d^2-3*b^2*c*d^
2+13*a*b*c-3*b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+
b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+
(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

Fricas [B] time = 2.76058, size = 9735, normalized size = 27.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + 2
*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^
2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2*c
- 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x + sqrt(1/2)*((a^2*b^2*c - 4*a^3
*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*
a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*
a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c
+ 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x +
((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4
*c)*d)*e*f^2)*sqrt(-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)
*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e
^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6
 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3
- 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 19
71*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b
^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4
- 1280*a^10*c^5)*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 25
50*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
- 64*a^13*c^3)*e^4*f^8)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 1054
9*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e*f^2)*sqrt(-((a^5*b^6
- 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*
b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*
a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c
^2 - 64*a^8*c^3)*e^2*f^4))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^
5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^
2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3
*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c -
 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*
a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sq
rt(-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((
a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7
 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c
+ 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4
 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5
625*a^2*b^2*c^5 - 2500*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8
*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)
*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e
^4*f^8)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 +
14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e*f^2)*sqrt(-((a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2
*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c
^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)
*e^2*f^4))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c
 - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*
c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^
3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4
+ 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (
a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sqrt(((a^5*b^6 - 1
2*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6
```

```
*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^1
1*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c -
385*a^2*b^3*c^2 + 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c
^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 -
 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*
c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*f^6*sqrt((81
*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + (27*b^
11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
 - 5200*a^5*b*c^5)*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 + 420*a^3*b*c^3
)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))) + sqrt
(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^
5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*
x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*
x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a
^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*
c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a
^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*
c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 +
420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*
f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)
*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d -
 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4
*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c
 + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*
b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + (27*b^11 - 486*a*b^9*c +
 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)
*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^
4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^
4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)
) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 + 420*a^3*b*c^3)/((a^5*b^6 - 12*a
^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))))/((a^2*b^2*c - 4*a^3*c^2
)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*
b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*
c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*
(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^
2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*
d)*e*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError
```

3.652 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=228

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3ef^3\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2ef^3\left(b^2-4ac\right)(d+ex)^2}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}-\frac{2b\log}{a}$$

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2)) + (b^2 - 2*a*c + b*c
*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)
/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e*f^3) - (2*b*Log[d + e*x])/(
a^3*e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e*f^3)
```

Rubi [A] time = 0.368873, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3ef^3\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2ef^3\left(b^2-4ac\right)(d+ex)^2}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}-\frac{2b\log}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2)) + (b^2 - 2*a*c + b*c
*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)
/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e*f^3) - (2*b*Log[d + e*x])/(
a^3*e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e*f^3)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntegerQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{ef^3}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2ef^3}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+e}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+e}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+e}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+e}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d\right.}$$

Mathematica [A] time = 0.492499, size = 287, normalized size = 1.26

$$\frac{\frac{\left(6a^2c^2+b^3\sqrt{b^2-4ac}-6ab^2c-4abc\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-6a^2c^2+b^3\sqrt{b^2-4ac}+6ab^2c-4abc\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right.}{\left(b^2-4ac\right)^{3/2}}}{2a^3ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*
x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d + e
*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[
b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)
^(3/2))/(2*a^3*e*f^3)
```

Maple [C] time = 0.03, size = 1047, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2/f^3/a^2/e/(e*x+d)^2-2*b*ln(e*x+d)/a^3/e/f^3-1/f^3/a/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*e
/(4*a*c-b^2)*x^2+1/2/f^3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e/(4*a*c-b^2)*x^2*b^2-2/f^3/a/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)*c^2*d/(4*a*c-b^2)*x+1/f^3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*
e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*d/(4*a*c-b^2)*x*b^
2-1/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c^2*d^2+1/2/f^3/a^2/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*
a*c-b^2)*b^2*c*d^2-3/2/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b*c+1/2/f^3/a^2/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)/e/(4*a*c-b^2)*b^3+1/f^3/a^3/(4*a*c-b^2)/e*sum((b*e^3*c*(4*a*c-b^2
)*_R^3+3*b*d*e^2*c*(4*a*c-b^2)*_R^2+e*(12*a*b*c^2*d^2-3*b^3*c*d^2-3*a^2*c^2
+5*a*b^2*c-b^4)*_R+4*a*b*c^2*d^3-b^3*c*d^3-3*a^2*c^2*d+5*a*b^2*c*d-b^4*d)/(
2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=Ro
otOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)
*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

Fricas [B] time = 10.3466, size = 9563, normalized size = 41.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] [-1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^
2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*
(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a
^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^
5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a
^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + ((b^4*c - 6*
a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*
x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b
^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5
 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)
```

```
*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^
2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2
*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*
e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^
3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^
4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 -
8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^
3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*
x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) +
4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c
+ 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*
b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2
*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*f^3
*x^6 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*f^3*x^5 + (a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*
d^2)*e^5*f^3*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3
*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*f^3*x^3 + (a^4*b^4 - 8*a^5*b^2*c
+ 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5
 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*f^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b
^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (
a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*e^2*f^3*x + ((a^3*b^4*c - 8*a^4*b^2*
c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*e*f^3), -1/2*(2*(a*b^4*c - 7*a^2*b^2*c^
2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^
3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^
3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b
^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)
*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*
b^3*c + 28*a^3*b*c^2)*d)*e*x + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6
 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b
^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5
 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^
4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*
c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 -
6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*sq
rt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2
+ 4*a*c)/(b^2 - 4*a*c)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2
*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 1
6*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2
```

```
*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 -
8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 +
 (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(
5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6
- 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*
b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*l
og(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*f^3*x^6 + 6*(a^3
*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*f^3*x^5 + (a^3*b^5 - 8*a^4*b^3*c
 + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*f^3*
x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*
b^3*c + 16*a^5*b*c^2)*d)*e^4*f^3*x^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2
+ 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*
c + 16*a^5*b*c^2)*d^2)*e^3*f^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (a^4*b^4 - 8*a
^5*b^2*c + 16*a^6*c^2)*d)*e^2*f^3*x + ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*
c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b^4 - 8*a^5*b^
2*c + 16*a^6*c^2)*d^2)*e*f^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.46304, size = 1160, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] 1/2*(a^3*b^6*f^3*e - 10*a^4*b^4*c*f^3*e + 30*a^5*b^2*c^2*f^3*e - 24*a^6*c^3
*f^3*e)*sqrt(b^2 - 4*a*c)*log(abs(8*a^4*c*f^3*e^4 + 4*(a^3*b*c + sqrt(b^2 -
 4*a*c)*a^3*c)*f^3*x^2*e^6 + 8*(a^3*b*c + sqrt(b^2 - 4*a*c)*a^3*c)*d*f^3*x*
e^5 + 4*(a^3*b*c + sqrt(b^2 - 4*a*c)*a^3*c)*d^2*f^3*e^4))/(a^6*b^6*f^6*e^2
- 12*a^7*b^4*c*f^6*e^2 + 48*a^8*b^2*c^2*f^6*e^2 - 64*a^9*c^3*f^6*e^2) - 1/2
*(a^3*b^6*f^3*e - 10*a^4*b^4*c*f^3*e + 30*a^5*b^2*c^2*f^3*e - 24*a^6*c^3*f^
3*e)*sqrt(b^2 - 4*a*c)*log(abs(-8*a^4*c*f^3*e^4 - 4*(a^3*b*c - sqrt(b^2 - 4
*a*c)*a^3*c)*f^3*x^2*e^6 - 8*(a^3*b*c - sqrt(b^2 - 4*a*c)*a^3*c)*d*f^3*x*e^
5 - 4*(a^3*b*c - sqrt(b^2 - 4*a*c)*a^3*c)*d^2*f^3*e^4))/(a^6*b^6*f^6*e^2 -
12*a^7*b^4*c*f^6*e^2 + 48*a^8*b^2*c^2*f^6*e^2 - 64*a^9*c^3*f^6*e^2) + 1/2*b
```

```
*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e +
 c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^3*f^3) - 2*b*e^(-1)*log(abs
(x*e + d))/(a^3*f^3) - 1/2*(2*a*b^2*c*d^4 - 6*a^2*c^2*d^4 + 2*a*b^3*d^2 - 7
*a^2*b*c*d^2 + 2*(a*b^2*c*e^4 - 3*a^2*c^2*e^4)*x^4 + a^2*b^2 - 4*a^3*c + 8*
(a*b^2*c*d*e^3 - 3*a^2*c^2*d*e^3)*x^3 + (12*a*b^2*c*d^2*e^2 - 36*a^2*c^2*d^
2*e^2 + 2*a*b^3*e^2 - 7*a^2*b*c*e^2)*x^2 + 2*(4*a*b^2*c*d^3*e - 12*a^2*c^2*
d^3*e + 2*a*b^3*d*e - 7*a^2*b*c*d*e)*x)*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3
+ 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a
)*(b^2 - 4*a*c)*(x*e + d)^2*a^3*f^3)
```

3.653 $\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=423

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3ef^4\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2\right.}{2\sqrt{2}a^3ef^4\left(b^2-4ac\right)^{3/2}\sqrt{}}$$

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3) + (b*(5*b^2 - 19*
a*c))/(2*a^3*(b^2 - 4*a*c)*e*f^4*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^
2)/(2*a*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)
) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) - (S
qrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sq
rt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4)
```

Rubi [A] time = 3.5503, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1121, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3ef^4\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2\right.}{2\sqrt{2}a^3ef^4\left(b^2-4ac\right)^{3/2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```


```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3) + (b*(5*b^2 - 19*
a*c))/(2*a^3*(b^2 - 4*a*c)*e*f^4*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^
2)/(2*a*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)
) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) - (S
qrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sq
rt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{ef^4}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^4(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)ef^4(d+ex)^3\left(a+b(d\right.}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)ef^4(d+ex)} + \frac{}{2a\left(b\right.}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)ef^4(d+ex)} + \frac{}{2a\left(b\right.}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)ef^4(d+ex)} + \frac{}{2a\left(b\right.}$$

Mathematica [A] time = 3.30551, size = 387, normalized size = 0.91

$$\frac{\frac{6(d+ex)\left(2a^2c^2-4ab^2c-3abc^2(d+ex)^2+b^3c(d+ex)^2+b^4\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} + \frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2+5b^3\sqrt{b^2-4ac}-29ab^2c-19abc\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(-28a^2\right.}{}}{12a^3ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2*
a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (
d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c +
 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c
- 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e*f^4)
```

Maple [C] time = 0.028, size = 1569, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/3/f^4/a^2/e/(e*x+d)^3+2/f^4/a^3*b/e/(e*x+d)+3/2/f^4/a^2/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*e
^2*c^2/(4*a*c-b^2)*x^3-1/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b^3*e^2*c/(4*a*c-b^2)*x^3+9
/2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)*d*b*e*c^2/(4*a*c-b^2)*x^2-3/2/f^4/a^3/(c*e^4*x^4+4*
c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*
d*b^3*e*c/(4*a*c-b^2)*x^2+9/2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*c^2*d^2-
3/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3*c*d^2-1/f^4/a/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*
c-b^2)*x*c^2+2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c-1/2/f^4/a^3/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)/(4*a*c-b^2)*x*b^4+3/2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^
2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b*c^2-1/
2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b^3*c-1/f^4/a/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*
a*c-b^2)*c^2+2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^2*c-1/2/f^4/a^3/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)*d/e/(4*a*c-b^2)*b^4+1/4/f^4/a^3/(4*a*c-b^2)/e*sum((b*e^2*c*(19*a*c-
5*b^2)*_R^2+2*b*d*e*c*(19*a*c-5*b^2)*_R+19*a*b*c^2*d^2-5*b^3*c*d^2-14*a^2*c
^2+24*a*b^2*c-5*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z
^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

Fricas [B] time = 3.81475, size = 13152, normalized size = 31.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 36*(5*b^3*c - 19*a*b*c^2)*d*e^5*x^
5 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19*a*b*c^2)*d^2)*e^
4*x^4 + 6*(5*b^3*c - 19*a*b*c^2)*d^6 + 8*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (
15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e^3*x^3 + 2*(15*b^4 - 62*a*b^2*c + 14*
a^2*c^2)*d^4 + 2*(45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6
*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 4*a^2*b^2 + 16*a^3*c + 2
0*(a*b^3 - 4*a^2*b*c)*d^2 + 4*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 6
2*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x - 3*sqrt(1/2)*(
(a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x
^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5
*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a
^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c
)*d^2)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*
b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2
)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x
+ ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a
^5*c)*d^3)*e*f^4)*sqrt(-((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10
*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^
3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^
6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + 25*b^9 - 31
5*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6
 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))*log((1125*b^8*c^4
 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*
e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c
^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*
b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12
*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 7
6686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b
^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) - (125*b^14 - 2425*a*b^12*
c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^
5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e*f^4)*sqrt(-((a^7*b^6 - 12*
a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b
^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a
^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 6
4*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3
*b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a
^10*c^3)*e^2*f^8))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*
(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*
c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b
^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4
*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*
```

a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*
f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*
(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^
3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)*sqrt(-((a^7*b^6 - 12*a
^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^
10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^
5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64
*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*
b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^
10*c^3)*e^2*f^8))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 -
 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 +
43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5
*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*
b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12*sqrt((625*b^12 - 8250*a*b^10*c + 39525*
a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2
401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^
4*f^16)) - (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c
^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^
7*c^7)*e*f^4)*sqrt(-((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3
)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^
6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 -
12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*
b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 1
2*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))) + 3*sqrt(1/2)*((a^3*
b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 +
(a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(
a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^
2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2
)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*
d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6
 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a
^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)
*d^3)*e*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*
e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*
c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12
*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) - 25*b^9 + 315*a*b^
7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*
a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))*log((1125*b^8*c^4 - 123
25*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x +
(1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9
604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^
2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12*sqrt(
(625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a
^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c +
 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + (125*b^14 - 2425*a*b^12*c + 18
940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*
c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4
*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c +
 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*
c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*
c^3)*e^4*f^16)) - 25*b^9 + 315*a*b^7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^
3 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3
)*e^2*f^8))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^
2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a
^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*
a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d
^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2
)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2
 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^

```
2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a
^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4*c
 + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 3
9525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^
5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^
3)*e^4*f^16)) - 25*b^9 + 315*a*b^7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3
- 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*
e^2*f^8))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a
^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^
2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5*a^7*b^1
1 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4
- 3328*a^12*b*c^5)*e^3*f^12*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*
c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*
c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16))
 + (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160
932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e
*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8
*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 7
6686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b
^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) - 25*b^9 + 315*a*b^7*c - 1
386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4
*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))))/((a^3*b^2*c - 4*a^4*c^2)*e^8
*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c +
 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)
*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*
b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a
^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a
^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*
b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*
d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2\left(efx+df\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^4), x)
```

3.654 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx$

Optimal. Leaf size=353

$$-\frac{f^4(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3\sqrt{c}f^4\left(-2b\sqrt{b^2-4ac}+4a\ldots\right.}{4\sqrt{2}e\left(b^2-4ac\right)^{5}\ldots}$$

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)^2) - (f^4*(d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))
/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(3*
b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*f^4*
ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]
*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.870027, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1120, 1178, 1166, 205}

$$-\frac{f^4(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3\sqrt{c}f^4\left(-2b\sqrt{b^2-4ac}+4a\ldots\right.}{4\sqrt{2}e\left(b^2-4ac\right)^{5}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)^2) - (f^4*(d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))
/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(3*
b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*f^4*
ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]
*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4\,\text{Subst}\left(\int \frac{2a-5bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d\right.}{4\left(b^2-4ac\right)e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-4ac+12bc(\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+\right.}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-4ac+12bc(\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+\right.}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-4ac+12bc(\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+\right.}$$

Mathematica [A] time = 4.61255, size = 331, normalized size = 0.94

$$\frac{f^4\left(\frac{(d+ex)\left(4ac-7b^2-12bc(d+ex)^2\right)}{\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3\sqrt{2}\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{2}\sqrt{c}\left(2b\sqrt{b^2-4a\right.}{\left(b^2-\right.}\right.}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/
((b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*
(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(8*e)
```

Maple [C] time = 0.04, size = 3432, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] -6*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*b*c+3/16*f^4/(16
*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*d*e-4*b*c*d^2+4*a*c
+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R
),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2
*b*d*e)*_Z+c*d^4+b*d^2+a))-5/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^3*b^3-15/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*d^2-
3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a^2*c-3/8*f^4/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*b^2-5/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/e/(16*a
^2*c^2-8*a*b^2*c+b^4)*b^3+5*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^2*a*c^2-95/4*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2
*b^2*c-6*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*b*c*d^2-2*f^4/(c
*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*
x+b*d^2+a)^2*d^3/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*b*c-21/2*f^4/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*c^2*d*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-63/2*f^4/(c*e^4*x^4+4*c*d*e^3*x
^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^4
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*d^2*b-105/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^3/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b+5/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^3/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^4*a-95/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^3/(16*a^2*c^2-8*a*b^2
*c+b^4)*x^4*b^2-105/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3
*b*c^2*d^4+5*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a*c^2*d^
2-95/4*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2*c*d^2-2*f^
4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a*b*c-63/2*f^4/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
```

```
2+a)^2*d^5*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2+1/2*f^4/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^
5/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*c^2+1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^4/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^5*a-19/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/e/(16*a^2*c^2-8*a*b^2
*c+b^4)*b^2*c-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*c-3
/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2*d/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*b^2-3/2*f^4/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)^2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-19/8*f^4/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*e
^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*b^2-15/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^2*b^3-21/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4
)*x*b*c^2*d^6+5/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*c^2*d^4
-95/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c*d^4-3/2*f^4/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)^2*d^7/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [B] time = 3.69373, size = 14596, normalized size = 41.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] -1/16*(24*b*c^2*e^7*f^4*x^7 + 168*b*c^2*d*e^6*f^4*x^6 + 2*(252*b*c^2*d^2 +
19*b^2*c - 4*a*c^2)*e^5*f^4*x^5 + 10*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2)*d
)*e^4*f^4*x^4 + 2*(420*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^
2)*d^2)*e^3*f^4*x^3 + 2*(252*b*c^2*d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5
*b^3 + 16*a*b*c)*d)*e^2*f^4*x^2 + 2*(84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*
d^4 + 3*a*b^2 + 12*a^2*c + 3*(5*b^3 + 16*a*b*c)*d^2)*e*f^4*x + 2*(12*b*c^2*
d^7 + (19*b^2*c - 4*a*c^2)*d^5 + (5*b^3 + 16*a*b*c)*d^3 + 3*(a*b^2 + 4*a^2*
c)*d)*f^4 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
```

*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4
+ 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2
+ 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3
*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 +
(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 +
16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*
a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*
f^8 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5
*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*
c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^1
0 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1
024*a^6*c^5)*e^2))*log(27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*f^12*x +
27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*f^12 + 27/2*sqrt(1/2)*((b^8 - 8*
a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e*f^8 - (a*b^13 - 8*a^2*b^11*c - 8
0*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 1
2288*a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640
*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^3)*sqrt(-((b^5 + 40
*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 6
40*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20
*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^
7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c
^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^
8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 +
16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3
+ 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32
*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^
3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c +
32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 +
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (
b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)
*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*
c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-
((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b
^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2
*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4
- 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640
*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(27*(5*b^4*c + 40*
a*b^2*c^2 + 16*a^2*c^3)*e*f^12*x + 27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)
*d*f^12 - 27/2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)
*e*f^8 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*
a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 2
0*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a
^7*c^5)*e^4))*e^3)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 -
20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024
*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b
^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*
c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^
2))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*

```
b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(
14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)
*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b
^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b
^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 -
 (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*
c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 -
 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 2
0*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a
^6*c^5)*e^2))*log(27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*f^12*x + 27*(5
*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*f^12 + 27/2*sqrt(1/2)*((b^8 - 8*a*b^6
*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e*f^8 + (a*b^13 - 8*a^2*b^11*c - 80*a^3
*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*
a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*
b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^3)*sqrt(-((b^5 + 40*a*b^
3*c + 80*a^2*b*c^2)*f^8 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^
4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*
b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5
)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 +
1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7
 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*
c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*
d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a
^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5
 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4
 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 -
 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*
x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 3
2*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5
 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^
2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10
 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 10
24*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*
b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(27*(5*b^4*c + 40*a*b^2
*c^2 + 16*a^2*c^3)*e*f^12*x + 27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*f^
12 - 27/2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e*f^
8 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b
^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 20*a^3
*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^
5)*e^4))*e^3)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 - (a*b^10 - 20*a
^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*
c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^
3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 1
60*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))))
```

```
/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6
 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 3
0*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (
b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*
(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c +
 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2
+ (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^
2)*d^2)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(efx+df\right)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^3, x)
```

3.655 $\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=159

$$-\frac{3bf^3\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^3\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3bcf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}$$

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)^2) - (3*b*f^3*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d
 + e*x)^2 + c*(d + e*x)^4)) + (3*b*c*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt
[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rubi [A] time = 0.19975, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1142, 1114, 638, 614, 618, 206}

$$-\frac{3bf^3\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^3\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3bcf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)^2) - (3*b*f^3*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d
 + e*x)^2 + c*(d + e*x)^4)) + (3*b*c*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt
[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+bx^2+cx^4\right)^3}\,dx,x,d+ex\right)}{e}$$

$$= \frac{f^3\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx,x,(d+ex)^2\right)}{2e}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{\left(3bf^3\right)\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx,x,(\right.}{4\left(b^2-4ac\right)e}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(\right.}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(\right.}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(\right.}$$

Mathematica [A] time = 0.217258, size = 149, normalized size = 0.94

$$\frac{f^3\left(\frac{\left(b^2-4ac\right)\left(2a+b(d+ex)^2\right)}{\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)^2}-\frac{12bc\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{3b\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}\right)}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^3*((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((
b^2 - 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2
 - (12*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c]))/(4*(b^2 - 4*a*c)^2*e)
```

Maple [C] time = 0.04, size = 2181, normalized size = 13.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] -3/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*f^3/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)^2*e^4*d*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-45/2*f^3/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
^2*b*c^2*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*d^2-9/4*f^3/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^2*
c*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-30*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^2*b/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^3-9*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^2*b^2/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^3-45/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
2*c^2*d^4-27/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c*d^
2-5/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*b*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*c-1/2*f^3/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)^2*b^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-9*f^3/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*b
/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^2-9*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*b^2/(16*a^2*c^2-
8*a*b^2*c+b^4)*x*c-5*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*
c-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2*d*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x-3/2*f^3/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2*d^6-9/4*f^3/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2
*c^2-8*a*b^2*c+b^4)*b^2*c*d^4-5/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*
c+b^4)*a*b*c*d^2-1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*d^
2-2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*c-1/4*f^3/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*b^2+3/2*f^3*b*c/(16*a^2*c^2-8*a*b^2*c+
b^4)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z
^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [B] time = 2.96641, size = 8080, normalized size = 50.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*f^3*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^5
*f^3*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*f^3*x
^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*f^3*
x^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 + 27*(
b^4*c - 4*a*b^2*c^2)*d^2)*e^2*f^3*x^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*
(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*f^3*x + (6*
(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4
*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)*f^3 - 6*(b*c^3*e^8*
f^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*f^3*x^6 +
4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f^3*x^5 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2
 + b^3*c + 2*a*b*c^2)*e^4*f^3*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3
*c + 2*a*b*c^2)*d)*e^3*f^3*x^3 + 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c
 + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*f^3*x^2 + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5
+ a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3*d^8 + 2*b^2*c^2*d^6
 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f^3)*sqrt(b^2 - 4*a*c
)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^
2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2
+ 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 +
c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c
^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 -
 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^
4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4
+ 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^
3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c
^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c
^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x +
 ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*
b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*d^2)*e), -1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*f^3*x^6 + 36*(b^3*c^2
 - 4*a*b*c^3)*d*e^5*f^3*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*
```

```
c^3)*d^2)*e^4*f^3*x^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b
^2*c^2)*d)*e^3*f^3*x^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*
a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*f^3*x^2 + 4*(9*(b^3*c^2 -
4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^
2)*d)*e*f^3*x + (6*(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3
*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)*
f^3 - 12*(b*c^3*e^8*f^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*d^2 + b^2
*c^2)*e^6*f^3*x^6 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f^3*x^5 + (70*b*c^3*
d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*f^3*x^4 + 4*(14*b*c^3*d^5 + 1
0*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3*f^3*x^3 + 2*(14*b*c^3*d^6 + 15*b
^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*f^3*x^2 + 4*(2*b*c^3*
d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3
*d^8 + 2*b^2*c^2*d^6 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f
^3)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 1
4*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(
14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c -
12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 1
28*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*
b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*
(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*
b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2
*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a
^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 -
10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7
- 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 11.6977, size = 979, normalized size = 6.16

$$-\frac{3\left(b^5cf^3e-8\,ab^3c^2f^3e+16\,a^2bc^3f^3e\right)\sqrt{b^2-4\,ac}\,\log\left(\left|\left(b+\sqrt{b^2-4\,ac}\right)x^2e^2+2\left(b+\sqrt{b^2-4\,ac}\right)dxe+\left(b+\sqrt{b^2-4\,a\cdots}\right.\right.\right.}{2\left(b^{10}e^2-20\,ab^8ce^2+160\,a^2b^6c^2e^2-640\,a^3b^4c^3e^2+1280\,a^4b^2c^4e^2-1024\,a^5c^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3/2*(b^5*c*f^3*e - 8*a*b^3*c^2*f^3*e + 16*a^2*b*c^3*f^3*e)*sqrt(b^2 - 4*a*
c)*log(abs((b + sqrt(b^2 - 4*a*c))*x^2*e^2 + 2*(b + sqrt(b^2 - 4*a*c))*d*x*
e + (b + sqrt(b^2 - 4*a*c))*d^2 + 2*a))/(b^10*e^2 - 20*a*b^8*c*e^2 + 160*a^
2*b^6*c^2*e^2 - 640*a^3*b^4*c^3*e^2 + 1280*a^4*b^2*c^4*e^2 - 1024*a^5*c^5*e
^2) + 3/2*(b^5*c*f^3*e - 8*a*b^3*c^2*f^3*e + 16*a^2*b*c^3*f^3*e)*sqrt(b^2 -
 4*a*c)*log(abs(-(b - sqrt(b^2 - 4*a*c))*x^2*e^2 - 2*(b - sqrt(b^2 - 4*a*c)
)*d*x*e - (b - sqrt(b^2 - 4*a*c))*d^2 - 2*a))/(b^10*e^2 - 20*a*b^8*c*e^2 +
160*a^2*b^6*c^2*e^2 - 640*a^3*b^4*c^3*e^2 + 1280*a^4*b^2*c^4*e^2 - 1024*a^5
*c^5*e^2) - 1/4*(6*b*c^2*f^3*x^6*e^6 + 36*b*c^2*d*f^3*x^5*e^5 + 90*b*c^2*d^
2*f^3*x^4*e^4 + 120*b*c^2*d^3*f^3*x^3*e^3 + 90*b*c^2*d^4*f^3*x^2*e^2 + 36*b
*c^2*d^5*f^3*x*e + 6*b*c^2*d^6*f^3 + 9*b^2*c*f^3*x^4*e^4 + 36*b^2*c*d*f^3*x
^3*e^3 + 54*b^2*c*d^2*f^3*x^2*e^2 + 36*b^2*c*d^3*f^3*x*e + 9*b^2*c*d^4*f^3
+ 2*b^3*f^3*x^2*e^2 + 10*a*b*c*f^3*x^2*e^2 + 4*b^3*d*f^3*x*e + 20*a*b*c*d*f
^3*x*e + 2*b^3*d^2*f^3 + 10*a*b*c*d^2*f^3 + a*b^2*f^3 + 8*a^2*c*f^3)/((c*x^
4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 +
 2*b*d*x*e + b*d^2 + a)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

3.656 $\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx$

Optimal. Leaf size=375

$$\frac{f^2(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{\sqrt{c}f^2\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+\right.}{8\sqrt{2}ae\left(b^2\right.}$$

```
[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)^2) + (f^2*(d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*
(d + e*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
 (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2
 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b
^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*e)
```

Rubi [A] time = 0.972237, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1119, 1178, 1166, 205}

$$\frac{f^2(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{\sqrt{c}f^2\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+\right.}{8\sqrt{2}ae\left(b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```



```
[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)^2) + (f^2*(d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*
(d + e*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
 (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2
 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b
^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*e)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
```

```
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2\,\text{Subst}\left(\int \frac{b-10cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+\right.}{4\left(b^2-4ac\right)e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b\left(b^2+8ac\right)+c\left(b^2+\right.\right.}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2\right.}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b\left(b^2+8ac\right)+c\left(b^2+\right.\right.}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2\right.}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b\left(b^2+8ac\right)+c\left(b^2+\right.\right.}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2\right.}$$

Mathematica [A] time = 4.96309, size = 385, normalized size = 1.03

$$\frac{f^2\left(\frac{2(d+ex)\left(8abc+20ac^2(d+ex)^2+b^2c(d+ex)^2+b^3\right)}{a\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{4\left(b(d+ex)+2c(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.}{16e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^2*((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 2
0*a*c^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]
*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c
)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(16*e)
```

Maple [C] time = 0.04, size = 4751, normalized size = 12.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] 5/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2*c*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4*b^3+35/8*f^
2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3*b^2*c^2*d^4+5/2*f^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3*b^3*c*d^2+21/8*f^2/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*d^5*e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2*b^2*c^2+5/2*f^2/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2*d^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2*b^3*c+7/8*f^2/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^
2*d*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6*b^2+21/8*f^2/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^
4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5*b^2*d^2+35/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*
e^3/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4*b^2+1/16*f^2/(16*a^2*c^2-8*a*b^2*c+b^4
)/a/e*sum((c*e^2*(20*a*c+b^2)*_R^2+2*c*d*e*(20*a*c+b^2)*_R+20*a*c^2*d^2+b^2
*c*d^2-16*a*b*c+b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z
^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))-1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+
6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*x*b^3+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c
*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^7/e/(16*a^2*c^2-8*a*b^2*c+b
^4)*c^3-1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*
x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3+5/2*f^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2*c^3*e^6/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+35/2*f^2/(c*e^4*x^4+4*
c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^
2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3*d^6+1/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*e^4/(16*a^2
*c^2-8*a*b^2*c+b^4)/a*x^5*b^3+27/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b
^2*c+b^4)*a*x^2*c^2+3/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b^4)/a
*x^2*b^4+7/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x*b^2*c^2*d^6+
1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
```

```
4+2*b*d*e*x+b*d^2+a)^2*c^2*e^6/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7*b^2+35/2*f^
2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2*c^2*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b+35*f^2/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b*c^2*d^2+35*f^2/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d
^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2+15/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c+5/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*
c+b^4)/a*x*b^3*c*d^4+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^7/e/(16*a^2*c^2-8*a*b^2*c+b^4
)/a*b^2*c^2+1/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*b^3*c
+2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2*d/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*b*c+105/2*f^2/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)^2*c^3*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*d^2+7/2*f^2/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*
c^2*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*b+175/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^3*e
^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+175/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^3*c^3*d^4+5/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^3*b^2*c+105/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^2*c^3+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3*b^4+27/2
*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*x*c^2*d^2+3/8*f^2/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x*b^4*d^2+2*f^2/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*a*x*b*c+9/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/e/(16*a^2*c^2-8*a*b^2
*c+b^4)*a*c^2+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*b^4
+9/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*x^3*c^2+35/2*f^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^2*d^4+15/8*f^2/(c*e^4*x^4+4*
c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^
2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c*d^2+7/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/e/(16*a
^2*c^2-8*a*b^2*c+b^4)*b*c^2+5/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^
2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/e/(16*a^2*c^2-8*a*b^
2*c+b^4)*b^2*c+35/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
```

```
)
```

```
[Out] Timed out
```

Fricas [B] time = 4.16709, size = 16847, normalized size = 44.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*e^7*f^2*x^7 + 14*(b^2*c^2 + 20*a*c^3)*d*e^6*f^
2*x^6 + 2*(2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*e^5*f^2*x^5
+ 10*(7*(b^2*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)*d)*e^4*f^2*x^4 +
2*(35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*c + 36*a^2*c^2 + 20*(b^3*c +
 14*a*b*c^2)*d^2)*e^3*f^2*x^3 + 2*(21*(b^2*c^2 + 20*a*c^3)*d^5 + 20*(b^3*c
+ 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d)*e^2*f^2*x^2 + 2*(7*
(b^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c
 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*f^2*x + 2*((b^2*c^2 + 20*a*c^3)*
d^7 + 2*(b^3*c + 14*a*b*c^2)*d^5 + (b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 - (a*
b^3 - 16*a^2*b*c)*d)*f^2 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c
^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b
^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 +
3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*
c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2
*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^
3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*
c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4
*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*
(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^
4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2
- 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*
(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^
4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 -
 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^
8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*
sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a
^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2
)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*
b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*
b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2
*b^2*c^4 + 10000*a^3*c^5)*d*f^6 + 1/2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a
^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e*f^4
- (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^
7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^
4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c
^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-
((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20
*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^
8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c
+ 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^
4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 12
```

80*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c
^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8
x^7 + 2(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^
2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6
 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d
^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^
4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*
a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5
 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)
d^6 + 15(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b
^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*
b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x +
 ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*
b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*
sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^1
0 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1
024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*
b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c
^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^
3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3
 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^
3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 - 1/2*sqrt(1/2)*((b^11 - 53*a*
b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*
b*c^5)*e*f^4 - (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*
c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*
c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c +
160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)
)*e^3)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (
a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*
c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 -
20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024
*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6
*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) + sqrt(1/2)*((a*b^4*c^2
 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c
^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^
2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6
*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4
 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*
b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3
 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3
 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*
b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*
c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a
*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^
2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a
*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*
c^2)*d^2)*e)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f
^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^
7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b
^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4
 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 6
40*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1
491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 -

1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 + 1/2*sqrt(1/2)*(
(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4
 - 25600*a^5*b*c^5)*e*f^4 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 -
2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6
+ 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20
*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a
^11*c^5)*e^4))*e^3)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b
*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 +
1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/
((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b
^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*
c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - sqrt(1/2)
*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3
+ 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c
^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*
b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a
^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*
d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b
^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^
3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8
*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3
)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2
 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*
c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c
+ 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 +
a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*
c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*
c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680
*a^3*b*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*
c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2
)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*
a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^
5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((3
5*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (
35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 - 1/
2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 2182
4*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e*f^4 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^
5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768
*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((
a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2
*c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2
 + 1680*a^3*b*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a
^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*
a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3
+ 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c +
 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))
))/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2
*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^
3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4
*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^
3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8
*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3
)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a
*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*
c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 -
 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c
^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c
^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*

```
b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*
c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8
+ a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*
b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^
3*c + 16*a^4*b*c^2)*d^2)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(efx+df\right)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^3, x)
```

3.657 $\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{6c^2f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3cf\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[Out] -(f*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)^2) + (3*c*f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) - (6*c^2*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rubi [A] time = 0.191942, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1142, 1107, 614, 618, 206}

$$-\frac{6c^2f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3cf\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```



```
[Out] -(f*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)^2) + (3*c*f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) - (6*c^2*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f\,\text{Subst}\left(\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx,x,d+ex\right)}{e}$$

$$= \frac{f\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx,x,(d+ex)^2\right)}{2e}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(3cf)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx,x,(\ldots}{2\left(b^2-4ac\right)e}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c\ldots\right.}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c\ldots\right.}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c\ldots\right.}$$

Mathematica [A] time = 0.182468, size = 148, normalized size = 0.97

$$\frac{f\left(\frac{24c^2\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{\left(b^2-4ac\right)\left(-b-2c(d+ex)^2\right)}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{6c\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}\right)}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f*(((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)
^4)^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (
24*c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
))/(4*(b^2 - 4*a*c)^2*e)
```

Maple [C] time = 0.042, size = 2132, normalized size = 13.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] 3*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*f/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
^2*e^4*d*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+45*f/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^3/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^4*d^2+9/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^3/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^4*b+60*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^3*e^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3+18*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b+45*f/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)^2*c^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*d^4+27*f/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*c^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*d^2*b+5*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^2*a+f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*e/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^2*b^2+18*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x+18*f/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*c^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b+10*f/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d/
(16*a^2*c^2-8*a*b^2*c+b^4)*x*a+2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d/(16*a^2*c^2-8*a*b^2*c
+b^4)*x*b^2+3*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*
x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*d^6+9/2*f/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2*d^4+5*f/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(
16*a^2*c^2-8*a*b^2*c+b^4)*a*c^2*d^2+f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*
c+b^4)*b^2*c*d^2+5/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*b*c-1/
4*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3+3*f*c^2/(16*a^2*c^2-8*a
*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^
3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e
^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.32785, size = 7892, normalized size = 51.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*f*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*f*x^5
 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*f*x^4 + 24*(10
*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*e^3*f*x^3 + 4*(b^4*c
+ a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b
*c^3)*d^2)*e^2*f*x^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^
3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*e*f*x + 12*(c^4*e^8*f*x^8 + 8*
c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*f*x^6 + 4*(14*c^4*d^3 + 3*b*c^
3*d)*e^5*f*x^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*f*x^4
+ 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*f*x^3 + 2*(14*c
^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*e^2*f*x^2 + 4*
(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*e*f*x + (c^
4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^4 + a^2*c^2)*f)
*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*
c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*
c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 +
 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*
e*x + a)) + (12*(b^2*c^3 - 4*a*c^4)*d^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 +
 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2)*f)/
((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a
*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2
*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^
5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 +
10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a
*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b
^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3
*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128
*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e
^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 +
48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 3
2*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2
 - 64*a^4*b*c^3)*d^2)*e), 1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*f*x^6 + 72*(b^2*c
^3 - 4*a*c^4)*d*e^5*f*x^5 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4
)*d^2)*e^4*f*x^4 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)
*d)*e^3*f*x^3 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*
d^4 + 27*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^2*f*x^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^
5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*e*f*x
 - 24*(c^4*e^8*f*x^8 + 8*c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*f*x^6
 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*f*x^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*
c^2 + 2*a*c^3)*e^4*f*x^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^
3)*d)*e^3*f*x^3 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a
*c^3)*d^2)*e^2*f*x^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 +
2*a*c^3)*d^3)*e*f*x + (c^4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2
*a*c^3)*d^4 + a^2*c^2)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e
*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (12*(b^2*c^3 - 4*a*c^
4)*d^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4
*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2)*f)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2
*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
```

```
 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^
7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8
- 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^
2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*
b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^
2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5
)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(
b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*
x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6
*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 +
 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^
6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4
+ 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 11.7623, size = 909, normalized size = 5.94

$$\frac{3\left(b^4c^2fe - 8ab^2c^3fe + 16a^2c^4fe\right)\sqrt{b^2-4ac}\log\left(\left|\left(b+\sqrt{b^2-4ac}\right)x^2e^2 + 2\left(b+\sqrt{b^2-4ac}\right)dxe + \left(b+\sqrt{b^2-4ac}\right)d^2\right.\right.}{b^{10}e^2 - 20ab^8ce^2 + 160a^2b^6c^2e^2 - 640a^3b^4c^3e^2 + 1280a^4b^2c^4e^2 - 1024a^5c^5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3*(b^4*c^2*f*e - 8*a*b^2*c^3*f*e + 16*a^2*c^4*f*e)*sqrt(b^2 - 4*a*c)*log(ab
s((b + sqrt(b^2 - 4*a*c))*x^2*e^2 + 2*(b + sqrt(b^2 - 4*a*c))*d*x*e + (b +
sqrt(b^2 - 4*a*c))*d^2 + 2*a))/(b^10*e^2 - 20*a*b^8*c*e^2 + 160*a^2*b^6*c^2
*e^2 - 640*a^3*b^4*c^3*e^2 + 1280*a^4*b^2*c^4*e^2 - 1024*a^5*c^5*e^2) - 3*(
b^4*c^2*f*e - 8*a*b^2*c^3*f*e + 16*a^2*c^4*f*e)*sqrt(b^2 - 4*a*c)*log(abs(-
(b - sqrt(b^2 - 4*a*c))*x^2*e^2 - 2*(b - sqrt(b^2 - 4*a*c))*d*x*e - (b - sq
rt(b^2 - 4*a*c))*d^2 - 2*a))/(b^10*e^2 - 20*a*b^8*c*e^2 + 160*a^2*b^6*c^2*e
^2 - 640*a^3*b^4*c^3*e^2 + 1280*a^4*b^2*c^4*e^2 - 1024*a^5*c^5*e^2) + 1/4*(
12*c^3*f*x^6*e^6 + 72*c^3*d*f*x^5*e^5 + 180*c^3*d^2*f*x^4*e^4 + 240*c^3*d^3
*f*x^3*e^3 + 180*c^3*d^4*f*x^2*e^2 + 72*c^3*d^5*f*x*e + 12*c^3*d^6*f + 18*b
*c^2*f*x^4*e^4 + 72*b*c^2*d*f*x^3*e^3 + 108*b*c^2*d^2*f*x^2*e^2 + 72*b*c^2*
d^3*f*x*e + 18*b*c^2*d^4*f + 4*b^2*c*f*x^2*e^2 + 20*a*c^2*f*x^2*e^2 + 8*b^2
```

```
*c*d*f*x*e + 40*a*c^2*d*f*x*e + 4*b^2*c*d^2*f + 20*a*c^2*d^2*f - b^3*f + 10
*a*b*c*f)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d
^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2
*e))
```

3.658 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=270

$$\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2ef\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3ef\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+b(d+ex)\right.}{4a^3e}$$

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)
*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e*f) + Log[d + e*x]/(a^3*e*f) - L
og[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e*f)
```

Rubi [A] time = 0.496373, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2ef\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3ef\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+b(d+ex)\right.}{4a^3e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)
*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e*f) + Log[d + e*x]/(a^3*e*f) - L
og[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e*f)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-2\left(b^2-4ac\right.}{x\left(a+bx+\right.}\right.}{4a\left(b^2\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16}{4a^2\left(b^2-4ac\right)^2 e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16}{4a^2\left(b^2-4ac\right)^2 e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16}{4a^2\left(b^2-4ac\right)^2 e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16}{4a^2\left(b^2-4ac\right)^2 e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16}{4a^2\left(b^2-4ac\right)^2 e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16}{4a^2\left(b^2-4ac\right)^2 e}$$

Mathematica [A] time = 3.99762, size = 394, normalized size = 1.46

$$\frac{\frac{a\left(16a^2c^2-15ab^2c-14abc^2(d+ex)^2+2b^3c(d+ex)^2+2b^4\right)}{\left(b^2-4ac\right)^2\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2+b^4\sqrt{b^2-4ac}-10ab^3c-8ab^2c\sqrt{b^2-4ac}+b^5\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{5/2}}}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] ((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d + e*
x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c*(d
 + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqr
t[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) + ((b^5
 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 -
 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d
+ e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e*f)
```

Maple [C] time = 0.063, size = 4606, normalized size = 17.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] 1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^5+1/f/a^2/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^5+1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16
*a^2*c^2-8*a*b^2*c+b^4)*b^5*d^2-1/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^
2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+
b^4)*x*b*c^2-1/2/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2*d^2+16
/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)^2*c^3*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+24/f/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^3*d^2-1/2/f/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^2*b*c^2-87/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2
*c+b^4)*x^2*b^2*c^2*d^2+6/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^2*b^4*c*d^2-21/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*c^3*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^5+3/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^3*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-10
5/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2*e^3*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b*d^2+15/2/f/
a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*e^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^3*d^2-70/f/a/(c
*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*
x+b*d^2+a)^2*c^3*d^3*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b+10/f/a^2/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)^2*c^2*d^3*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^3-29/f/a/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*c^2*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2+4/f/a^2/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^
2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4-105/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^2*b*c^3*d^4+15/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^2*b^3*c^2*d^4+4/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*
d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^3/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^4+16/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3+4/f/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*d^4-21/4/f/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*
a^2*c^2-8*a*b^2*c+b^4)*b^2*c+6/f*a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b
^4)*c^2+3/4/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*
x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4+ln(e*x+d)/a
^3/e/f-3/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c-21/f/a/(
```

```
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^3+3/f/a^2/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c^2-29/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2
*c^2-8*a*b^2*c+b^4)*x*b^2*c^2+4/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^
2*c+b^4)*x*b^4*c-6/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c-
7/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^3*d^6-7/2/f/a/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*c^3*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2/f/a^2/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*c^2*e^5*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*
a^2*c^2-8*a*b^2*c+b^4)*b^3*c^2*d^6-29/4/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*
a*b^2*c+b^4)*b^2*c^2*d^4+1/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4
)*b^4*c*d^4-3/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c*d^2-29/
4/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2*e^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2+1/f/a^2/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)^2*e^3*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^4-1/2/f/a^3/(16*a^2*c^2-
8*a*b^2*c+b^4)/e*sum((c*e^3*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*c*d*e^2*(16*a
^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*c^2*d^2+3*b^4*c*d^2+2
3*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*a*b^2*c^2*d^3+b^4*c*d^3+23*a
^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c
*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+
b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [B] time = 14.791, size = 20941, normalized size = 77.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2
 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d*e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 +
```

132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^2)*e^4*x^4 + 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)*e*x + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a

^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*
c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 4
8*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*
a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*
a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 -
 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3
 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*
c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 -
 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b
^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^
2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3
*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a
^4*b*c^3)*d)*e*x)*log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2
*c^4 - 64*a^6*c^5)*e^9*f*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2
*c^4 - 64*a^6*c^5)*d*e^8*f*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3
*c^3 - 64*a^6*b*c^4 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 6
4*a^6*c^5)*d^2)*e^7*f*x^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^
2*c^4 - 64*a^6*c^5)*d^3 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 -
64*a^6*b*c^4)*d)*e^6*f*x^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*
a^6*b^2*c^3 - 128*a^7*c^4 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c
^4 - 64*a^6*c^5)*d^4 + 30*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64
*a^6*b*c^4)*d^2)*e^5*f*x^4 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b
^2*c^4 - 64*a^6*c^5)*d^5 + 10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3
 - 64*a^6*b*c^4)*d^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2
*c^3 - 128*a^7*c^4)*d)*e^4*f*x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c
^2 - 64*a^7*b*c^3 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*
a^6*c^5)*d^6 + 15*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c
^4)*d^4 + 3*(a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128
*a^7*c^4)*d^2)*e^3*f*x^2 + 4*(2*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*
c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64
*a^6*b*c^4)*d^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3
 - 128*a^7*c^4)*d^3 + (a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c
^3)*d)*e^2*f*x + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a
^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^7
*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a^
4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7 -
 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d^2)*e*f), 1/4*(2*(a*b^5*c^2
 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2 - 11*a^2*b^3*c^3
 + 28*a^3*b*c^4)*d*e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 -
 64*a^4*c^4 + 30*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^2)*e^4*x^4 +
 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^2 - 1
1*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*
a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4
)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*
d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3 + 15*(a*b^5*c^
2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 45*a^2*b^4*c^2 + 13
2*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3
*b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c
^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^3 +
 (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)*e*x + 2*((b^5*c^2
 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^
2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2

- 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*

```
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a
^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*d)*e*x)*log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^
2*c^4 - 64*a^6*c^5)*e^9*f*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^
2*c^4 - 64*a^6*c^5)*d*e^8*f*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^
3*c^3 - 64*a^6*b*c^4 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 -
64*a^6*c^5)*d^2)*e^7*f*x^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b
^2*c^4 - 64*a^6*c^5)*d^3 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 -
 64*a^6*b*c^4)*d)*e^6*f*x^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32
*a^6*b^2*c^3 - 128*a^7*c^4 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*
c^4 - 64*a^6*c^5)*d^4 + 30*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 6
4*a^6*b*c^4)*d^2)*e^5*f*x^4 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*
b^2*c^4 - 64*a^6*c^5)*d^5 + 10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3
 - 64*a^6*b*c^4)*d^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^
2*c^3 - 128*a^7*c^4)*d)*e^4*f*x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*
c^2 - 64*a^7*b*c^3 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64
*a^6*c^5)*d^6 + 15*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*
c^4)*d^4 + 3*(a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 12
8*a^7*c^4)*d^2)*e^3*f*x^2 + 4*(2*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2
*c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 6
4*a^6*b*c^4)*d^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^
3 - 128*a^7*c^4)*d^3 + (a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*
c^3)*d)*e^2*f*x + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (
a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^
7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a
^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7
- 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d^2)*e*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 12.0452, size = 1887, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^3*b^9*f*e - 18*a^4*b^7*c*f*e + 126*a^5*b^5*c^2*f*e - 400*a^6*b^3*c^
3*f*e + 480*a^7*b*c^4*f*e)*sqrt(b^2 - 4*a*c)*log(abs(2*(a^3*b^3*c - 4*a^4*b
*c^2 + (a^3*b^2*c - 4*a^4*c^2)*sqrt(b^2 - 4*a*c))*f*x^2*e^6 + 4*(a^3*b^3*c
- 4*a^4*b*c^2 + (a^3*b^2*c - 4*a^4*c^2)*sqrt(b^2 - 4*a*c))*d*f*x*e^5 + 2*(a
^3*b^3*c - 4*a^4*b*c^2 + (a^3*b^2*c - 4*a^4*c^2)*sqrt(b^2 - 4*a*c))*d^2*f*e
^4 + 4*(a^4*b^2*c - 4*a^5*c^2)*f*e^4))/(a^6*b^10*f^2*e^2 - 20*a^7*b^8*c*f^2
*e^2 + 160*a^8*b^6*c^2*f^2*e^2 - 640*a^9*b^4*c^3*f^2*e^2 + 1280*a^10*b^2*c^
```

```
4*f^2*e^2 - 1024*a^11*c^5*f^2*e^2) + 1/4*(a^3*b^9*f*e - 18*a^4*b^7*c*f*e +
126*a^5*b^5*c^2*f*e - 400*a^6*b^3*c^3*f*e + 480*a^7*b*c^4*f*e)*sqrt(b^2 - 4
*a*c)*log(abs(-2*(a^3*b^3*c - 4*a^4*b*c^2 - (a^3*b^2*c - 4*a^4*c^2)*sqrt(b^
2 - 4*a*c))*f*x^2*e^6 - 4*(a^3*b^3*c - 4*a^4*b*c^2 - (a^3*b^2*c - 4*a^4*c^2
)*sqrt(b^2 - 4*a*c))*d*f*x*e^5 - 2*(a^3*b^3*c - 4*a^4*b*c^2 - (a^3*b^2*c -
4*a^4*c^2)*sqrt(b^2 - 4*a*c))*d^2*f*e^4 - 4*(a^4*b^2*c - 4*a^5*c^2)*f*e^4))
/(a^6*b^10*f^2*e^2 - 20*a^7*b^8*c*f^2*e^2 + 160*a^8*b^6*c^2*f^2*e^2 - 640*a
^9*b^4*c^3*f^2*e^2 + 1280*a^10*b^2*c^4*f^2*e^2 - 1024*a^11*c^5*f^2*e^2) - 1
/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e
 + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^3*f) + e^(-1)*log(abs(x*e
 + d))/(a^3*f) + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3*d^6 + 4*a*b^4*c*d^4 -
29*a^2*b^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 - 12*a^2*b^3*c*d^2 - 2*a^
3*b*c^2*d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^6 + 3*a^2*b^4 - 21*a^3*
b^2*c + 24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c^3*d*e^5)*x^5 + (30*a*b
^3*c^2*d^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e^4 - 29*a^2*b^2*c^2*e^4
 + 16*a^3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 70*a^2*b*c^3*d^3*e^3 + 4
*a*b^4*c*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d*e^3)*x^3 + 2*(15*a*b^3
*c^2*d^4*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^2*e^2 - 87*a^2*b^2*c^2*
d^2*e^2 + 48*a^3*c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3*c*e^2 - a^3*b*c^2*e^2)
*x^2 + 4*(3*a*b^3*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4*a*b^4*c*d^3*e - 29*a^2
*b^2*c^2*d^3*e + 16*a^3*c^3*d^3*e + a*b^5*d*e - 6*a^2*b^3*c*d*e - a^3*b*c^2
*d*e)*x)*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e
 + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^2 - 4*a*c)^2*a^3*f)
```

3.659 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=499

$$\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2ef^2\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}}{8\sqrt{2}a^3ef^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*f^2*(d + e*x))
 + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a +
b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*
(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*f^2*(d + e*x)*(a + b
*(d + e*x)^2 + c*(d + e*x)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c)
 + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*
a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b
^2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*
(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rubi [A] time = 1.09359, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1142, 1121, 1277, 1281, 1166, 205}

$$\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2ef^2\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}}{8\sqrt{2}a^3ef^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*f^2*(d + e*x))
 + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a +
b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*
(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*f^2*(d + e*x)*(a + b
*(d + e*x)^2 + c*(d + e*x)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c)
 + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*
a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b
^2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*
(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
```

```
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{ef^2}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{5b^4 -}{8a^2\left(b^2\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2ef^2(d+ex)} + \frac{b^2-2ac+bc(d+}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2ef^2(d+ex)} + \frac{b^2-2ac+bc(d+}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2ef^2(d+ex)} + \frac{b^2-2ac+bc(d+}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d\right.}$$

Mathematica [A] time = 6.21918, size = 575, normalized size = 1.15

$$\frac{-3abc(d+ex)-2ac^2(d+ex)^3+b^2c(d+ex)^3+b^3(d+ex)}{4a^2ef^2\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{-84a^2bc^2(d+ex)-52a^2c^3(d+ex)^3+47ab^2c^2(d+ex)^3 -}{8a^3ef^2\left(4ac-b^2\right)^2\left(a+b(d+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -(1/(a^3*e*f^2*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d
+ e*x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*f^2*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*a^2
*b*c^2*(d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*
c^3*(d + e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*f^2*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)) - (3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2
- 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^
3*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*(-5*b
^5 + 47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt
[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Maple [C] time = 0.059, size = 7019, normalized size = 14.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

Fricas [B] time = 7.83583, size = 23385, normalized size = 46.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="frica
s")
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 48*(5*b^4*c^2 -
37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^
2*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 12*(28*
(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 3
92*a^2*b*c^3)*d)*e^5*x^5 + 6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 +
2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37
*a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3
)*d^2)*e^4*x^4 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 8*(42*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 +
392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d
)*e^3*x^3 + 16*a^2*b^4 - 128*a^3*b^2*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4
*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + 2*(84*(5*b^4*c^2 - 37*a*b^2*c^3 +
60*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + 2*(25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^
2)*d^2 + 4*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^
2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x -
3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a
^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a
^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d
^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a
^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a^4*
b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 4
2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 +
16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4 +
 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3
```

+ 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(
a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^2 -
8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b
*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5
*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
+ 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4
*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 +
6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 -
8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c
^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3
*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2)*sqr
t(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^
4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 -
640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^1
2 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^
4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 1
60*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4
*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 12
80*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4 - 77825*a
*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 -
810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6
*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d + 27/2
sqrt(1/2)((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b
^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 -
323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12250*a*b^1
0*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*
a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2
- 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)) - (125*b
^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^
4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7
+ 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^
2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 2
0*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024
*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 35
1310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)
/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a
^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9
*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4)))
+ 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9
*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c -
8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4
)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 +
(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a
^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4
+ 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^
3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2
+ 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^
4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 +
5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^
5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*
a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*
c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*
a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^
4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2
- 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*
b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*
b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2)*

sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720
*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2
- 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*
b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4
*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c
+ 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*
e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 +
1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4 - 7782
5*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^
8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*
b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d - 2
7/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^1
0*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6
- 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12250*a*
b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 3123
00*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*
c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)) - (12
5*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534
*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*
c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7
*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10
- 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1
024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 -
351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c
^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 128
0*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*
a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4
))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9
+ 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c
- 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^
3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 -
6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d
^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*
(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*
c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2
x^4 + 2(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^
2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4
+ 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4
*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16
*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 -
8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a
^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c -
8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)
d^4 + 6(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*
c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a
^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a
^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^
2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27
720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*
c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((6
25*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*
a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8
*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^
5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^
3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4 - 7
7825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2
*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a
^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d

```
+ 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*
a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*
c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12250
*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 3
12300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b
^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)) +
(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623
534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b
^3*c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*
b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^
10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4
- 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^
2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^
6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 +
1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 1
60*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*
f^4))) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x
^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^
5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a
^5*c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)
*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6
 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4
)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (1
26*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b
^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*
f^2*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4
*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*
d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*
b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 +
 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^
5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*
c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c
^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 1
6*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 -
8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e
*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 +
 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b
^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt
((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 5918
86*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*
b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19
*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4
*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4
- 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*
b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 57103
0*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)
*d - 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 149
60*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b
^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12
250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4
- 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^1
6*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8))
 + (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1
623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^
7*b^3*c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a
^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7
*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c
```

```
^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8
*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625
*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3
 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c
+ 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e
^2*f^4))))/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a^
3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^
4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^
2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^
3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a^4*b
^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42
*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^3*b^
4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 1
6*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4 +
2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a
^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^2 -
8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*
c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*
b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
+ 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 +
6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 - 8
*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^
3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError
```

3.660 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=343

$$\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{4a^2ef^3\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4ef^3\left(b^2-4ac\right)^{5/2}}$$

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2)
+ (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b
*c*(b^2 - 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 -
20*a^3*c^3)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 -
 4*a*c)^(5/2)*e*f^3) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3*b*Log[a + b*(d +
 e*x)^2 + c*(d + e*x)^4])/(4*a^4*e*f^3)
```

Rubi [A] time = 0.588706, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{4a^2ef^3\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4ef^3\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2)
+ (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b
*c*(b^2 - 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 -
20*a^3*c^3)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 -
 4*a*c)^(5/2)*e*f^3) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3*b*Log[a + b*(d +
 e*x)^2 + c*(d + e*x)^4])/(4*a^4*e*f^3)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
```

```
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{ef^3}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2ef^3}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3b}{4a^2\left(b^2\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3b}{4a^2\left(b^2\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(\right.}$$

<u>Mathematica</u> [A] time = 6.16765, size = 509, normalized size = 1.48

$$\frac{-3abc-2ac^2(d+ex)^2+b^2c(d+ex)^2+b^3}{4a^2ef^3\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{-46a^2bc^2-28a^2c^3(d+ex)^2+26ab^2c^2(d+ex)^2+29ab^3c-4b^4c(d}{4a^3ef^3\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -1/(2*a^3*e*f^3*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2
*(d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*f^3*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)^2) + (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26*a*b^
2*c^2*(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*f^3*(
a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3*(
b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*
a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3) + (3*(-b^6
 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b
^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3)
```

Maple [C] time = 0.076, size = 5737, normalized size = 16.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

Fricas [B] time = 37.7185, size = 32292, normalized size = 94.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="frica
s")
```

```
[Out] [-1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*e^8*x^8
 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*e^7*x^7
+ 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b
^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*e^6*x^6 + 6*(56
*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^
7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d)*e^5*x^5 + 2*a^3*
b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b
^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60*a^2*b^6*c + 158*a
^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3
 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*
a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 +
 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^
3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 7
9*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^2*b^7 - 104*a^3*b^
5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 3
8*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*
b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7 - 104*a^3*b^5*c +
 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3
```

*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8 + a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^

3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b
^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6
4*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d
^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(
b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 1
2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (
a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2
 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*
c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b
^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48
*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c
^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2
 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12*a*b^
5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c
^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^
3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a
^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^
3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b
*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2
)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 +
 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(
b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a
*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b
^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*
a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3
 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b
^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a
^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c
^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 +
48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*
c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b
^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64
*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^
4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 +
45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c
 - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7
*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*
a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a
^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^
3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b
^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^
3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 -
 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*
c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
 - 64*a^7*c^5)*e^11*f^3*x^10 + 10*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2
*c^4 - 64*a^7*c^5)*d*e^10*f^3*x^9 + (2*a^4*b^7*c - 24*a^5*b^5*c^2 + 96*a^6*
b^3*c^3 - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
 - 64*a^7*c^5)*d^2)*e^9*f^3*x^8 + 8*(15*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*
a^6*b^2*c^4 - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*
c^3 - 64*a^7*b*c^4)*d)*e^8*f^3*x^7 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c
^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*
a^6*b^2*c^4 - 64*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3
*c^3 - 64*a^7*b*c^4)*d^2)*e^7*f^3*x^6 + 2*(126*(a^4*b^6*c^2 - 12*a^5*b^4*c^
3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*
a^6*b^3*c^3 - 64*a^7*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^
2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d)*e^6*f^3*x^5 + (2*a^5*b^7 - 24*a^6*b^5*
c + 96*a^7*b^3*c^2 - 128*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48

*a^6*b^2*c^4 - 64*a^7*c^5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^2)*e^5*f^3*x^4 + 4*(30*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d)*e^4*f^3*x^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^4 + 12*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^2)*e^3*f^3*x^2 + 2*(5*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d)*e^2*f^3*x + ((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^2)*e*f^3), -1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*e^8*x^8 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*e^7*x^7 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*e^6*x^6 + 6*(56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d)*e^5*x^5 + 2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60*a^2*b^6*c + 158*a^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*c^2 - 10*a*b^

4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b^5*c^2 + 30*a ^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40* a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2 *b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20 *a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40 *a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d) *e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8 + a^2*b^6 - 1 0*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a ^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40* a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20 *a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5) *d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^7 + 3*(b^ 8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^5 + 4*(a*b^ 7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b^6 - 10*a^3*b ^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c *e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^ 7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5* c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b ^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48* a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c ^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2* b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 6 4*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64 *a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3 *b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d ^3 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4) *d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^ 8 + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c ^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a* b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24* a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^ 7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6 *c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2* b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 4 8*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a ^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^ 5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7 *c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a* b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a ^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b ^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c *d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12* a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b ^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b ^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^ 2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^

3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a
^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)
*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^
10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 +
56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 -
10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 +
2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 -
24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*
c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a
^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 +
32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b
^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^
3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*
b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a
^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2
- 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 6
4*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^
3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b
^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a
*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 -
12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 -
12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^
5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a
^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 3
2*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c
^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^
5*b*c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*
c^4 - 64*a^7*c^5)*e^11*f^3*x^10 + 10*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6
*b^2*c^4 - 64*a^7*c^5)*d*e^10*f^3*x^9 + (2*a^4*b^7*c - 24*a^5*b^5*c^2 + 96*
a^6*b^3*c^3 - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2
*c^4 - 64*a^7*c^5)*d^2)*e^9*f^3*x^8 + 8*(15*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 +
48*a^6*b^2*c^4 - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*
b^3*c^3 - 64*a^7*b*c^4)*d)*e^8*f^3*x^7 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b
^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 +
48*a^6*b^2*c^4 - 64*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6
*b^3*c^3 - 64*a^7*b*c^4)*d^2)*e^7*f^3*x^6 + 2*(126*(a^4*b^6*c^2 - 12*a^5*b^
4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 +
48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^
4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d)*e^6*f^3*x^5 + (2*a^5*b^7 - 24*a^6*
b^5*c + 96*a^7*b^3*c^2 - 128*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3
+ 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a
^6*b^3*c^3 - 64*a^7*b*c^4)*d^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^
2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^2)*e^5*f^3*x^4 + 4*(30*(a^4*b^6*c^2 - 1
2*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^5*b
^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c + 2
4*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*b^5
*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d)*e^4*f^3*x^3 + (a^6*b^6 - 12*a^7*b^4*
c + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6
*b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^
3 - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^
7*b^2*c^3 - 128*a^8*c^4)*d^4 + 12*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2
- 64*a^8*b*c^3)*d^2)*e^3*f^3*x^2 + 2*(5*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*
a^6*b^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*
c^3 - 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a
^7*b^2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2
- 64*a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)*d)*e^2*f^3*x + ((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c
^5)*d^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d
^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^

```
4)*d^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^4 + (
a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^2)*e*f^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out
```

Giac [B] time = 16.4778, size = 2847, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac"
)

[Out] 3/4*(a^4*b^10*f^3*e - 18*a^5*b^8*c*f^3*e + 126*a^6*b^6*c^2*f^3*e - 420*a^7*
b^4*c^3*f^3*e + 640*a^8*b^2*c^4*f^3*e - 320*a^9*c^5*f^3*e)*sqrt(b^2 - 4*a*c
)*log(abs(2*(a^4*b^3*c - 4*a^5*b*c^2 + (a^4*b^2*c - 4*a^5*c^2)*sqrt(b^2 - 4
*a*c))*f^3*x^2*e^6 + 4*(a^4*b^3*c - 4*a^5*b*c^2 + (a^4*b^2*c - 4*a^5*c^2)*s
qrt(b^2 - 4*a*c))*d*f^3*x*e^5 + 2*(a^4*b^3*c - 4*a^5*b*c^2 + (a^4*b^2*c - 4
*a^5*c^2)*sqrt(b^2 - 4*a*c))*d^2*f^3*e^4 + 4*(a^5*b^2*c - 4*a^6*c^2)*f^3*e^
4))/(a^8*b^10*f^6*e^2 - 20*a^9*b^8*c*f^6*e^2 + 160*a^10*b^6*c^2*f^6*e^2 - 6
40*a^11*b^4*c^3*f^6*e^2 + 1280*a^12*b^2*c^4*f^6*e^2 - 1024*a^13*c^5*f^6*e^2
) - 3/4*(a^4*b^10*f^3*e - 18*a^5*b^8*c*f^3*e + 126*a^6*b^6*c^2*f^3*e - 420*
a^7*b^4*c^3*f^3*e + 640*a^8*b^2*c^4*f^3*e - 320*a^9*c^5*f^3*e)*sqrt(b^2 - 4
*a*c)*log(abs(-2*(a^4*b^3*c - 4*a^5*b*c^2 - (a^4*b^2*c - 4*a^5*c^2)*sqrt(b^
2 - 4*a*c))*f^3*x^2*e^6 - 4*(a^4*b^3*c - 4*a^5*b*c^2 - (a^4*b^2*c - 4*a^5*c
^2)*sqrt(b^2 - 4*a*c))*d*f^3*x*e^5 - 2*(a^4*b^3*c - 4*a^5*b*c^2 - (a^4*b^2*
c - 4*a^5*c^2)*sqrt(b^2 - 4*a*c))*d^2*f^3*e^4 - 4*(a^5*b^2*c - 4*a^6*c^2)*f
^3*e^4))/(a^8*b^10*f^6*e^2 - 20*a^9*b^8*c*f^6*e^2 + 160*a^10*b^6*c^2*f^6*e^
2 - 640*a^11*b^4*c^3*f^6*e^2 + 1280*a^12*b^2*c^4*f^6*e^2 - 1024*a^13*c^5*f^
6*e^2) - 1/4*(6*b^4*c^2*x^8*e^8 - 42*a*b^2*c^3*x^8*e^8 + 60*a^2*c^4*x^8*e^8
 + 48*b^4*c^2*d*x^7*e^7 - 336*a*b^2*c^3*d*x^7*e^7 + 480*a^2*c^4*d*x^7*e^7 +
 168*b^4*c^2*d^2*x^6*e^6 - 1176*a*b^2*c^3*d^2*x^6*e^6 + 1680*a^2*c^4*d^2*x^
6*e^6 + 336*b^4*c^2*d^3*x^5*e^5 - 2352*a*b^2*c^3*d^3*x^5*e^5 + 3360*a^2*c^4
*d^3*x^5*e^5 + 420*b^4*c^2*d^4*x^4*e^4 - 2940*a*b^2*c^3*d^4*x^4*e^4 + 4200*
a^2*c^4*d^4*x^4*e^4 + 336*b^4*c^2*d^5*x^3*e^3 - 2352*a*b^2*c^3*d^5*x^3*e^3
+ 3360*a^2*c^4*d^5*x^3*e^3 + 168*b^4*c^2*d^6*x^2*e^2 - 1176*a*b^2*c^3*d^6*x
^2*e^2 + 1680*a^2*c^4*d^6*x^2*e^2 + 48*b^4*c^2*d^7*x*e - 336*a*b^2*c^3*d^7*
x*e + 480*a^2*c^4*d^7*x*e + 6*b^4*c^2*d^8 - 42*a*b^2*c^3*d^8 + 60*a^2*c^4*d
^8 + 12*b^5*c*x^6*e^6 - 87*a*b^3*c^2*x^6*e^6 + 138*a^2*b*c^3*x^6*e^6 + 72*b
^5*c*d*x^5*e^5 - 522*a*b^3*c^2*d*x^5*e^5 + 828*a^2*b*c^3*d*x^5*e^5 + 180*b^
5*c*d^2*x^4*e^4 - 1305*a*b^3*c^2*d^2*x^4*e^4 + 2070*a^2*b*c^3*d^2*x^4*e^4 +
 240*b^5*c*d^3*x^3*e^3 - 1740*a*b^3*c^2*d^3*x^3*e^3 + 2760*a^2*b*c^3*d^3*x^
3*e^3 + 180*b^5*c*d^4*x^2*e^2 - 1305*a*b^3*c^2*d^4*x^2*e^2 + 2070*a^2*b*c^3
*d^4*x^2*e^2 + 72*b^5*c*d^5*x*e - 522*a*b^3*c^2*d^5*x*e + 828*a^2*b*c^3*d^5
*x*e + 12*b^5*c*d^6 - 87*a*b^3*c^2*d^6 + 138*a^2*b*c^3*d^6 + 6*b^6*x^4*e^4
```

```
- 36*a*b^4*c*x^4*e^4 + 14*a^2*b^2*c^2*x^4*e^4 + 100*a^3*c^3*x^4*e^4 + 24*b^
6*d*x^3*e^3 - 144*a*b^4*c*d*x^3*e^3 + 56*a^2*b^2*c^2*d*x^3*e^3 + 400*a^3*c^
3*d*x^3*e^3 + 36*b^6*d^2*x^2*e^2 - 216*a*b^4*c*d^2*x^2*e^2 + 84*a^2*b^2*c^2
*d^2*x^2*e^2 + 600*a^3*c^3*d^2*x^2*e^2 + 24*b^6*d^3*x*e - 144*a*b^4*c*d^3*x
*e + 56*a^2*b^2*c^2*d^3*x*e + 400*a^3*c^3*d^3*x*e + 6*b^6*d^4 - 36*a*b^4*c*
d^4 + 14*a^2*b^2*c^2*d^4 + 100*a^3*c^3*d^4 + 9*a*b^5*x^2*e^2 - 68*a^2*b^3*c
*x^2*e^2 + 122*a^3*b*c^2*x^2*e^2 + 18*a*b^5*d*x*e - 136*a^2*b^3*c*d*x*e + 2
44*a^3*b*c^2*d*x*e + 9*a*b^5*d^2 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^2*d^2 + 2
*a^2*b^4 - 16*a^3*b^2*c + 32*a^4*c^2)/((a^3*b^4*f^3*e - 8*a^4*b^2*c*f^3*e +
 16*a^5*c^2*f^3*e)*(c*x^5*e^5 + 5*c*d*x^4*e^4 + 10*c*d^2*x^3*e^3 + 10*c*d^3
*x^2*e^2 + 5*c*d^4*x*e + c*d^5 + b*x^3*e^3 + 3*b*d*x^2*e^2 + 3*b*d^2*x*e +
b*d^3 + a*x*e + a*d)^2) + 3/4*b*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 +
6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))
/(a^4*f^3) - 3*b*e^(-1)*log(abs(x*e + d))/(a^4*f^3)
```

3.661 $\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$

Optimal. Leaf size=340

$$\frac{(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}}{e^2\sqrt{a+b(d+ex)}}$$

```
[Out] -((d*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-
2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2
- 4*a*c])])/(e^2*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])) + ((d + e*x)^2*S
qrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*e
^2*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])
```

Rubi [A] time = 0.691003, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1389, 1790, 1348, 429, 1385, 510}

$$\frac{(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}}{e^2\sqrt{a+b(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

```
[Out] -((d*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-
2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2
- 4*a*c])])/(e^2*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])) + ((d + e*x)^2*S
qrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*e
^2*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])
```

Rule 1389

```
Int[((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIntegrand[(x -
 Coefficient[v, x, 0])^m*(a + b*x^n + c*x^(2*n))^p, x], x], x, v], x] /; Fr
eeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[v, x] && IntegerQ[m] &&
NeQ[v, x]
```

Rule 1790

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \frac{\text{Subst}\left(\int \frac{-d+x}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^2}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{d}{\sqrt{a+bx^3+cx^6}}+\frac{x}{\sqrt{a+bx^3+cx^6}}\right)dx, x, d+ex\right)}{e^2}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^2} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^2}$$

$$= \frac{\left(\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx, x, d+e\right.}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

$$= -\frac{d(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

Mathematica [F] time = 1.13568, size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

```
[Out] Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x \frac{1}{\sqrt{a+b\left(ex+d\right)^{3}+c\left(ex+d\right)^{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

```
[Out] int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(ex+d)^{6}c+(ex+d)^{3}b+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\sqrt{ce^{6}x^{6}+6\,cde^{5}x^{5}+15\,cd^{2}e^{4}x^{4}+cd^{6}+\left(20\,cd^{3}+b\right)e^{3}x^{3}+3\left(5\,cd^{4}+bd\right)e^{2}x^{2}+bd^{3}+3\left(2\,cd^{5}+bd^{2}\right)ex+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(c*e^6*x^6 + 6*c*d*e^5*x^5 + 15*c*d^2*e^4*x^4 + c*d^6 + (20*
c*d^3 + b)*e^3*x^3 + 3*(5*c*d^4 + b*d)*e^2*x^2 + b*d^3 + 3*(2*c*d^5 + b*d^2
)*e*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bd^{3}+3bd^{2}ex+3bde^{2}x^{2}+be^{3}x^{3}+cd^{6}+6cd^{5}ex+15cd^{4}e^{2}x^{2}+20cd^{3}e^{3}x^{3}+15cd^{2}e^{4}x^{4}+6cde^{5}x^{5}+ce^{6}x^{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**3 +
 c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*c*d
**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

3.662 $\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$

Optimal. Leaf size=398

$$\frac{d^2(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+}}{e^3\sqrt{a+b(d+e}}$$

```
[Out] (d^2*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-
2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2
- 4*a*c])])/(e^3*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) - (d*(d + e*x)^2*
Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3
)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(e^
3*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) + ArcTanh[(b + 2*c*(d + e*x)^3)/
(2*Sqrt[c]*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])]/(3*Sqrt[c]*e^3)
```

Rubi [A] time = 0.687049, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1389, 1790, 1348, 429, 1385, 510, 1352, 621, 206}

$$\frac{d^2(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+}}{e^3\sqrt{a+b(d+e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```


```
[Out] (d^2*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-
2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2
- 4*a*c])])/(e^3*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) - (d*(d + e*x)^2*
Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3
)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(e^
3*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) + ArcTanh[(b + 2*c*(d + e*x)^3)/
(2*Sqrt[c]*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])]/(3*Sqrt[c]*e^3)
```


Rule 1389

```
Int[((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIntegrand[(x -
 Coefficient[v, x, 0])^m*(a + b*x^n + c*x^(2*n))^p, x], x], x, v], x] /; Fr
eeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[v, x] && IntegerQ[m] &&
NeQ[v, x]
```

Rule 1790

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \frac{\text{Subst}\left(\int \frac{(-d+x)^2}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{d^2}{\sqrt{a+bx^3+cx^6}} - \frac{2dx}{\sqrt{a+bx^3+cx^6}} + \frac{x^2}{\sqrt{a+bx^3+cx^6}}\right) dx, x, d+ex\right)}{e^3}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^3} - \frac{(2d)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^3} + \frac{d^2\,\text{S}\cdots}{\cdots}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, (d+ex)^3\right)}{3e^3} - \frac{\left(2d\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\right)\text{Subst}\cdots}{e^3\sqrt{a+b(d+ex)\cdots}}$$

$$= \frac{d^2(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} - \cdots$$

$$= \frac{d^2(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} - \cdots$$

Mathematica [F] time = 0.430273, size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

```
[Out] Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{a+b\,(ex+d)^3+c\,(ex+d)^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

```
[Out] int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{ce^6x^6 + 6\,cde^5x^5 + 15\,cd^2e^4x^4 + cd^6 + \left(20\,cd^3 + b\right)e^3x^3 + 3\left(5\,cd^4 + bd\right)e^2x^2 + bd^3 + 3\left(2\,cd^5 + bd^2\right)ex + a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(c*e^6*x^6 + 6*c*d*e^5*x^5 + 15*c*d^2*e^4*x^4 + c*d^6 + (2
0*c*d^3 + b)*e^3*x^3 + 3*(5*c*d^4 + b*d)*e^2*x^2 + b*d^3 + 3*(2*c*d^5 + b*d
^2)*e*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + bd^3 + 3bd^2ex + 3bde^2x^2 + be^3x^3 + cd^6 + 6cd^5ex + 15cd^4e^2x^2 + 20cd^3e^3x^3 + 15cd^2e^4x^4 + 6cde^5x^5 + ce^6x^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**
3 + c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*
c*d**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{(ex+d)^6c + (ex+d)^3b + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

3.663 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) dx$

Optimal. Leaf size=34

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Rubi [A] time = 0.0356623, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1390, 14}

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Rule 1390

```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n + c*x^(
2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && L
inearPairQ[u, v, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) dx &= \frac{1}{3}\text{Subst}\left(\int x^6\left(1+x^7+x^{14}\right) dx, x, 2+3x\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(x^6+x^{13}+x^{20}\right) dx, x, 2+3x\right)\\
&= \frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}
\end{aligned}$$

Mathematica [A] time = 0.0132572, size = 34, normalized size = 1.

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Maple [B] time = 0.002, size = 105, normalized size = 3.1

$$\frac{1162261467\,x^{21}}{7} + 2324522934\,x^{20} + 15496819560\,x^{19} + 65431015920\,x^{18} + 196293047760\,x^{17} + 444930908256\,x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x)
```

```
[Out] 1162261467/7*x^21+2324522934*x^20+15496819560*x^19+65431015920*x^18+1962930
47760*x^17+444930908256*x^16+790988281344*x^15+15819767221203/14*x^14+13183
14865122*x^13+1269491970942*x^12+1015602174288*x^11+677082445416*x^10+37617
4427616*x^9+173635132896*x^8+66158154783*x^7+20588764518*x^6+5149786572*x^5
+1010576952*x^4+149902032*x^3+15808800*x^2+1056832*x
```

Maxima [B] time = 0.969965, size = 140, normalized size = 4.12

$$\frac{1162261467}{7}\,x^{21} + 2324522934\,x^{20} + 15496819560\,x^{19} + 65431015920\,x^{18} + 196293047760\,x^{17} + 444930908256\,x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="maxima")
```

```
[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18 +
 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 15819767221203
/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11 + 6
77082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^7 +
20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 1580880
0*x^2 + 1056832*x
```

Fricas [B] time = 1.07722, size = 532, normalized size = 15.65

$$\frac{1162261467}{7}x^{21} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="fricas")
```

```
[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18 +
 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 15819767221203
/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11 + 6
77082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^7 +
20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 1580880
0*x^2 + 1056832*x
```

Sympy [B] time = 0.092468, size = 107, normalized size = 3.15

$$\frac{1162261467x^{21}}{7} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14),x)
```

```
[Out] 1162261467*x**21/7 + 2324522934*x**20 + 15496819560*x**19 + 65431015920*x**
18 + 196293047760*x**17 + 444930908256*x**16 + 790988281344*x**15 + 1581976
7221203*x**14/14 + 1318314865122*x**13 + 1269491970942*x**12 + 101560217428
8*x**11 + 677082445416*x**10 + 376174427616*x**9 + 173635132896*x**8 + 6615
8154783*x**7 + 20588764518*x**6 + 5149786572*x**5 + 1010576952*x**4 + 14990
2032*x**3 + 15808800*x**2 + 1056832*x
```

Giac [A] time = 1.08124, size = 38, normalized size = 1.12

$$\frac{1}{63}(3x+2)^{21} + \frac{1}{42}(3x+2)^{14} + \frac{1}{21}(3x+2)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="giac")
```

```
[Out] 1/63*(3*x + 2)^21 + 1/42*(3*x + 2)^14 + 1/21*(3*x + 2)^7
```

3.664 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{1}{105}(3x+2)^{35}+\frac{1}{42}(3x+2)^{28}+\frac{1}{21}(3x+2)^{21}+\frac{1}{21}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/21 + (2 + 3*x)^21/21 + (2 + 3*x)^28/42 + (2 +
 3*x)^35/105
```

Rubi [A] time = 0.0957482, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1390, 1352, 611}

$$\frac{1}{105}(3x+2)^{35}+\frac{1}{42}(3x+2)^{28}+\frac{1}{21}(3x+2)^{21}+\frac{1}{21}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/21 + (2 + 3*x)^21/21 + (2 + 3*x)^28/42 + (2 +
 3*x)^35/105
```

Rule 1390

```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n + c*x^(
2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && L
inearPairQ[u, v, x]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\begin{aligned}
\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx &= \frac{1}{3}\text{Subst}\left(\int x^6 \left(1+x^7+x^{14}\right)^2 \, dx, x, 2+3x\right)\\
&= \frac{1}{21}\text{Subst}\left(\int \left(1+x+x^2\right)^2 \, dx, x, (2+3x)^7\right)\\
&= \frac{1}{21}\text{Subst}\left(\int \left(1+2x+3x^2+2x^3+x^4\right) dx, x, (2+3x)^7\right)\\
&= \frac{1}{21}(2+3x)^7+\frac{1}{21}(2+3x)^{14}+\frac{1}{21}(2+3x)^{21}+\frac{1}{42}(2+3x)^{28}+\frac{1}{105}(2+
\end{aligned}$$

Mathematica [B] time = 0.0116218, size = 188, normalized size = 3.36

$$\frac{16677181699666569x^{35}}{35} + 11118121133111046x^{34} + 126005372841925188x^{33} + 924039400840784712x^{32} + 4928210137$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] 17451466816*x + 443569828128*x^2 + 7299544818384*x^3 + 87406679578680*x^4 +
 (4057390785756924*x^5)/5 + 6077684727888102*x^6 + 37727143432895007*x^7 +
197897276851452864*x^8 + 889942562270387136*x^9 + (17344958593049772048*x^1
0)/5 + 11821487501620716192*x^11 + 35454069480572048124*x^12 + 940692639189
29616324*x^13 + 221699757548270194389*x^14 + 465517091041681015296*x^15 + 8
72775774067455498528*x^16 + 1463104032160519033200*x^17 + 21945771660147522
40080*x^18 + 2945285062308448290360*x^19 + 3534290697929473864098*x^20 + (2
6506949038858918036881*x^21)/7 + 3614565944605222108800*x^22 + 306451507651
2846852480*x^23 + 2298383223254096766840*x^24 + (7584660010542711771792*x^2
5)/5 + 875152864622814086340*x^26 + 437576396725285446564*x^27 + (262545832
6972530284475*x^28)/14 + 67899784121041365504*x^29 + (101849676181562048256
*x^30)/5 + 4928210137817518464*x^31 + 924039400840784712*x^32 + 12600537284
1925188*x^33 + 11118121133111046*x^34 + (16677181699666569*x^35)/35
```

Maple [B] time = 0.003, size = 175, normalized size = 3.1

$$17451466816\,x + 11118121133111046\,x^{34} + 126005372841925188\,x^{33} + 924039400840784712\,x^{32} + 49282101378175184$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x)
```

```
[Out] 17451466816*x+11118121133111046*x^34+126005372841925188*x^33+92403940084078
4712*x^32+4928210137817518464*x^31+101849676181562048256/5*x^30+67899784121
041365504*x^29+2625458326972530284475/14*x^28+437576396725285446564*x^27+87
5152864622814086340*x^26+7584660010542711771792/5*x^25+2298383223254096766 8
40*x^24+3064515076512846852480*x^23+3614565944605222108800*x^22+26506949038
858918036881/7*x^21+3534290697929473864098*x^20+2945285062308448290360*x^19
+2194577166014752240080*x^18+1463104032160519033200*x^17+872775774067455498
528*x^16+465517091041681015296*x^15+17344958593049772048/5*x^10+19789727685
1452864*x^8+37727143432895007*x^7+221699757548270194389*x^14+11821487501620
716192*x^11+94069263918929616324*x^13+35454069480572048124*x^12+88994256227
0387136*x^9+7299544818384*x^3+6077684727888102*x^6+4057390785756924/5*x^5+4
43569828128*x^2+87406679578680*x^4+16677181699666569/35*x^35
```

Maxima [B] time = 1.01913, size = 235, normalized size = 4.2

$$\frac{16677181699666569}{35}\,x^{35} + 11118121133111046\,x^{34} + 126005372841925188\,x^{33} + 924039400840784712\,x^{32} + 4928210137$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="maxima")
```

```
[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^3
3 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 101849676181562048
256/5*x^30 + 67899784121041365504*x^29 + 2625458326972530284475/14*x^28 + 4
37576396725285446564*x^27 + 875152864622814086340*x^26 + 758466001054271177
1792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^23 + 3
614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290697929
473864098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*x^18
+ 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 46551709104168
1015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x^13 + 354
54069480572048124*x^12 + 11821487501620716192*x^11 + 17344958593049772048/5
*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 37727143432895007
*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 +
 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

<u>Fricas</u> [B] time = 1.00882, size = 1289, normalized size = 23.02

$$\frac{16677181699666569}{35}x^{35} + 11118121133111046x^{34} + 126005372841925188x^{33} + 924039400840784712x^{32} + 4928210$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="fricas")
```


```
[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^3
3 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 101849676181562048
256/5*x^30 + 67899784121041365504*x^29 + 2625458326972530284475/14*x^28 + 4
37576396725285446564*x^27 + 875152864622814086340*x^26 + 758466001054271177
1792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^23 + 3
614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290697929
473864098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*x^18
+ 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 46551709104168
1015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x^13 + 354
54069480572048124*x^12 + 11821487501620716192*x^11 + 17344958593049772048/5
*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 37727143432895007
*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 +
 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

<u>Sympy</u> [B] time = 0.134396, size = 187, normalized size = 3.34

$$\frac{16677181699666569x^{35}}{35} + 11118121133111046x^{34} + 126005372841925188x^{33} + 924039400840784712x^{32} + 4928210$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14)**2,x)
```

```
[Out] 16677181699666569*x**35/35 + 11118121133111046*x**34 + 126005372841925188*x
**33 + 924039400840784712*x**32 + 4928210137817518464*x**31 + 1018496761815
62048256*x**30/5 + 67899784121041365504*x**29 + 2625458326972530284475*x**2
8/14 + 437576396725285446564*x**27 + 875152864622814086340*x**26 + 75846600
10542711771792*x**25/5 + 2298383223254096766840*x**24 + 3064515076512846852
480*x**23 + 3614565944605222108800*x**22 + 26506949038858918036881*x**21/7
+ 3534290697929473864098*x**20 + 2945285062308448290360*x**19 + 21945771660
14752240080*x**18 + 1463104032160519033200*x**17 + 872775774067455498528*x*
*16 + 465517091041681015296*x**15 + 221699757548270194389*x**14 + 940692639
```

```
18929616324*x**13 + 354540694805720481 24*x**12 + 118214875016207161 92*x**11
 + 17344958593049772048*x**10/5 + 889942562270387136*x**9 + 197897276851452
864*x**8 + 37727143432895007*x**7 + 6077684727888102*x**6 + 405739078575692
4*x**5/5 + 87406679578680*x**4 + 7299544818384*x**3 + 443569828128*x**2 + 1
7451466816*x
```

Giac [A] time = 1.10208, size = 62, normalized size = 1.11

$$\frac{1}{105}(3x+2)^{35}+\frac{1}{42}(3x+2)^{28}+\frac{1}{21}(3x+2)^{21}+\frac{1}{21}(3x+2)^{14}+\frac{1}{21}(3x+2)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="giac")
```

```
[Out] 1/105*(3*x + 2)^35 + 1/42*(3*x + 2)^28 + 1/21*(3*x + 2)^21 + 1/21*(3*x + 2)
^14 + 1/21*(3*x + 2)^7
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
              print("both result and optimal complex");
           fi;
           #both result and optimal complex
           if leaf_count_result<=2*leaf_count_optimal then
              return "A";
           else
              return "B";
           end if
        else #result contains complex but optimal is not
           if debug then
              print("result contains complex but optimal is not");
           fi;
           return "C";
        end if
     else # result do not contain complex
          # this assumes optimal do not as well
          if debug then
              print("result do not contain complex, this assumes optimal do not as well");
          fi;
          if leaf_count_result<=2*leaf_count_optimal then
             if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
             fi;
             return "A";
          else
             if debug then
                print("leaf_count_result>2*leaf_count_optimal");
             fi;
             return "B";
          end if
     end if
  else #ExpnType(result) > ExpnType(optimal)
     if debug then
        print("ExpnType(result) > ExpnType(optimal)");
     fi;
     return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                    #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```